ANGULAR MOTION

Objectives….

- **Define and apply concepts of angular displacement, velocity, and acceleration.**
- **Draw analogies relating rotational-motion parameters (θ, ω, α) to linear (*x*, *v*, *a*) and solve rotational problems.**
- **Write and apply relationships between linear and angular parameters.**

- **Define moment of inertia and apply it for several regular objects in rotation.**
- **Apply the following concepts to rotation:**

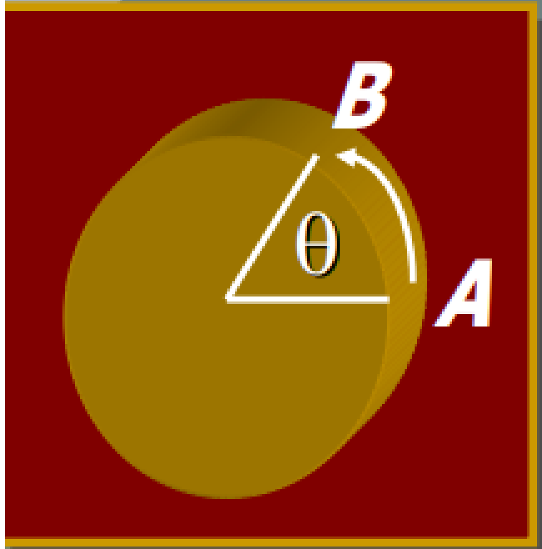
1. Rotational work, energy, and power

2. Rotational kinetic energy and momentum

3. Conservation of angular momentum

Rotational Displacement, θ

Consider a disk that rotates from A to B:



Angular displacement θ:

Measured in revolutions, degrees, or radians.

$$1 \text{ rev} = 360^{0} = 2\pi \text{ rad}$$

The best measure for rotation of rigid bodies is the **radian.**

Angular Displacement

- Every point on the object undergoes circular motion about the point O
- Angles generally need to be measured in *radians*

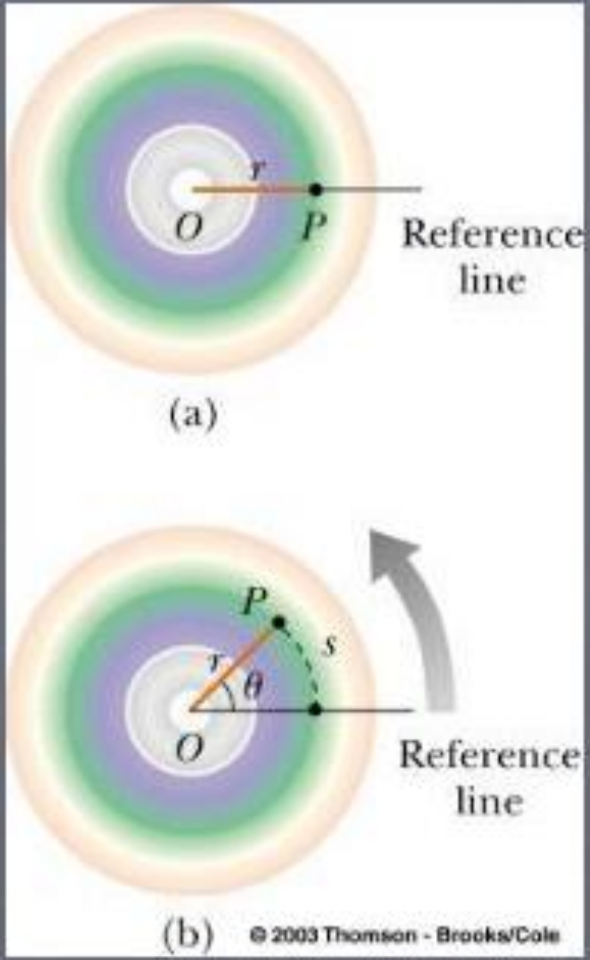
$$\boxed{\theta = \frac{s}{r}}$$

length of arc

radius

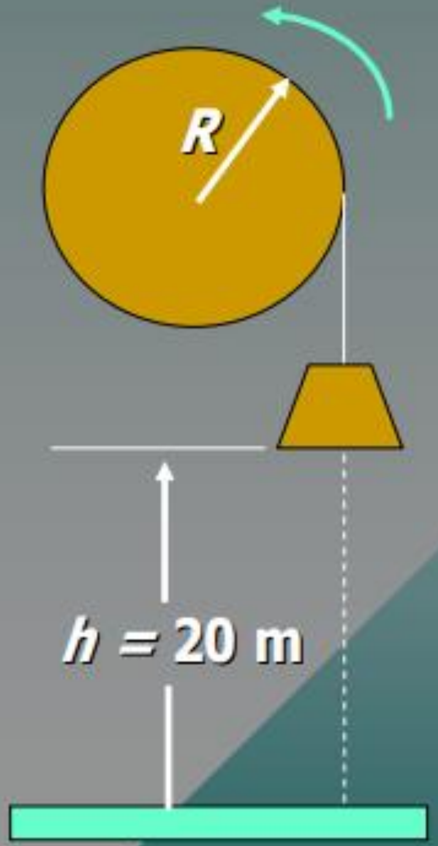
- Note: $1\,rad = \frac{360°}{2\pi} = 57.3°$

$$\theta\,[\text{rad}] = \frac{\pi}{180°}\,\theta\,[\text{degrees}]$$



Example 1: A rope is wrapped many times around a drum of radius **50 cm**. How many revolutions of the drum are required to raise a bucket to a height of **20 m**?

$$\theta = \frac{s}{R} = \frac{20 \text{ m}}{0.50 \text{ m}} \qquad \theta = 40 \text{ rad}$$



Now, **1 rev = 2π rad**

$$\theta = (40 \cancel{\text{rad}})\left(\frac{1 \text{ rev}}{2\pi \cancel{\text{rad}}}\right)$$

$$\boxed{\theta = 6.37 \text{ rev}}$$

Example 2: A bicycle tire has a radius of 25 cm. If the wheel makes 400 rev, how far will the bike have traveled?

$$\theta = (400\ \cancel{\text{rev}})\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right)$$

$$\theta = 2513\ \text{rad}$$

$$s = \theta R = 2513\ \text{rad}\ (0.25\ \text{m})$$

$$\boxed{s = 628\ \text{m}}$$

A wheel has a radius of 4.1 m. How far (path length) does a point on athe circumference travel if the wheel is rotated through angles 30°, 30 rad and 30 rev respectively?

The distance traveled is $s = r\theta$, where θ is in radians.

For 30°, $$s = r\theta = (4.1\text{ m})\left[30°\left(\frac{\pi\text{ rad}}{180°}\right)\right] = \boxed{2.1\text{ m}}.$$

For 30 radians, $$s = r\theta = (4.1\text{ m})(30\text{ rad}) = \boxed{1.2\times10^2\text{ m}}.$$

For 30 revolutions, $$s = r\theta = (4.1\text{ m})\left[30\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\right] = \boxed{7.7\times10^2\text{ m}}.$$

Angular Velocity

Angular velocity, ω, is the rate of change in angular displacement. (radians per second.)

$$\omega = \frac{\Delta\theta}{\Delta t}$$ Angular velocity in rad/s.

Angular velocity can also be given as the frequency of revolution, f (rev/s or rpm):

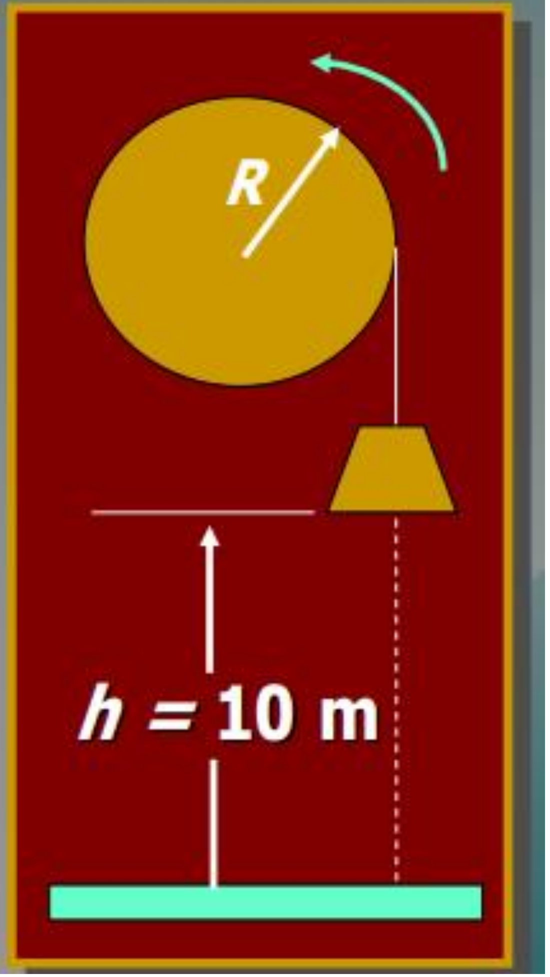
$$\omega = 2\pi f$$ Angular frequency f (rev/s).

Example 3: A rope is wrapped many times around a drum of radius 20 cm. What is the angular velocity of the drum if it lifts the bucket to 10 m in 5 s?

$$\theta = \frac{s}{R} = \frac{10\text{ m}}{0.20\text{ m}} \qquad \theta = 50\text{ rad}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\textbf{50 rad}}{\textbf{5 s}}$$

$$\boxed{\omega = 10.0\text{ rad/s}}$$



Example 4: In the previous example, what is the frequency of revolution for the drum? Recall that $\omega = 10.0$ rad/s.

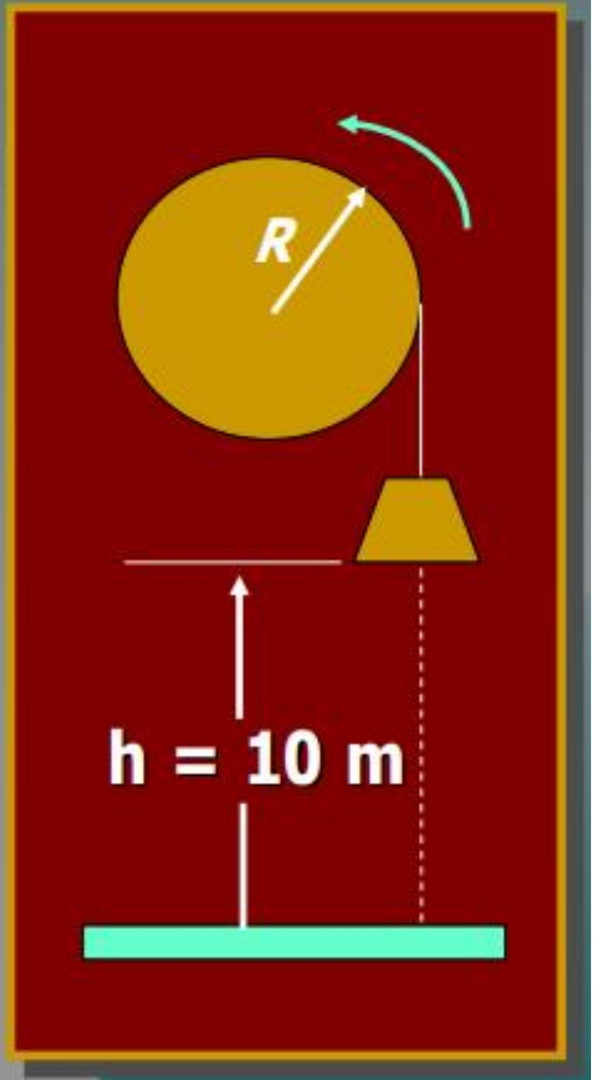
$$\omega = 2\pi f \quad \text{or} \quad f = \frac{\omega}{2\pi}$$

$$f = \frac{10.0 \text{ rad/s}}{2\pi \text{ rad/rev}} = 1.59 \text{ rev/s}$$

Or, since 60 s = 1 min:

$$f = 1.59 \frac{\text{rev}}{\cancel{\text{s}}}\left(\frac{60\ \cancel{\text{s}}}{1 \text{ min}}\right) = 95.5 \frac{\text{rev}}{\text{min}}$$

$$\boxed{\boldsymbol{f} = 95.5 \text{ rpm}}$$



Angular Acceleration

Angular acceleration is the rate of change in angular velocity. (Radians per sec per sec.)

$$\alpha = \frac{\Delta\omega}{\Delta t} \quad \text{Angular acceleration (rad/s}^2\text{)}$$

The angular acceleration can also be found from the change in frequency, as follows:

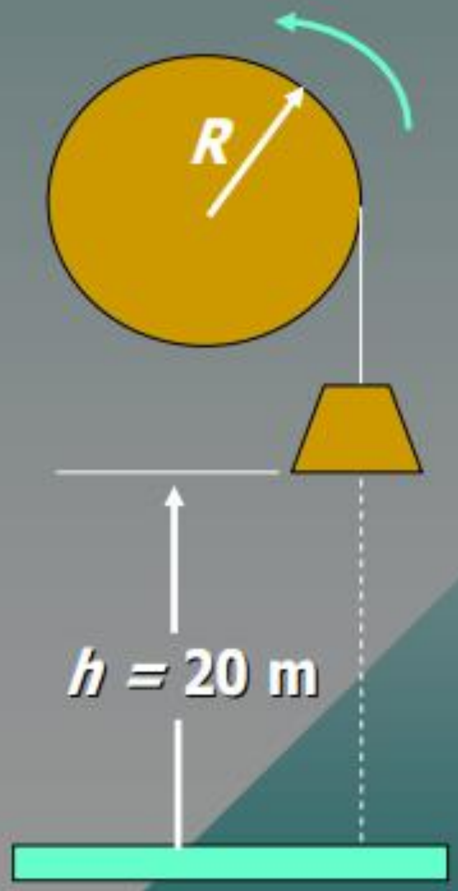
$$\alpha = \frac{2\pi(\Delta f)}{t} \quad Since \quad \omega = 2\pi f$$

Example 5: The block is lifted from rest until the angular velocity of the drum is **16 rad/s** after a time of **4 s**. What is the average angular acceleration?

$$\alpha = \frac{\omega_f - \cancel{\omega_o}^{\,0}}{t} \quad or \quad \alpha = \frac{\omega_f}{t}$$

$$\alpha = \frac{16 \text{ rad/s}}{4 \text{ s}} = 4.00 \frac{\text{rad}}{\text{s}^2}$$

$$\boxed{\alpha = 4.00 \text{ rad/s}^2}$$



1. Bicycle wheel turns 240 revolutions/min. What is its angular velocity in radians/second?

$$\omega = 240\frac{\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{sec}} \times \frac{2\pi\ \text{rads}}{1\,\text{rev}} = 8\pi\ \text{radians/sec} \approx 25.1\,\text{radians/sec}$$

2. If wheel slows down uniformly to rest in 5 seconds, what is the angular acceleration?

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 - 25\,\text{rad/sec}}{5\,\text{sec}} = -5\,\text{rad/sec}^2$$

Angular and Linear Speed

From the definition of angular displacement:

$s = \theta R$ ***Linear vs. angular displacement***

$$v = \frac{\Delta s}{\Delta t} = \left(\frac{\Delta\theta \cdot R}{\Delta t}\right) = \left(\frac{\Delta\theta}{\Delta t}\right)R$$

$$\boxed{v = \omega R}$$

Linear speed = angular speed x radius

Angular and Linear Acceleration:

From the velocity relationship we have:

$v = \omega R$ Linear vs. angular velocity

$$v = \frac{\Delta v}{\Delta t} = \left(\frac{\Delta v \cdot R}{\Delta t}\right) = \left(\frac{\Delta v}{\Delta t}\right) R$$

$$\boxed{a = \alpha R}$$

Linear accel. = angular accel. x radius

Relationship Between Angular and Linear Quantities

- Displacements

$$s = \theta r$$

- Speeds

$$v = \omega r$$

- Accelerations

$$a = \alpha r$$

- Every point on the rotating object has **the same angular motion**
- Every point on the rotating object does ***not*** have the same **linear motion**

Angular vs. Linear Parameters

Recall the definition of **linear acceleration** a from kinematics.

$$\boxed{a = \frac{v_f - v_0}{t}}$$

But, $a = \alpha R$ and $v = \omega R$, so that we may write:

$$a = \frac{v_f - v_0}{t} \quad \textbf{\textit{becomes}} \quad \alpha \cancel{R} = \frac{\cancel{R}\omega_f - \cancel{R}\omega_0}{t}$$

Angular acceleration is the time rate of change in angular velocity.

$$\boxed{\alpha = \frac{\omega_f - \omega_0}{t}}$$

Linear equations	Angular equations
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$

Moment of Inertia

Moment of inertia is defined as **the quantity expressed by the body resisting angular acceleration** which is the sum of the product of the mass of every particle with its square of a distance from the axis of rotation.

Formula

$$I = \frac{L}{\omega}$$

I = inertia

L = angular momentum

ω = angular velocity

Moment of Inertia of Different objects