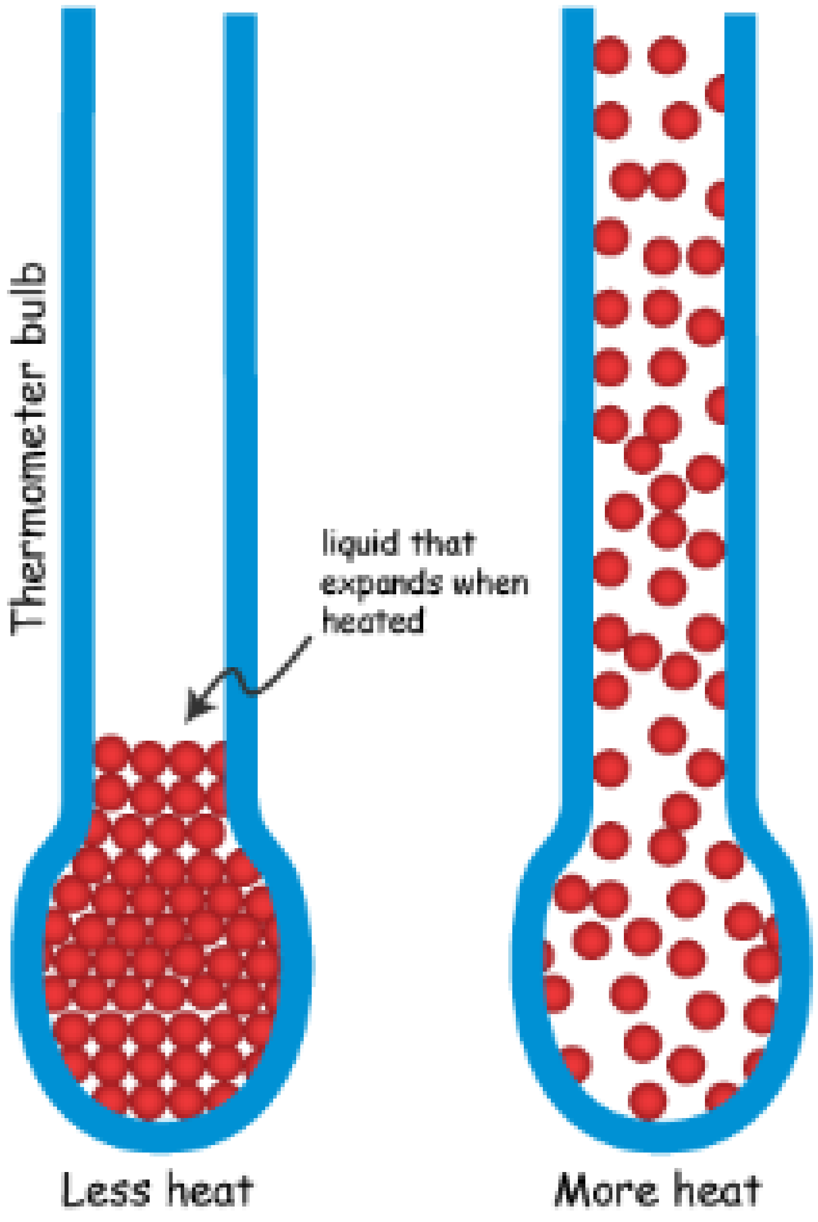
# Thermal Expansion & Contraction

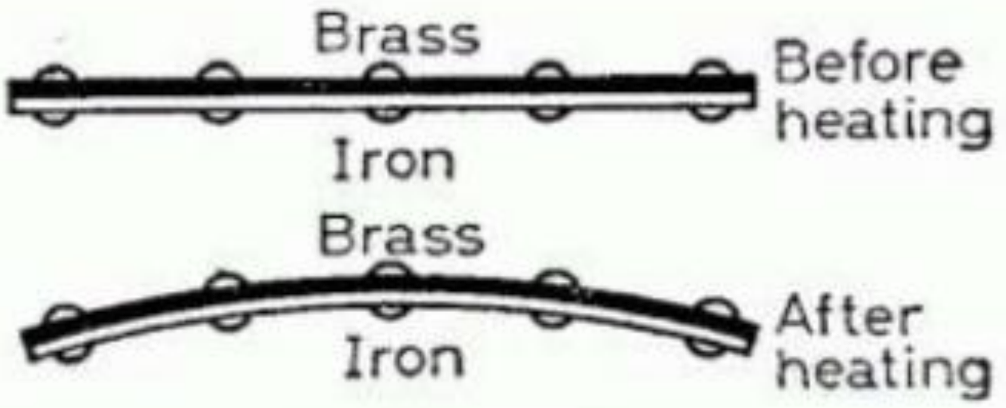

Thermometer bulb
liquid that expands when heated
Less heat
More heat

# CONSIDERATION DURING CONSTRUCTIONS

The expansion or contraction of a materials must be kept in mind while designing large structures.

# THE BIMETALLIC STRIP

A bimetallic strip can be used to control thermostats and can also be used in fire alarms.




Bimetallic strip

## . THERMOMETERS

A thermometer is also constructed on the principle of thermal expansion.

# ENGINE COOLANT

An engine's coolant overflows due to thermal expansion.

## JAR LIDS AND POWER LINES

It is an everyday experience that tight metal lids are easy to remove after passage of hot water over them.

Sagging of electrical power lines is another example of thermal expansion.

# Thermal Expansion and contraction

- Temperature- the average amount of kinetic energy (energy of motion) in an object
- Thermal expansion- when an object is heated, molecules move further apart due to their increased kinetic energy.

- **Thermal contraction**- when an object cools, molecules move closer together due to their decreased kinetic energy. **Cooled objects CONTRACT- volume gets smaller**

- IMPORTANT:
- Solids, liquids and gases all show thermal expansion/contraction, but gases expand/contract the most.
- Thermal expansion is a CHARACTERISTIC PROPERTY of solids and liquids- this means solids/liquids expand differently; depending on what material they are made of.

- All gases expand and contract equally, regardless of what they are, so thermal expansion is **NOT** a characteristic property of gasses.

# Thermal contraction/expansion in weather

- Air pressure: weight of air molecules
- Low air pressure: less air molecules, less dense-warm air
- High air pressure: more air molecules, more dense-cold air
- High pressure air masses move to areas of low pressure (molecules always move from more crowded areas to less crowded areas)

# Thermal Expansion in Solids

- **THERMAL EXPANSION** refers to the spreading out of particles in a substance as the thermal energy increases.
- Almost all materials take up more space when heated **(expansion)** and less a space when cooled **(contraction).**

- Quantified by a constant value for coefficient of thermal expansion for some materials
- The higher the coefficient, the higher the expansion

## 14.3 Thermal expansion



- Thermal expansion is defined **as the change in dimensions of a body accompanying a change in temperature.**

- 3 types of thermal expansion :
  - Linear expansion
  - Area expansion
  - Volume expansion

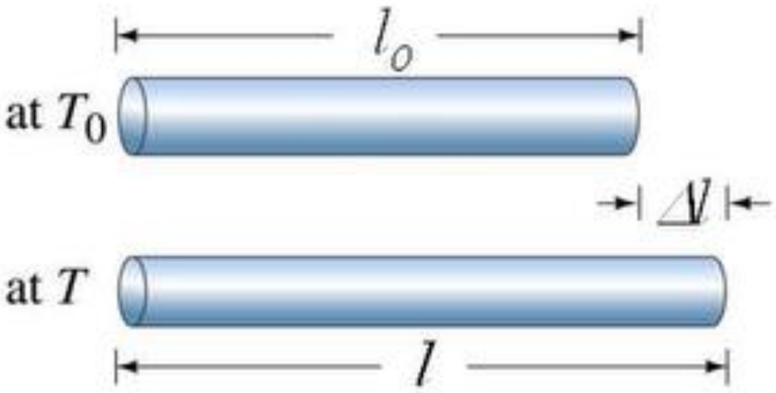
- In **solid**, **all types of thermal expansion** are occurred.
- In **liquid and gas**, only **volume expansion** is occurred.

- At the same temperature, the gas expands greater than liquid and solid.

# Linear expansion



•Consider a thin rod of initial length, $l_0$ at temperature, $T_0$ is heated to a new uniform temperature, $T$ and acquires length, $l$ as shown in figure below.




• If $\Delta T$ is not too large (< 100° C)

$$\Delta l \propto \Delta T \quad \text{and} \quad \Delta l \propto l_0$$

$$\boxed{\Delta l = \alpha l_0 \Delta T}$$

$\Delta l$ : change in length $= l - l_0$

$\Delta T$ : change in temperature $= T - T_0$

$\alpha$ : coefficient of linear expansion

$$\boxed{\Delta l = \alpha l_0 \Delta T}$$

$$\boxed{\alpha = \frac{\Delta l}{l_0 \Delta T}}$$

•Unit of $\alpha$ is °C$^{-1}$ or K$^{-1}$.

- The **coefficient of linear expansion,** $\alpha$ is defined as ***the change in length of a solid per unit length per unit rise change in temperature.***
- If the length of the object at a temperature $T$ is $l$,

$$\Delta l = l - l_o$$

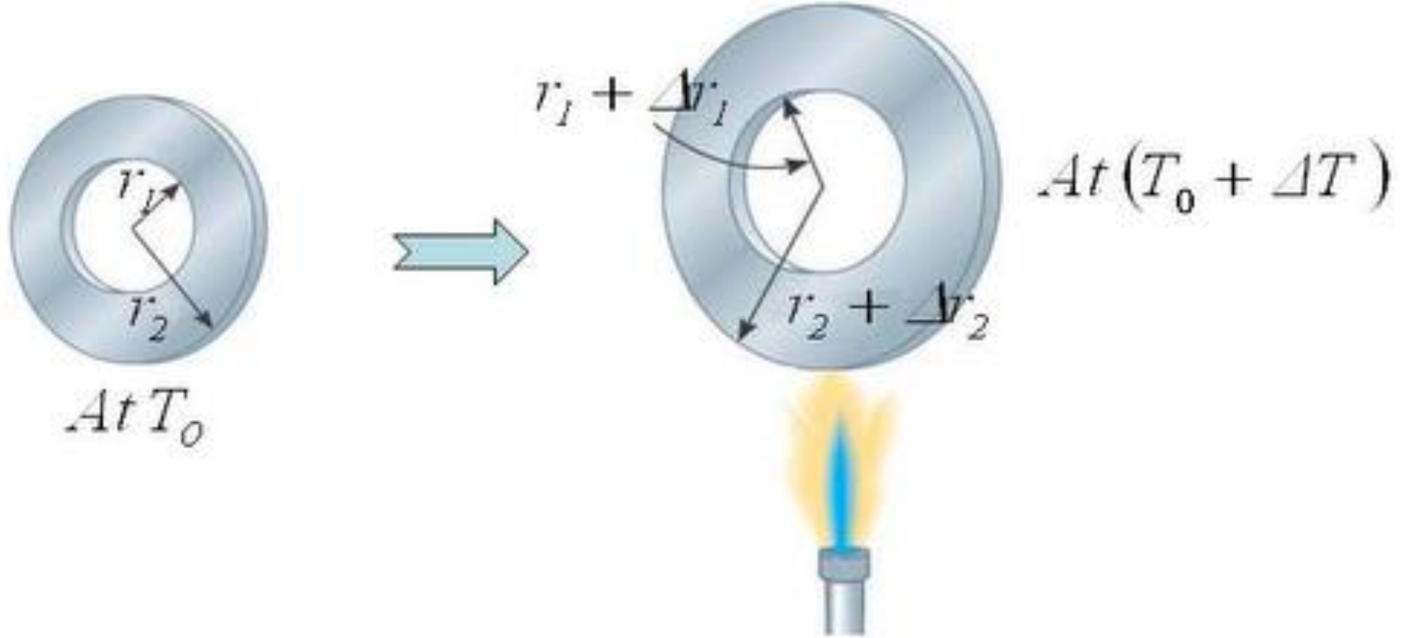
$$l = \Delta l + l_o$$

$$l = \alpha l_o \Delta T + l_o$$

$$\boxed{l = l_o(\alpha \Delta T + 1)}$$

- For many materials, every linear dimension changes according to both equations above. Thus, $l$ could be the **length of a rod**, the **side length of a square plate** or the **diameter (radius) of a hole**.

- For example, as a metal washer is heated, all dimensions including the radius of the hole increase as shown in figure below.

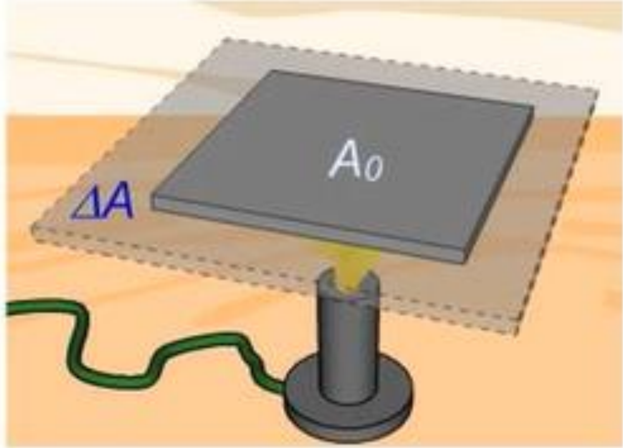


## Area expansion



- This expansion involving the expansion of a surface area of an object.
- Consider a plate with initial area, $A_0$ at temperature $T_0$ is heated to a new uniform temperature, $T$ and expands by $\Delta A$, as shown in figure below.
- From this experiment, we get

$$\Delta A \propto A_0 \quad \text{and} \quad \Delta A \propto \Delta T$$




$$\boxed{\Delta A = \beta A_0 \Delta T}$$

$\Delta A$ : change in area $= A - A_0$

$\Delta T$ : change in temperature $= T - T_0$

$\beta$ : coefficient of area expansion

$$\boxed{\Delta A = \beta A_0 \Delta T}$$

$$\boxed{\beta = \frac{\Delta A}{A_0 \Delta T}}$$

- Unit of $\beta$ is °C$^{-1}$ or K$^{-1}$

- The **coefficient of area expansion,** $\beta$ is defined as ***the change in area of a solid surface per unit area per unit rise in temperature.***

- The **area** of the of the surface of object at a temperature $T$ can be written as,

$$\boxed{A = A_0 (1 + \beta \Delta T)}$$

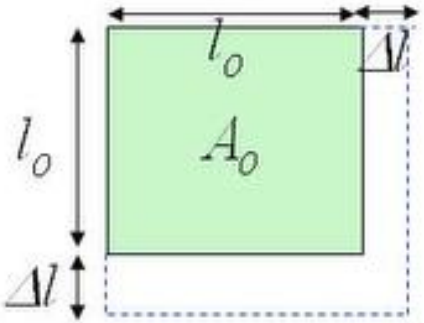
- For isotropic material (solid) , the area expansion is uniform in all direction, thus the relationship between $\alpha$ and $\beta$ is given by

$$\boxed{\beta = 2\alpha}$$

## Derivation

$$\boxed{\beta = 2\alpha}$$

Consider a square plate with side length, $l_0$ is heated and expands uniformly as shown in figure below.




$$A_0 = l_0^2 \qquad A = l^2 \qquad l = l_0 + \Delta l$$

$$A = (l_0 + \Delta l)^2$$

$$A = l_0^2 + 2l_0\Delta l + (\Delta l)^2$$

$$A = l_0^2\left[1 + 2\frac{\Delta l}{l_0} + \left(\frac{\Delta l}{l_0}\right)^2\right]$$ because $$\left(\frac{\Delta l}{l_0}\right)^2 \approx 0$$

$$A = l_0^2\left(1 + 2\frac{\Delta l}{l_0}\right)$$ where $l_0^2 = A_0$ and $\frac{\Delta l}{l_0} = \alpha\Delta T$

$$A = A_0(1 + 2\alpha\Delta T)$$ compare with $$A = A_0(1 + \beta\Delta T)$$

$$\boxed{\beta = 2\alpha}$$

## Volume expansion



Consider a metal cube with side length, $l_0$ is heated and expands uniformly. From the experiment, we get

$$\Delta V \propto V_0 \quad \text{and} \quad \Delta V \propto \Delta T$$

$$\boxed{\Delta V = \gamma V_0 \Delta T}$$

$\Delta V$ : change in volume $= V - V_0$

$\Delta T$ : change in temperature $= T - T_0$

$\gamma$ : coefficient of volume expansion

$$\boxed{\gamma = \frac{\Delta V}{V_0 \Delta T}}$$

Unit of $\gamma$ is $°C^{-1}$ or $K^{-1}$.

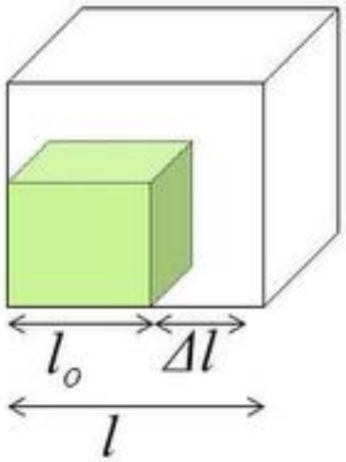
The **coefficient of volume expansion,** $\gamma$ is defined as ***the change in volume of a solid per unit volume per unit rise in temperature.***

- The **volume** of an object at a temperature $T$ can be written as, $\boxed{V = V_0(1+\gamma\Delta T)}$

- For isotropic material (solid), the volume expansion is uniform in all direction, thus the relationship between $\alpha$ and $\gamma$ is given by $\boxed{\gamma = 3\alpha}$

## Derivation

Consider a metal cube with side length, $l_0$ is heated and expands uniformly.




$\boxed{V_0 = l_0^3}$ $\boxed{V = l^3}$ $\boxed{l = l_0 + \Delta l}$

$$V = (l_0 + \Delta l)^3$$
$$V = l_0^3 + 3l_0^2\Delta l + 3l_0\Delta l^2 + (\Delta l)^3$$

$$V = l_0^3\left[1 + 3\frac{\Delta l}{l_0} + 3\left(\frac{\Delta l}{l_0}\right)^2 + \left(\frac{\Delta l}{l_0}\right)^3\right]$$ because $$3\left(\frac{\Delta l}{l_0}\right)^2 + \left(\frac{\Delta l}{l_0}\right)^3 \approx 0$$

$$V = l_0^3\left(1 + 3\frac{\Delta l}{l_0}\right)$$ where $l_0^3 = V_0$ and $$\frac{\Delta l}{l_0} = \alpha \Delta T$$

$V = V_0(1 + 3\alpha\Delta T)$ compare with $V = V_0(1 + \gamma\Delta T)$

$$\boxed{\gamma = 3\alpha}$$

## Example 13.3

The length of metal rod is 30.000 cm at 20°C and 30.019 cm at 45°C, respectively. Calculate the coefficient of linear expansion for the rod.

## Solution

$l_0 = 30.000$ cm, $T_0 = 20°C$ , $l = 30.019$ cm, $T = 45°C$

## Example 13.4

A steel ball is 1.900 cm in diameter at 20.0°C. Given that the coefficient of linear expansion for steel is $1.2 \times 10^{-5}$ °C$^{-1}$, calculate the diameter of the steel ball at

a) 57.0°C

b) –66.0°C

**Solution** $d_0 = 1.900$ cm, $T_0 = 20.0$°C, $\alpha = 1.2\times10^{-5}$°C$^{-1}$

$$l = l_0(1+\alpha\Delta T) \rightarrow \boxed{d = d_0(1+\alpha\Delta T)}$$

a)

b)

## Example 13.3

A cylinder of radius 18.0 cm is to be inserted into a brass ring of radius 17.9 cm at 20.0°C. Find the temperature of the brass ring so that the cylinder could be inserted.

(Given the coefficient of area expansion for brass is $4.0 \times 10^{-5}\ ^{\circ}C^{-1}$)

## Solution

$r_c = 18.0$ cm, $r_{0b} = 17.9$ cm, $T_0 = 20.0°C$, $\beta = 4.0 \times 10^{-5}\ ^{\circ}C^{-1}$

When the cylinder pass through the brass ring, thus

$$\beta = 2\alpha$$

$$\alpha = \frac{\beta}{2} = 2\times10^{\ 5}\ ^{\circ}C^{-1}$$

## Example 13.6

Determine the change in volume of block of cast iron 5.0 cm x 10 cm x 6.0 cm, when the temperature changes from 15 °C to 47 °C. ($\alpha_{\text{cast iron}} = 0.000010\ °C^{-1}$)

## Solution

# Exercise

1. A rod 3.0 m long is found to have expanded 0.091 cm in length after a temperature rise of 60 ° C. What is the coefficient of linear expansion for the material of the rod ?

**$5.1 \times 10^{-6}\ °C^{-1}$**

2. The length of a copper rod is 2.001 m and the length of a wolfram rod is 2.003 m at the same temperature. Calculate the change in temperature so that the two rods have the same length where the final temperature for both rods is equal.
(Given the coefficient of linear expansion for copper is $1.7 \times 10^{-5}\ °C^{-1}$ and the coefficient of linear expansion for wolfram is $0.43 \times 10^{-5}\ °C^{-1}$)

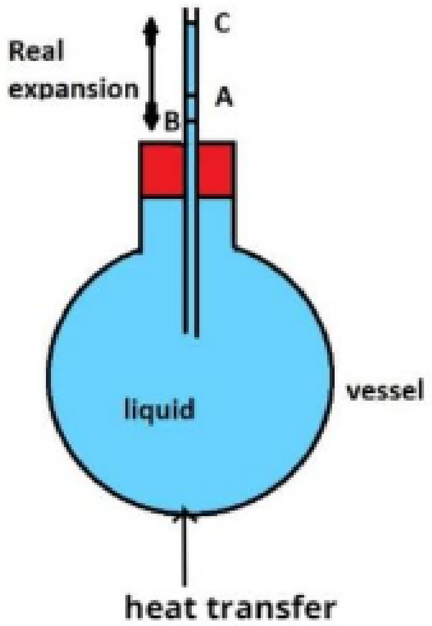
**78.72°C**

# Expansion of Liquids

- **Liquids just like solids** expand on heating and contract on cooling. The expansion and contraction in liquids is **more than** those in solids.
- The reason for this is that the **intermolecular force in** liquids is less than that in solids.
- Hence, the molecules in a liquid **vibrate more than** the molecules in a solid when they **get thermal energy**.
- **Since the liquids do not have a shape** of their own but possess a fixed volume, we have to consider their **co-efficient of cubical expansion** while studying their expansion.
- **When a liquid is heated** in a vessel, the vessel also expands on heating. Therefore, **while studying** the cubical expansion of liquids, **the expansion of container** should also be taken into account.
- The reading of expansions of liquids would have been a little more, had there been no expansion of the vessel.
- **So in case of liquids**, we define **two coefficients of cubical** expansion.

*Real and apparent expansion of a liquid*

- The expansion of the liquid due **to increase in temperature** is always associated by **expansion of containing vessel.**
- The expansion which we observe is called **apparent expansion** which is always less than **real expansion of the liquid**.
- Let us take a test tube filled with coloured water. Its mouth is closed with a rubber cork having a hole. **A capillary tube** is inserted into the test tube through the hole in the cork as in figure.



**Initially, the water rises** in the capillary tube. Let the water rises up-to **point A**. The test tube is placed in a beaker in which water is boiled. Initially, the water level in the capillary falls. Let it falls to **point B**. After that, water level begins rising and it rises above the initial level. Let it rises up-to **point C**.

**The fall in water** level in the capillary **in the beginning of the heating** is due to the **expansion of the capillary**. It is because **the capillary tube which** is made up of glass expands more rapidly than water. So in the beginning, the capillary expands but the water does not. Therefore, **we observe fall in water level**.

**After sometime**, the water in the capillary **also gets enough heat** to expand. When the water starts expanding, **the water level rises** in the capillary. Since the expansion of the water in the capillary **is more** than the expansion of the capillary, the water level **rises above the initial level** in the capillary.

**From this experiment**, it is clear that **a liquid like water** expands on heating. This experiment shows that

1. Volume of water from **A to B in** the capillary represents **expansion of the capillary.**
2. Volume of water from **A to C** in the capillary represents **apparent (or observed) expansion of water.**

Hence, the real expansion of water is given by

**Real expansion** = expansion of the capillary + apparent expansion of water

**For any liquid,**

Real expansion of liquid = expansion of vessel + apparent expansion of liquid

*Coefficient of real expansion ($\gamma_r$)*

- It is defined as **the real increase in volume** of the liquid per unit volume **per unit rise** in temperature.
- That is, $\gamma_r$ = Real increase in volume/( Original volume × rise in temperature)

Let $V_0$ and $V_\theta$ be the volumes of a liquid **at $0^oC$** and at **$\theta^oC$** respectively, then

$\gamma_r = (V_\theta - V_0)/(V_0 \times \theta)$................................. (i)

$V_\theta = V_0 (1 + \gamma_r \theta)$......................................... (ii)

*Coefficient of apparent expansion ($\gamma_a$)*

- It is defined as **the apparent (or observed) increase in volume** per unit volume of the liquid per unit rise of temperature.
- That is, $\gamma_a$ = Apparent increase in volume/ (Original volume × rise in temperature)

***Relation between Coefficient of Real and Apparent Expansion***

- Let us consider some liquid contained in a capillary tube at room temperature **$\theta_1$°C** with level of liquid **at A** as shown in figure. Let **V** be the original volume of the liquid. Suppose the system is heated to temperature **$\theta_2$°C**. As heat is supplied to the system, **the capillary expands first** and the level of liquid falls to the **level B**. Thus, volume **$\Delta V_g$** in between levels **A** and **B** gives the expansion of the capillary tube. Therefore,

**$\Delta V_g = V\ \gamma_g(\theta_2 - \theta_1)$**........................................ (iii)

where '**$\gamma_g$**' is coefficient of cubical expansion of the glass.

**On heating**, the liquid expands and its level moved up from **B to C**.

Thus, the real increase in volume **$\Delta V_r$** is equal to the level between **B and C**.

Therefore,

**$\Delta V_r = V\ \gamma_r(\theta_2 - \theta_1)$**........................................ (iv)

where '**$\gamma_r$**' is the coefficient of real expansion of liquid.

Apparently, the liquid in the capillary tube rises from **A to C**.

Thus, the apparent increase in volume is given by,

$\Delta V_a = V \gamma_a(\theta_2 - \theta_1)$.......................................... (v)

where '$\gamma_a$' is the coefficient of apparent expansion of liquid.

From the figure it is clear that,

**BC = AB + AC**

$\Delta V_r = \Delta V_g + \Delta V_a$

$V \gamma_r(\theta_2 - \theta_1) = V \gamma_g(\theta_2 - \theta_1) + V \gamma_a(\theta_2 - \theta_1)$

$\gamma_r = \gamma_g + \gamma_a$ ....................................................... (vi)

Thus, the **coefficient of real expansion** of the liquid is equal to **the sum of the coefficient** of cubical expansion of the container and the coefficient of apparent expansion of the liquid.

If '$\alpha$' is the coefficient of **linear expansion** of the material of the containing vessel,

Then, $\gamma_g = 3\alpha$

The equation (vi) becomes

$\gamma_r = 3\alpha + \gamma_a$....................................................... (vii)