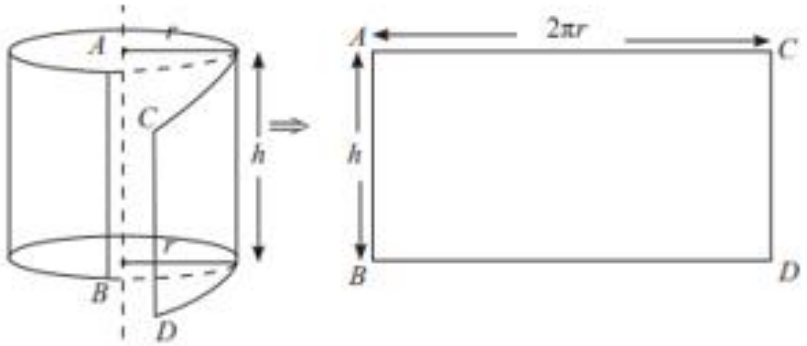
Science For Technology

Grade 12 : Mathematics

AREA & VOLUME

Recall Your Memory from O/L Mathematics…..

Surface Area of a Right Circular Cylinder

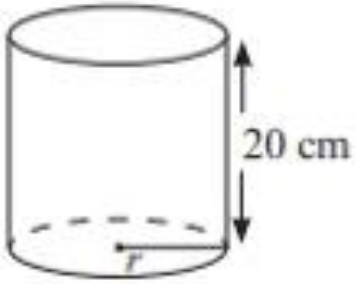


Total surface area of the cylinder	=	Area of top face	+	Area of bottom face	+	Area of curved surface

$$A = \pi r^2 + \pi r^2 + 2\pi rh$$

$$\boxed{A = 2\pi r^2 + 2\pi rh}$$

Examples.....



The circumference of the base of a cylindrical vessel without a lid, of height 20 cm is 88 cm.

(i) Find the radius of the base.

(ii) Find the total exterior surface area.

The surface area of a solid metal cylinder is 2442 cm^2 while the sum of its radius and height is 37 cm.

(i) Find the radius of the cylinder.

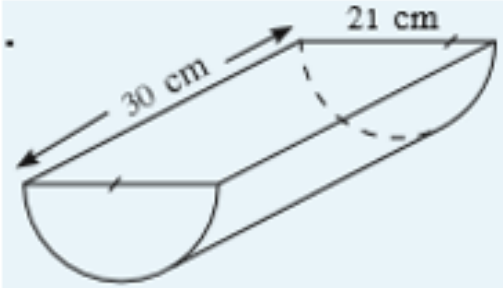
(ii) Find the area of the curved surface of the cylinder.

Volume of a Right Circular Cylinder

The volume of a cylindrical vessel of base area 346.5 cm^2 is 6930 cm^3.

(i) Find the radius of the cylinder.

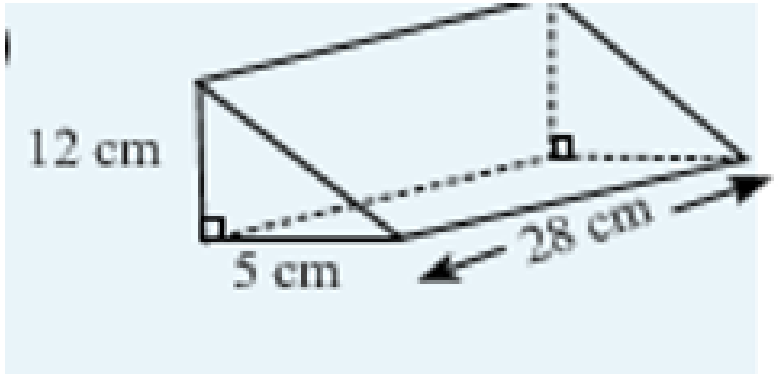
(ii) Find the height of the cylinder.

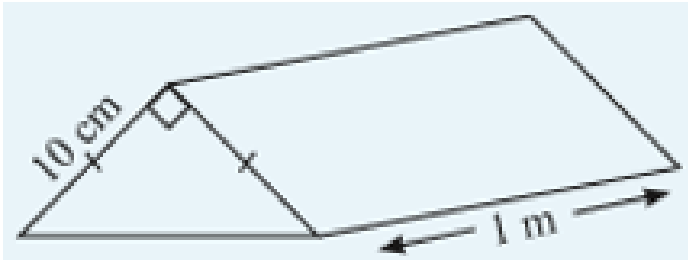


Calculate how many solid metal cylinders of height 21 cm and radius 3.5 cm can be made without wastage by heating the solid semi-cylindrical metal object with the measurements given in the figure.

A cylindrical vessel of radius 14 cm has been filled with water to a height of 30 cm. What is the minimum number of times that a cylindrical vessel of radius 7 cm and height 10 cm should be used to remove all the water in the given vessel?

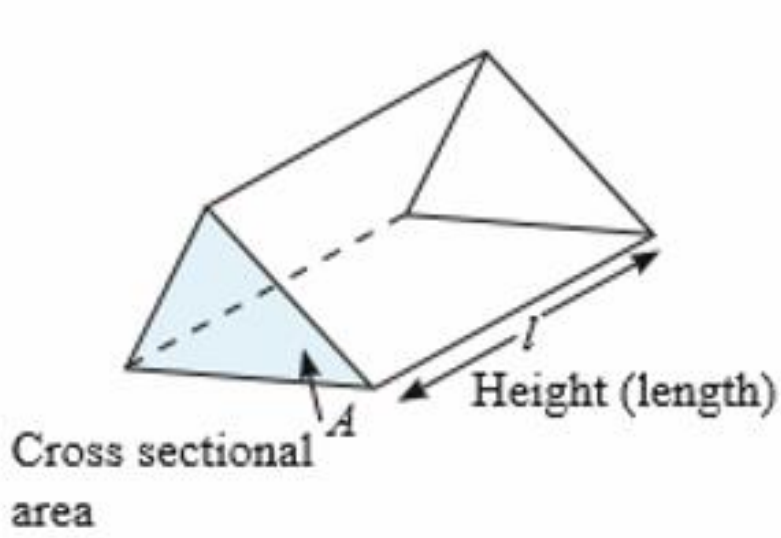
Surface Area of a Rectangular Prism





Find the surface area of the prism in the figure.

Volume of a rectangular Prism



Volume of the prism = Area of the cross section × Perpendicular height (length)

$$V = Al$$

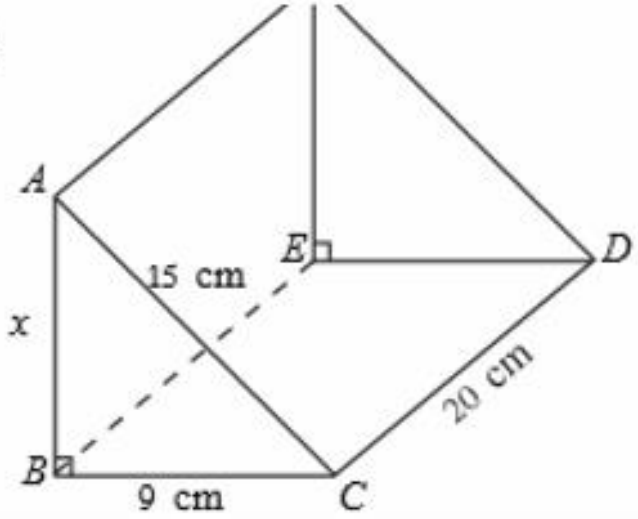
Examples ……

A prism with a cross section the shape of a right angled triangle is shown in the figure.

(i) Find the length denoted by x in the cross section.

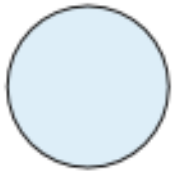
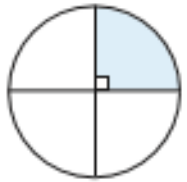
(ii) Find the area of the cross section.

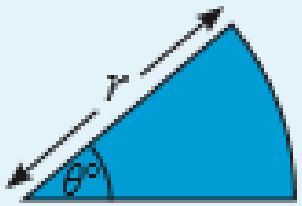
(iii) Find the volume of the prism.



Water has been filled to a height of 8 cm of a cuboid shaped vessel of length 30 cm and breadth 20 cm. If the level of the water in the vessel rises by 2 cm when a right triangular prism of cross sectional area 60 cm^2 is dropped carefully into the water, find the perpendicular height of the prism.

Area of a Circle

Sector	Shaded Sector as a fraction of the circle	Area of the Sector
	1	πr^2
	$\frac{1}{2}$	$\frac{1}{2} \times \pi r^2$
	$\frac{1}{4}$	$\frac{1}{4} \times \pi r^2$
	$\frac{10}{360}$	$\frac{10}{360} \times \pi r^2$
	$\frac{\theta}{360}$	$\frac{\theta}{360} \times \pi r^2$



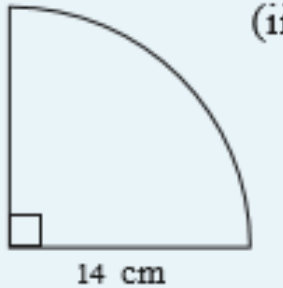
According to the pattern in the table, the area of the sector of radius r and angle at the centre θ° is $\frac{\theta}{360} \times \pi r^2$

Let us consider (through the following examples) how the area of a sector is found using this result. In the examples and exercises of this chapter, the value of π is taken as $\frac{22}{7}$.

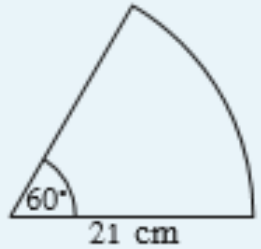
1. Find the area of each sector.

(i)

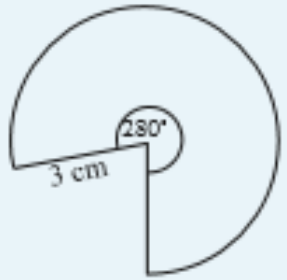
(ii)



(iii)

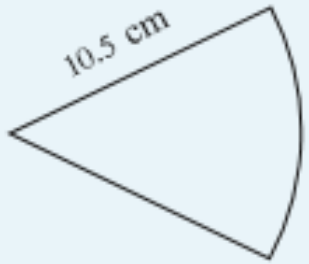


(iv)

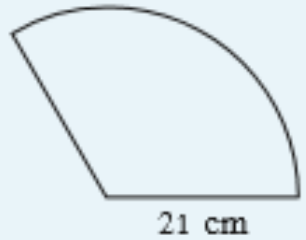


2. The areas of the two sectors of circles given below are 77 cm^2 and 462 cm^2 respectively. Find the angle at the centre of each sector.

(i)

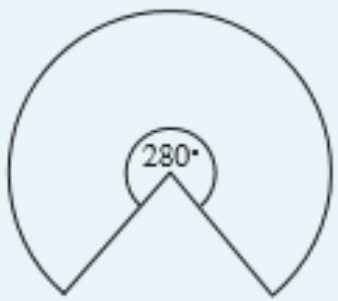


(ii)

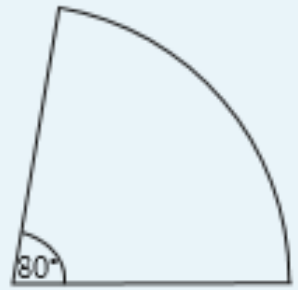


3. The areas of the two sectors of circles given below are 792 cm^2 and $6\frac{2}{7}cm^2$ respectively. For each sector, find the radius of the corresponding circle.

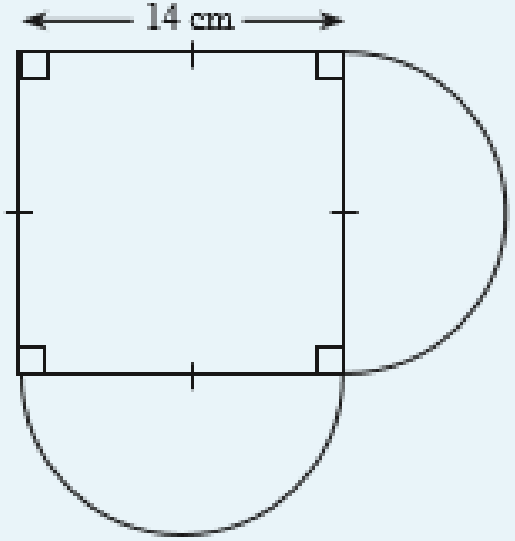
(i)



(ii)

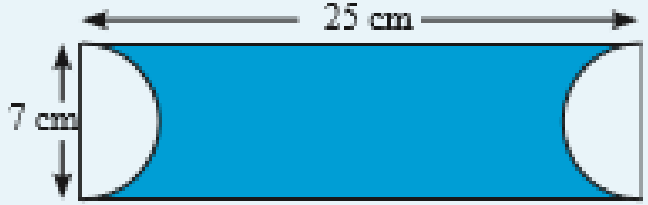


1. The following is a compound plane figure consisting of a square and two semi-circles.



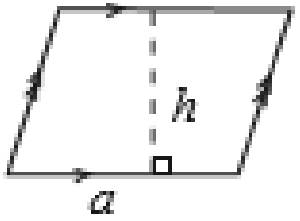
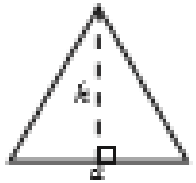
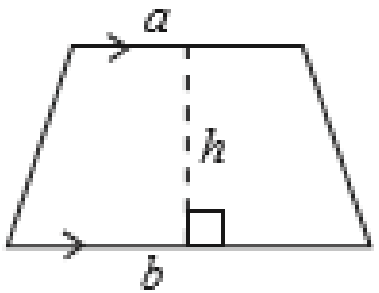
(i) Find the area of the square.
(ii) Find the radius of a semi-circular portion.
(iii) Find the total area of the two semi-circular portions.
(iv) Find the area of the compound plane figure.

2. The shaded portion in the figure was obtained by cutting out two semi-circular parts from a rectangular piece of paper.

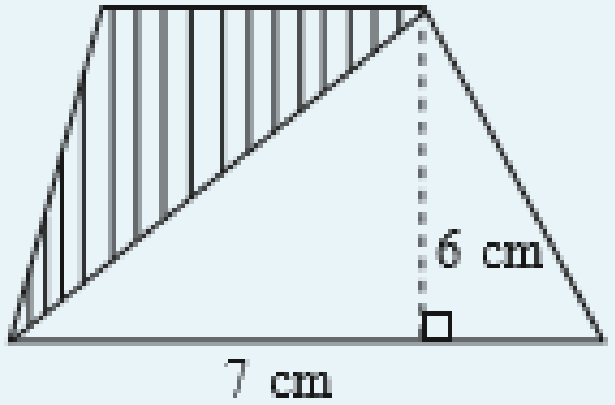


(i) Find the area of the rectangle.
(ii) Find the total area of the two semi-circular parts.
(iii) Find the area of the shaded portion.

SURFACE AREA OF OTHER GEOMETRICAL SHAPES

Parallelogram		base × altitude	$A = a \times h$
Triangle		$\frac{1}{2}$ × base × altitude	$A = \frac{1}{2} \times a \times h$
Trapezium		$\frac{1}{2}$ × sum of the lengths of the parallel sides × altitude	$A = \frac{1}{2}(a+b) \times h$

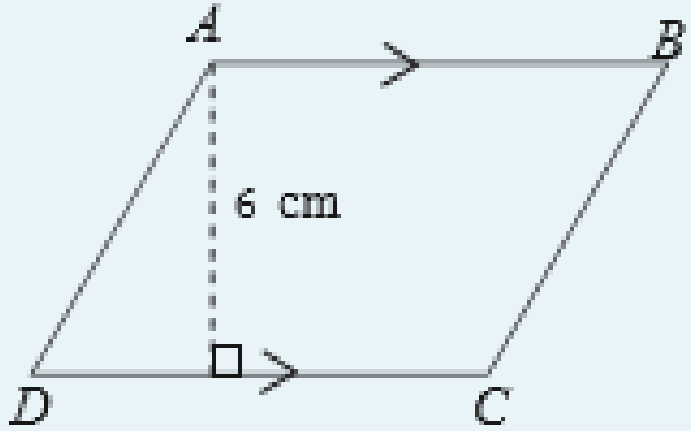
The figure denotes a trapezium of area 33 cm^2 that has been formed by joining two triangles together. Find the area of the triangle which is shaded.



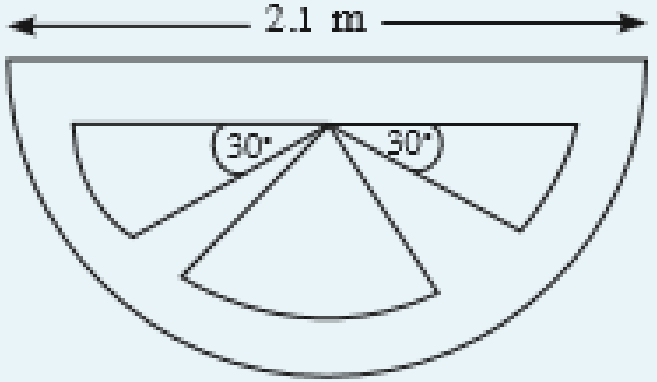
The figure denotes a parallelogram of area 120 cm^2. Its perimeter is 64 cm. Determine the following based on the information that is given.

(i) The length of the side CD.

(ii) The length of the side BC.



. A sketch of the area in front of a commemorative plaque is given in the figure. Grass has been grown in the three portions within the semi-circle which are in the shape of sectors of circles, while white sand has been spread in the other regions. The radius of each sector within the semi-circle is 84 cm.



(i) What is the radius of the semi-circle in centimetres?

(ii) Find the area of the semi-circular part in cm^2.

(iii) Find the area of one of the sectors with angle at the centre equal to 30°.

(iv) Find the angle at the centre of the large sector if its area is 1848 square centimetres more than the sum of the areas of the other two sectors.

SURFACE AREA OF RIGHT PYRAMID WITH SQUARE BASE

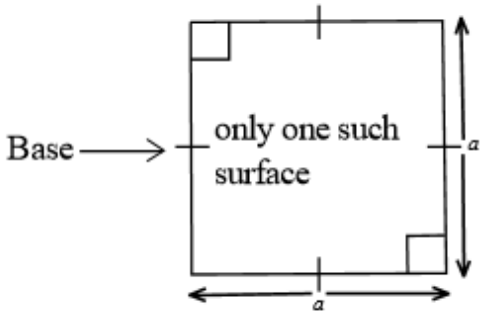


Figure I

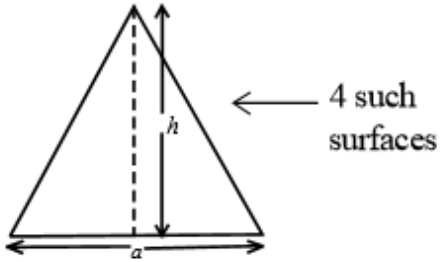
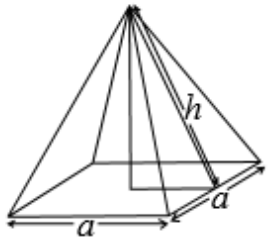


Figure II

Surface area of figure I and figure II can be calculated as follows.



Area of the base of the pyramid $= a \times a = a^2$

Area of a triangular face $= \frac{1}{2} \times a \times h$

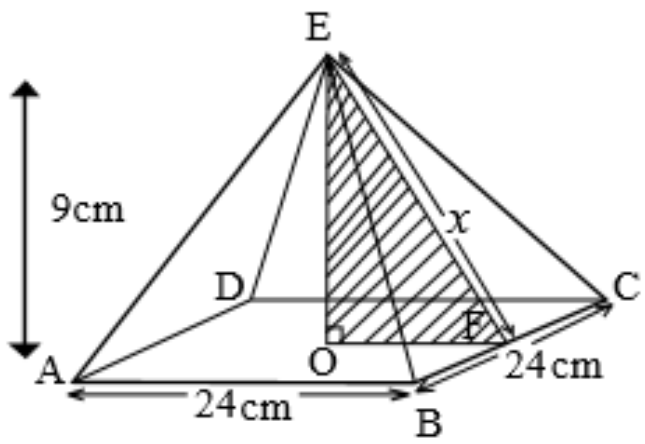
Area of 4 triangular faces $= \frac{1}{2} \times a \times h \times 4$

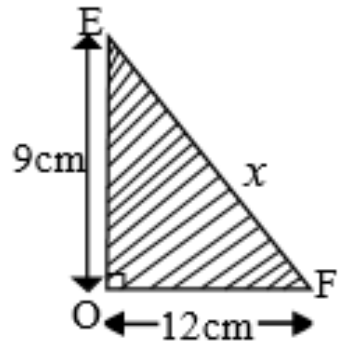
$= 2ah$

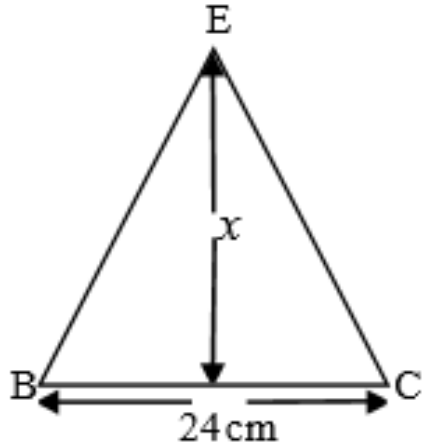
$\therefore$ The total surface area $= a^2 + 2ah$

$= a(a + 2h)$

The perpendicular height of a square based pyramid is 9 cm. The length of a side of the base is 24 cm. Find the surface area of the pyramid.







$$x^2 = 9^2 + 12^2$$

$$x^2 = 81 + 144 = 225$$

$$x = 15\,\text{cm}$$

Area of a triangular face $= \frac{1}{2} \times 24 \times 15 = 180\,\text{cm}^2$

Area of 4 triangular faces $= 180 \times 4\,\text{cm}^2 = 720\,\text{cm}^2$

Area of the base $= 24 \times 24\,\text{cm}^2 = 576\,\text{cm}^2$

Surface area of the pyramid $= \underline{\underline{1296\,\text{cm}^2}}$

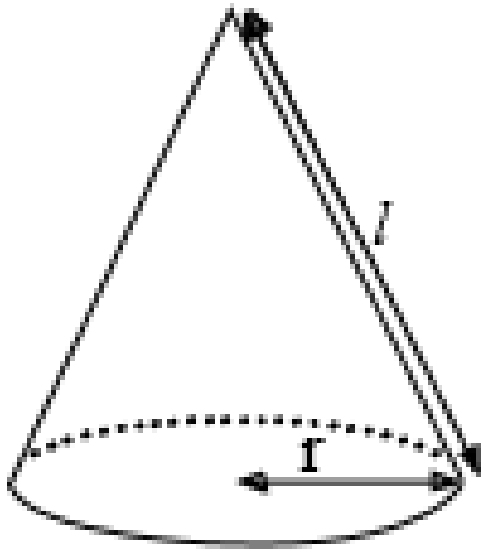
Ajith is planning to build a tent in the shape of a pyramid. For the construction, he has 4 sticks each of length 6 m for the base and 4 sticks each of length 5m for the slant edges. Find the area of the cloth needed to cover the pyramid completely.

The surface area of a square based right pyramid is 340 cm^2. A side of the base is 10 cm.

(i) Find the perpendicular height of a triangular face.

(ii) Find the length of a slant edge of a triangular face.

SURFACE AREA OF A CONE



The Total surface area of a Cone = area of the circular base + area of the curved surface

$= \pi r^2 + \pi rl$ Square Units

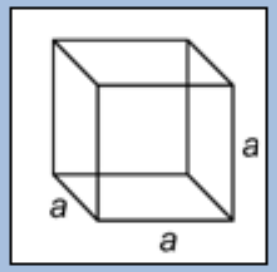
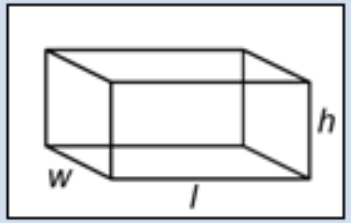
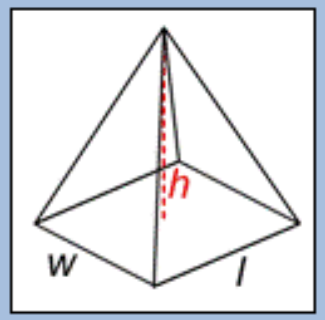
$= \pi r(r + l)$ Square Units

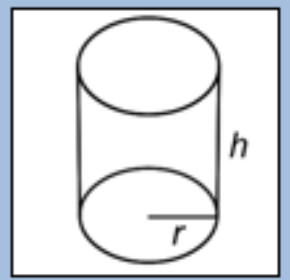
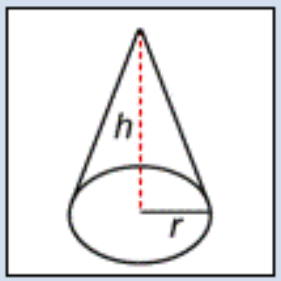
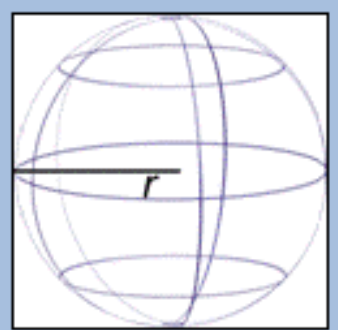
The circumference of the base of a right conical shaped heap of sand is 528cm. The slant height of the heap of sand is 140 cm Find an approximate value for the area on which the heap of sand is spread. Find the area of the curved surface.

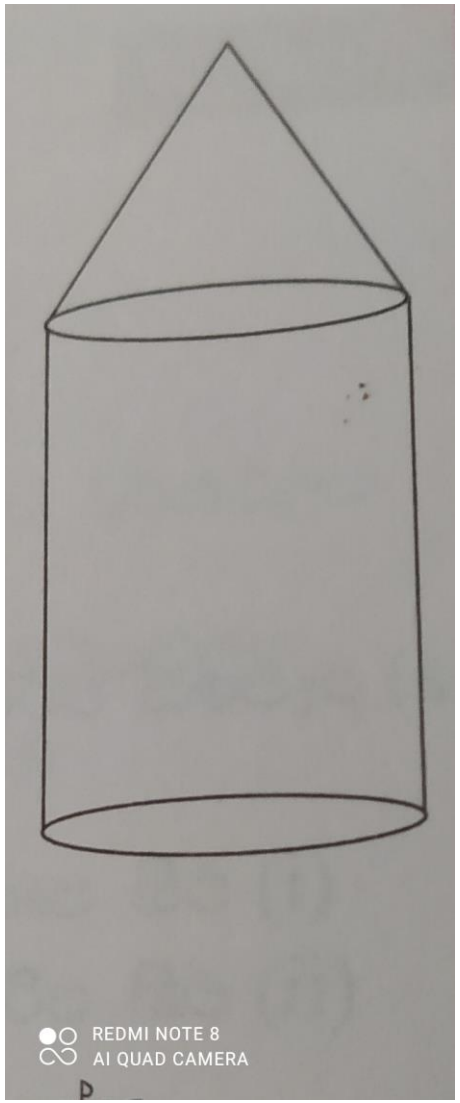
Kamal wishes to make a conical tent. He decides that the height of the tent should be 200 cm, and the radius 1.5 m. Find the area of the canvas required. to make the tent.

(Ignore canvas used for joining)

Volume of Different Geometrical Shapes

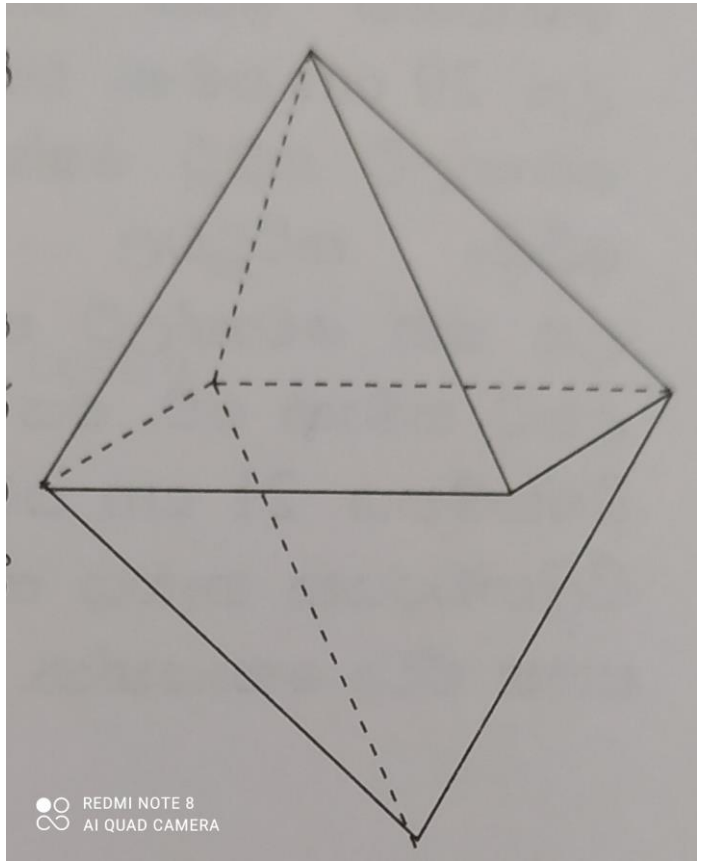
Name	Transparent Form	Volume Formula
Cube		$V = a \cdot a \cdot a = a^3$ a = the length of one side
Rectangular prism		$V = l \cdot w \cdot h$ l = length w = width h = height
Pyramid		$V = \frac{l \cdot w \cdot h}{3}$ l = length w = width h = height

Name	Transparent Form	Volume Formula
Cylinder		$V = \pi \cdot r^2 \cdot h$ r = radius h = height
Cone		$V = \frac{\pi \cdot r^2 \cdot h}{3}$ r = radius h = height
Sphere		$V = \frac{4}{3}\pi r^3$ r = radius

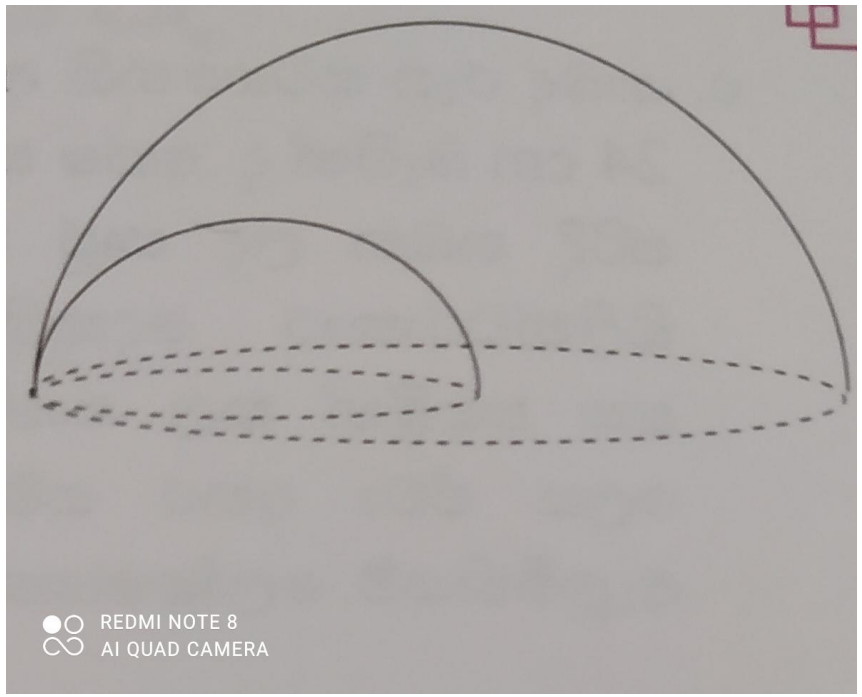


The figure shows the candle, upper part is conical shape and latter part is cylindrical shape. The height of the cone is given as 3 cm and the cylinder is 10 cm. the radius of the cylinder is 10 cm .

i. If the cylinder should rapped, the find the minimum area required for the rapping paper.
ii. Find the amount of wax needed for produce this candle.



Two pyramids having square base with 20 cm length, joined together, as shown in figure. The height of pyramids are 30 cm & 25 cm.
Find the area and volume of combined object.



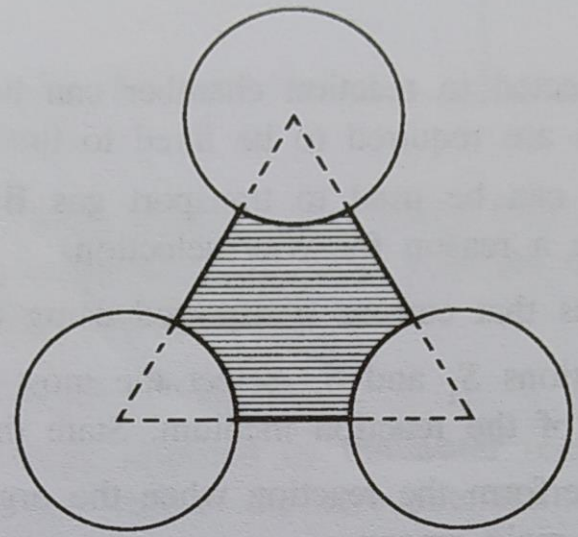
The volume of half sphere, having a radius of 10 cm, cut and removed from the large semi sphere of radius 20 cm. Find ,

I. Volume of remaining part of large semi sphere
II. Ratio of volume of remaining large semi sphere to small semi sphere.

6. (a) Show that the volumes of a closed spherical tank of radius 3 m and a closed right circular cylinder of radius 3 m and height 4 m are equal.

(b) The expected production cost of spherical tank and right circular cylindrical tank per square metre are Rs 20 000/= and Rs 15 000/= respectively. Show that it is cheaper to build the cylindrical tank by calculating the production cost of each of these two tanks.

(c) Three right circular cylindrical tanks of radius 3 m and height 4 m are to be placed on a flat land with the centres of the circular bottom of the tanks at the vertices of an equilateral triangular area of side 7 m as shown in the figure.



(i) Calculate the area of this triangle.

(ii) The area of this triangle not covered by the cylindrical tanks has to be calculated. This is show by the shaded area in the above figure. Calculate this area.

[see page ei