FLUID DYNAMICS

Fluid Flow

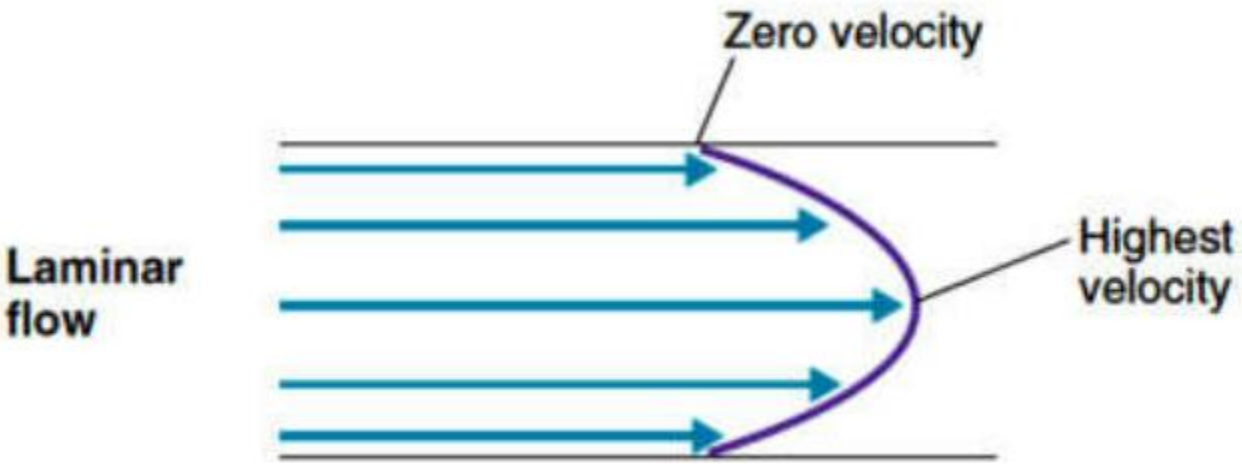
Up till now, we have pretty much focused on fluids at rest. Now let's look at fluids in <u>**motion**</u>

It is important that you understand that an **IDEAL FLUID:**

- **Is non viscous (meaning there is NO internal friction)**
- **Is incompressible (meaning its Density is constant)**
- **Its motion is steady and NON – TURBULENT**

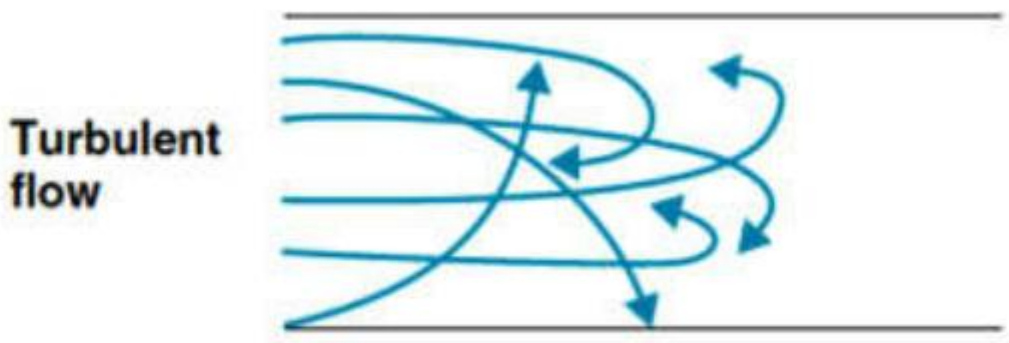
A fluid's motion can be said to be STREAMLINE, or LAMINAR. The path itself is called the streamline. By Laminar, we mean that every particle moves exactly along the smooth path as every particle that follows it. If the fluid DOES NOT have Laminar Flow it has TURBULENT FLOW in which the paths are irregular and called EDDY CURRENTS.

laminar flow, type of fluid (gas or liquid) flow in which the fluid travels smoothly or in regular paths, in contrast to turbulent flow, in which the fluid undergoes irregular fluctuations and mixing. In laminar flow, sometimes called streamline flow, the velocity, pressure, and other flow properties at each point in the fluid remain constant. Laminar flow over a horizontal surface may be thought of as consisting of thin layers, or laminae, all parallel to each other. ~~The fluid in contact with the horizontal~~



turbulent flow, type of fluid (gas or liquid) flow in which the fluid undergoes irregular fluctuations, or mixing, in contrast to laminar

flow, in which the fluid moves in smooth paths or layers. In turbulent flow the speed of the fluid at a point is continuously undergoing changes in both magnitude and direction. The flow of wind and rivers is generally turbulent in this sense, even if the currents are gentle. The air or water swirls and eddies while its overall bulk moves along a specific direction.



4.3 Liquid flow

4.3.1 Flow rate versus flow velocity

The flow rate is the volume of fluid that moves through the system in a given period of time.

Flow rates determine the speed at which the output device (e.g., a cylinder) will operate.

The flow velocity of a fluid is the distance the fluid travels in a given period of time.

These two quantities are often confused, so care should be taken to note the distinction. The following equation relates the flow rate and flow velocity of a liquid to the size (area) of the conductors (pipe, tube or hose) through which it flows.

$$Q = V \times A$$

Where:

Q= flow rate (m^3/s)

V= flow velocity (m/s)

A= area (m^2)

4.3 Liquid flow

This is shown graphically in Fig. 1.11. Arrows are used to represent the fluid flow. It is important to note that the area of the pipe or tube being used.

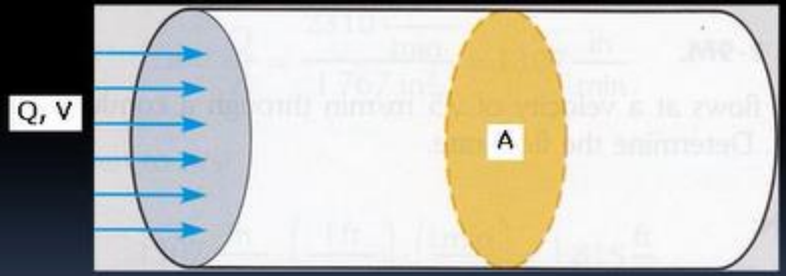


Fig.1.11: Flow velocity and flow rate

4.3.2 The continuity equation

Hydraulic systems commonly have a pump that produces a constant flow rate. If we assume that the fluid is incompressible (oil), this situation is referred to as steady flow. This simply means that whatever volume of fluid flows through one section of the system must also flow through any other section. Fig. 1.12 shows a system where flow is constant and the diameter varies

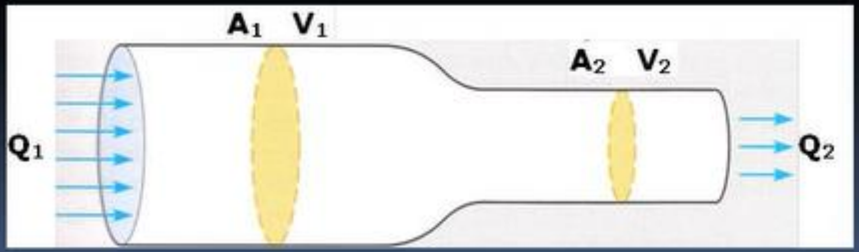
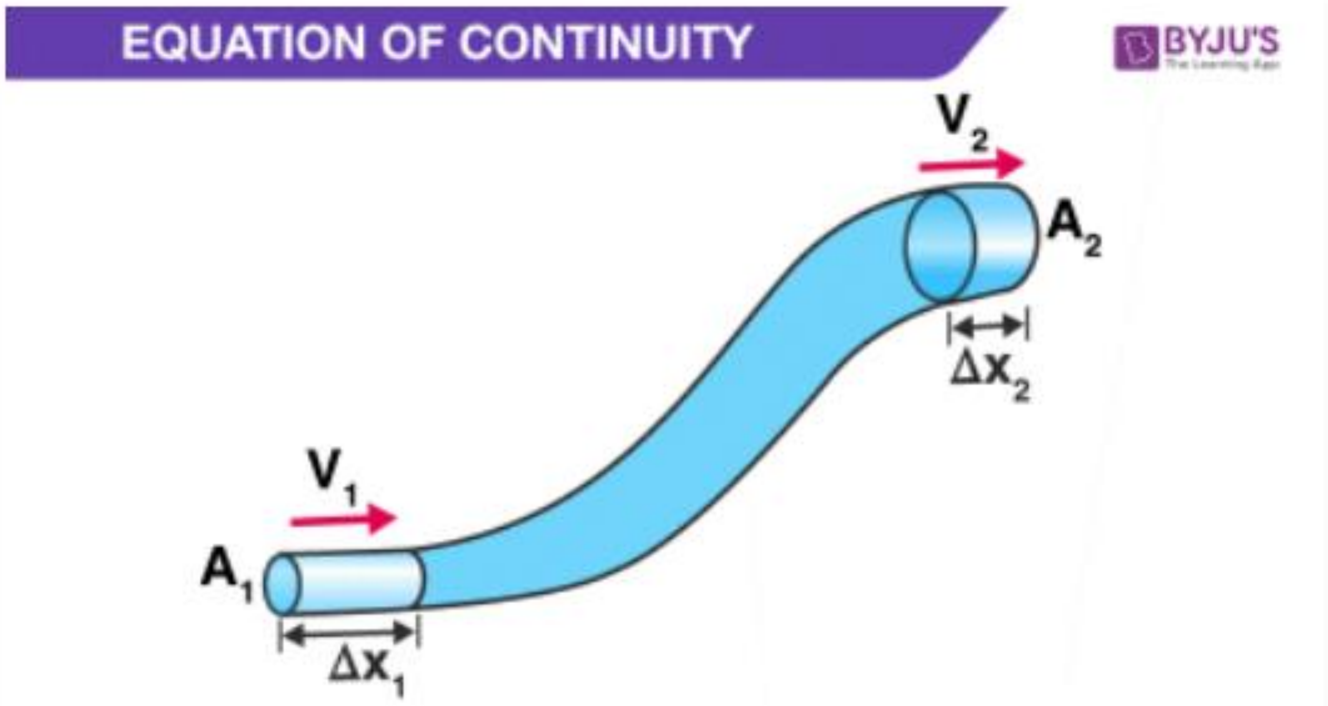


Fig.1.12: Continuity of flow.



Assumption of Continuity Equation

Following are the assumptions of continuity equation:

- The tube is having a single entry and single exit
- The fluid flowing in the tube is non-viscous
- The flow is incompressible
- The fluid flow is steady

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ ---*(Equation 4)*

This can be written in a more general form as:

$\rho A v = \text{constant}$

The equation proves the law of conservation of mass in fluid dynamics. Also, if the fluid is incompressible, the density will remain constant for steady flow. So, $\rho_1 = \rho_2$.

Thus, ***Equation 4*** can be now written as:

$A_1 v_1 = A_2 v_2$

This equation can be written in general form as:

$A v = \text{constant}$

4.3.2 The continuity equation

The following equation applies in this system:

$$\boxed{Q_1 = Q_2}$$

Therefore,

$$\boxed{V_1 \times A_1 = V_2 \times A_2}$$

The following example illustrates the significance of the continuity equation shown above.

4.3.2 The continuity equation

- **Example 1-5.**
- A fluid flows at a velocity of 0.2 m/s at point 1 in the system shown in Fig. 1.12. The diameter at point 1 is 50mm and the diameter at point 2 is 30 mm. Determine the flow velocity at point 2. Also determine the flow rate in m/s.
- 1. Calculate the areas

$$A_1 = \pi \times \frac{D_1}{4}^2 = 3.14 * \frac{(50 \times 10^{-3})^2}{4} = 1.963 \times 10^{-3} m^2$$

$$A_2 = \pi \times \frac{D_2}{4}^2 = 3.14 * \frac{(30 \times 10^{-3})^2}{4} = 7.068 \times 10^{-4} m^2$$

Example - Equation of Continuity

$10\ m^3/h$ of water flows through a pipe with *$100\ mm$* inside diameter. The pipe is reduced to an inside dimension of *$80\ mm$*.

Using equation *(2)* the velocity in the *$100\ mm$* pipe can be calculated

$$(10\ m^3/h)\ (1\ /\ 3600\ h/s) = v_{100}\ (3.14\ (0.1\ m)^2\ /\ 4)$$

or

$$v_{100} = (10\ m^3/h)\ (1\ /\ 3600\ h/s)\ /\ (3.14\ (0.1\ m)^2\ /\ 4)$$

$$= \underline{0.35}\ m/s$$

Using equation *(2)* the velocity in the *$80\ mm$* pipe can be calculated

$$(10\ m^3/h)\ (1\ /\ 3600\ h/s) = v_{80}\ (3.14\ (0.08\ m)^2\ /\ 4)$$

or

$$v_{80} = (10\ m^3/h)\ (1\ /\ 3600\ h/s)\ /\ (3.14\ (0.08\ m)^2\ /\ 4)$$

$$= \underline{0.55}\ m/s$$

4.3.2 The continuity equation

The example shows that in a system with a steady flow rate, a reduction in area (pipe size) corresponds to an increase in flow velocity by the same factor. If the pipe diameter increases, the flow velocity is reduced by the same factor. This is an important concept to understand because in an actual hydraulic system, the pipe size changes repeatedly as the fluid flows through hoses, fittings, valves, and other devices.

Bernoulli’s Principle

Bernoulli’s principle states that

> The total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion, remains constant.

Bernoulli’s Principle Formula

Bernoulli’s equation formula is a relation between pressure, kinetic energy, and gravitational potential energy of a fluid in a container.

The formula for Bernoulli’s principle is given as follows:

$$p + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Where p is the pressure exerted by the fluid, v is the velocity of the fluid, ρ is the density of the fluid and h is the height of the container.

Bernoulli’s equation gives great insight into the balance between pressure, velocity and elevation.

Bernoulli’s Equation at Constant Depth

When the fluid moves at a constant depth that is when $h_1 = h_2$, then Bernoulli’s equation is given as:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Bernoulli’s Equation for Static Fluids

When the fluid is static, then $v_1 = v_2 = 0$, then Bernoulli’s equation is given as:

When $v_1 = v_2 = 0$	$P_1 + \rho g h_1 = P_2 + \rho g h_2$
When $h_2 = 0$	$P_2 = P_1 + \rho g h_1$

Bernoulli's Principle Example

Q1. Calculate the pressure in the hose whose absolute pressure is 1.01 x 105 N.m-2 if the speed of the water in the hose increases from 1.96 m.s-1 to 25.5 m.s-1. Assume that the flow is frictionless and density 10^3 kg.m^{-3}

Ans: Given,

Pressure at point 2, $p_2 = 1.01 \times 10^5$ N.m^{-2}

Density of the fluid, $\rho = 10^3$ kg.m^{-3}

Velocity of the fluid at point 1, $v_1 = 1.96$ m.s^{-1}

Velocity of the fluid at point 2, $v_2 = 25.5$ m.s^{-1}

From Bernoulli's principle for p_1,

$$p_1 = p_2 \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

Substituting the values in the above equation, we get

$$p_1 = (1.01 \times 10^5) + \frac{1}{2}(10^3)[(25.5)^2 - (1.96)^2]$$

$p_1 = 4.24 \times 10^5$ N.m^{-2}