Multi-channel Kondo model URG

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1 Introduction

$$H = \sum_{k,\sigma,\gamma} \epsilon_k^\gamma \hat{n}_{k\sigma}^\gamma + J \sum_{kk',\gamma} \vec{S}_d \cdot \vec{s}_{\alpha\alpha'} {c_{k\alpha}^\gamma}^\dagger c_{k'\alpha'}^\gamma \tag{1}$$

$$\Delta H = c^\dagger T \frac{1}{\hat{\omega} - H_D} T^\dagger c + T^\dagger c \frac{1}{\hat{\omega} - H_D} c^\dagger T \tag{2}$$

2 Leading order renormalization

Second term.

$$\begin{aligned}
& T^\dagger c \frac{1}{\hat{\omega} - H_D} c^\dagger T \\
&= J^2 \sum_{q\beta kk'\alpha\alpha'} c_{k'\alpha'}^\dagger c_{q\beta} \vec{S}_d \cdot \vec{s}_{\alpha'\beta} \frac{1}{\omega - \epsilon_q \tau_{q\beta} - J S_d^z s_q^z} c_{q\beta}^\dagger c_{k\alpha} \vec{S}_d \cdot \vec{s}_{\beta\alpha} \\
&= J^2 \sum_{q\beta kk'\alpha\alpha',a,b} c_{k'\alpha'}^\dagger c_{q\beta} S_d^a s_{\alpha'\beta}^a \frac{1}{\omega - \frac{1}{2}D - J S_d^z s_q^z} c_{q\beta}^\dagger c_{k\alpha} S_d^b s_{\beta\alpha}^b && \left[\tau = \frac{1}{2}, \epsilon_q = D\right] \\
&= J^2 \sum_{q\beta kk'\alpha\alpha',a,b} c_{k'\alpha'}^\dagger c_{q\beta} S_d^a s_{\alpha'\beta}^a \frac{\omega - \frac{1}{2}D + J S_d^z s_q^z}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} c_{q\beta}^\dagger c_{k\alpha} S_d^b s_{\beta\alpha}^b \\
&= J^2 \sum_{q\beta kk'\alpha\alpha',a,b} c_{k'\alpha'}^\dagger c_{k\alpha} S_d^a s_{\alpha'\beta}^a \frac{\left(\omega - \frac{1}{2}D\right)\left(1 - \hat{n}_{q\beta}\right) + J S_d^z c_{q\beta} s_q^z c_{q\beta}^\dagger}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} S_d^b s_{\beta\alpha}^b \\
&= J^2 \sum_{q\beta kk'\alpha\alpha',a,b} c_{k'\alpha'}^\dagger c_{k\alpha} S_d^a s_{\alpha'\beta}^a \frac{\left(\omega - \frac{1}{2}D\right)\left(1 - \hat{n}_{q\beta}\right) + J S_d^z \left(s_q^z + \beta\right)\left(1 - \hat{n}_{q\beta}\right)}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} S_d^b s_{\beta\alpha}^b && \left[\left[c_{q\beta}, s_q^z\right] = \beta c_{q\beta}\right] \\
&= J^2 |\delta D| \sum_{q\beta kk'\alpha\alpha',a,b} c_{k'\alpha'}^\dagger c_{k\alpha} S_d^a s_{\alpha'\beta}^a s_{\beta\alpha}^b \frac{\left(\omega - \frac{1}{2}D\right) + J S_d^z \left(s_q^z + \beta\right)}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} S_d^b \\
&= \frac{J^2\left(\omega - \frac{1}{2}D\right)}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} |\delta D| \sum_{\beta kk'\alpha\alpha',a,b} c_{k'\alpha'}^\dagger c_{k\alpha} S_d^a S_d^b \left(s^a s^b\right)_{\alpha'\alpha} && \text{[term 1]} \\
&+ \frac{J^3}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} |\delta D| \sum_{\beta kk'\alpha\alpha',a,b} c_{k'\alpha'}^\dagger c_{k\alpha} S_d^a s_{\alpha'\beta}^a S_d^z S_d^b s_{\beta\alpha}^b \left(s_q^z + \beta\right) && \text{[term 2]}
\end{aligned} \tag{3}$$

First term.

$$
\begin{aligned}
& c^\dagger T \frac{1}{\hat{\omega} - H_D} T^\dagger c \\
&= J^2 \sum_{q\beta kk'\alpha\alpha'} c^\dagger_{q\beta} c_{k\alpha} \vec{S}_d \cdot \vec{s}_{\beta\alpha} \frac{1}{\omega' - \epsilon_q \tau_{q\beta} - J S^z_d s^z_q} c^\dagger_{k'\alpha'} c_{q\beta} \vec{S}_d \cdot \vec{s}_{\alpha'\beta} \\
&= J^2 \sum_{q\beta kk'\alpha\alpha' ab} c^\dagger_{q\beta} c_{k\alpha} S^a_d s^a_{\beta\alpha} \frac{1}{\omega' - \frac{1}{2}D - J S^z_d s^z_q} c^\dagger_{k'\alpha'} c_{q\beta} S^b_d s^b_{\alpha'\beta} && \left[\tau = -\frac{1}{2}, \epsilon_q = -D\right] \\
&= J^2 \sum_{q\beta kk'\alpha\alpha' ab} c^\dagger_{q\beta} c_{k\alpha} S^a_d s^a_{\beta\alpha} \frac{1}{-\omega + \frac{1}{2}D - J S^z_d s^z_q} c^\dagger_{k'\alpha'} c_{q\beta} S^b_d s^b_{\alpha'\beta} && [\omega + \omega' = D] \\
&= J^2 \sum_{q\beta kk'\alpha\alpha' ab} c^\dagger_{q\beta} c_{k\alpha} S^a_d s^a_{\beta\alpha} \frac{\left(-\omega + \frac{1}{2}D\right) + J S^z_d s^z_q}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} c^\dagger_{k'\alpha'} c_{q\beta} S^b_d s^b_{\alpha'\beta} \\
&= J^2 \sum_{q\beta kk'\alpha\alpha' ab} c^\dagger_{q\beta} c^\dagger_{k'\alpha'} c_{k\alpha} S^a_d s^a_{\beta\alpha} \frac{\left(\omega - \frac{1}{2}D\right) - J S^z_d s^z_q}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} c_{q\beta} S^b_d s^b_{\alpha'\beta} && \left[c_k c^\dagger_{k'} \sim -c^\dagger_{k'} c_k\right] \\
&= J^2 \sum_{q\beta kk'\alpha\alpha' ab} c^\dagger_{k'\alpha'} c_{k\alpha} S^a_d s^a_{\beta\alpha} \frac{\left(\omega - \frac{1}{2}D\right) \hat{n}_{q\beta} - J S^z_d c^\dagger_{q\beta} s^z_q c_{q\beta}}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} S^b_d s^b_{\alpha'\beta} \\
&= \frac{J^2 \left(\omega - \frac{1}{2}D\right)}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} \sum_{q\beta kk'\alpha\alpha' ab} c^\dagger_{k'\alpha'} c_{k\alpha} S^a_d s^a_{\beta\alpha} S^b_d s^b_{\alpha'\beta} \hat{n}_{q\beta} && [\text{term 3}] \\
&- \frac{J^3}{\left(\omega - \frac{1}{2}D\right)^2 - \frac{1}{16}J^2} \sum_{q\beta kk'\alpha\alpha' ab} c^\dagger_{k'\alpha'} c_{k\alpha} S^a_d s^a_{\beta\alpha} S^z_d S^b_d s^b_{\alpha'\beta} c^\dagger_{q\beta} s^z_q c_{q\beta} && [\text{term 4}]
\end{aligned} \tag{4}
$$

Adding the two terms gives

$$
\Delta H = \text{term 1} + \text{term 2} + \text{term 3} + \text{term 4} \tag{5}
$$

The odd terms together give

$$
\text{term 1} + \text{term 3} = \tag{6}
$$