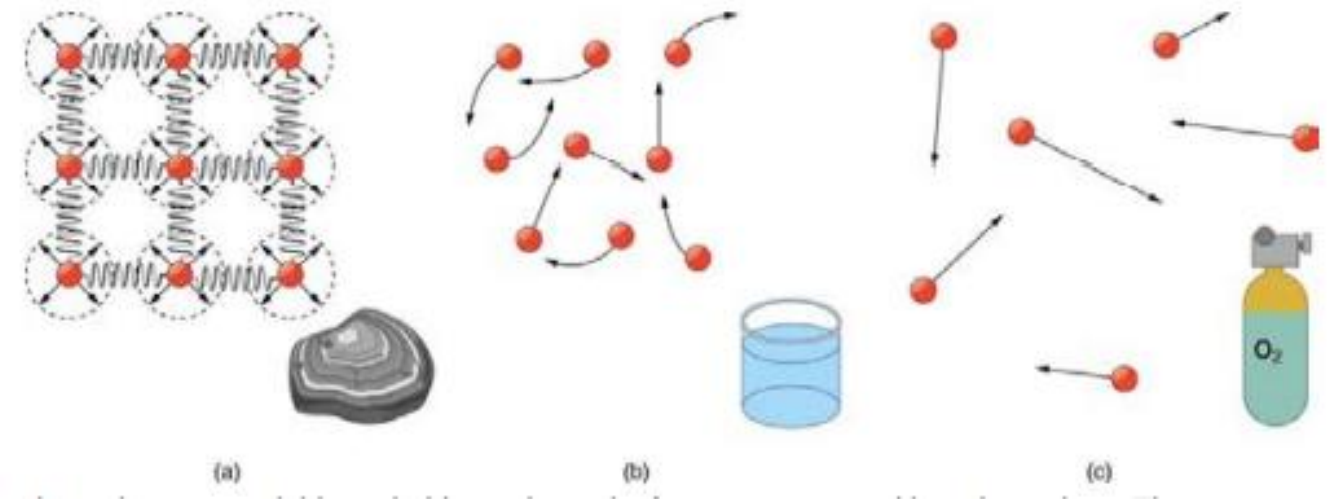
FLUID STATICS



Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common *phases of matter*. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, and gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held. ~~(See Figure 11.2.)~~ Liquids and gases are considered to be fluids because they yield to shearing forces, whereas solids resist them. ~~Note that the extent to which fluids yield to~~

Density, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object. In equation form, density is defined as

$$\rho = \frac{m}{V}, \qquad \cancel{(11.1)}$$

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume occupied by the substance.

~~is much lower~~. The SI unit of density is $\mathrm{kg/m^3}$, representative values are given in Table 11.1. The metric system was originally devised so that water would have a density of $1\ \mathrm{g/cm^3}$, equivalent to $10^3\ \mathrm{kg/m^3}$. Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 cm^3.

Table 11.1 Densities of Various Substances

Substance	$\rho(10^3\ \mathrm{kg/m^3}$ or g/mL)	Substance	$\rho(10^3\ \mathrm{kg/m^3}$ or g/mL)	Substance	$\rho(10^3\ \mathrm{kg/m^3}$ or g/mL)
Solids		**Liquids**		**Gases**	
Aluminum	2.7	Water (4ºC)	1.000	Air	1.29×10^{-3}
Brass	8.44	Blood	1.05	Carbon dioxide	1.98×10^{-3}
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	1.25×10^{-3}

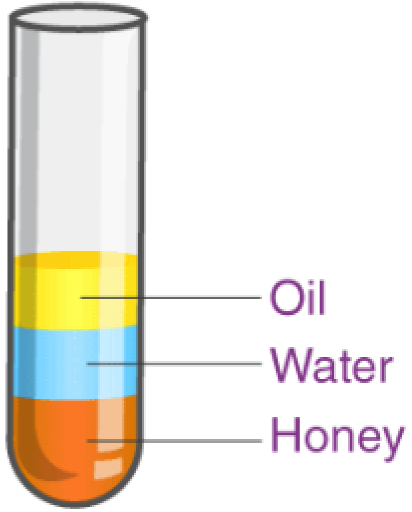
Relative Density

Density is the amount of mass in a unit volume of matter, every substance has a different density, to understand the idea of density let's conduct an experiment, we will need a tall glass cup, honey, water, coconut oil, and food coloring,

Step1: Pour a one-quarter cup of honey,

Step2: Pour a one-quarter cup of colored water gently on top of the honey.

Step3: pour a one-quarter cup of coconut oil on top of the colored water.



Notice how the different liquids from different layers, why is it so? The different substance has a different density, which means for the same volume different substances weigh differently, as they weigh differently heavier substances tend to settle at the bottom, like honey and lighter material like oil tend to float at the top which means.

What is Relative Density?

The difference between the specific gravity and density is that at room temperature and pressure is 1gram per 1 cubic cm is the density of water this density is treated as a standard and the density of any other material (usual liquids) is calculated relative to this is called **relative density** or specific gravity.

Formula

$$RD = \frac{\rho_{\text{substance}}}{\rho_{\text{reference}}}$$

RD = relative density

$\rho_{\text{substance}}$ = density of the substance being measured

$\rho_{\text{reference}}$ = Density of the reference

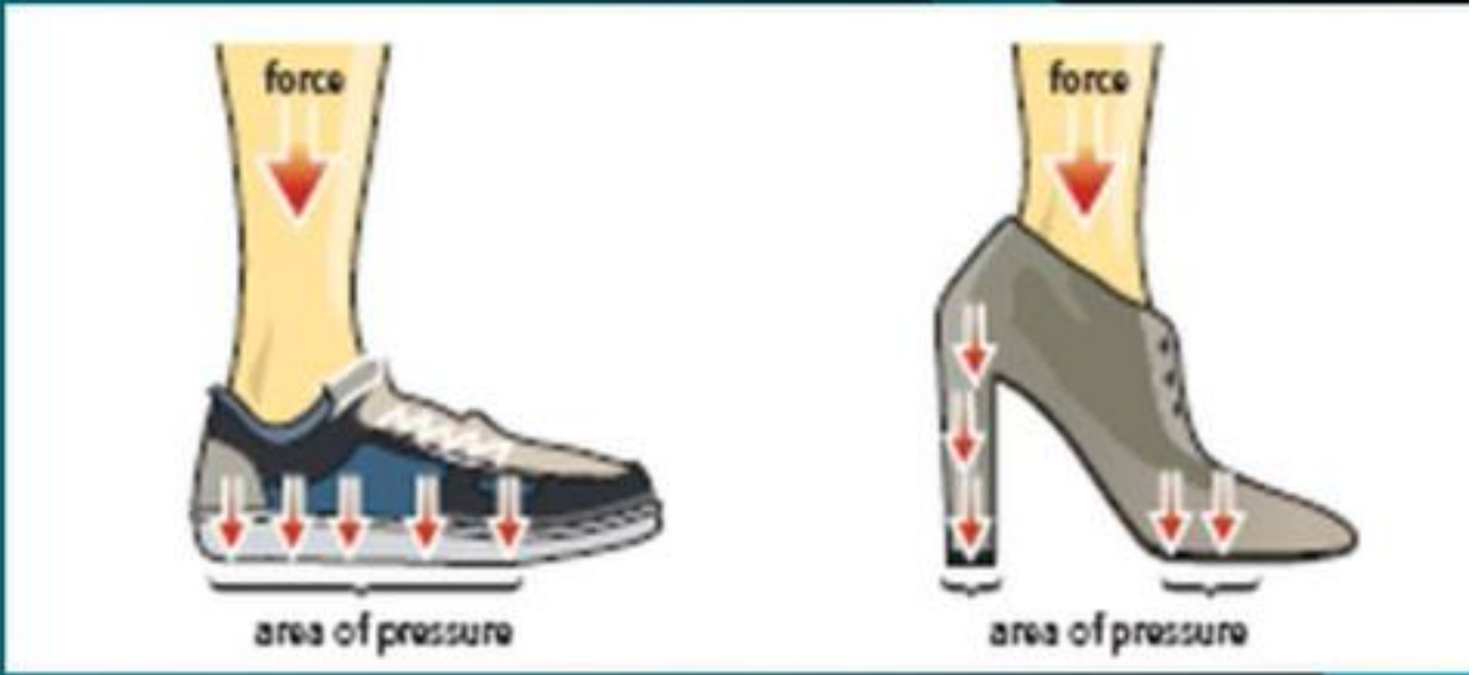
Pressure

Pressure is the force acting on a certain area of a surface.

When you press your hand against a wall, you are applying pressure on that particular area of the wall. If you increase the force, the pressure will also increase.

Pressure

High heels exert more pressure on the ground because the pressure is concentrated into a smaller area than a flat shoe.



Calculating Pressure

- Remember that **force** is measured in Newtons (N) and **area** is often measured in square metres (m^2).
- The unit for pressure, therefore, is newtons per square metre (N/m2). This unit is also called a **pascal** (Pa).
- One pascal is a very small amount of pressure, 1 Pa=1N/m2. This is the equivalent of 100 grams of force spread over a 1m by 1m surface.

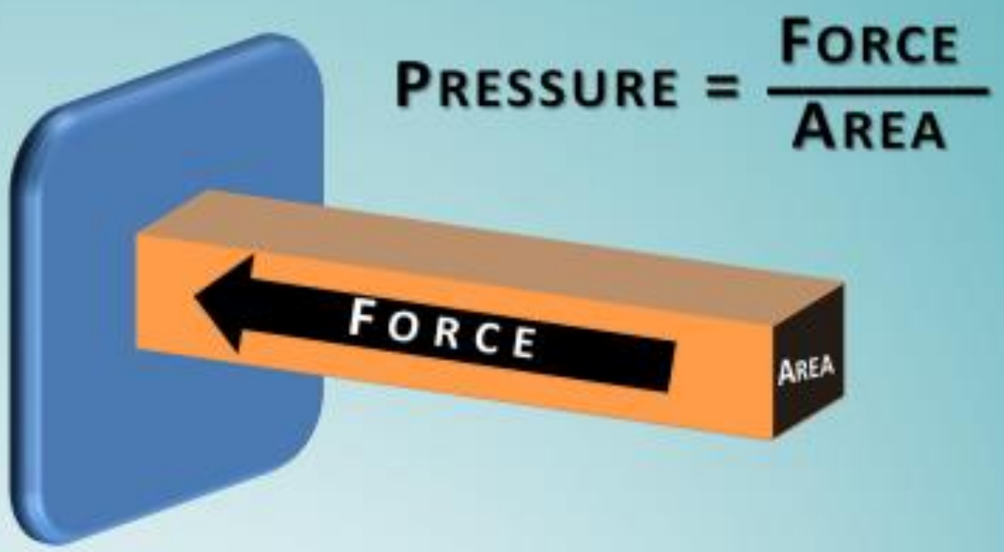
Atmospheric Pressure

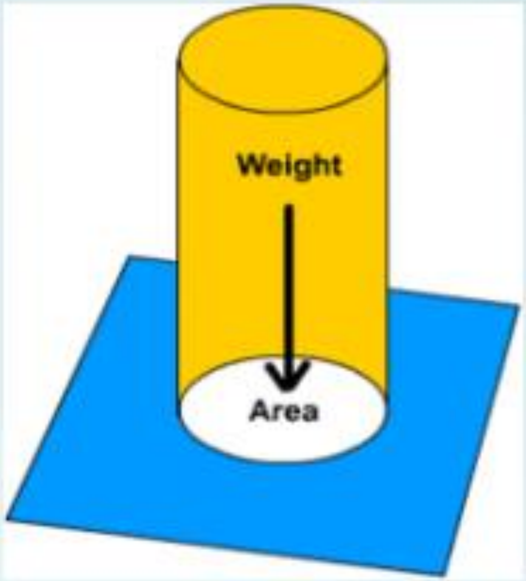
- **Atmospheric pressure** is the amount of force that is exerted by the weight of the atmosphere.

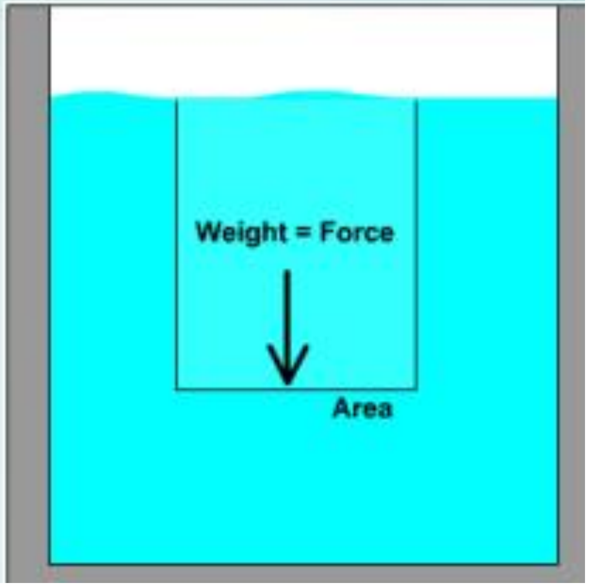
- As you climb higher in the atmosphere, the amount of air above you decreases. Therefore, the air exerts less pressure on you. The air pressure inside your body, however, does not change as quickly.

Pressure Definition

Pressure is defined as a normal force exerted by a fluid divided by the area that the force acts over and therefore has units of F/A . It can be a result of an applied force (for example pumping) or hydrostatic (weight of a column of fluid).







Pressure Units

Unit for pressure is the Pascal (**Pa**), which equal (**N/m²** or **kg/m.s²**).

1 bar = 10^5 Pa = 0.1 MPa = 100 kPa

1 atm = 101,325 Pa

1 atm = 101.325 kPa

1 atm = 1.01325 bars

Units for Expressing Pressure

Unit	Value
Atmosphere	1 atm
Pascal (Pa)	1 atm = 1.01325×10^5 Pa
Kilopascal (kPa)	1 atm = 101.325 kPa
mmHg	1 atm = 760 mmHg
Torr	1 atm = 760 torr
Bar	1 atm = 1.01325 bar
mbar	1 atm = 1013.25 mbar
psi	1 atm = 14.7 psi

Pressure

Hydrostatic Pressure in a Liquid

The pressure at a given depth in a static liquid is a result of the weight of the liquid acting on a unit area at that depth **plus** any pressure acting on the surface of the liquid.

The ***pressure due to the liquid alone*** (i.e. the ***<u>Gauge Pressure</u>***) at a given depth depends only upon the density of the liquid $\boldsymbol{\rho}$ and the distance below the surface of the liquid $\boldsymbol{h}$.

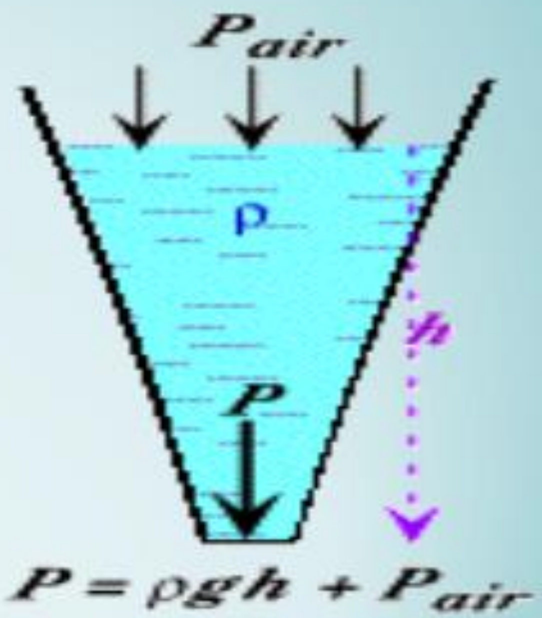
$$\mathbf{P = \rho\, g\, h}$$

$$\mathbf{P = F/A \quad i.e: m^{*}g/A}$$

But $\mathbf{m = V\rho}$

$$\mathbf{P = V\, \rho\, g\, /\, A = \rho\, g\, (V/A)}$$

$$\mathbf{P = \rho\, g\, h}$$



Hydrostatic Pressure in a Liquid

Hydrostatic pressure in a liquid can determined using the following equation:

$$P = \rho g h$$

Where

P = *Pressure;*
(*N/m²*, *or Pa*, or kg/m.s²)

ρ = *Density of liquid* (*kg/m³*)

g = *The gravitational gravity, constant* (*9.807 m/s²*)

h = *Depth at which the pressure is measured* (*m*)

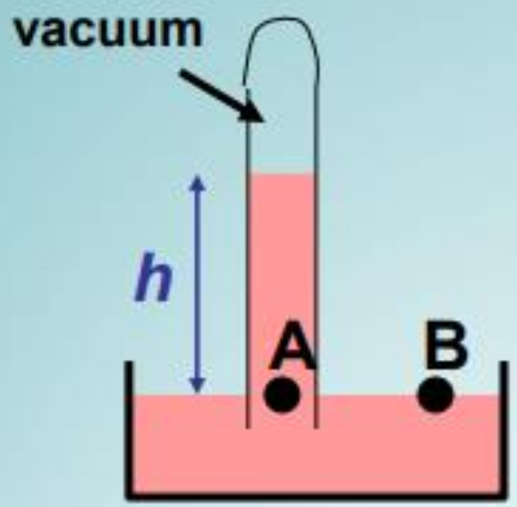
Hydrostatic pressure in a water column at (ρ =1000 kg/m³) is indicated below:

Height of Water Column		Pressure		
(m)	*(ft)*	*(kPa)*	*(bar)*	*(psi)*
1	3.3	9.8	0.1	1.4
2	6.6	19.6	0.2	2.8
3	9.8	29	0.3	4.3
4	13.1	39	0.4	5.7
5	16.4	49	0.5	7.1
6	19.7	59	0.6	8.5
7	23	69	0.7	10.0
8	26	78	0.8	11.4
9	30	88	0.9	12.8
10	33	98	1.0	14.2
12	39	118	1.2	17.1
14	46	137	1.4	19.9
16	52	157	1.6	23
18	59	177	1.8	26
20	66	196	2.0	28
25	82	245	2.5	36
30	98	294	2.9	43
35	115	343	3.4	50
40	131	392	3.9	57
50	164	491	4.9	71
60	197	589	5.9	85
70	230	687	6.9	100
80	262	785	7.8	114
90	295	883	8.8	128
100	328	981	9.8	142

Measuring Pressure

Barometer – an instrument that measures pressure

Mercury barometer

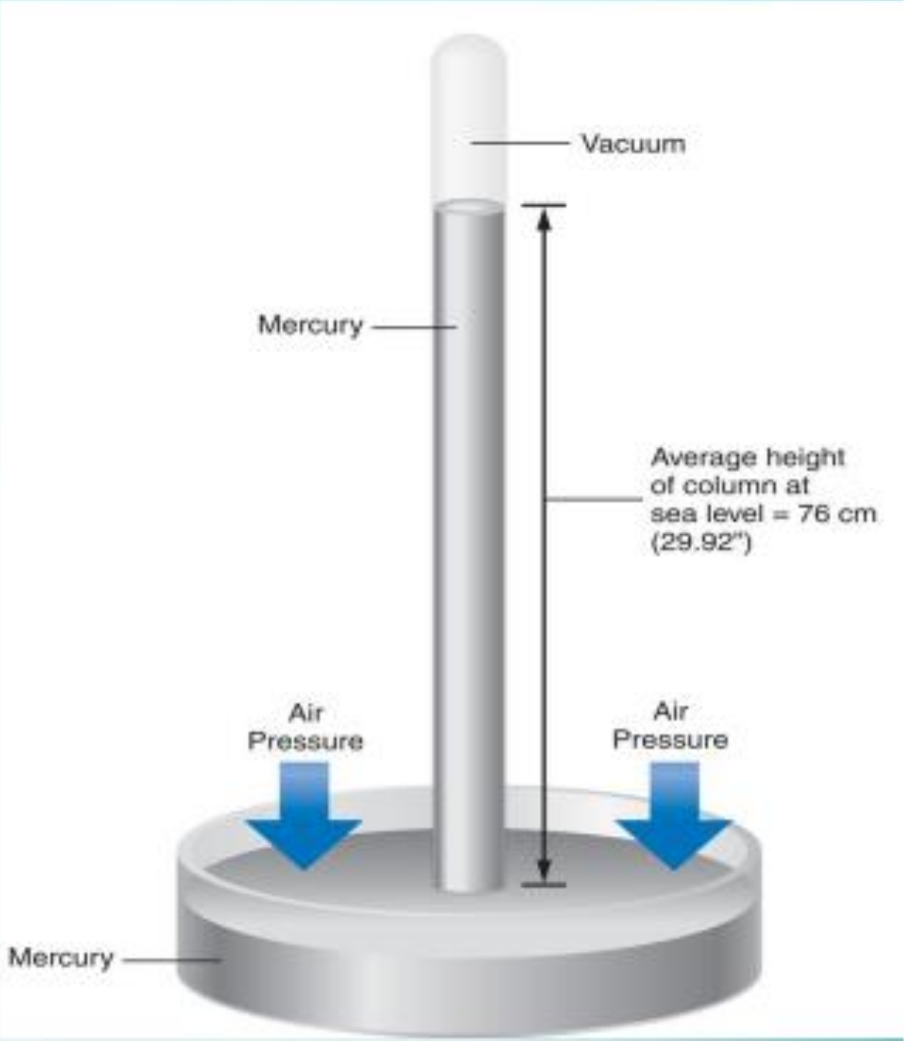


Atmospheric Pressure $P = P_A = P_B = \rho_{Hg}\, g\, h$

Mean sea-level pressure:

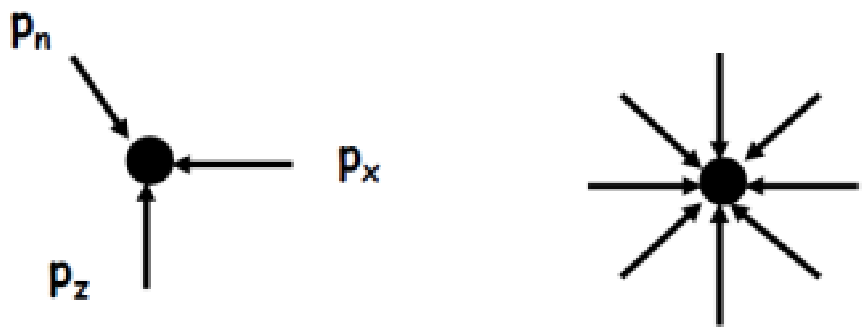
P = 1.01325x10^5 Pa = 101.325 kPa
= 0.101325 MPa
= 1.01325 bars
= 1 atm

P = 1013.25 hecto Pa = 1013.25 hPa



Pressure at a Point

- Pressure at any point in a fluid is the same in all directions.
- Pressure has a magnitude, but not a specific direction, and thus it is a scalar quantity.



$p_x = p_z = p_n$ =constant

p =constant

Pressure acts equally in all directions

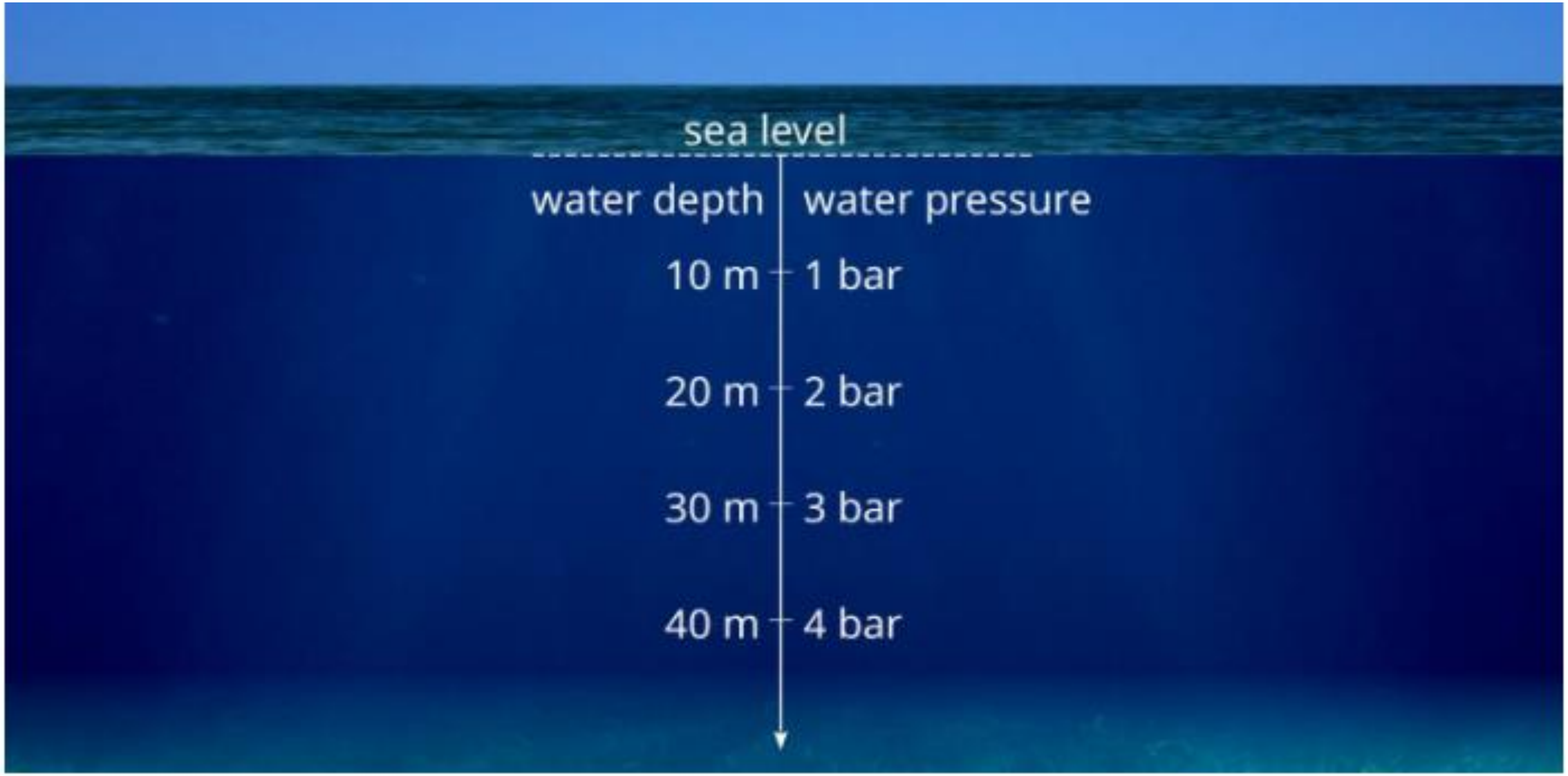


Figure: Water pressure as a function of water depth below sea level

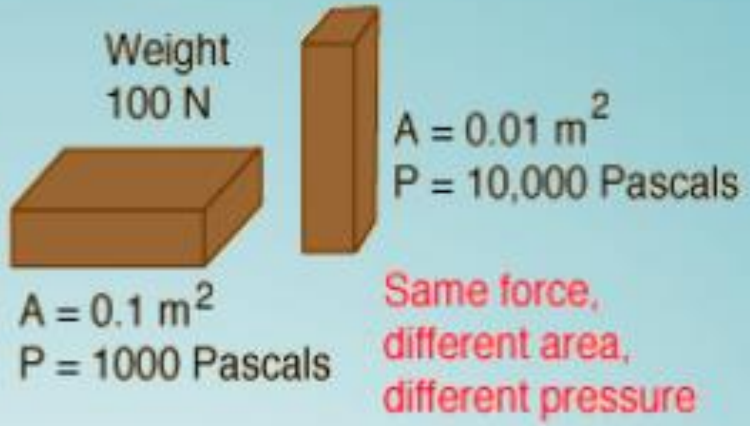
The water pressure increases by about 1 bar per 10 meters of water depth!

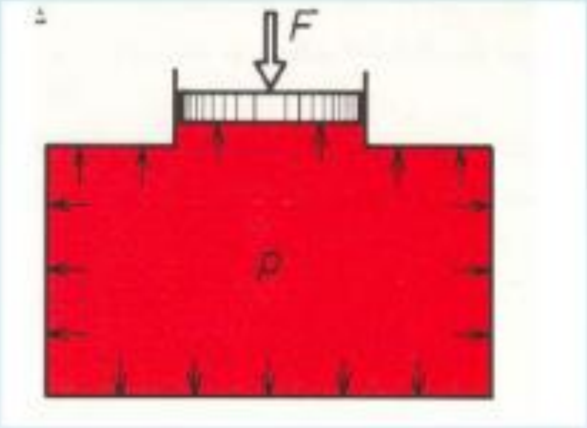
Pascal’s Law

Pascal’s law states that pressure applied to an enclosed fluid is transmitted with equal force throughout the entire container.

Pressure at a Point: Pascal's Law

$$Pressure = \frac{Force}{Area} = \frac{F}{A}$$





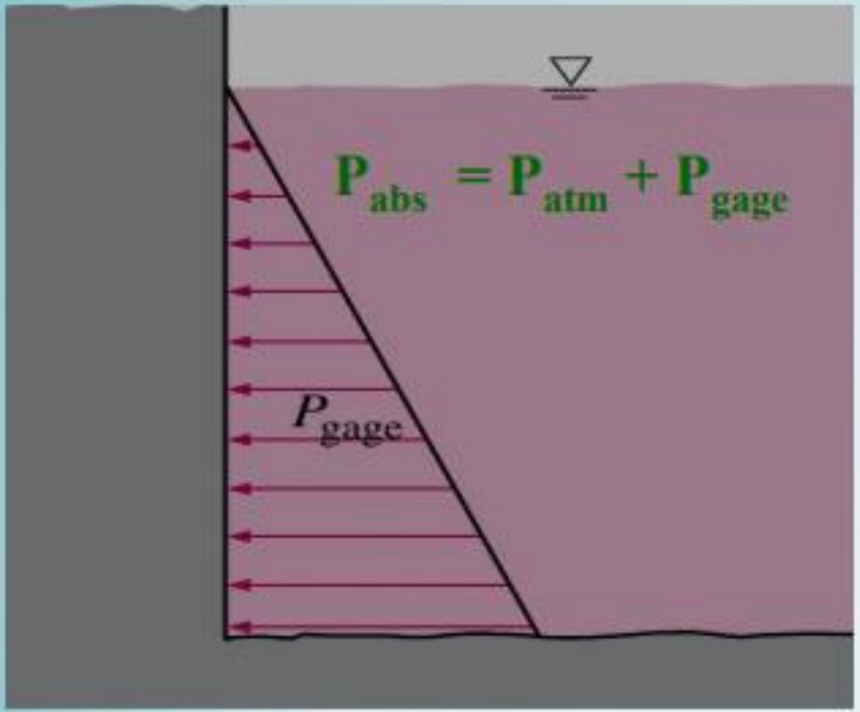
Pascal's low: The pressure at point is the same in all directions and normal to surface.

Variation of Pressure with Depth

$$\Delta P = P_2 - P_1 = \rho g \, \Delta z = \gamma_s \, \Delta z$$

$$P_{below} = P_{above} + \rho g |\Delta z| = P_{above} + \gamma_s |\Delta z|$$

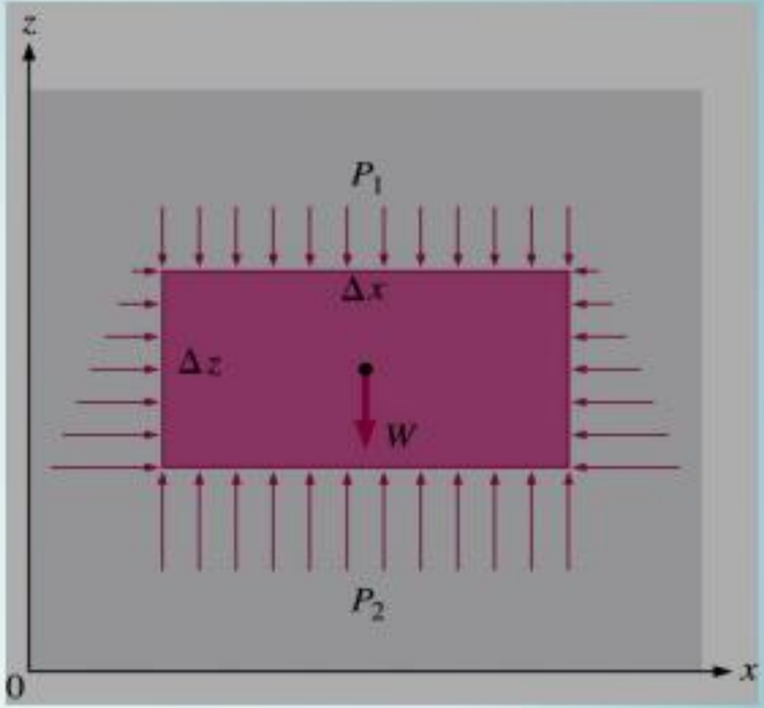
$$P = P_{atm} + \rho g h \quad \text{or} \quad P_{gage} = \rho g h$$



The pressure of a fluid at rest increases with depth (as a result of added weight).

When the variation of density with elevation is known

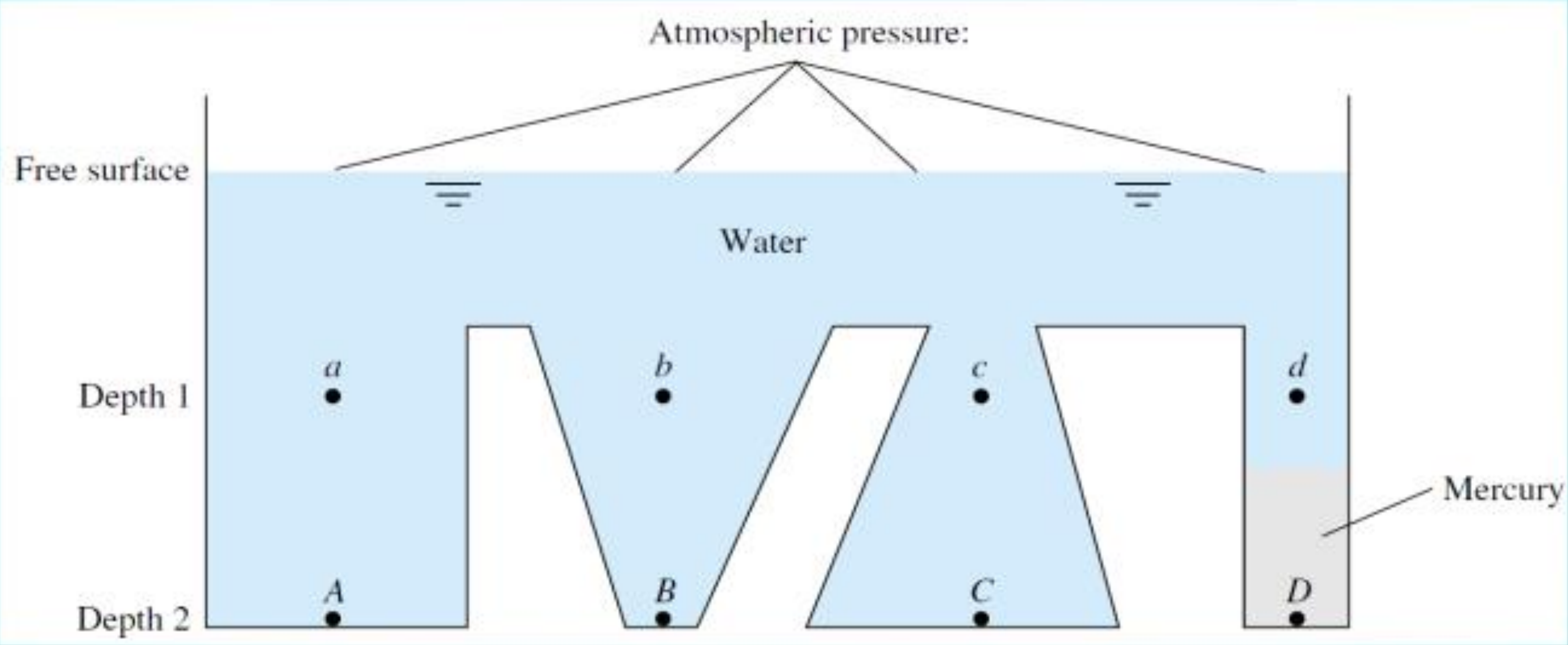
$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g \, dz$$



Free-body diagram of a rectangular fluid element in equilibrium.

Hydrostatic Pressure Distribution

Points a, b, c, and d are at equal depths in water and therefore have identical pressures. Points A, B, and C are also at equal depths in water and have identical pressures higher than a, b, c, and d. Point D has a different pressure from A, B, and C because it is not connected to them by a water path.

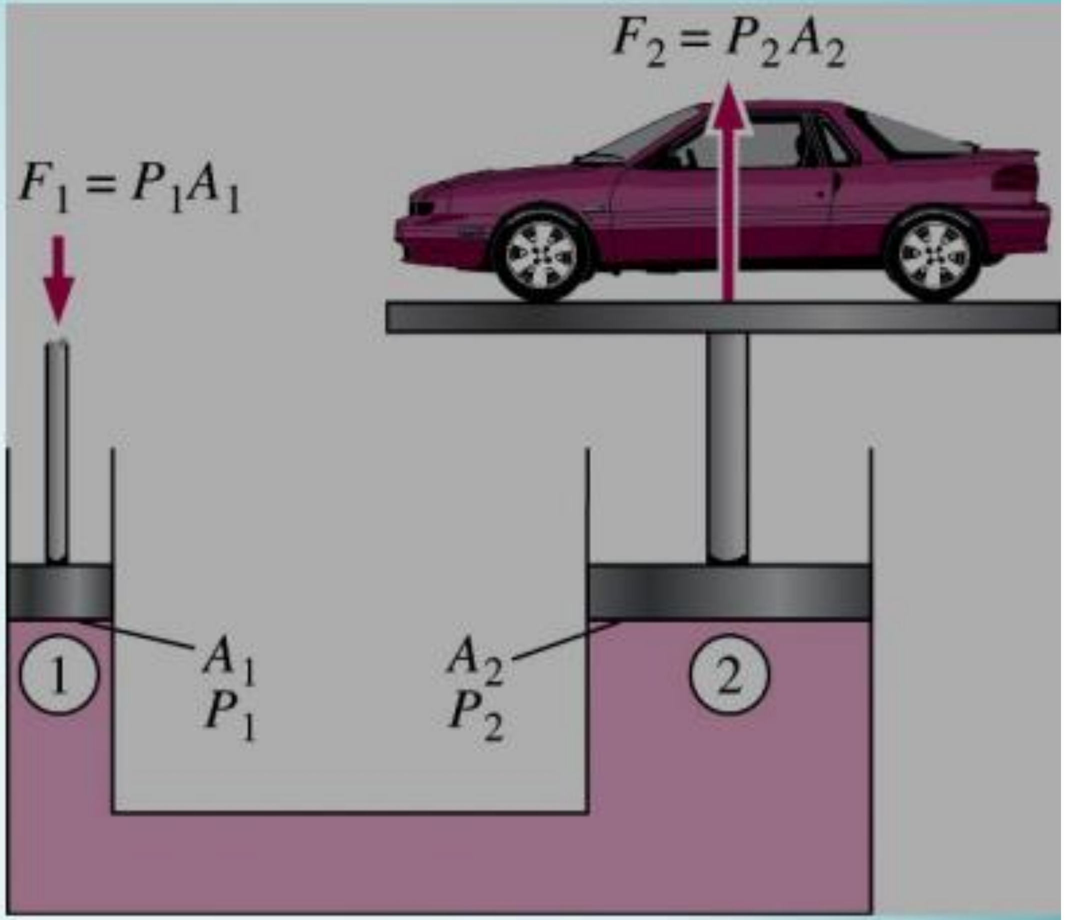


The pressure is the same at all points on a horizontal plane in a given fluid regardless of geometry, provided that the points are interconnected by the same fluid.

Pascal's law: The pressure applied to a confined fluid increases the pressure throughout by the same amount.

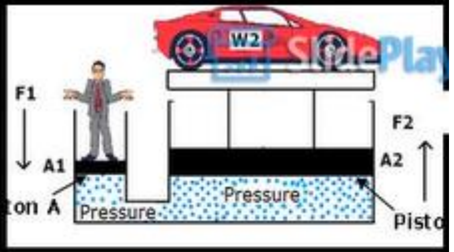
$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

The area ratio $\boldsymbol{A_2/A_1}$ is called the ***ideal mechanical advantage*** of the hydraulic lift.



Lifting of a large weight by a small force by the application of Pascal's law.

4.2 Pascal's Law



- **Example 1-3.**

  In Fig 1.11, if the weight of the car is 10,000 N, the diameter of piston A is 0.01 m, and the force applied on piston A is 250 N. Calculate the area of piston B.

- **Solution:**

  1. Calculate the area of piston A, the piston shape is circular as shown in Fig. 1.10a, accordingly the area will be calculated using the following formula.

$$A_1 = \pi \frac{D^2}{4} = 3.14 \times \frac{(0.01)^2}{4} = 0.0000785 \text{ m}^2$$

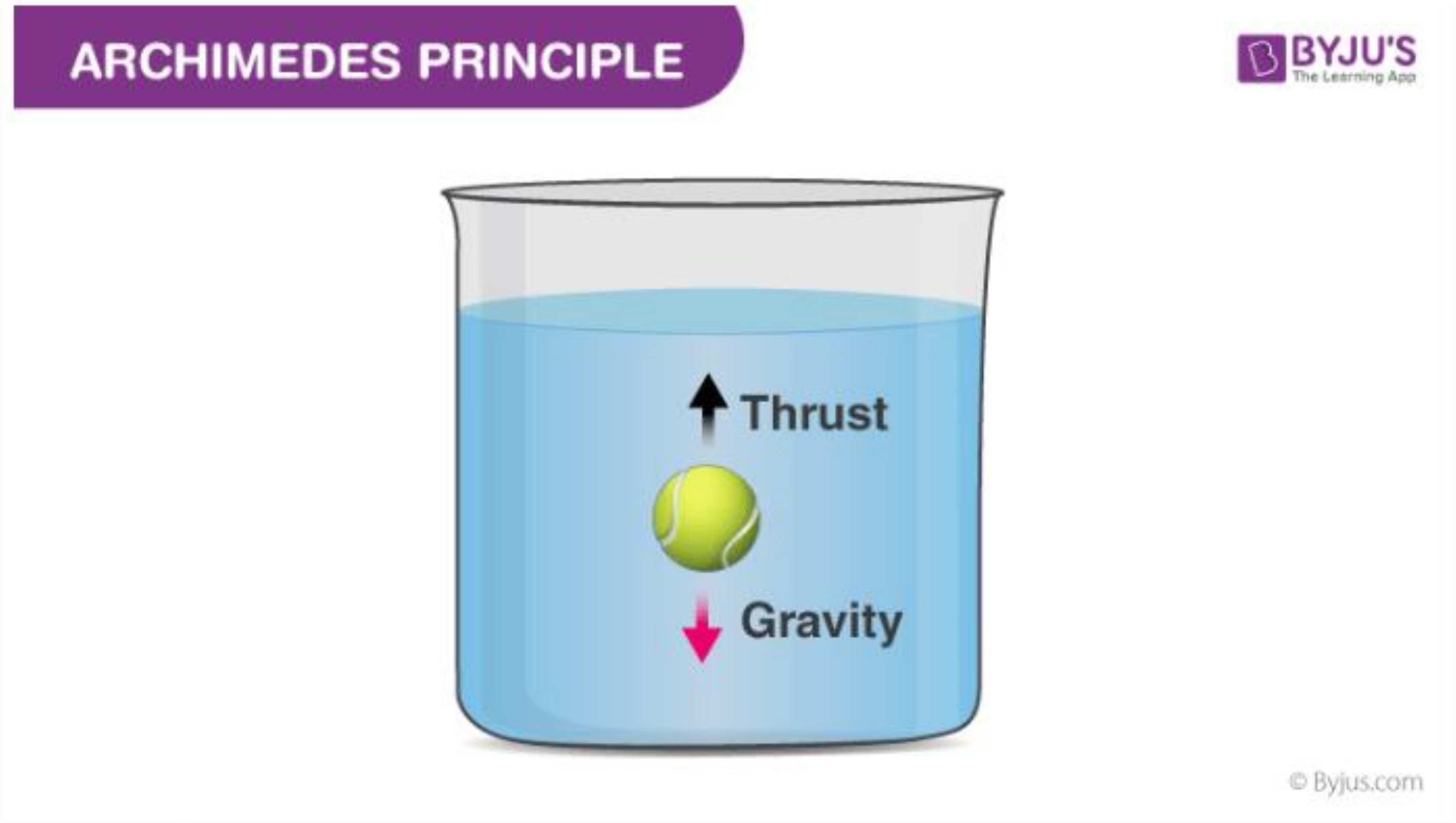
$$F_1 = 250\ N \qquad F_2 = 10{,}000\ N$$

What is Archimedes Principle?

Archimedes' principle states that:

> **"The upward buoyant force that is exerted on a body immersed in a fluid, whether partially or fully submerged, is equal to the weight of the fluid that the body displaces and acts in the upward direction at the center of mass of the displaced fluid".**

The value of thrust force is given by the Archimedes law which Archimedes of Syracuse of Greece discovered. When an object is partially or fully immersed in a liquid, the apparent loss of weight is equal to the weight of the liquid displaced by it.



If you look at the figure, the weight due to gravity is opposed by the thrust provided by the fluid. The object inside the liquid only feels the total force acting on it as the weight. Because the actual gravitational force is decreased by the liquid's upthrust, the object feels as though its weight is reduced. The apparent weight is thus given by:

> Apparent weight= Weight of object (in the air) – Thrust force (buoyancy)

Archimedes Principle Formula

In simple form, the Archimedes law states that the buoyant force on an object is equal to the weight of the fluid displaced by the object. Mathematically written as:

$$F_b = \rho \times g \times V$$

Where F_b is the buoyant force, ρ is the density of the fluid, V is the submerged volume, and g is the acceleration due to gravity.

Archimedes Principle Derivation

The mass of the liquid displaced is.

$$Mass$$
=
$$Density \times Volume$$
=
$$\rho \times V$$

This is because density (ρ) is defined as

$$Density, \rho$$
=
$$\frac{Mass}{Volume}$$
=
$$\frac{M}{V}$$

Thus the weight of that displaced liquid is:

$$Weight$$
=
$$Mass \times Acceleration\ due\ to\ gravity$$

$$W$$
=
$$M \times g$$
=
$$\rho \times V \times g$$

Thus, from the Archimedes principle, we can write:

The apparent loss of weight = weight of water displaced = $\rho \times V \times g$

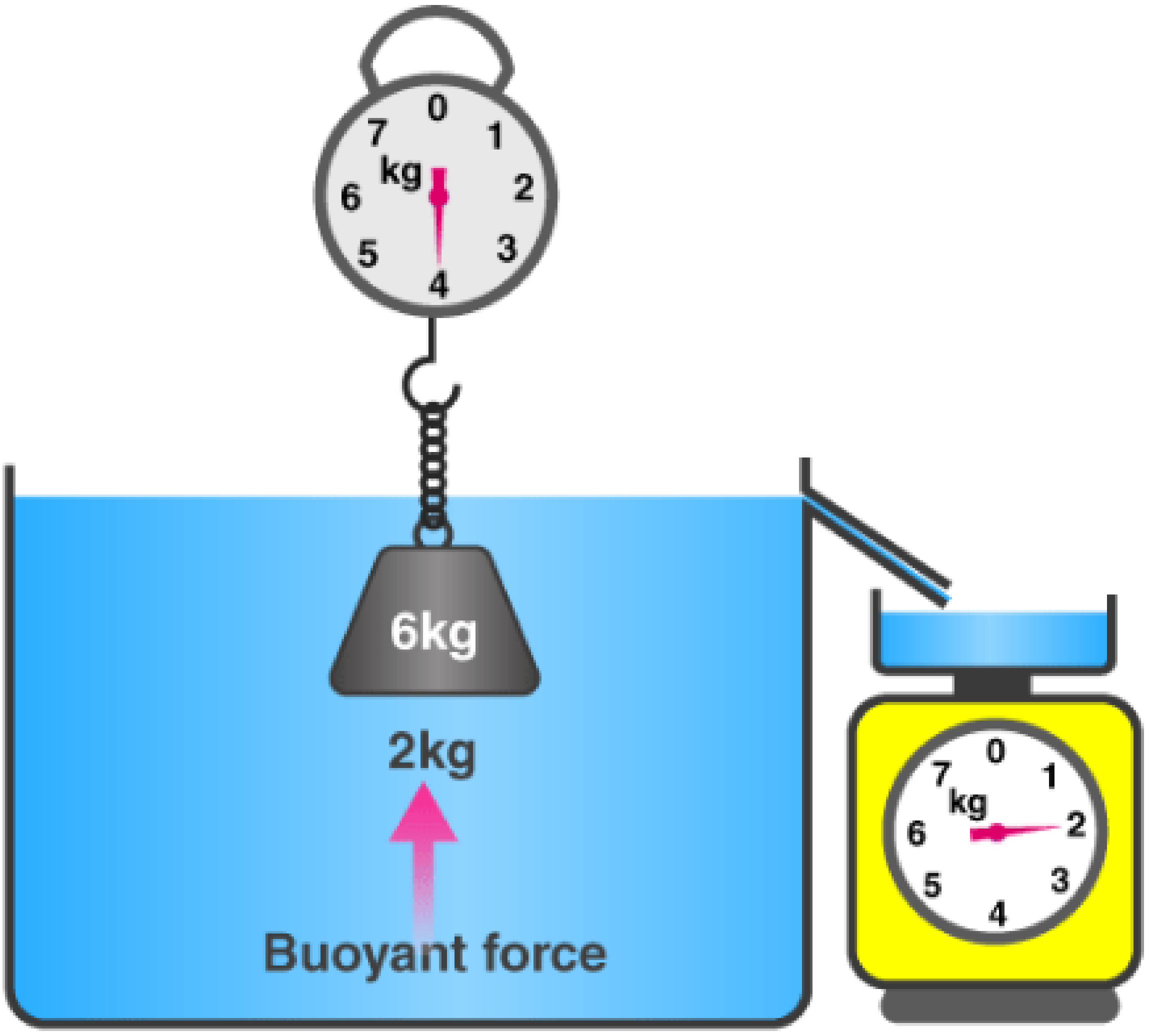
Thus, the Thrust force is,

$$Thrust = \rho \times V \times g$$

Where ρ is the density of the liquid and V is the volume of liquid displaced.

The thrust force is also called the buoyant force because it is responsible for objects to float. Thus, this equation is also called the law of buoyancy.

Archimedes Principle Experiment



Archimedes Principle Applications

Following are the applications of Archimedes principle:

Submarine:

The reason why submarines are always underwater is that they have a component called ballast tank which allows the water to enter making the submarine be in its position underwater as the weight of the submarine is greater than the buoyant force.

Hot-air balloon:

The reason why hot-air balloons rise and float in mid-air is because the buoyant force of the hot-air balloon is less than the surrounding air. When the buoyant force of the hot-air balloon is more, it starts to descend. This is done by varying the quantity of hot air in the balloon.

Hydrometer:

A hydrometer is an instrument used for measuring the relative density of liquids. Hydrometer consists of lead shots which makes them float vertically on the liquid. The lower the hydrometer sinks, the lesser is the density of the liquid.

Archimedes Principle Examples

Q1. Calculate the resulting force, if a steel ball of radius 6 cm is immersed in water.

Ans: Given,

Radius of steel ball = 6 cm = 0.06 m

Volume of steel ball, V =

$$\frac{4}{3}\pi r^3$$

V =

$$\frac{4}{3}\pi 0.06^3$$

$\therefore V = 9.05 \times 10^{-4}\ m^3$

Density of water, $\rho = 1000\ kg.m^{-3}$

Acceleration due to gravity, $g = 9.8\ m.s^{-2}$

From Archimedes principle formula,

$F_b = \rho \times g \times V$

$F_b = (1000\ kg.m^{-3})(9.8\ m.s^{-2})(9.05 \times 10^{-4}\ m^3)$

$\therefore F_b = 8.87\ N$

Q2. Calculate the buoyant force, if a floating body is 95% submerged in water. The density of water is 1000 kg.m^{-3}.

Ans: Given,
Density of water, p = 1000 kg.m^{-3}
From Archimedes principle formula,
$F_b = \rho \times g \times V$

or

$V_b \times \rho_b \times g = \rho \times g \times V$

Where,

ρ,g, and V are the density, acceleration due to gravity, and volume of the water

V_b, ρ_b, and g are the volume, density, and acceleration due to gravity of body immersed

Rearranging the equation,

$$\rho_b = \frac{V\rho}{V_b}$$

Since 95% of the body is immersed,

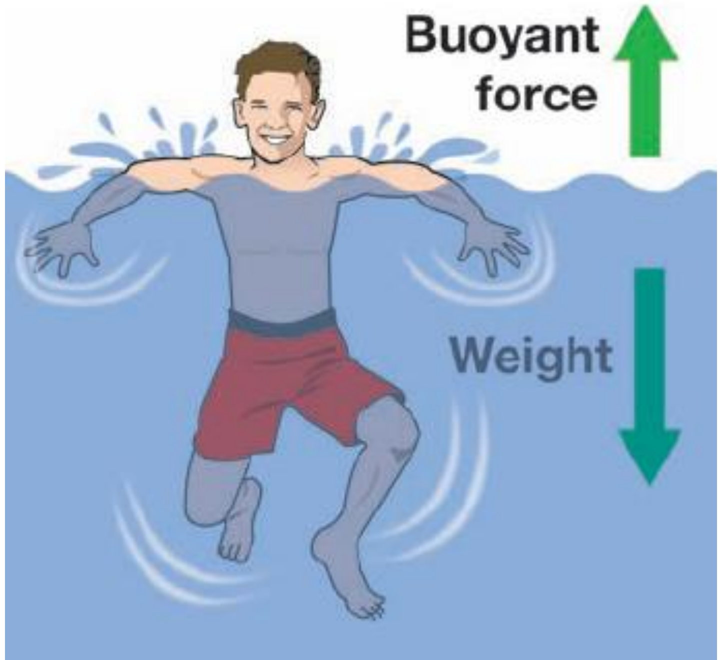
$0.95 \times V_b = V$

$\therefore \rho_b = 950$ kg.m^{-3}

Buoyancy is a force

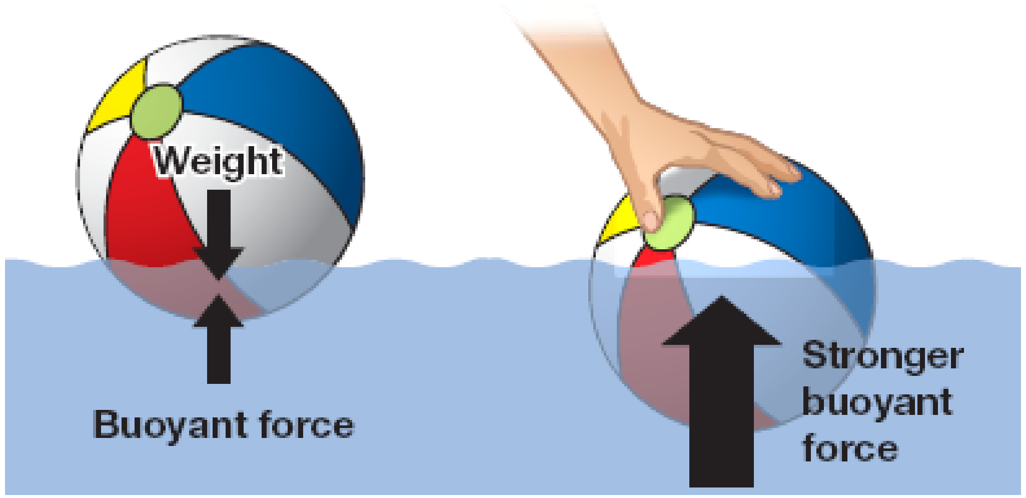
- *Buoyancy* is a measure of the upward force a fluid exerts on an object that is submerged.

The water in the pool exerts an <u>upward force</u> that acts in a direction opposite to the boy's weight.



Volume and buoyancy

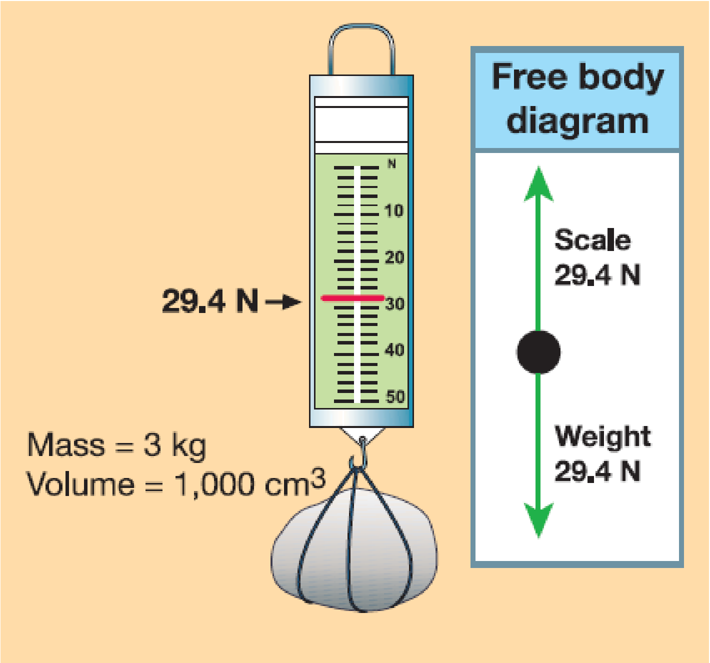
- The strength of the buoyant force on an object in water depends on the volume of the object that is underwater.



As you keep pushing downward on the ball, the buoyant force gets stronger and stronger.

Weight and buoyancy

- *Weight* is a force, like any other pushing or pulling force, and is caused by Earth's gravity.
- It is easy to confuse mass and weight, but they are not the same.
- Weight is the downward force of gravity acting on mass.



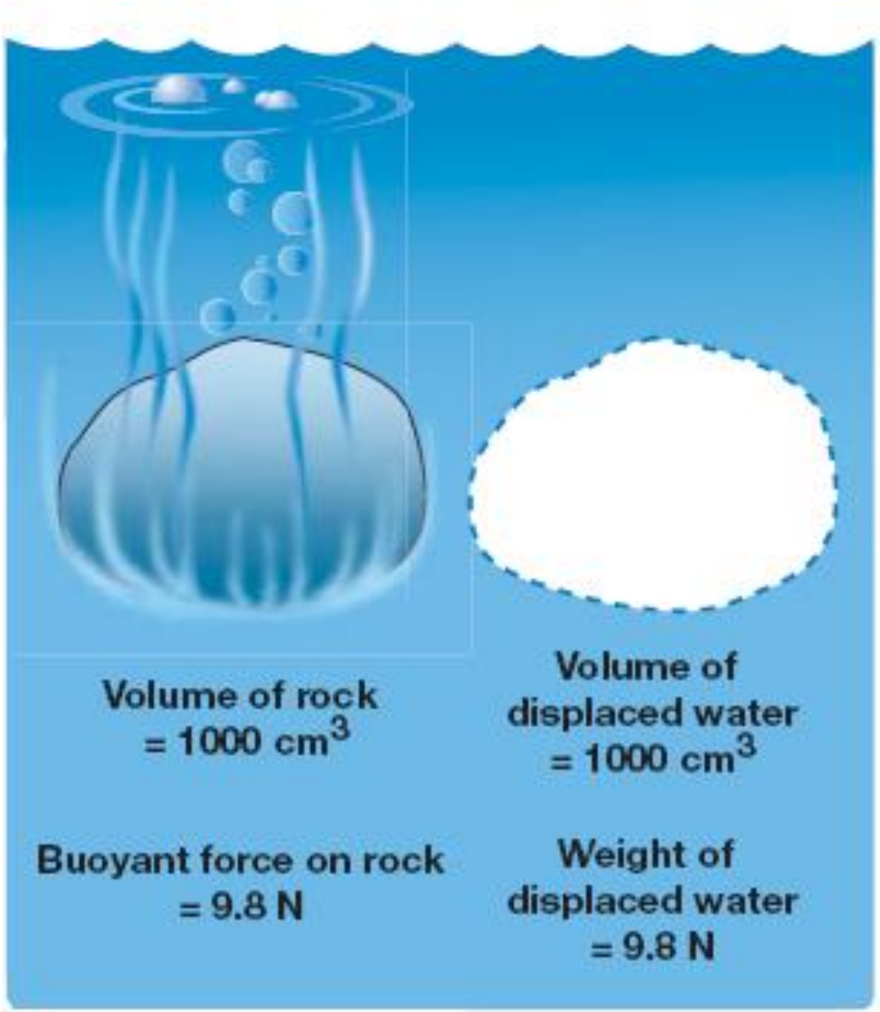
What is the rock's weight?

What is the rock's mass?

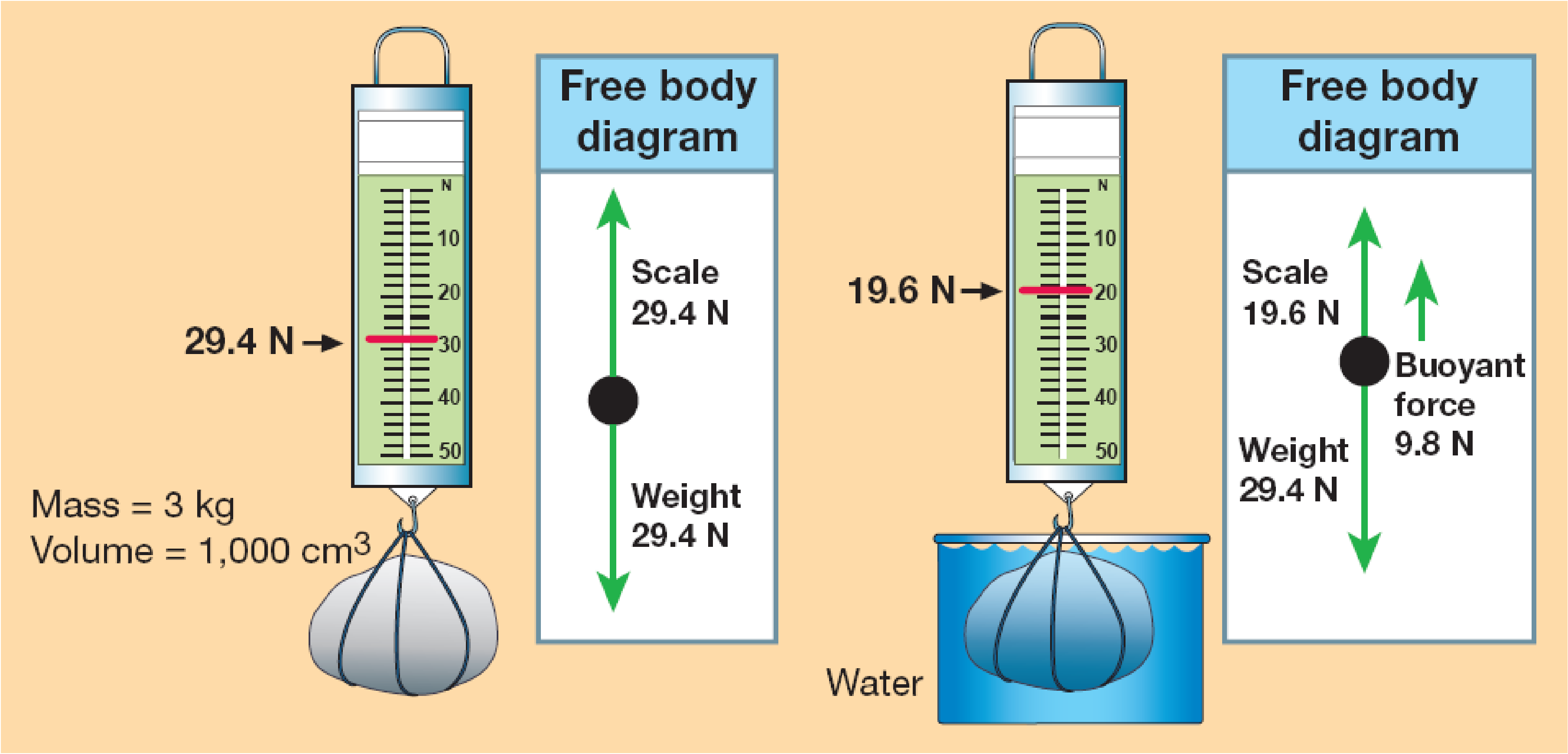
Sinking and floating

- In air the buoyant force on the rock is 29.4 N.

- **When the rock was submerged, the scale read 19.6 N.**
- **The difference is a force of 9.8 N, exactly the amount of force the displaced water exerts.**



Archimedes' Principle



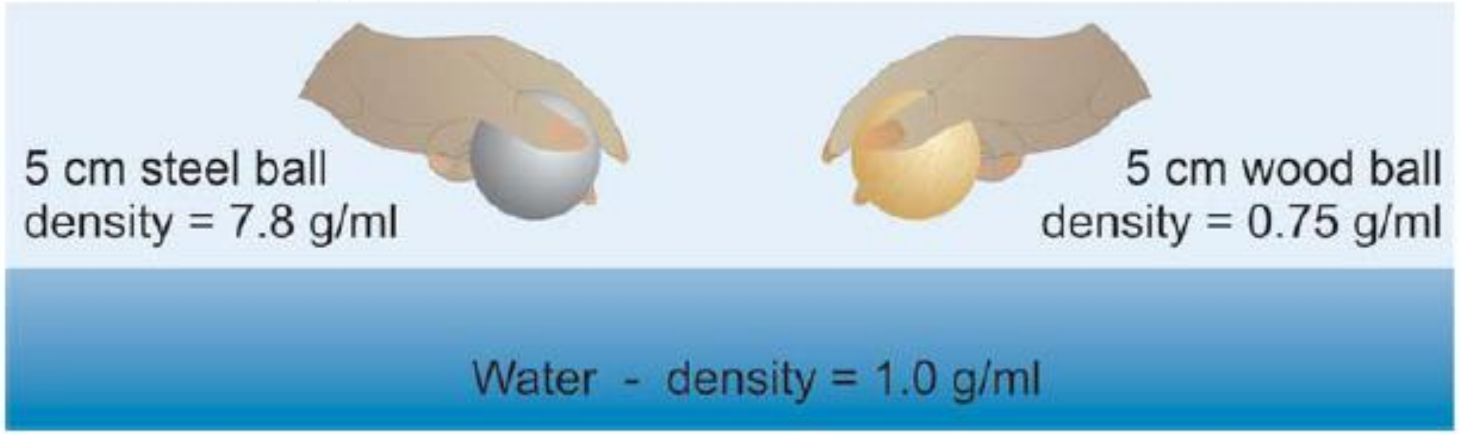
Sinking and floating

- **Buoyancy explains why some objects sink and others float.**
- **Whether an object sinks or floats depends on how the buoyant force compares with the weight.**
- **If an object weighs more than the weight of the water it displaces, it will sink. If the object weighs less, it will float**

http://videos.howstuffworks.com/discovery/6540-mythbusters-lets-talk-buoyancy-video.htm

Density and buoyancy

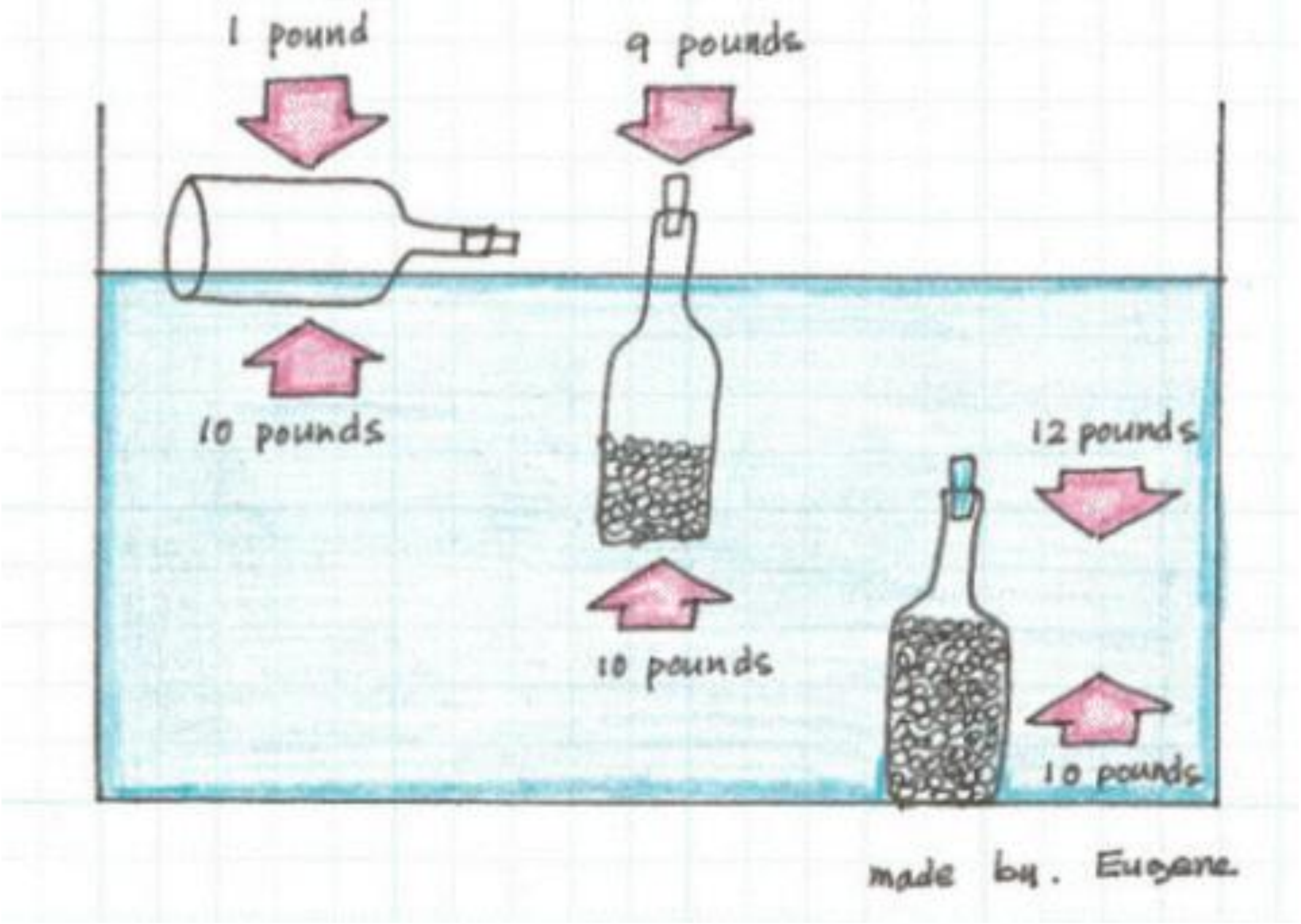
- If you know an object's density you can quickly predict whether it will sink or float.



Which ball will sink in water?
Which ball will float in water?

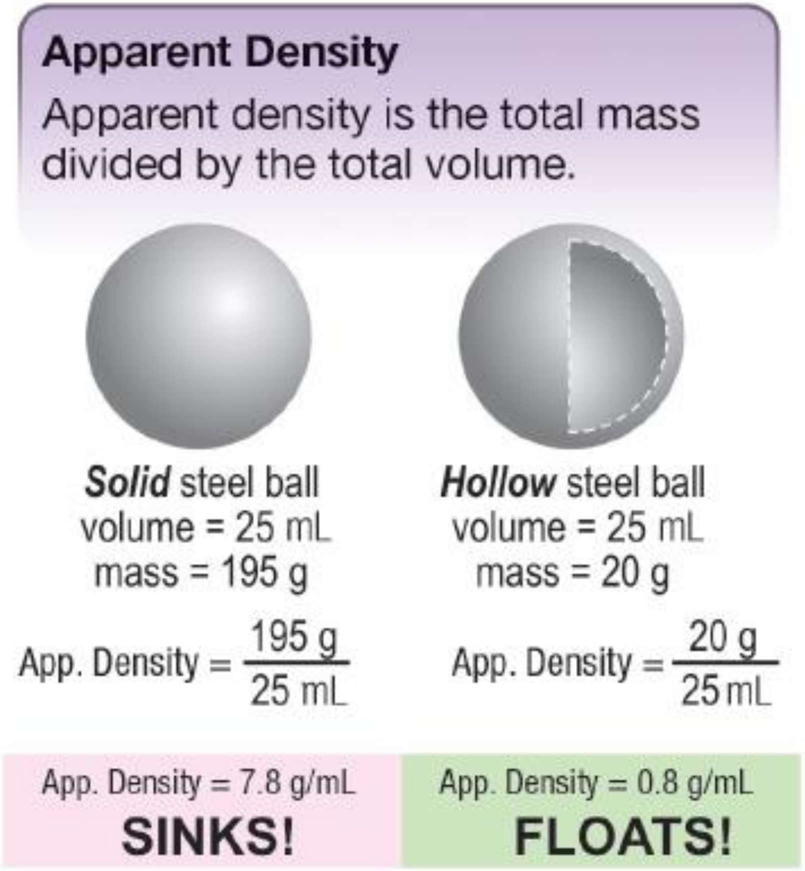
Density and buoyancy

- **When they are completely underwater, both balls have the same buoyant force because they displace the same volume of water.**
- However, the steel ball has more weight since it has a higher density.

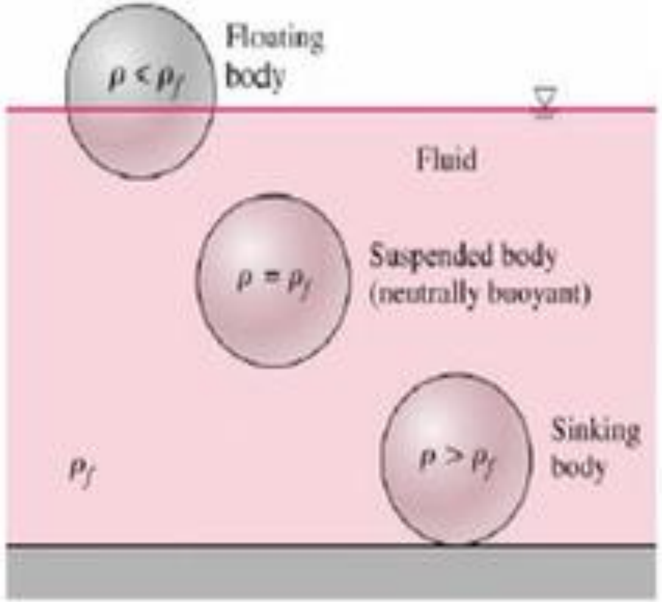

1 pound
9 pounds
10 pounds
12 pounds
10 pounds
10 pounds
made by. Eugene

Apparent Density

- An object with an apparent density GREATER than the density of water will sink.
- An object with an apparent density LESS than the density of water will float.

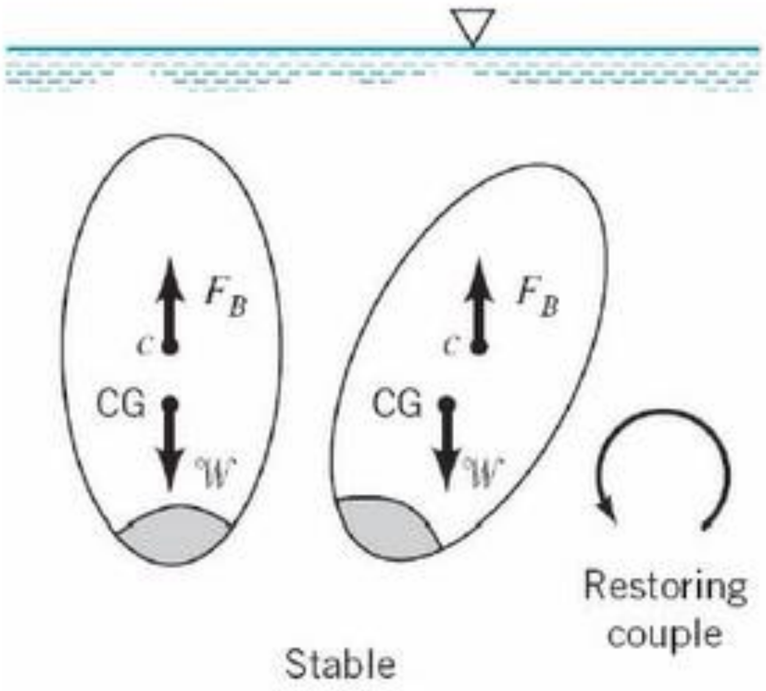


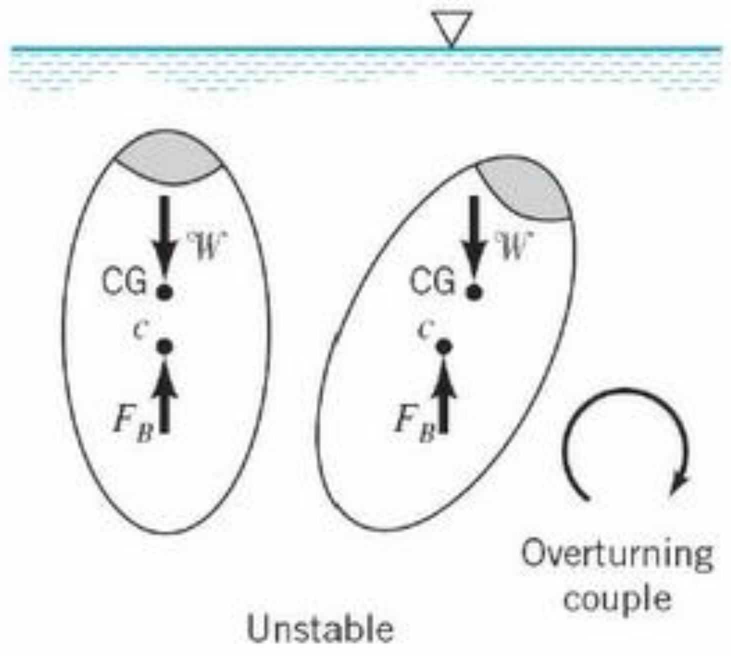
Buoyancy, Flotation and Stability



- When a stationary body is completely submerged in a fluid, or floating (partially submerged), the resultant fluid force on the body is the buoyant force.
- A net upward force results because
- Buoyant force has a magnitude equal to the weight of the fluid displaced by body and is directed vertically upward.

Stability of an immersed body





Stability of a completely immersed body – center of gravity below entroid.

Stability of a completely immersed body – center of gravity above centroid.

Stability of a floating body