QUIZ-2

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Download latex-tikz code from

https://github.com/abhiroopchintalapudi03/EE3900/QUIZ−2

1 Problem 3.9(c)

A casual LTI system has an impulse response$h[n]$, for which z-transform is

$$H(z) = \frac{1 + z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})} \tag{1.0.1}$$

(c) Find the z-transform $X(z)$ of an input $x[n]$ that will produce the output

$$y[n] = -\frac{1}{3}(-\frac{1}{4})^n u[n] - \frac{4}{3}(2)^n u[-n-1] \tag{1.0.2}$$

2 Solution

From z-transform of basic signals

$$\text{u[n]} \overset{\text{z}}{\rightleftharpoons} \frac{\text{z}}{\text{z-1}} \tag{2.0.1}$$

$$\tag{2.0.2}$$

$$(-\frac{1}{4})^n\text{u[n]} \overset{\text{z}}{\rightleftharpoons} \frac{\text{z}}{\text{z-}(-\frac{1}{4})} \tag{2.0.3}$$

$$-\frac{1}{3}(-\frac{1}{4})^n\text{u[n]} \overset{\text{z}}{\rightleftharpoons} -\frac{1}{3}(\frac{\text{z}}{\text{z-}(-\frac{1}{4})}) \tag{2.0.4}$$

$$-\frac{1}{3}(-\frac{1}{4})^n\text{u[n]} \overset{\text{z}}{\rightleftharpoons} -\frac{1}{3}(\frac{4\,\text{z}}{4\,\text{z}+1}) \tag{2.0.5}$$

$$\text{u[}-\text{n-1]} \overset{\text{z}}{\rightleftharpoons} -\frac{\text{z}}{\text{z-1}} \tag{2.0.6}$$

$$-\frac{4}{3}(2)^{\,n}\text{u[}-\text{n-1]} \overset{\text{z}}{\rightleftharpoons} \frac{4}{3}(\frac{\text{z}}{\text{z-2}}) \tag{2.0.7}$$

$$-\frac{1}{3}(-\frac{1}{4})^n\text{u[n]} - \frac{4}{3}(2)^{\,n}\text{u[}-\text{n-1]} \overset{\text{z}}{\rightleftharpoons} -\frac{1}{3}(\frac{4\,\text{z}}{4\,\text{z}+1}) + \frac{4}{3}(\frac{\text{z}}{\text{z-2}}) \tag{2.0.8}$$

$$\Rightarrow Y(z) = \frac{4z(z+1)}{(z-2)(4z+1)} \tag{2.0.9}$$

From (1.0.1) we also know that,

$$H(z) = \frac{1 + z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})} = \frac{8z(z+1)}{(2Z-1)(4z+1)} \tag{2.0.10}$$

From transfer function we know that,

$$H(z) = \frac{Y(z)}{X(z)} \Rightarrow X(z) = \frac{Y(z)}{H(z)} \tag{2.0.11}$$

$$\Rightarrow X(z) = \frac{\frac{4z(z+1)}{(z-2)(4z+1)}}{\frac{8z(z+1)}{(2Z-1)(4z+1)}}$$

$$\Rightarrow X(z) = \frac{2z-1}{2(z-2)} = \frac{1-\frac{1}{2}z^{-1}}{1-2z^{-1}}$$

$\Rightarrow$ The z-transform $X(z)$ of an input $x[n]$ that will produce the given output is

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - 2z^{-1}} \tag{2.0.12}$$