

QUIZ-2

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Download latex-tikz code from

<https://github.com/abhiroopchintalapudi03/EE3900/tree/main/QUIZ-2>

1 PROBLEM 3.9(c)

A casual LTI system has an impulse response $h[n]$, for which z -transform is

$$H(z) = \frac{1 + z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})} \quad (1.0.1)$$

(c) Find the z -transform $X(z)$ of an input $x[n]$ that will produce the output

$$y[n] = -\frac{1}{3}(-\frac{1}{4})^n u[n] - \frac{4}{3}(2)^n u[-n-1] \quad (1.0.2)$$

2 SOLUTION

From z -transform of basic signals

$$u[n] \xLeftrightarrow{z} \frac{z}{z-1} \quad (2.0.1)$$

$$(2.0.2)$$

$$(-\frac{1}{4})^n u[n] \xLeftrightarrow{z} \frac{z}{z - (-\frac{1}{4})} \quad (2.0.3)$$

$$-\frac{1}{3}(-\frac{1}{4})^n u[n] \xLeftrightarrow{z} -\frac{1}{3} \left(\frac{z}{z - (-\frac{1}{4})} \right) \quad (2.0.4)$$

$$-\frac{1}{3}(-\frac{1}{4})^n u[n] \xLeftrightarrow{z} -\frac{1}{3} \left(\frac{4z}{4z + 1} \right) \quad (2.0.5)$$

$$u[-n-1] \xLeftrightarrow{z} -\frac{z}{z-1} \quad (2.0.6)$$

$$-\frac{4}{3}(2)^n u[-n-1] \xLeftrightarrow{z} \frac{4}{3} \left(\frac{z}{z-2} \right) \quad (2.0.7)$$

$$-\frac{1}{3}(-\frac{1}{4})^n u[n] - \frac{4}{3}(2)^n u[-n-1] \xLeftrightarrow{z} -\frac{1}{3} \left(\frac{4z}{4z + 1} \right) + \frac{4}{3} \left(\frac{z}{z-2} \right) \quad (2.0.8)$$

$$\Rightarrow Y(z) = \frac{4z(z+1)}{(z-2)(4z+1)} \quad (2.0.9)$$

From (1.0.1) we also know that,

$$H(z) = \frac{1 + z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})} = \frac{8z(z+1)}{(2Z-1)(4z+1)} \quad (2.0.10)$$

From transfer function we know that,

$$H(z) = \frac{Y(z)}{X(z)} \Rightarrow X(z) = \frac{Y(z)}{H(z)} \quad (2.0.11)$$

$$\Rightarrow X(z) = \frac{\frac{4z(z+1)}{(z-2)(4z+1)}}{\frac{8z(z+1)}{(2Z-1)(4z+1)}} \\ \Rightarrow X(z) = \frac{2z-1}{2(z-2)} = \frac{1 - \frac{1}{2}z^{-1}}{1 - 2z^{-1}}$$

\Rightarrow The z -transform $X(z)$ of an input $x[n]$ that will produce the given output is

$$X(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - 2z^{-1}} \quad (2.0.12)$$