Key Notes

Chapter-3

Matrices

- A matrix is an ordered rectangular array of numbers or functions.

- A matrix having m rows and n columns is called a matrix of order m × n.

- $[a_{ij}]_{m\times l}$ is a column matrix.

- $[a_{ij}]_{l\times n}$ is a row matrix.

- An m × n matrix is a square matrix if m = n.

- A = $A = [a_{ij}]_{m\times n}$ is a diagonal matrix if $a_{ij} = 0$, when $i \neq j$

- $A = \left[a_{ji}\right]_{n\times n}$ is a scalar matrix if $a_{ij} = 0$ when $i \neq j$, $a_{ij} = k$ (k is *some constant), when I=j.*

- $A = \left[a_{ij}\right]_{n\times n}$ is an identity matrix, if $._{ij} = 1, when\ i = j,\ a_{ij} = 0,\ when\ i \neq j.$

- A zero matrix has all its elements as zero.

- $A = \left[a_{ij}\right] = \left[b_{ij}\right] = B$ if (i) A and B are of same order, (ii) for all possible values of $i\ and\ j.$

- $kA = k\left[a_{ij}\right]_{m\times n} = \left[k\left(a_{ij}\right)\right]_{m\times n}$

- – A = (–1)A

- A – B = A + (–1) B

- A + B = B + A

- (A + B) + C = A + (B + C), where A, B and C are of same order.

- k(A + B) = kA + kB, where A and B are of same order, k is constant.

- (k + l) A = kA + lA, where k and l are constant.

- $If A = [a_{ij}]_{m\times n}\ and\ B = [b_{jk}]_{n\times p}, then\ AB = C = [c_{ik}]_{m\times p},$ where $C_{tl} = \sum_{j=i}^{n} a_{ij}b_{jk}$

    (i)   A(BC) = (AB)C,

(ii)   A(B + C) = AB + AC,

(iii)   (A + B)C = AC + BC

- $If\ A = \left[a_{ij}\right]_{m\times n},\ then\ A'\ or\ A^{\mathrm{T}} = \left[a_{ji}\right]_{n\times m}$

- (i) $(A')' = A$,

- (ii) $(kA)' = kA'$,

- (iii) $(A + B)' = A' + B'$,

- (iv) $(AB)' = B'A'$

- A is a symmetric matrix if $A' = A$.

- A is a skew symmetric matrix if $A' = -A$.

- Any square matrix can be represented as the sum of a symmetric and a skew symmetric matrix.

- Elementary operations of a matrix are as follows:

- (i) $R_1 \leftrightarrow R_j$ or $C_1 \leftrightarrow C_j$

  (i) $R_1 \rightarrow kR_i$ or $C_1 \leftrightarrow kC_1$

  (i) $R_1 \leftrightarrow R_j + kR_j$ or $C_1 + kC_j$

- If A and B are two square matrices such that AB = BA = I, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.

- Inverse of a square matrix, if it exists, is unique.