Mechanical Properties of matter

Introduction

- This lesson is focussed on how we stretch, compress and bend a body.
- We have seen that if we stretch a rubber and let it go it deforms but by applying the force, it again regains its original shape.
- If we take an aluminium rod and apply very strong external force on it, the shape of the rod might change but it does not regain its original position.
- We will take a look why some materials are elastic and why some are plastic.

Ball of rubber bands

Aluminium rod

Solids and their mechanical properties

- Mechanical Properties of solids describe characteristics such as their strength and resistance to deformation.

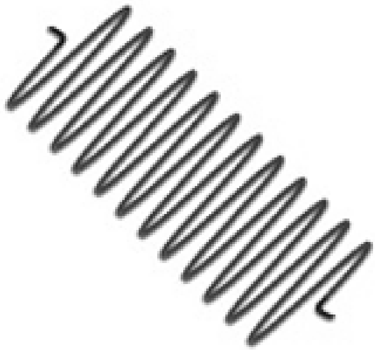
- It describes about the ability of an object to withstand the stress applied to that object. Objects also resist changing their shape.
- For example:- Objects such as clay can be easily deformed so they have less resistance to deformation but objects like iron don't change their shapes easily. When heated they change their shapes which means they have very high resistance to deformation.

Clay can be moulded in the shape of an earthen pot.

Mechanical properties:-

1. **Elasticity**: - Elasticity is a property by virtue of which original shape is regained once the external force is removed.

- This means it tells us how much elastic a body is.
- For example:- A spring .If we stretch a spring it changes its shape and when the external force is removed spring comes back to its original position.



Spring

Rubber Band

2. Plasticity: - Plasticity is reverse of elasticity.

- Property means permanent deformation.
- The object never regains its original shape even when the external force is removed. These types of objects are called as plastics.
- For example:- Toys, Buckets made up of plastics.

Plastic bucket

3. Ductility: - Property of being drawn into thin wires or sheets.

- For Example: - Small chains of gold and silver.

Chain made of gold.

4. **Strength**: - Ability to withstand applied stress without failure.

Stress:-

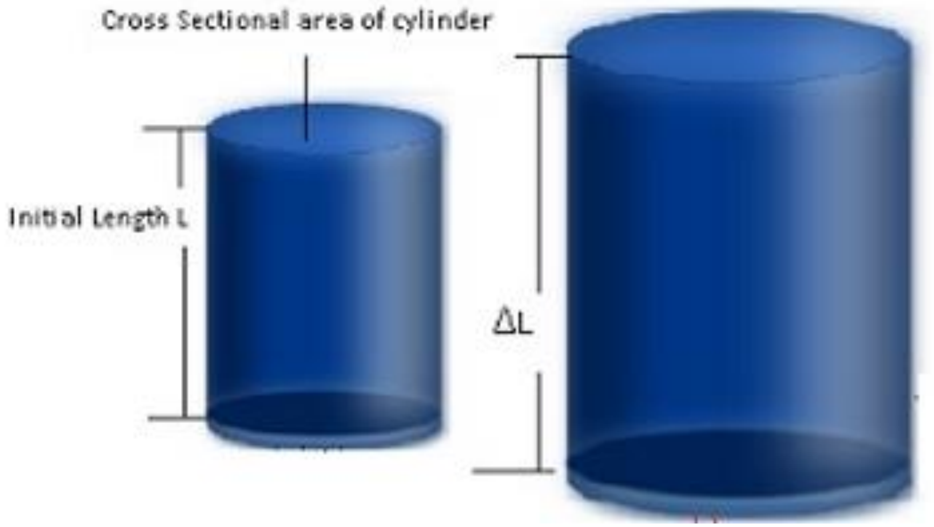
- Stress is the restoring force per unit area.
- Whenever we apply an external force on the body to change its shape there is a restoring force that develops in the body in the opposite direction.
- For example:-
    - When we apply an external force to a rubber ball at the same instant of time some force develops in the ball which acts in the opposite direction.
    - This opposite force which develops in the ball when an external force is applied is known as restoring force.
    - Both the forces are equal in magnitude.

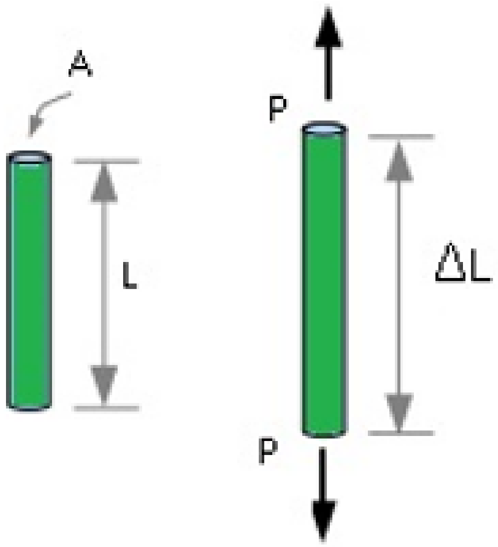
- Mathematically:-
- **Stress = F/A**
- Where F= restoring force develops in the body because of force we apply.
- A=area
- S.I. Unit :- N/m^2 or Pascal(Pa)
- Dimensional formula is $[ML^{-1}T^{-2}]$.

Types of Stress: **Longitudinal stress**

- Longitudinal stress is defined as restoring force per unit area when the force is applied to the cross-sectional area of the cylindrical body.

- Consider a cylinder which we have to deform. If we apply the force perpendicular to the cross-sectional area, there will be a restoring force that develops in the cylinder in the opposite direction.
- This restoring force per unit area is known as longitudinal stress.
- Experimentally we can observe the increase in length.
- If we tie a heavy object to the cylinder with the help of threads.
- Let Initial length of the cylinder is L.
- After it gets stretched its length increases by ΔL due to the stress.
- As there is change in the length therefore this type of stress is known as longitudinal stress.
- In the below figure if we attach a box to the cylinder, a force is applied on the cross-sectional area of cylinder due to which it gets stretched and as a result there is change in the length of the cylinder.


Cross Sectional area of cylinder
Initial Length L
ΔL


A
L
P
ΔL
P

Types of Longitudinal Stress:-

1. Tensile Stress
2. Compressive Stress

Tensile Stress

- Tensile stress is a longitudinal stress when the length of the cylinder increases.

For example:-

- When the force is applied to both sides of the cylinder, the cylinder gets stretched. As a result there will be increase in its length.

$$\textit{tensile stress} \quad \sigma_T = \frac{\text{magnitude of perpendicular force}}{\text{area of cross section}} = \frac{F_\perp}{A}$$

Force is applied on both the sides as a result length of cylinder increases

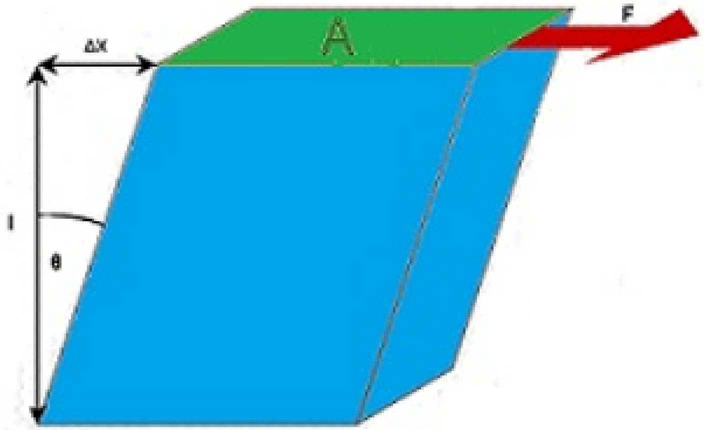
Compressive Stress

- Compressive stress is a longitudinal stress where the force is applied to compress the cylinder.

Compressing the cylinder

Tangential or Shearing Stress

- Restoring force per unit area when the force applied is parallel to the cross sectional area of the body.
- Relative displacement occurs between the opposite faces of the body.
- For example:-
- Consider a cube. If we apply force parallel to the cross sectional area there will be movement which takes place between the opposite faces of the cube as they have relative motion with each other.
- This type of stress is known as tangential or shearing stress.



Strain

- Strain is a measure of deformation representing the displacement between particles in the body to a reference length.
- It tells us how and what changes takes place when a body is subjected to strain.
- Mathematically:- **Strain = $\Delta L/L$,** where ΔL=change in length L= original length
- It is dimensionless quantity because it is a ratio of two quantities.
- For example: - If we have a metal beam and we apply force from both sides the shape of the metal beam will get deformed.
- This change in length or the deformation is known as Strain.

Strain = Change in the configuration / Original configuration

It has no unit and it is a dimensionless quantity.

According to the change in configuration, the strain is of three types

(1) Longitudinal strain= Change in length / Original length

(2) Volumetric strain = Change in volume / Original volume

(iii) Shearing strain = Angular displacement of the plane perpendicular to the fixed surface.

$$\textit{tensile strain} \quad \varepsilon_T = \text{fractional change in length} = \frac{\Delta l}{l}$$

Since Δl and l have the same dimensions, strain is a dimensionless quantity, normally quoted either as a decimal fraction or as a percentage. Typical values for strain in a metallic rod might be of the order of magnitude of 10^{-4}, or 0.01%.

The usual way of denoting this proportionality is by means of the **Young's modulus** of the material Y, which is defined as the ratio of tensile stress to tensile strain. So we have:

$$\textit{Young's modulus} \quad Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{\sigma_T}{\varepsilon_T}$$

The Young's modulus of a material allows us to find what *stress* is required to produce a given *strain*. Since ε_T is dimensionless, Y must have the same units as σ_T, or N m^{-2}. Typical values of Young's modulus for a metal are of the order of 10^{10} to $10^{11}\,\text{N m}^{-2}$. Table 1 gives approximate values of Young's modulus for some common materials. These should be taken only as a rough guide, since the precise values will depend on the composition and on the mechanical and thermal history of the material.

Table 1 Approximate values of Young's modulus (Y) for some common materials at 20 °C.

Material	Y / 10^{10} N m^{-2}
diamond	83
steel	20.0
copper	11.0
glass (fused quartz)	7.1
aluminium	7.0
concrete	1.7
graphite	1.0
nylon	0.36
natural rubber	0.0007

Using the definitions of σ_T and ε_T,

$$\textit{tensile stress} \quad \sigma_T = \frac{\text{magnitude of perpendicular force}}{\text{area of cross section}} = \frac{F_\perp}{A}$$

$$\textit{tensile strain} \quad \varepsilon_T = \text{fractional change in length} = \frac{\Delta l}{l}$$

show that Equation 3

$$\textit{Young's modulus} \quad Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{\sigma_T}{\varepsilon_T}$$

can be reformulated to resemble the force law for a stretched wire, which can be written as $F_x = kx$, where F_x is the force required to extend the wire along its axis by an amount x and where k is the Hooke's law constant of the wire.

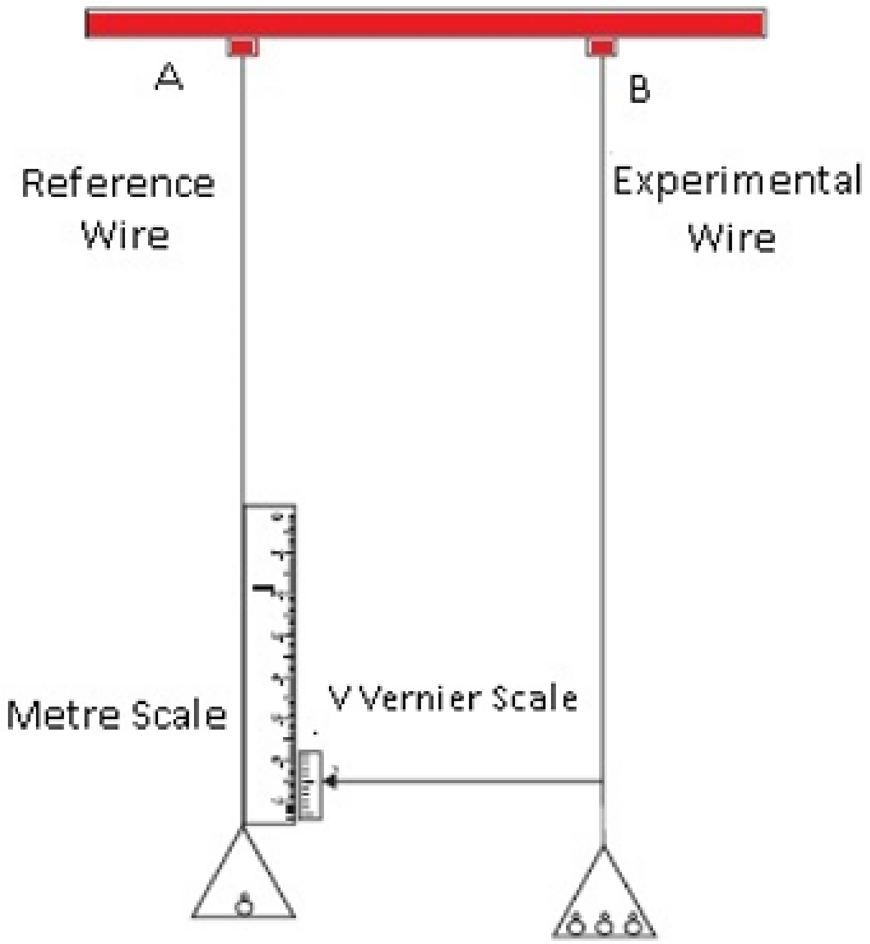
Express the constant k in terms of the Young's modulus of the material.

Young's Modulus: Application

- In industrial constructions steel is preferred over copper. The reason behind this is steel is more elastic than copper.
- If there is slight deformation in steel due to contraction and expansion it will come back to its original position.
- Steel is preferred over copper to construct bridges.

Determination of Young's Modulus of the material of the wire

Experimental set up:-



- Two strings were hung from a support and two pans were attached to both the strings.
- Weights are kept on both the pans.
- When the number of weights in second pan was increased, the string got stretched and moved in downward direction.
- The change in length was measured by the metre scale which was kept on reference wire.
- Using this experiment, the Young's modulus value was calculated
- Y= longitudinal stress/ longitudinal strain = σ / ε
- $= (F/A)/ (\Delta L/L)$
- Where original length = L and ΔL = change in length, F=mg (acting downwards) and A (area of cross-section of wire) = πr^2
- $= (mg/ \pi r^2)/ (\Delta L/L)$
- **$Y= mgL/ \pi r^2 \Delta L$**
- This is the way to calculate the Young's modulus.

2.3 Brittleness and fracture

A material can be characterized by the nature of its loading curve, which depends critically not only on the chemical composition, but also on the existing microstructure. This is the actual structure of the material at the microscopic level and includes not only the idealized positions of the atoms but also the presence of impurity atoms and defects (missing atoms or holes in the atomic lattice). This microstructure can be drastically altered by treatment of the material such as cold working (repeated controlled stressing of the sample) or annealing (controlled heating and cooling of the sample).

Exercises

Elastic Potential Energy in a Stretched Wire

The work done in stretching a wire is stored in form of potential energy of the wire.

Potential energy U = Average force * Increase in length

$= 1 / 2 \, F\Delta l$

= 1 / 2 Stress * Strain * Volume of the wire

Elastic potential energy per unit volume

U = 1 / 2 * Stress * Strain

= 1 / 2 (Young's modulus) * $(\text{Strain})^2$

Elastic potential energy of a stretched spring = $1 / 2 \, kx^2$

where, k = Force constant of spring and x = Change in length.

Thermal Stress

When temperature of a rod fixed at its both ends is changed, then the produced stress is called thermal stress.

Thermal stress = $F / A = y\alpha\Delta\theta$

where, α = coefficient of linear expansion of the material of the rod.

When temperature of a gas enclosed in a vessel is changed, then the thermal stress produced is equal to change in pressure (Δp)of the gas.

Thermal stress = $\Delta p = Ky \Delta \theta$

where, K = bulk modulus of elasticity and

γ = coefficient of cubical expansion of the gas.

Interatomic force constant

$K = Yr_o$

where, r_o = interatomic distance.

Poisson's Ratio

When a deforming force is applied at the free end of a suspended wire of length 1 and radius R, then its length increases by dl but its radius decreases by dR. Now two types of strains are produced by a single force.

(i) Longitudinal strain = &DElta;U l

(ii) Lateral strain = – Δ R/ R

∴ Poisson's Ratio (σ) = Lateral strain / Longitudinal strain = – Δ R/ R / ΔU l

The theoretical value of Poisson's ratio lies between – 1 and 0.5. Its practical value lies between 0 and 0.5.

Important Points

- Coefficient of elasticity depends upon the material, its temperature and purity but not on stress or strain.
- For the same material, the three coefficients of elasticity γ, η and K have different magnitudes.
- Isothermal elasticity of a gas $E_T = \rho$ where, ρ = pressure of the gas.
- Adiabatic elasticity of a gas $E_s = \gamma\rho$

where, $\gamma = C_p / C_v$ ratio of specific heats at constant pressure and at constant volume.

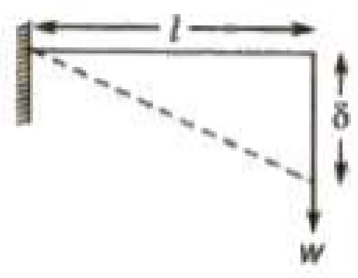
- Ratio between isothermal elasticity and adiabatic elasticity $E_s / E_T = \gamma = C_p / C_v$

Cantilever

A beam clamped at one end and loaded at free end is called a cantilever.

Depression at the free end of a cantilever is given by

$\delta = wl^3 / 3YI_G$



where, w = load, 1 = length of the cantilever,

y = Young's modulus of elasticity, and I_G = geometrical moment of inertia.

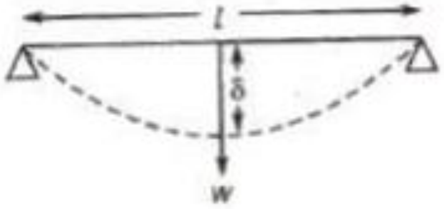
For a beam of rectangular cross-section having breadth b and thickness d.

$I_G = bd^3 / 12$

For a beam of circular cross-section area having radius r,

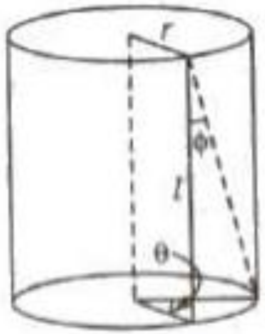
$$I_G = \pi r^4 / 4$$

Beam Supported at Two Ends and Loaded at the Middle



Depression at middle $\delta = wl^3 / 48YI_G$

Torsion of a Cylinder



Couple per unit twist $C = \dfrac{\pi\eta r^4}{2l}$

where, η = modulus of rigidity of the material of cylinder,

r = radius of cylinder,

and l = length of cylinder,

Work done in twisting the cylinder through an angle θ

$W = 1 / 2\, C\theta^2$

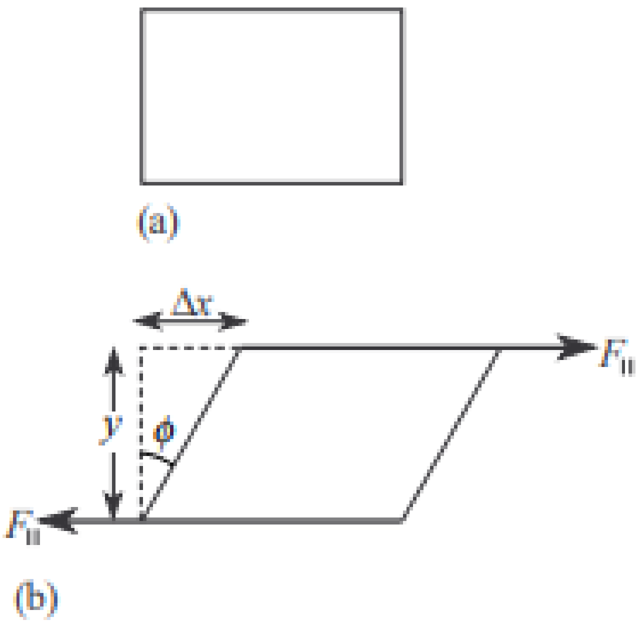
Relation between angle of twist (θ) and angle of shear (φ)

$r\theta = l\varphi$ or $\varphi = r / l = \theta$

2.4 Shear modulus and bulk modulus

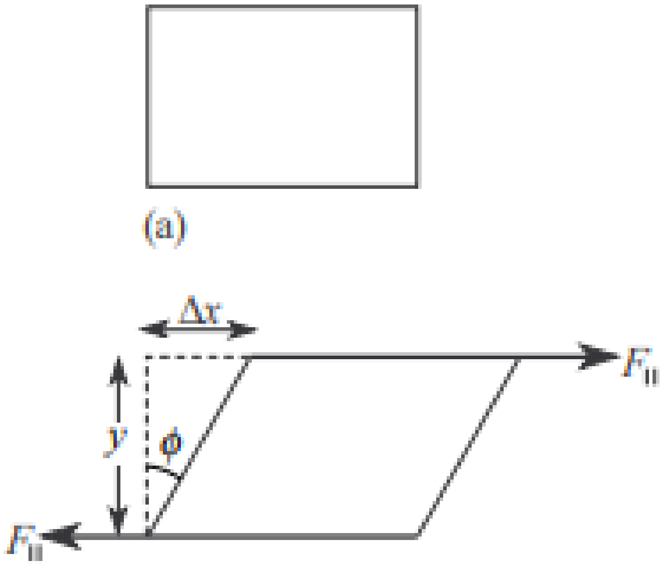
There is another class of stress on a body, in which non-collinear forces twist the body and change its shape. This is called a **shear stress** and is illustrated in Figure 5. Here a rectangular section block is subject to two balanced forces of equal magnitude, applied uniformly to two separated surfaces. The forces are applied parallel to the surfaces, rather than perpendicular to the surfaces as in tensile stress, and this twists the block and changes its shape.

Just as for tensile stress, the shear stress is defined as the ratio of the applied force to the area over which it is applied, remembering that in this case the forces are applied *parallel* to the surfaces, we represent it as $F_{\|}$ and write:



$$\textit{shear stress} \quad \sigma_S = \frac{\text{magnitude of parallel force}}{\text{area of surface of application}} = \frac{F_{\|}}{A}$$

The **shear strain** is defined as the ratio of the shear distance between the two surfaces, measured parallel to the applied stress, to the perpendicular distance between the two surfaces. In Figure 5b, these are denoted by Δx and y, respectively. So, in this case the shear strain is calculated as:



$$\textit{shear strain} \quad \varepsilon_S = \frac{\text{shear between the two surfaces}}{\text{separation between the two surfaces}} = \frac{\Delta x}{y} \quad (5)$$

Notice that the shear strain is dimensionless, since it is the ratio of two distances. Also, since these strains are generally very small, we can say that the angle of the shear (denoted by ϕ) is equal to the shear strain, since $\Delta x / y = \tan \phi = \phi$ (measured in radians) for small angles. As with tensile stresses and strains, for small stresses and deformations the shear strain is linearly proportional to the shear stress and we can define a **shear modulus** G by:

$$\textit{shear modulus} \quad G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\sigma_S}{\varepsilon_S}$$

Finally, we will consider a third type of stress, a **uniform stress** or a **volume stress**. In this case the force is applied to the body uniformly, perpendicular to every surface, as shown in Figure 6. The magnitude of such a stress is usually called a *pressure*. The pressure on a body tends to compress it and reduce its volume. Pressure itself is a scalar quantity which cannot be negative, so it is incorrect to speak of the *direction* of a pressure; however, the *forces* applied to a body due to pressure on the body *do* have a direction — and this is into the body. The pressure is numerically equal to the perpendicular force per unit area of the surface of the body.

volume stress (pressure)

$$\sigma_{\text{vol}} = \frac{\text{magnitude of perpendicular force}}{\text{area of surface of application}} = P \tag{7}$$

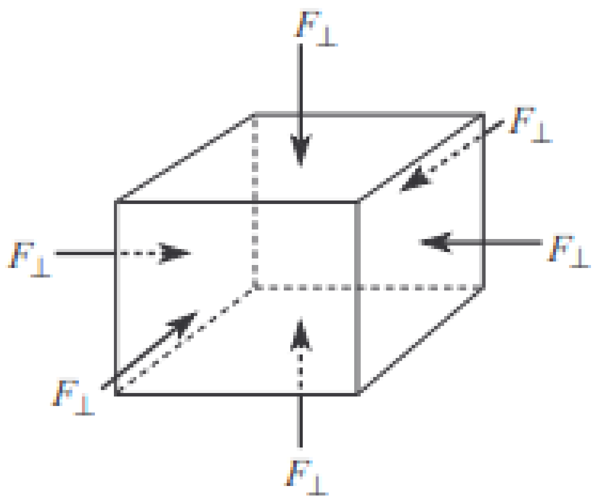


Figure 6 A uniform volume stress (uniform pressure) applied to a body.

The resulting strain is normally a **volume strain**, and is calculated as:

$$\textit{volume strain} \quad \varepsilon_{\text{vol}} = \frac{\text{change in volume}}{\text{total volume}} = \frac{\Delta V}{V}$$

The volume always decreases with increase of pressure and so ΔV is negative, as is the volume strain. The elastic modulus corresponding to volume stress and strain is called the **bulk modulus** K, which is defined as:

$$\textit{bulk modulus} \quad K = \frac{\sigma_{\text{vol}}}{\varepsilon_{\text{vol}}} = -\frac{\text{pressure change}}{\text{fractional volume change}} = -V\frac{\Delta P}{\Delta V}$$

Notice that, by convention, we use ΔP for the stress, since it is normally taken that the reference state is atmospheric pressure (rather than zero pressure) and we require the minus sign since pressure is *positive*.

These different elastic moduli represent the response of a material to different forms of external stress, but all are ultimately based on the same interatomic forces, and so one would expect their values to be linked.