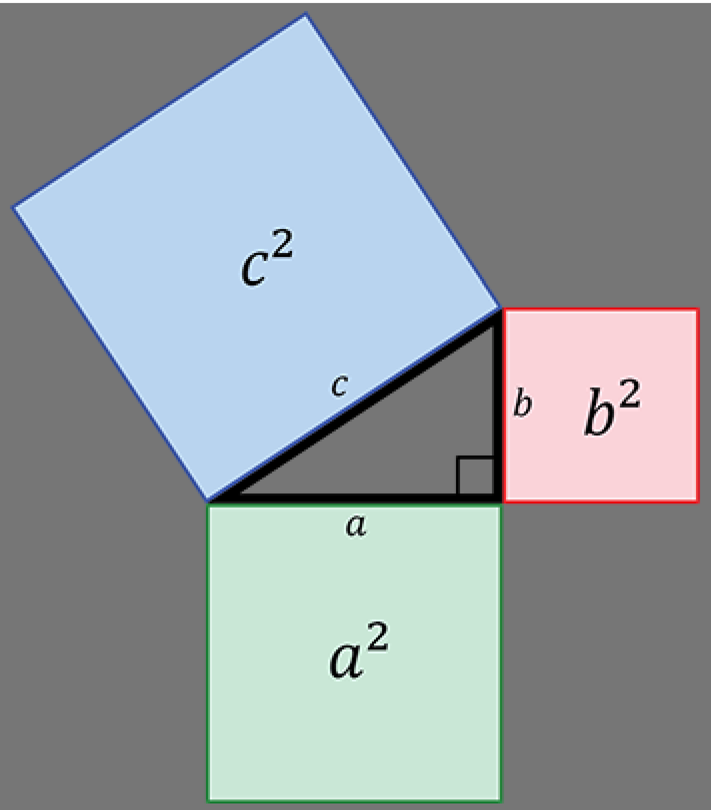
TRIGNOMETRY

Pythagoras’ Theorem vs. Trigonometry

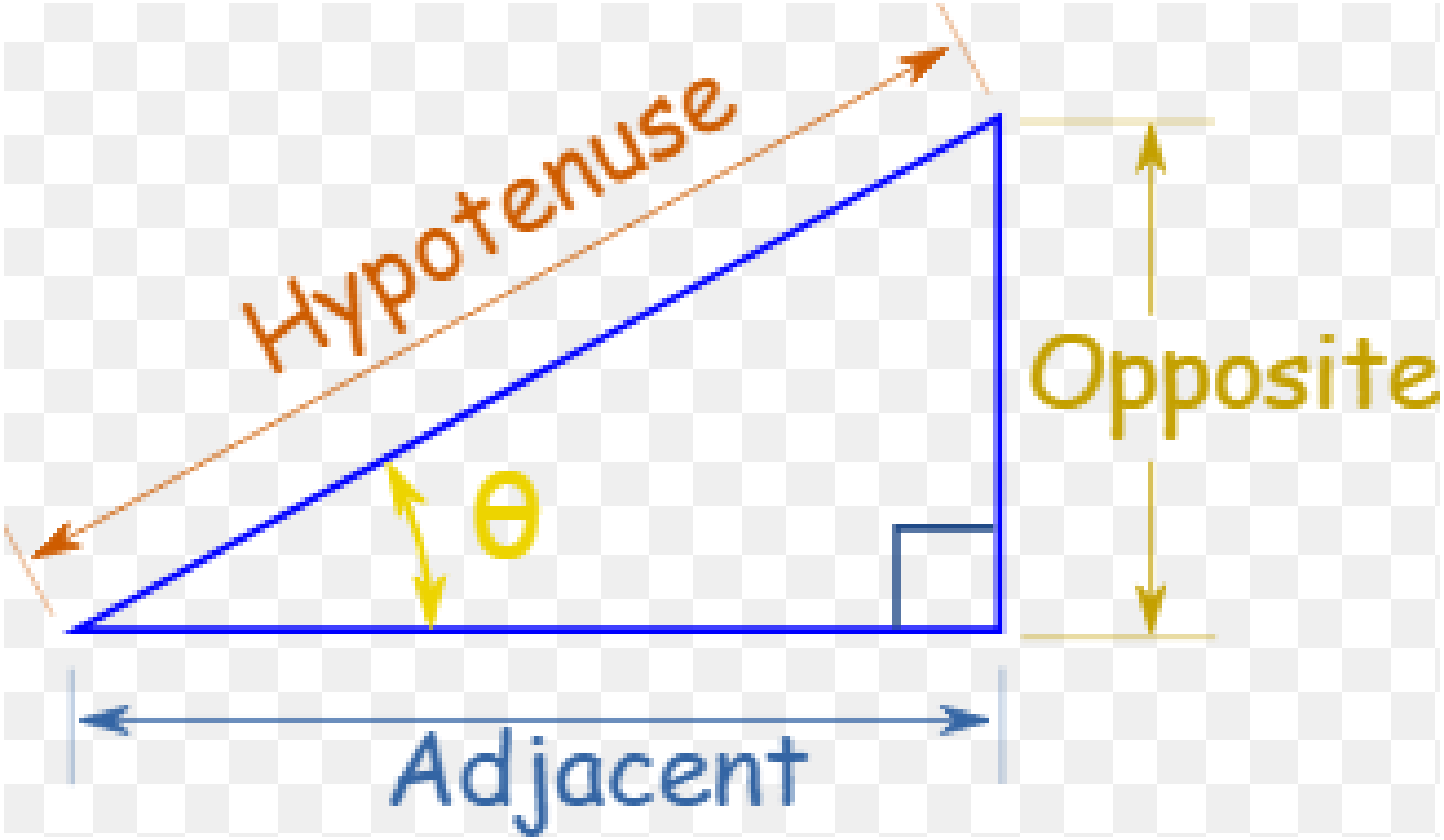
It is an important rule that applies *only to right-angled triangles*. It says that **‘the square on the hypotenuse is equal to the sum of the squares on the other two sides.’**



Pythagoras’ Theorem says :

$$a^2 + b^2 = c^2$$

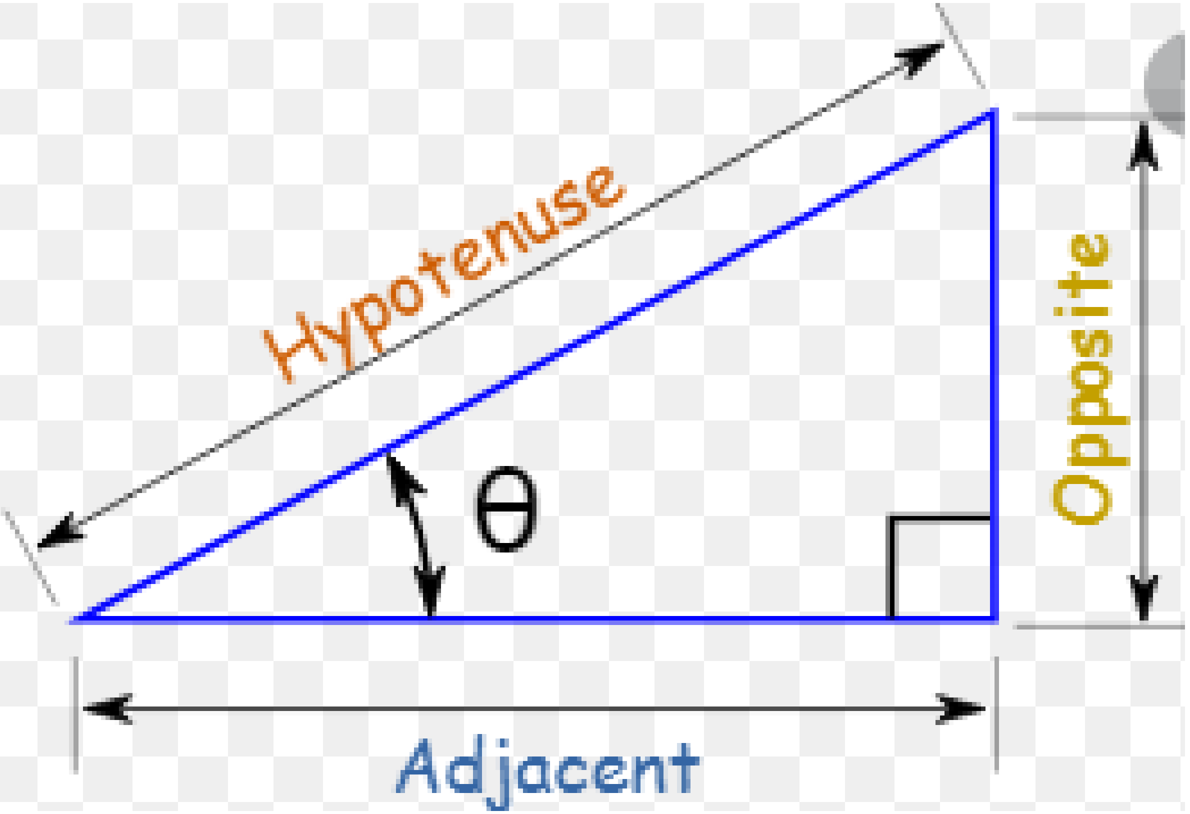
So, if we know the length of two sides of a triangle and we need to calculate the third, we can use Pythagoras’ Theorem.


Hypotenuse
Opposite
θ
Adjacent

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

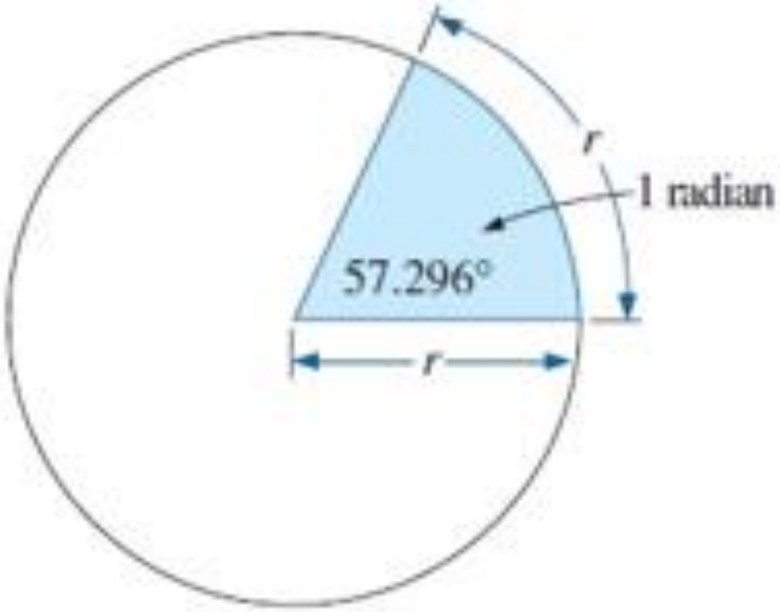
$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$



Angles in degrees	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not defined

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	අර්ථ නොදැක්වේ

Angular Measurement - Radian



Defining the radian.

$$\boxed{1 \text{ rad} = 57.296^\circ \cong 57.3^\circ}$$

$$\boxed{2\pi \text{ rad} = 360^\circ}$$

$$\boxed{\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees})}$$

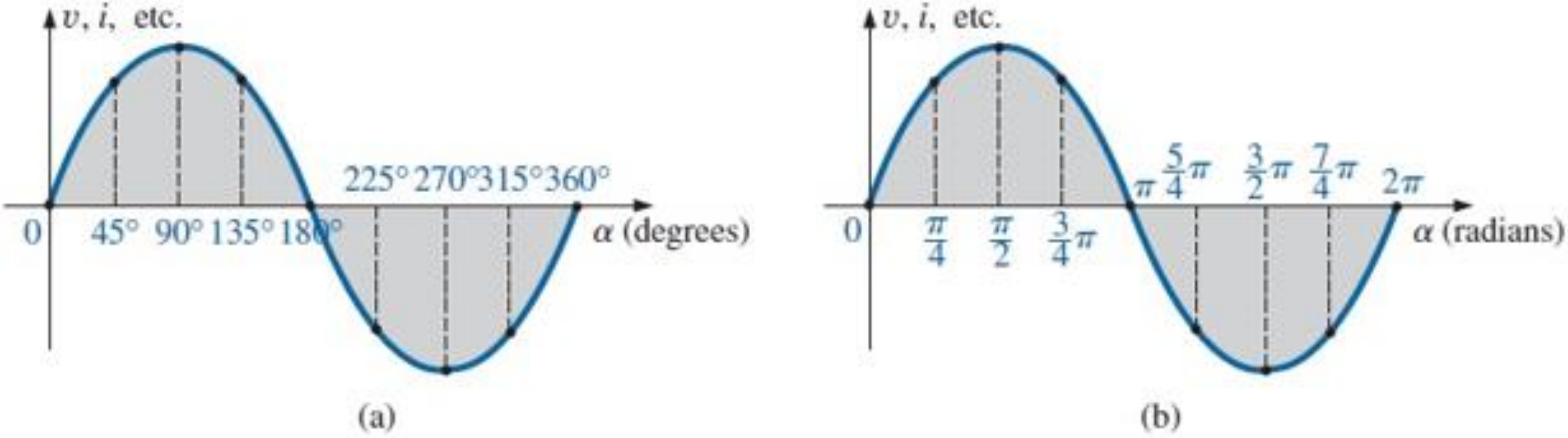
$$\boxed{\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians})}$$

$$\mathbf{90^\circ{:}}\quad \text{Radians} = \frac{\pi}{180^\circ}(90^\circ) = \frac{\pi}{2}\ \textbf{rad}$$

$$\mathbf{30^\circ{:}}\quad \text{Radians} = \frac{\pi}{180^\circ}(30^\circ) = \frac{\pi}{6}\ \textbf{rad}$$

$$\frac{\pi}{3}\ \textbf{rad}{:}\quad \text{Degrees} = \frac{180^\circ}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60^\circ}$$

$$\frac{3\pi}{2}\ \textbf{rad:}\quad \text{Degrees} = \frac{180^\circ}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270^\circ}$$

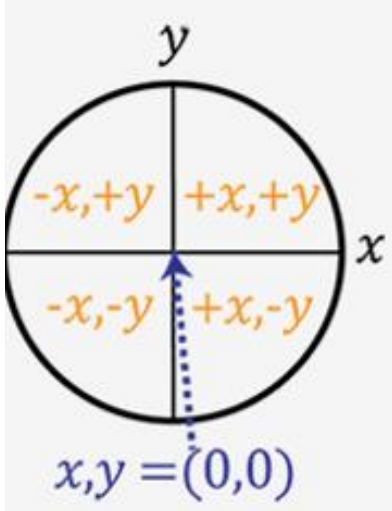


Plotting a sine wave versus (a) degrees and (b) radians.

Trigonometry in a Circle

When considering triangles, we are limited to angles less than 90°. However, trigonometry is equally applicable to all angles, from 0 to 360°. To understand how the trigonometric functions work with angles greater than 90°, it is helpful to think about triangles constructed within a circle.

Consider a circle, divided into four quadrants.

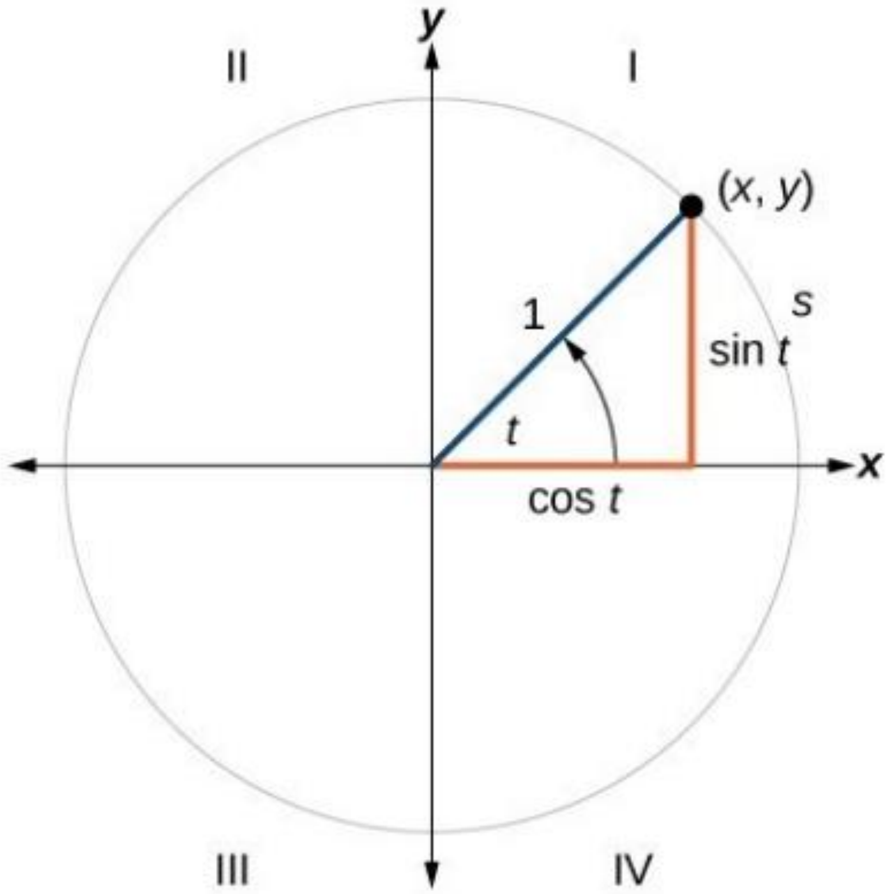


Anything to the left of the centre has an x value of less than 0, or is negative, while anything to the right has a positive value.

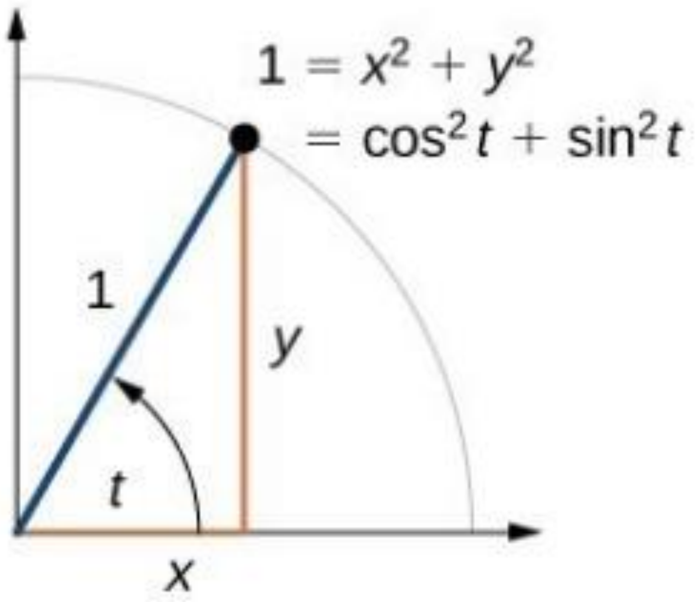
Similarly anything below the centre point has a y value of less than 0, or is negative and any point in the top of the circle has a positive y value.

For any angle t, we can label the intersection of the terminal side and the unit circle as by its coordinates, (x, y).

This means x = cos t and y = sin t.

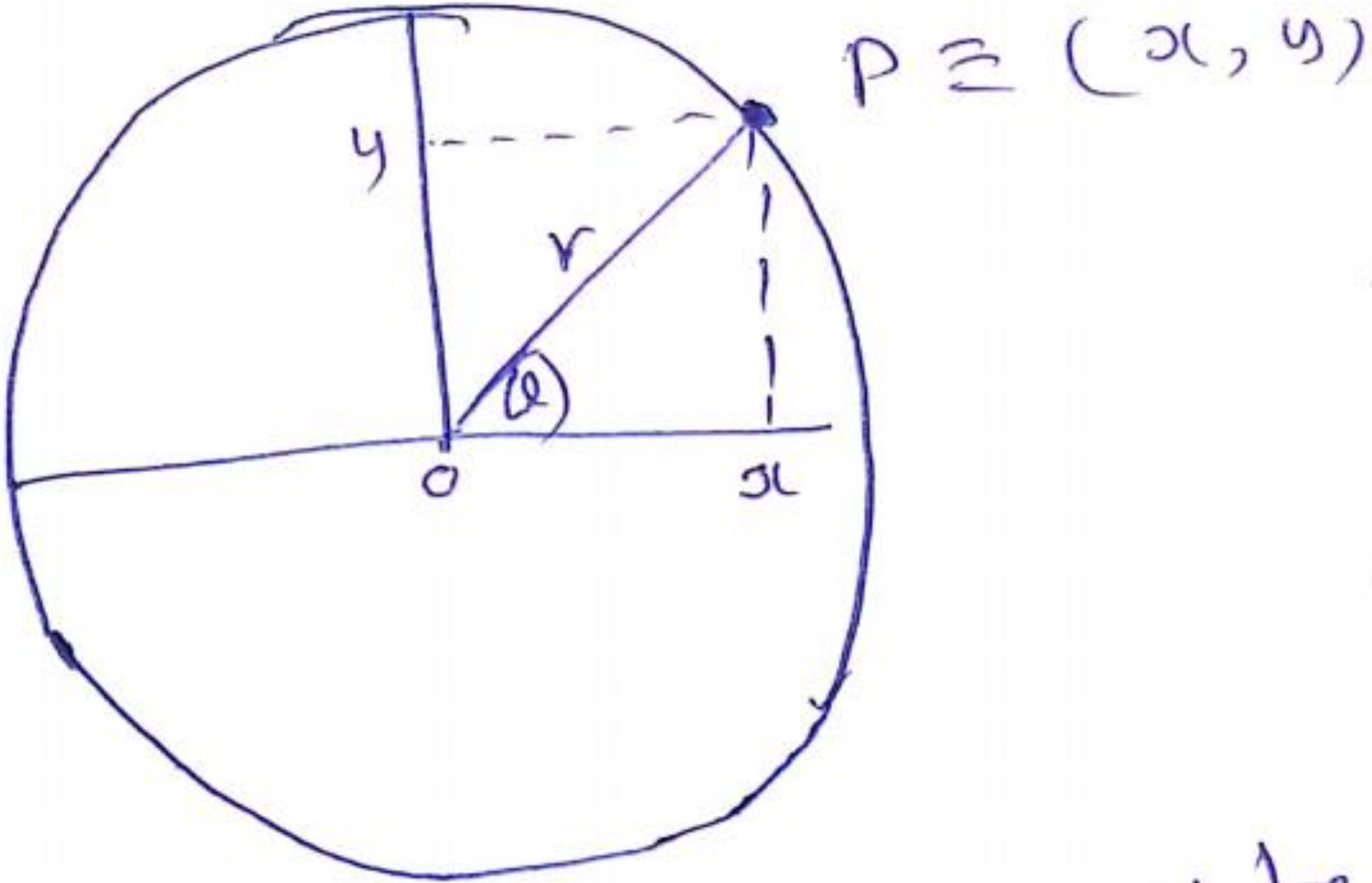


The Pythagorean Identity



$\cos^2 t + \sin^2 t = 1$, is known as the **Pythagorean Identity**.

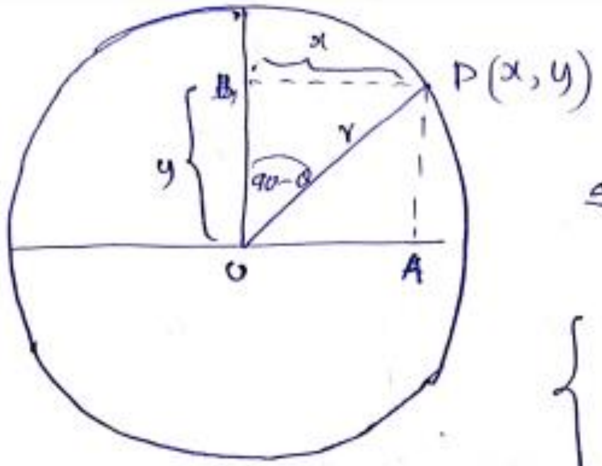
Some Important Trigonometric Functions….



$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\therefore \tan\theta = \frac{y}{x}$$



$$\sin(90-\theta) = \frac{BP}{OP}$$

$$\left\{ \begin{aligned} \text{But } BP &= OA \\ &= x \\ OP &= r \end{aligned} \right\}$$

$$\therefore \sin(90-\theta) = \frac{x}{r}$$

$$= \cos\theta$$

$$\therefore \boxed{\sin(90-\theta) = \cos\theta}$$

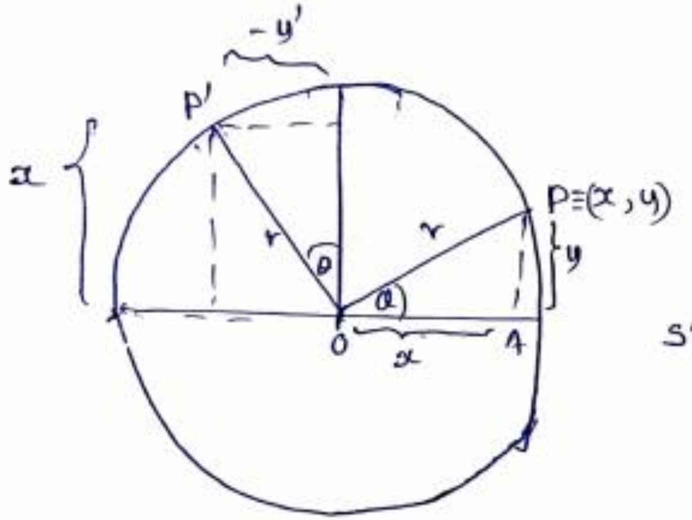
similarly;

$$\cos(90-\theta) = \frac{OB}{OP}$$

$$= \frac{AP}{OP}$$

$$= y/r = \sin\theta$$

$$\therefore \boxed{\cos(90-\theta) = \sin\theta}$$



$\sin\theta = \dfrac{y}{r}$; $\cos\theta = \dfrac{x}{r}$

$$\sin\left(\frac{\pi}{2}+\theta\right) = \frac{\text{Projection of } OP' \text{ on } y\text{-axis}}{OP'}$$

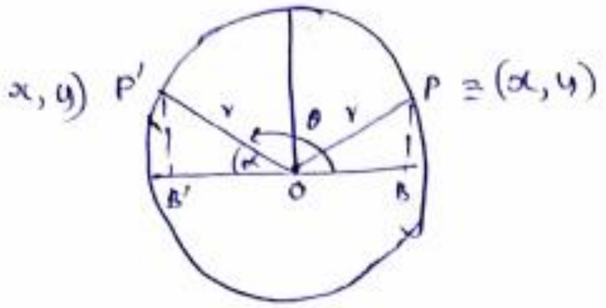
$$= \frac{x}{r} = \cos\theta$$

$$\therefore \boxed{\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta}$$

$$\cos\left(\frac{\pi}{2}+\theta\right) = \frac{\text{Projection of } OP' \text{ on } x\text{-axis}}{OP'}$$

$$= \frac{-y}{r} = -\left(\frac{y}{r}\right) = -\sin\theta$$

$$\therefore \boxed{\left(\cos \frac{\pi}{2}+\theta\right) = -\sin\theta}$$



$\alpha = 180 - \theta$

considering $OP'B'$ triangle,

$\sin\alpha = \frac{y}{r}$.

$\cos\alpha = \frac{-x}{r}$.

But $\alpha = 180 - \theta$;

$\therefore \sin(180-\theta) = \frac{y}{r} = \sin\theta$.

$\cos(180-\theta) = -\frac{x}{r} = -\cos\theta$

$\therefore$

$$\boxed{\begin{aligned} \sin(180-\theta) &= \sin\theta \\ \cos(180-\theta) &= -\cos\theta \\ \therefore \tan(180-\theta) &= \frac{\sin\theta}{-\cos\theta} = -\tan\theta \end{aligned}}$$

Exercises ……

Find the values of following;

- Sin (120^0)
- Cos (150^0)
- Tan (135^0)

Use of Trigonometric tables

- Find the value of Sin (44)

ප්‍රකෘති සයින இயற்கைச் சைன்கள் **NATURAL SINES**

	0'	10'	20'	30'	40'	50'	60'		1'	2'	3'	4'	5'	6'	7'	8'	9'
									මධ්‍යක අන්තරය இடை வித்தியாசங்கள் Mean Differences								
41	.6561	.6583	.6604	.6626	.6648	.6670	.6691	48	2	4	7	9	11	13	15	17	20
42	.6691	.6713	.6734	.6756	.6777	.6799	.6820	47	2	4	6	9	11	13	15	17	19
43	.6820	.6841	.6862	.6884	.6905	.6926	.6947	46	2	4	6	8	11	13	15	17	19
44	$\boxed{.6947}$	.6967	.6988	.7009	.7030	.7050	.7071	45	2	4	6	8	10	12	15	17	19
	60'	50'	40'	30'	20'	10'	0'		1'	2'	3'	4'	5'	6'	7'	8'	9'

$\sin 44^\circ = 0.6947$ වේ.

Find Sin (67.15’)

ප්‍රකෘති සයින   இயற்கைச் சைன்கள்   **NATURAL SINES**

	0'	10'	20'	30'	40'	50'	60'		මධ්‍යක අන්තර இடை வித்தியாசங்கள் Mean Differences 1'	2'	3'	4'	5'	6'	7'	8'	9'
45°	0.7071	0.7092	0.7112	0.7133	0.7153	0.7173	0.7193	44°	2	4	6	8	10	12	14	16	18
46	.7193	.7214	.7234	.7254	.7274	.7294	.7314	43	2	4	6	8	10	12	14	16	18
66	.9135	.9147	.9159	.9171	.9182	.9194	.9205	23	1	2	3	5	[illegible]	7	8	9	10
67	.9205	$\boxed{.9216}$	.9228	.9239	.9250	.9261	.9272	22	1	2	3	[illegible]	$\boxed{6}$	7	8	9	10
68	.9272	.9283	.9293	.9304	.9315	.9325	.9336	21	1	2	3	4	5	6	7	9	10
69	.9336	.9346	.9356	.9367	.9377	.9387	.9397	20°	1	2	3	4	5	6	7	8	9

එනම්, $\sin 67^{\circ}\ 15' = 0.9216 + 0.0006 = 0.9222$

Find Sin (114 30’)

$$\sin 114^\circ\ 30' = \sin(180^\circ - 65^\circ\ 30')$$
$$= \sin 65^\circ\ 30'$$

ප්‍රකෘති සයින
இயற்கைச் சைன்கள்
NATURAL SINES

	0′	10′	20′	30′	40′	50′	60′		1′	2′	3′	4′	5′	6′	7′	8′	9′
									මධ්‍යන්‍ය අන්තර / இடை வித்தியாசங்கள் / Mean Differences								
65	0.9063	0.9075	0.9088	$\boxed{0.9100}$	0.9112	0.9124	0.9135	24	1	2	4	5	6	7	8	10	11
66	.9135	.9147	.9159	.9171	.9182	.9194	.9205	23	1	2	3	5	6	7	8	9	10
67	.9205	.9216	.9228	.9239	.9250	.9261	.9272	22	1	2	3	4	6	7	8	9	10
68	.9272	.9283	.9293	.9304	.9315	.9325	.9336	21	1	2	3	4	5	6	7	9	10
69	.9336	.9346	.9356	.9367	.9377	.9387	.9397	20	1	2	3	4	5	6	7	8	9

වගුවට අනුව $\sin 114^\circ\ 30' = 0.9100$

Find Cos (133)

ප්‍රකෘති සයින இயற்கைச் சைன்கள் **NATURAL SINES**

°	0'	10'	20'	30'	40'	50'	60'		1'	2'	3'	4'	5'	6'	7'	8'	9'
40	0.6428	0.6450	0.6472	0.6494	0.6517	0.6539	0.6561	49	2	4	7	9	11	13	15	18	20
41	.6561	.6583	.6604	.6626	.6648	.6670	.6691	48	2	4	7	9	11	13	15	17	20
42	.6691	.6713	.6734	.6756	.6777	.6799	$\boxed{.6820}$	47	2	4	6	9	11	13	15	17	19
43	$\boxed{6820}$	.6841	.6862	.6848	.6905	.6926	.6947	46	2	4	6	8	11	13	15	17	19
44	.6947	.6967	.6988	.7009	.7030	.7050	.7071	45	2	4	6	8	10	12	15	17	19
	60'	50'	40'	30'	20'	10'	0'		1'	2'	3'	4'	5'	6'	7'	8'	9'

Mean differences heading (above columns 1'–9'):

මධ්‍යන්‍ය අන්තරය
இடை வித்தியாசங்கள்
Mean Differences

ප්‍රකෘති කෝසයින
இயற்கைக் கோசைன்கள்
NATURAL COSINES

Find Tan (72 15’)

ප්‍රකෘති ටැංජන இயற்கைத் தாள்சன்கள் **NATURAL TANGENTS**

	0'	10'	20'	30'	40'	50'	60'		1'	2'	3'	4'	5'	6'	7'	8'	9'
									මධ්‍යන්‍ය අන්තරය இடை வித்தியாசங்கள் Mean Differences								
45°	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44°	6	12	18	24	30	36	41	47	53
46	.0355	.0416	.0477	.0538	.0599	.0661	.0724	43	6	12	18	25	31	37	43	49	55
70°	2.747	2.773	2.798	2.824	2.850	2.877	2.904	19	3	5	8	10	13	16	18	21	23
71	2.904	2.932	2.960	2.989	3.018	3.047	3.078	18	3	6	9	12	14	17	20	23	26
72	3.078	$\boxed{3.108}$	3.140	3.172	3.204	3.237	3.271	17	3	6	10	13	$\boxed{16}$	19	23	26	29
73	3.271	3.305	3.340	3.376	3.412	3.450	3.487	16	4	7	11	14	18	22	25	29	32

Finding the Inverse value…

$\sin\theta = 0.5327$ නම් θ හි අගය සොයන්න.

ප්‍රකෘති සයින இயற்கைச் சைன்கள் NATURAL SINES

									இடை வித்தியாசங்கள் Mean Differences								
	0'	10'	20'	30'	40'	50'	60'		1'	2'	3'	4'	5'	6'	7'	8'	9'
0°	0.0000	0.0029	0.0058	0.0087	0.0116	0.0145	0.0175	89°	3	6	9	12	15	17	20	23	26
1	.0175	.0204	.0233	.0262	.0291	.0320	.0349	88	3	6	9	12	15	17	20	23	26
2	.0349	.0378	.0407	.0436	.0465	.0494	.0523	87	3	6	9	12	15	17	20	23	26
3	.0523	.0552	.0581	.0610	.0640	.0669	.0698	86	3	6	9	12	15	17	20	23	26
30°	0.5000	0.5025	0.5050	0.5075	0.5100	0.5125	0.5150	59	3	5	8	10	13	15	18	20	23
31	.5150	.5175	.5200	.5225	.5250	.5275	.5299	58	2	5	7	10	12	15	17	20	22
32	.5299	$\boxed{.5324}$	.5348	.5373	.5398	.5422	.5446	57	$\boxed{2}$	5	7	10	12	15	17	20	22
33	.5446	.5471	.5495	.5519	.5544	.5568	.5592	56	2	5	7	10	12	15	17	19	22

$\sin\theta = 0.9474$ නම් θහි අගය සොයන්න. මෙහි θ කෝණය මහා කෝණයක් බව සලකන්න.

ප්‍රකෘති සයින இயற்கைச் சைன்கள் NATURAL SINES

	0'	10'	20'	30'	40'	50'	60'		මධ්‍යන්‍ය අන්තරය இடை வித்தியாசங்கள் Mean Differences 1'	2'	3'	4'	5'	6'	7'	8'	9'
70°	0.9397	0.9407	0.9417	0.9426	0.9436	0.9446	0.9455	19	1	2	3	4	5	6	7	8	9
71	.9455	.9465	.9474	.9483	.9492	.9502	.9511	18	1	2	3	4	5	6	6	7	8
72	.9511	.9520	.9528	.9537	.9546	.9555	.9563	17	1	2	3	4	4	5	6	7	8
73	.9563	.9572	.9580	.9588	.9596	.9605	.9613	16	1	2	2	3	4	5	6	7	7
87	.9986	.9988	.9989	.9990	.9992	.9993	.9994	2	0	0	0	1	1	1	1	1	1
88	.9994	.9995	.9996	.9997	.9997	.9998	.9998	1									
89	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	0°									
	60'	50'	40'	30'	20'	10'	0'		1'	2'	3'	4'	5'	6'	7'	8'	9'

$\cos\theta = 0.9435$ නම් θ හි අගය සොයන්න.

ප්‍රකෘති සයින   இயற்கைச் சைன்கள்   NATURAL SINES

	0'	10'	20'	30'	40'	50'	60'		මධ්‍යක අන්තර / இடை வித்தியாசங்கள் / Mean Differences 1'	2'	3'	4'	5'	6'	7'	8'	9'
70°	0.9397	0.9407	0.9417	$\boxed{0.9426}$	0.9436	0.9446	0.9455	19	1	2	3	4	5	6	7	8	$\boxed{9}$
71	.9455	.9465	.9474	.9483	.9492	.9502	.9511	18	1	2	3	4	5	6	6	7	8
72	.9511	.9520	.9528	.9537	.9546	.9555	.9563	17	1	2	3	4	4	5	6	7	8
73	.9563	.9572	.9580	.9588	.9596	.9605	.9613	16	1	2	2	3	4	5	6	7	7
87	.9986	.9988	.9989	.9990	.9992	.9993	.9994	2	0	0	0	1	1	1	1	1	1
88	.9994	.9995	.9996	.9997	.9997	.9998	.9998	1									
89	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	0°									
	60'	50'	40'	30'	20'	10'	0'		1'	2'	3'	4'	5'	6'	7'	8'	9'

$\tan\theta = 0.3589$ නම් θහි අගය සොයන්න.

ප්‍රකෘති ටැංජන   இயற்கைத் தாள்சன்கள்   **NATURAL TANGENTS**

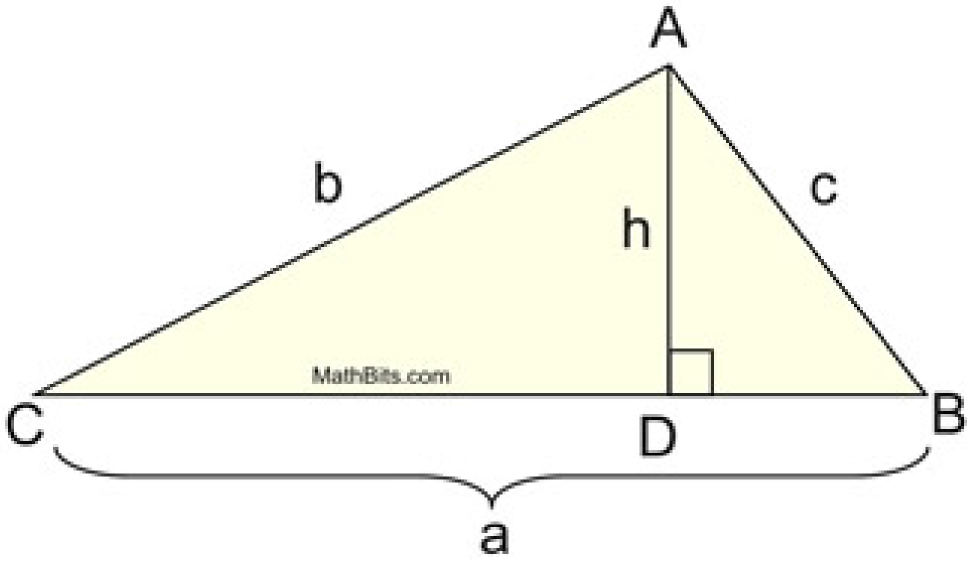
	0'	10'	20'	30'	40'	50'	60'		මධ්‍යක අන්තරය இடை வித்தியாசங்கள் Mean Differences 1'	2'	3'	4'	5'	6'	7'	8'	9'
0°	0.0000	0.0029	0.0058	0.0087	0.0116	0.0145	0.0175	89°	3	6	9	12	15	17	20	23	26
1	.0175	.0204	.0233	.0262	.0291	.0320	.0349	88	3	6	9	12	15	17	20	23	26
2	.0349	.0378	.0407	.0437	.0466	.0495	.0524	87	3	6	9	12	15	18	20	23	26
3	.0524	.0553	.0582	.0612	.0641	.0670	.0699	86	3	6	9	12	15	18	20	23	26
17	.3057	.3089	.3121	.3153	.3185	.3217	.3249	72	3	6	10	13	16	19	22	26	29
18	.3249	.3281	.3314	.3346	.3378	.3411	.3443	71	3	6	10	13	16	19	23	26	29
19	.3443	.3476	.3508	.3541	$\boxed{.3574}$	.3607	.3640	70°	3	7	10	13	$\boxed{16}$	20	23	26	29

A Trigonometric Formula for the Area of a Triangle:

The area of ΔABC can be expressed as:

$$A_{\Delta ABC} = \frac{1}{2}ah$$

where a represents the side (base)
and h represents the height drawn to that side.



Using trigonometry, let's take another look at this diagram.
In the right triangle CDA, we can state that:

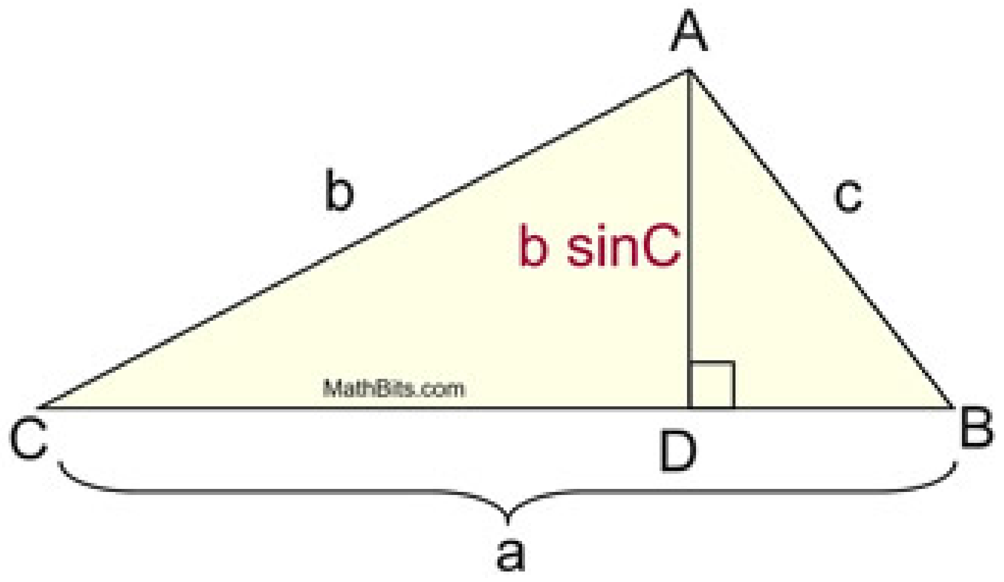
$$\sin C = \frac{h}{b} \qquad \text{(and multiplying by } b \text{ gives)} \qquad b \sin C = h$$

The height, ***h***, of the triangle can be expressed as ***b* sin *C***.

Substituting this new expression for the height, h, into the general formula for the area of a triangle gives:

$$A_{\Delta ABC} = \frac{1}{2} ab \sin C$$

where a and b can be any two sides and C is the included angle.



The **area of a triangle** can be expressed using the lengths of two sides and the sine of the included angle. **Area$_\Delta$ = ½ *ab* sin *C*.**

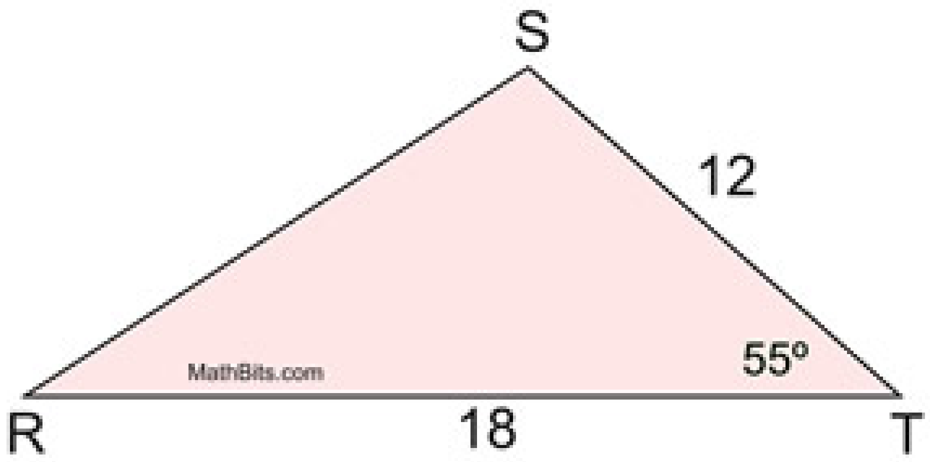
Given the triangle at the right, find its area. Express the answer to the *nearest hundredth* of a square unit.

$$A = \frac{1}{2} ab \sin C$$

$$A = \frac{1}{2}(12)(18) \sin 55^{\circ}$$

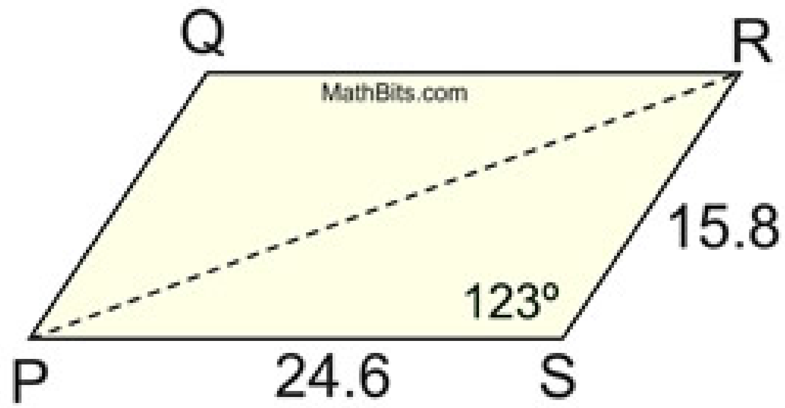
$$A \approx 88.46842078$$

$$A \approx 88.47 \text{ sq. units}$$



Let $a = ST$, $b = RT$, and $C = \angle RTS$.

Given the parallelogram shown at the right, find its area to the *nearest square unit.*



The diagonal of a parallelogram divides it into two congruent triangles. So the total area of the parallelogram will be TWICE the area of one of the triangles formed by the diagonal.

Let a = PS, b - RS, and C =$\angle PSR$.

$$A_{\Delta PSR} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(24.6)(15.8)\sin 123^\circ$$

$$\approx 162.9872382$$

$$A_{\square PQRS} \approx 2(162.9872382)$$

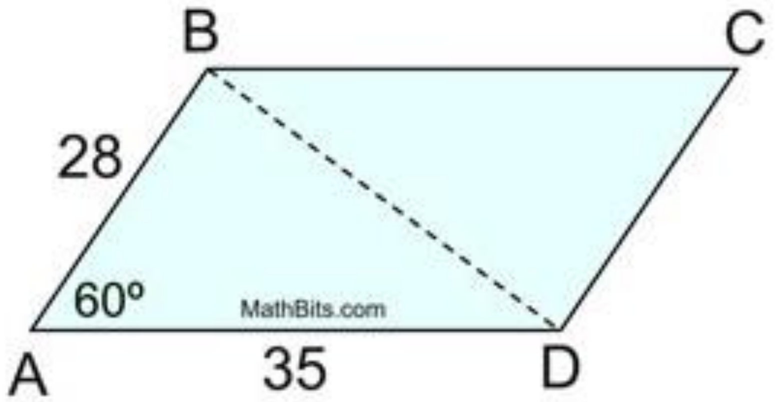
$$\approx 325.9744763 \approx 326 \text{ sq. units}$$

finding the area of a parallelogram, given 2 adjacent sides (a and b) and the included angle, C.

$$A_{\square} = ab\sin C$$

Area of Parallelogram

Given the parallelogram shown at the right, find its EXACT area.



Let $a = AD$, $b = AB$, and $C = \angle BAD$.

$$A_{\square ABCD} = ab\sin C$$

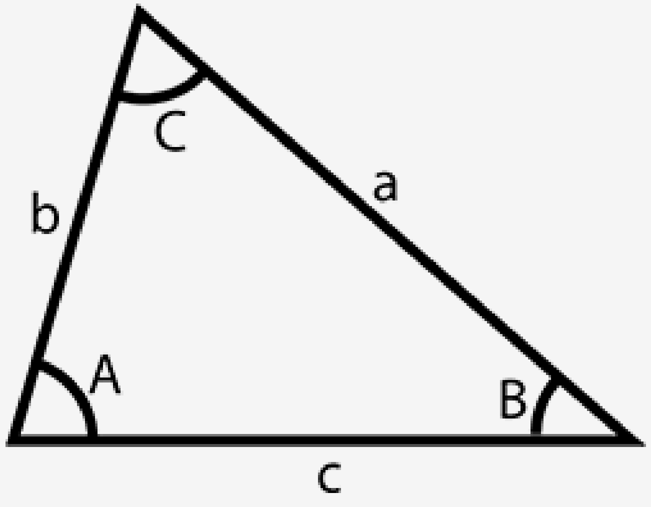
$$= (35)(28)\sin 60^\circ$$

$$= (35)(28)\left(\frac{\sqrt{3}}{2}\right)$$

$$= 490\sqrt{3} \text{ sq. units}$$

Other Triangles and Trigonometry

Trigonometry also works for other triangles, just not in quite the same way. Instead there are two rules based on a triangle like this:



The Sine rule is:

$^{a}/_{\sin A} = {}^{b}/_{\sin B} = {}^{c}/_{\sin C}$

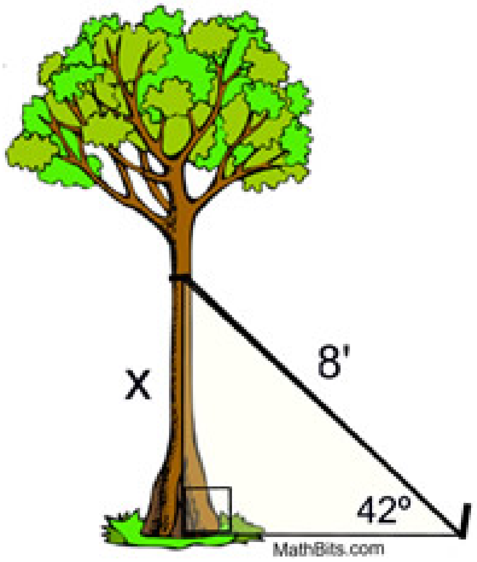
The Cosine Rule is:

$c^2 = a^2 + b^2 – 2ab \cos (C)$

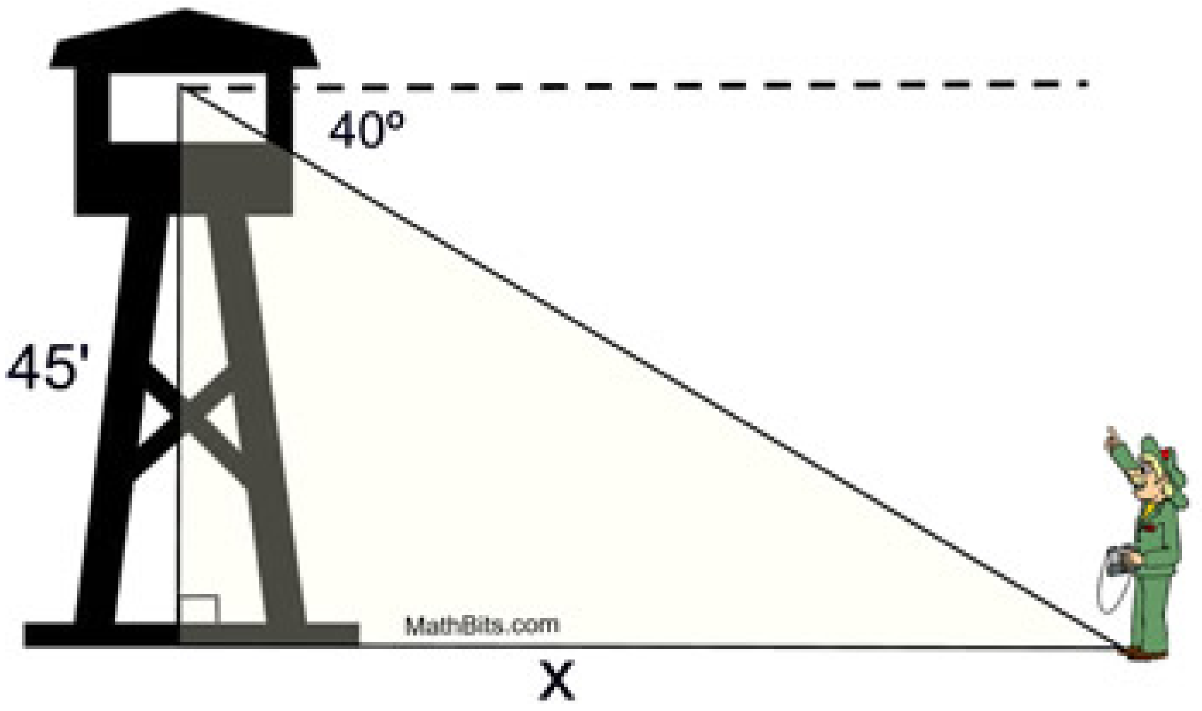
Pythagoras Theorem Applications with Trigonometric base

APPLICATIONS

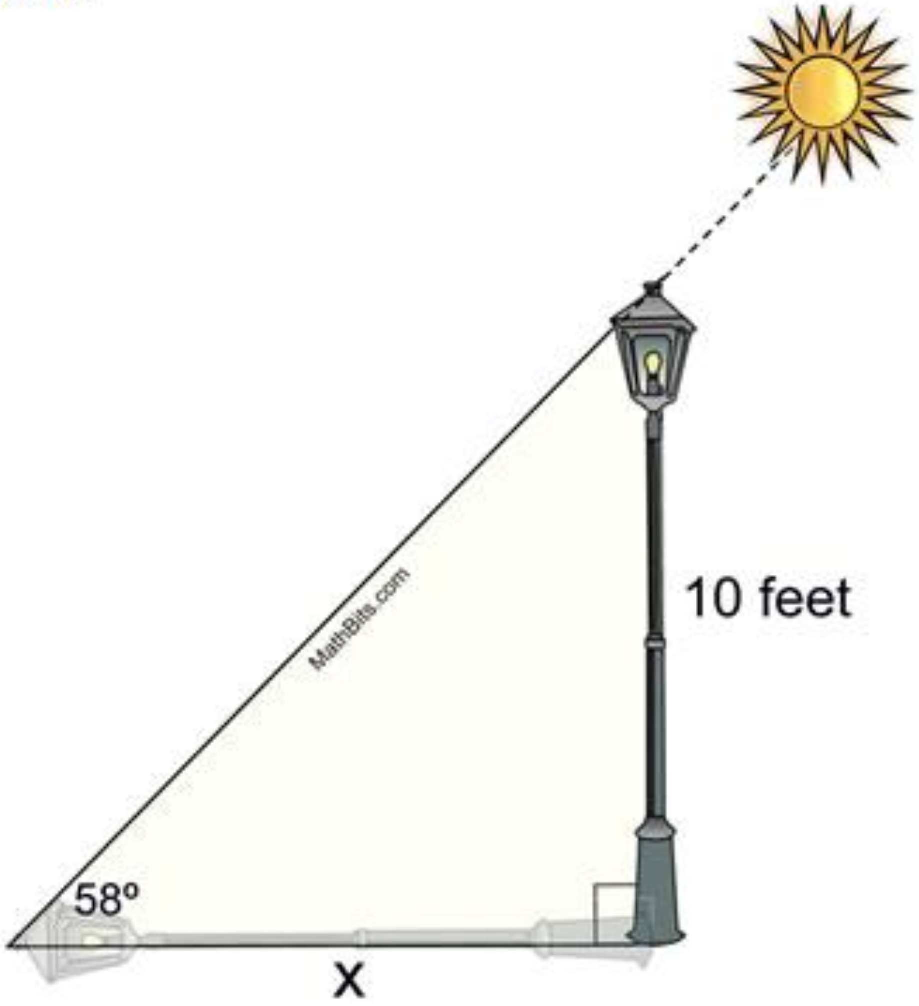
A nursery plants a new tree and attaches a guy wire to help support the tree while its roots take hold. An eight foot wire is attached to the tree and to a stake in the ground. From the stake in the ground the angle of elevation of the connection with the tree is 42º. Find to the *nearest tenth of a foot*, the height of the connection point on the tree.



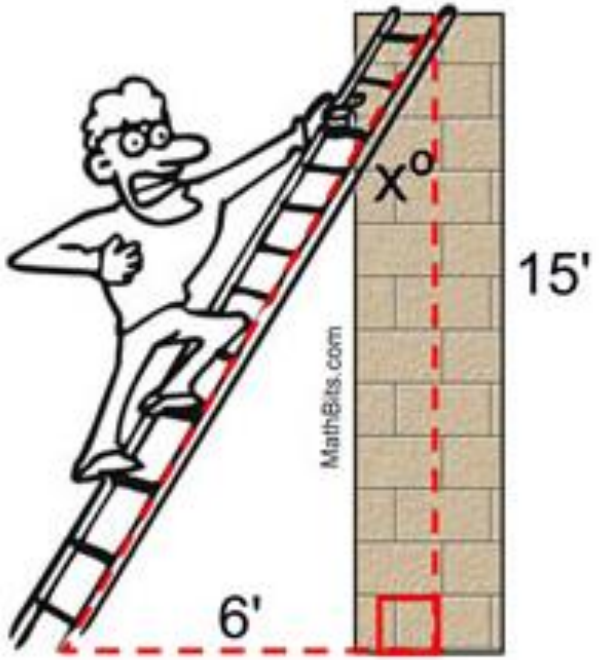
From the top of a fire tower, a forest ranger sees his partner on the ground at an angle of depression of 40°. If the tower is 45 feet in height, how far is the partner from the base of the tower, to the *nearest tenth of a foot*?



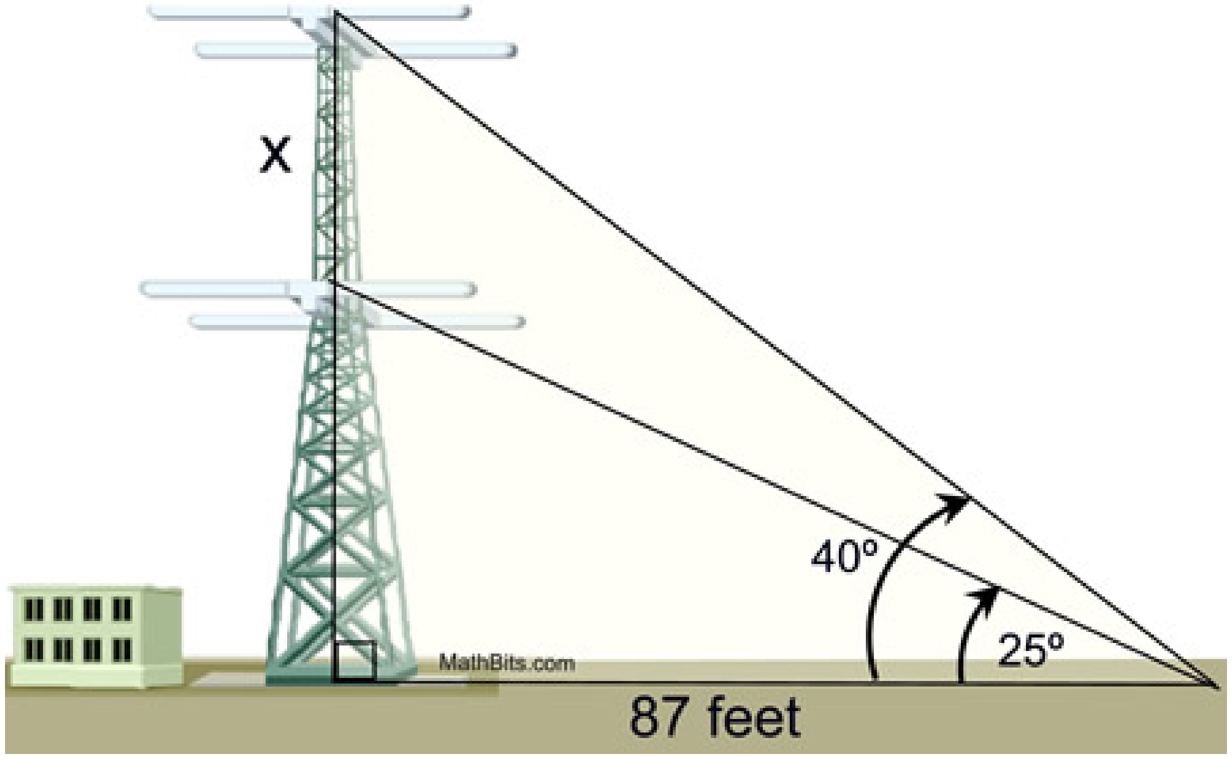
Find the shadow cast by a 10 foot lamp post when the angle of elevation of the sun is 58º. Find the length to the *nearest tenth of a foot.*



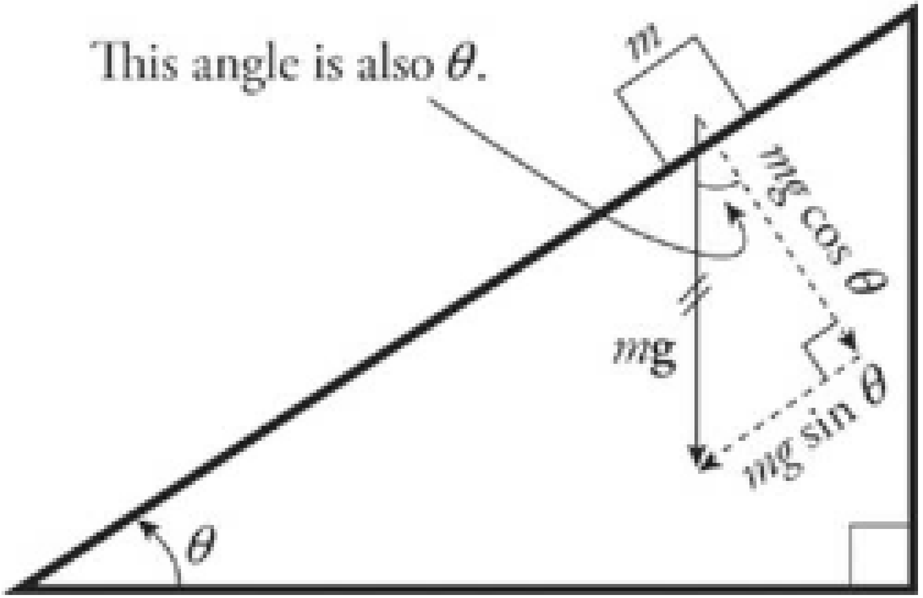
A ladder leans against a brick wall. The foot of the ladder is 6 feet from the wall. The ladder reaches a height of 15 feet on the wall. Find to the *nearest degree*, the angle the ladder makes with the wall.



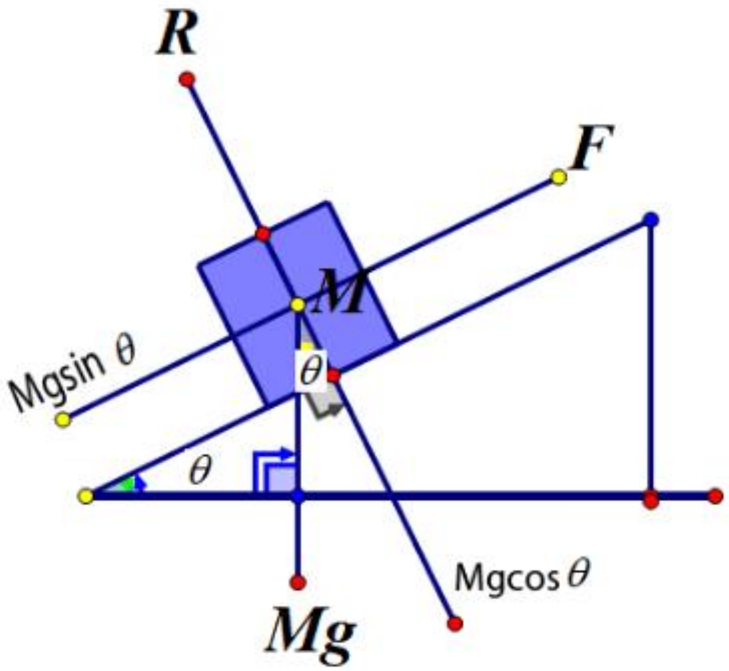
A radio station tower was built in two sections. From a point 87 feet from the base of the tower, the angle of elevation of the top of the first section is 25º, and the angle of elevation of the top of the second section is 40º. To the *nearest foot*, what is the height of the top section of the tower?

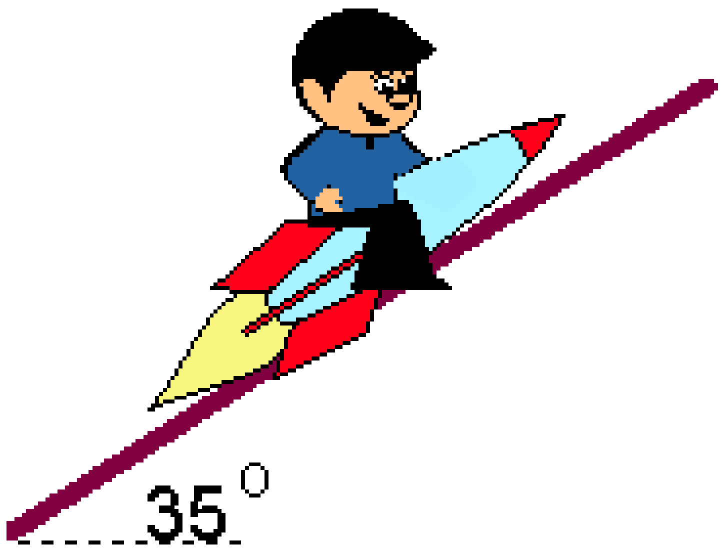


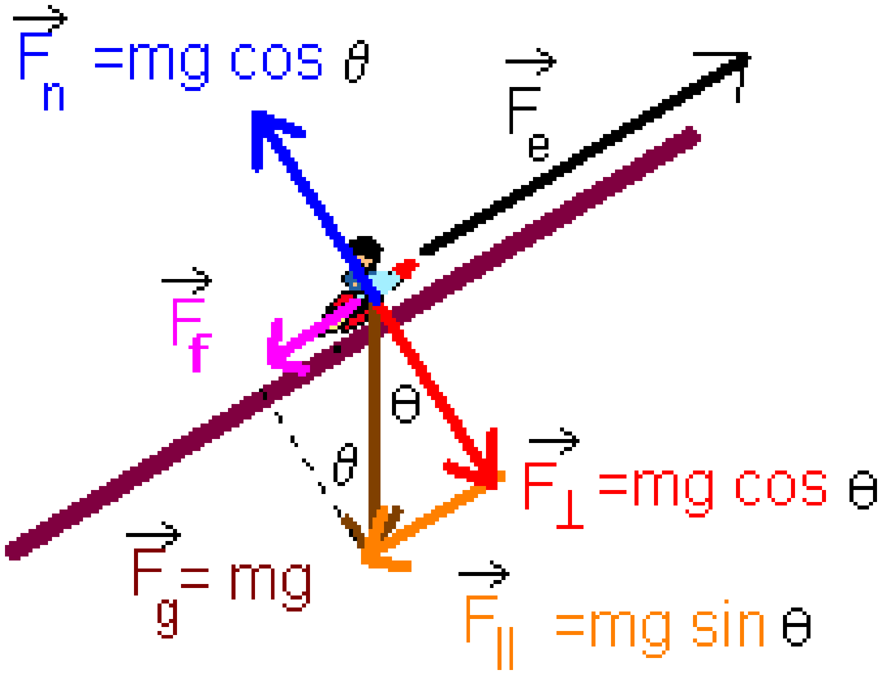
Trigonometry for Physics

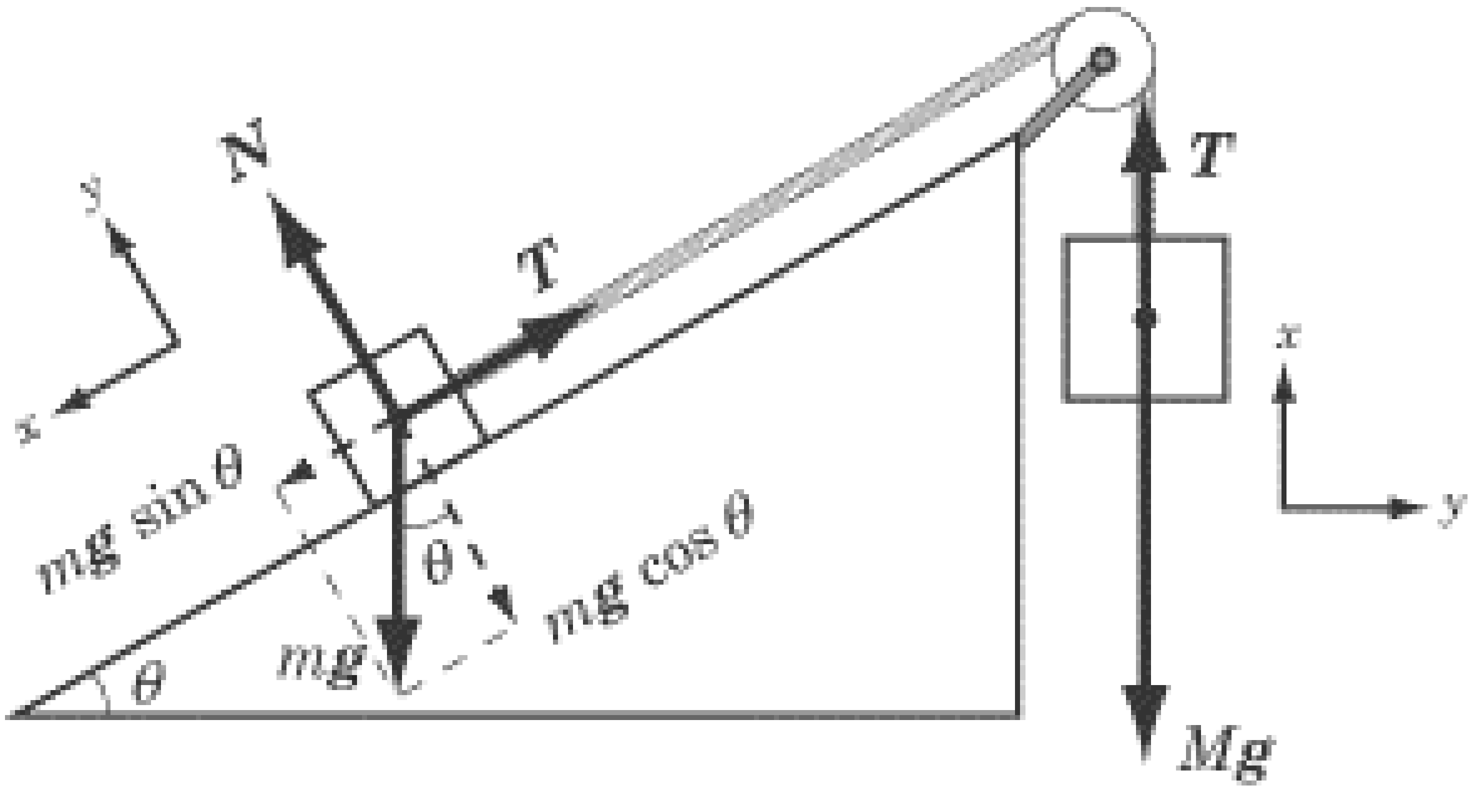

This angle is also θ.
m
$mg \cos \theta$
mg
θ
$mg \sin \theta$
θ

FORCE DIAGRAM ON INCLINED PLANE




35°


$\vec{F}_n$ =mg cos θ
$\vec{F}_e$
$\vec{F}_f$
θ
θ
$\vec{F}_\perp$ =mg cos θ
$\vec{F}_g$= mg
$\vec{F}_{||}$ =mg sin θ



N
y
x
T
$mg \sin\theta$
θ
$mg \cos\theta$
mg
θ
T
x
y
Mg