URG analysis of the extended Kondo model with interactions in various angular momentum channels

Abhirup Mukherjee, Siddhartha Lal

I. HAMILTONIAN

We consider an impurity spin $\vec{S}_d$ interacting with a two-dimensional tight-binding conduction bath through a general interaction of the form

$$\sum_{\mathbf{k}_1,\mathbf{k}_2,\sigma_1,\sigma_2} J_{\mathbf{k}_1,\mathbf{k}_2}\vec{S}_d\cdot\frac{1}{2}\vec{\tau}_{\sigma_1,\sigma_2}c^\dagger_{\mathbf{k}_1,\sigma_1}c_{\mathbf{k}_2,\sigma_2} \tag{1}$$

where $\vec{\tau}$ is the vector of sigma matrices and $\mathbf{k}_1,\mathbf{k}_2$ are momentum states of the conduction bath. The precise form of $J_{\mathbf{k}_1,\mathbf{k}_2}$ depends on the symmetry of the symmetry of the impurity-bath interaction. We consider the following three cases:

(i) a d-wave interaction, where the impurity couples with a coherent d-wave combination f_σ of the bath sites closest to it: $f_\sigma \equiv \frac{1}{2}\left(c^\dagger_{L,\sigma}+c^\dagger_{R,\sigma}-c^\dagger_{U\sigma}-c^\dagger_{D\sigma}\right)$, where L,R,U and D indicate electrons at the positions $(x,y)=(-a,0),(a,0),(0,a)$ and $(0,-a)$ respectively, a being the lattice spacing of the conduction bath lattice. The corresponding interaction (in real space) is of the form $\vec{S}_d\cdot\sum_{\sigma_1,\sigma_2}\frac{1}{2}\vec{\tau}_{\sigma_1,\sigma_2}f^\dagger_{\sigma_1}f_{\sigma_2}$. When fourier-transformed to momentum space, it gives rise to the momentum-dependent Kondo coupling

$$J_{\mathbf{k}_1,\mathbf{k}_2} = J\prod_{i=1,2}\left[\cos\left(ak_i^x\right)-\cos\left(ak_i^y\right)\right] \ . \tag{2}$$

For reference, we define the Fourier transforms as $c^\dagger_{L(R),\sigma}=\sum_{\mathbf{k}}c^\dagger_{\mathbf{k},\sigma}e^{-(+)ik^xa}$, $c^\dagger_{U(D),\sigma}=\sum_{\mathbf{k}}c^\dagger_{\mathbf{k},\sigma}e^{-(+)ik^ya}$.

(ii) a p-wave interaction, where the impurity couples with the p-wave electron $p_\sigma\equiv\frac{1}{2}\left(c^\dagger_{L,\sigma}+c^\dagger_{R,\sigma}+c^\dagger_{U\sigma}+c^\dagger_{D\sigma}\right)$. The momentum-dependent Kondo coupling in this case is

$$J_{\mathbf{k}_1,\mathbf{k}_2} = J\prod_{i=1,2}\left[\cos\left(ak_i^x\right)+\cos\left(ak_i^y\right)\right] \tag{3}$$

Since we are considering a 2-dimensional conduction bath in the tight-binding limit, such a Kondo coupling vanishes close to the Fermi surface. As a result, this case is not interesting for our purpose.

(iii) a p-wave interaction, but without the off-site terms in the bath. This amounts to considering the Kondo interaction term $J\vec{S}_d\cdot\sum_{\sigma_1,\sigma_2}\frac{1}{2}\vec{\tau}_{\sigma_1,\sigma_2}\frac{1}{4}\sum_{i=L,R,U,D}c^\dagger_{i,\sigma_1}c_{i,\sigma_2}$. The momentum-dependent Kondo coupling in this case is

$$J_{\mathbf{k}_1,\mathbf{k}_2} \equiv \frac{1}{2}J\cos\left[a\left(k_1^x-k_2^x\right)+x\to y\right] \tag{4}$$

This does not vanish identically on the Fermi surface, and is therefore of potential interest to us.

We also consider correlation terms on the bath sites L,R,U and D. In momentum space, it is of the general form

$$-\frac{1}{2}\sum_{1,2,3,4}\sum_\sigma W_{1,2,3,4}\ c^\dagger_{\mathbf{k}_1,\sigma}c_{\mathbf{k}_2,\sigma}\left(c^\dagger_{\mathbf{k}_3,\sigma}c_{\mathbf{k}_4,\sigma}-c^\dagger_{\mathbf{k}_3,\bar{\sigma}}c_{\mathbf{k}_4,\bar{\sigma}}\right) \ , \tag{5}$$

where the form of $W_{\{\mathbf{k}_i\}}$ is again determined by the orbitals participating in the interaction. We ignore the p-wave interaction (for the same reason as above) and consider just the d-wave interaction and a p-wave interaction without off-site terms. These two cases lead to the following Hamiltonian structures: (i) a d-wave interaction, of the form $-\frac{W}{2}\left(f^\dagger_\uparrow f_\uparrow - f^\dagger_\downarrow f_\downarrow\right)^2$ in real space, leading to the momentum-dependence

$$W_{1,2,3,4} = W\prod_{i=1,2,3,4}\left[\cos\left(ak_i^x\right)-\cos\left(ak_i^y\right)\right] \ . \tag{6}$$

(ii) a p-wave interaction without off-site terms, $-\frac{W}{2}\sum_{i=L,R,U,D}\left(c^{\dagger}_{i,\uparrow}c_{i,\uparrow}-c^{\dagger}_{i,\downarrow}c_{i,\downarrow}\right)^2$, leading to the following form in momentum space:

$$W_{\{\mathbf{k}_i\}}=\frac{1}{4}W\left[\cos\left(a\left(k_1^x-k_2^x+k_3^x-k_4^x\right)\right)+x\to y\right] \ . \tag{7}$$

To summarise, the total Hamiltonian we consider is of the form

$$\begin{aligned}H=-2t\sum_{kx,ky}\left[\cos(ak_x)+\cos(ak_y)\right]c^{\dagger}_{\mathbf{k},\sigma}c_{\mathbf{k},\sigma}+\sum_{\mathbf{k}_1,\mathbf{k}_2,\sigma_1,\sigma_2}J_{\mathbf{k}_1,\mathbf{k}_2}\vec{S}_d\cdot\frac{1}{2}\vec{\tau}_{\sigma_1,\sigma_2}c^{\dagger}_{\mathbf{k}_1,\sigma_1}c_{\mathbf{k}_2,\sigma_2}\\-\frac{1}{2}\sum_{1,2,3,4}\sum_{\sigma}W_{1,2,3,4}\ c^{\dagger}_{\mathbf{k}_1,\sigma}c_{\mathbf{k}_2,\sigma}\left(c^{\dagger}_{\mathbf{k}_3,\sigma}c_{\mathbf{k}_4,\sigma}-c^{\dagger}_{\mathbf{k}_3,\bar{\sigma}}c_{\mathbf{k}_4,\bar{\sigma}}\right) \ ,\end{aligned} \tag{8}$$

where $J_{\{\mathbf{k}_i\}}$ takes one of the two forms in eqs. 2 and 4, and $W_{\{\mathbf{k}_i\}}$ can similarly take either of the two forms in eqs. 6 and 7.

II. RG SCHEME

At any given step j of the RG procedure, we decouple the states $\{\mathbf{q}\}$ on the isoenergetic surface of energy ε_j. The diagonal Hamiltonian H_D for this step consists of all terms that do not change the occupancy of the states $\{\mathbf{q}\}$:

$$H_D^{(j)}=\varepsilon_j\sum_{q,\sigma}\tau_{q,\sigma}+\frac{1}{2}\sum_{\mathbf{q}}J_{\mathbf{q},\mathbf{q}}S_d^z\left(\hat{n}_{\mathbf{q},\uparrow}-\hat{n}_{\mathbf{q},\downarrow}\right)-\frac{1}{2}\sum_{\mathbf{q}}W_{\mathbf{q}}\left(\hat{n}_{\mathbf{q},\uparrow}-\hat{n}_{\mathbf{q},\downarrow}\right)^2 \ , \tag{9}$$

where $\tau=\hat{n}-1/2$ and $W_{\mathbf{q}}$ is a shorthand for $W_{\mathbf{q},\mathbf{q},\mathbf{q},\mathbf{q}}$. The three terms, respectively, are the kinetic energy of the momentum states on the isoenergetic shell that we are decoupling, the spin-correlation energy between the impurity spin and the spins formed by these momentum states and, finally, the local correlation energy associated with these states arising from the W term. The off-diagonal part of the Hamiltonian on the other hand leads to scattering in the states $\{\mathbf{q}\}$. We now list these terms, classified by the coupling they originate from.

a. Arising from the Kondo spin-exchange term

$$\begin{aligned}T^{\dagger}_{KZ1}+T_{KZ1}&=\frac{1}{2}\sum_{\mathbf{k},\mathbf{q},\sigma}\sigma J_{\mathbf{k},\mathbf{q}}S_d^z\left[c^{\dagger}_{\mathbf{q}\sigma}c_{\mathbf{k},\sigma}+\text{h.c.}\right],\\T^{\dagger}_{KZ2}+T_{KZ2}&=\frac{1}{2}\sum_{\mathbf{q},\sigma}\sigma J_{\mathbf{q},\bar{\mathbf{q}}}S_d^z\left[c^{\dagger}_{\mathbf{q}\sigma}c_{\bar{\mathbf{q}},\sigma}+\text{h.c.}\right],\\T^{\dagger}_{KT1}+T_{KT1}&=\frac{1}{2}\sum_{\mathbf{k},\mathbf{q}}J_{\mathbf{k},\mathbf{q}}\left[S_d^+\left(c^{\dagger}_{\mathbf{q}\downarrow}c_{\mathbf{k}\uparrow}+c^{\dagger}_{\mathbf{k}\downarrow}c_{\mathbf{q}\uparrow}\right)+\text{h.c.}\right],\\T^{\dagger}_{KT2}+T_{KT2}&=\frac{1}{2}\sum_{\mathbf{q}}J_{\mathbf{q},\bar{\mathbf{q}}}\left[S_d^+\left(c^{\dagger}_{\mathbf{q}\downarrow}c_{\bar{\mathbf{q}}\uparrow}+c^{\dagger}_{\bar{\mathbf{q}}\downarrow}c_{\mathbf{q}\uparrow}\right)+\text{h.c.}\right],\end{aligned} \tag{10}$$

b. Arising from spin-preserving scattering within conduction bath

$$\begin{aligned}T^{\dagger}_{P1}+T_{P1}&=-\sum_{\mathbf{q}\in\varepsilon_j}\sum_{\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4<\varepsilon_j}\sum_{\sigma}\left[W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\sigma}c_{\mathbf{k}_4,\sigma}+\text{h.c.}\right]\\T^{\dagger}_{P2}+T_{P3}&=-\sum_{\mathbf{q}\in\varepsilon_j}\sum_{\mathbf{k}_2<\varepsilon_j}\sum_{\sigma}W_{\mathbf{q},\mathbf{k}_2,\bar{\mathbf{q}},\bar{\mathbf{q}}}c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{k}_2,\sigma}n_{\bar{\mathbf{q}},\sigma}-\sum_{\mathbf{q}\in\varepsilon_j}\sum_{\mathbf{k}_1<\varepsilon_j}\sum_{\sigma}W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}}c^{\dagger}_{\mathbf{k}_1,\sigma}c_{\mathbf{q},\sigma}n_{\mathbf{q},\sigma}\end{aligned} \tag{11}$$

c. Arising from spin-flip scattering within conduction bath

$$
\begin{aligned}
T^{\dagger}_{F1}+T_{F1} &= \sum_{\mathbf{q}\in\varepsilon_j}\sum_{\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4<\varepsilon_j}\sum_{\sigma}\left[W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\bar{\sigma}}c_{\mathbf{k}_4,\bar{\sigma}}+\text{h.c.}\right]\\
T_{F2} &= \sum_{\mathbf{q},\mathbf{q}'\in\varepsilon_j}\sum_{\mathbf{k}_2,\mathbf{k}_3<\varepsilon_j}\sum_{\sigma}W_{\mathbf{q},\mathbf{q}',\mathbf{k}_2,\mathbf{k}_3}c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{q}',\sigma}c^{\dagger}_{\mathbf{k}_2,\bar{\sigma}}c_{\mathbf{k}_3,\bar{\sigma}}\\
T_{F3} &= \sum_{\mathbf{q},\mathbf{q}'\in\varepsilon_j}\sum_{\mathbf{k}_2,\mathbf{k}_3<\varepsilon_j}\sum_{\sigma}W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{q}'}c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\bar{\sigma}}c_{\mathbf{q}',\bar{\sigma}}\\
T^{\dagger}_{F4}+T_{F4} &= \sum_{\mathbf{q},\mathbf{q}',\in\varepsilon_j}\sum_{\mathbf{k}_1<\varepsilon_j}\sum_{\sigma}\left[W_{\mathbf{q},\mathbf{q},\mathbf{q}',\mathbf{k}_1}n_{\mathbf{q},\sigma}c^{\dagger}_{\mathbf{q}',\bar{\sigma}}c_{\mathbf{k}_1,\bar{\sigma}}+\text{h.c.}\right]
\end{aligned}
\tag{12}
$$

In all of the terms $T_{P[i]}$ and $T_{F[i]}$, the factor of $1/2$ in front has been cancelled out by a factor of 2 coming from the multiple possibilities of arranging the momentum labels.

The renormalisation of the Hamiltonian is constructed from the general expression

$$
\Delta H^{(j)} = H_X\frac{1}{\omega-H_D}H_X\ . \tag{13}
$$

The states on the isoenergetic shell $\pm|\varepsilon_j|$ come in particle-hole pairs $(\mathbf{q},\bar{\mathbf{q}})$ with energies of opposite signs (relative to the Fermi energy). If $\mathbf{q}$ is defined as the hole state (unoccupied in the absence of quantum fluctuations), it will have positive energy, while the particle state $\bar{\mathbf{q}}$ will be of negative energy and hence below the Fermi surface. To be more specific, given a state $\mathbf{q}$ with energy $\pm|\varepsilon_j|$, we define its particle-hole transformed counterpart as the state $\bar{\mathbf{q}}=\boldsymbol{\pi}+\mathbf{q}$, having energy $\mp|\varepsilon_j|$ and residing in the opposite quadrant of the Brillouin zone. Given this definition, we have the important property that

$$
\begin{aligned}
J_{\mathbf{k},\bar{\mathbf{q}}} &= -J_{\mathbf{k},\mathbf{q}},\\
W_{\{\mathbf{k}\},\bar{\mathbf{q}}} &= -W_{\{\mathbf{k}\},\mathbf{q}}\ .
\end{aligned}
\tag{14}
$$

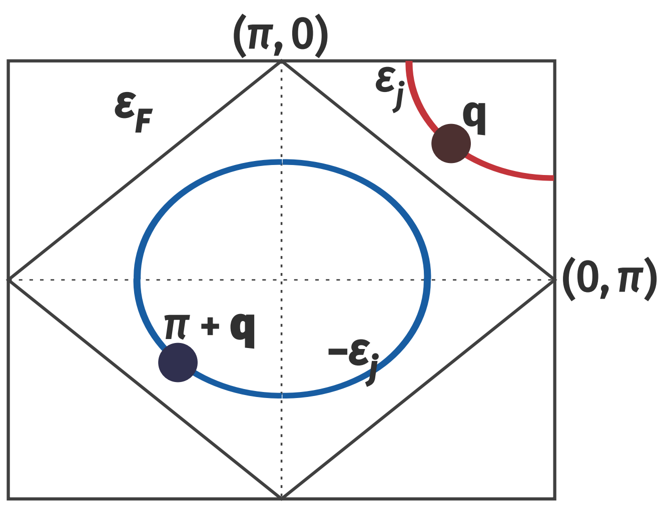

FIG. 1. Particle and hole states.

III. RENORMALISATION OF THE BATH CORRELATION TERM W

The bath correlation term W can undergo renormalisation only via scattering processes arising from itself. Irrespective of whether the state $\mathbf{q}$ being decoupled is in a particle or hole configuration in the initial many-body state, the propagator $G=1/(\omega-H_D)$ of the intermediate excited state is uniform, and equal to

$$
G_W = 1/\left(\omega-|\varepsilon_j|/2+W_{\mathbf{q}}/2)\right)\ , \tag{15}
$$

where $W_{\mathbf{q}}$ is the same whether $\mathbf{q}$ is above or below the Fermi surface. The $|\varepsilon_j|/2$ in H_D arises from the excited nature of the state after the initial scattering process.

A. Scattering arising purely from spin-preserving processes

In this subsection, we calculate the renormalisation to W arising from the terms T_{P1}, T_{P2} and T_{P3}. The first term is

$$\begin{aligned} T_{P1}^{\dagger}G_W T_{P3} &= \sum_{\sigma}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}\sum_{\mathbf{q}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\sigma}c_{\mathbf{k}_4,\sigma} G_W W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}} c^{\dagger}_{\mathbf{k}_1,\sigma}c_{\mathbf{q},\sigma}n_{\mathbf{q},\sigma} \\ &= -\sum_{\sigma}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} c^{\dagger}_{\mathbf{k}_1,\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\sigma}c_{\mathbf{k}_4,\sigma}\sum_{\mathbf{q}\in \mathrm{PS}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}G_W W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}}\ . \end{aligned} \tag{16}$$

The operators acting on the states being decoupled contract to form a number operator $n_{\mathbf{q},\sigma}$ which projects the sum over $\mathbf{q}$ into the states that are initial occupied (particle sector, PS).

The second such contribution is obtained by flipping the sequence of scattering processes:

$$\begin{aligned} T_{P3}G_W T_{P1}^{\dagger} &= \sum_{\sigma}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}\sum_{\mathbf{q}} W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}} c^{\dagger}_{\mathbf{k}_1,\sigma}c_{\mathbf{q},\sigma}n_{\mathbf{q},\sigma} G_W W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\sigma}c_{\mathbf{k}_4,\sigma} \\ &= \sum_{\sigma}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} c^{\dagger}_{\mathbf{k}_1,\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\sigma}c_{\mathbf{k}_4,\sigma}\sum_{\mathbf{q}\in \mathrm{HS}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}G_W W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}}\ . \end{aligned} \tag{17}$$

By virtue of eq. 14, the product of couplings $W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}G_W W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}}$ is the same irrespective of whether $\mathbf{q}$ belongs to the particle or hole sector. The two contributions therefore cancel each other. Moreover, the remaining contributions $T_{P3}^{\dagger}G_W T_{P1}$ and $T_{P1}G_W T_{P2}^{\dagger}$ are effectively hermitian conjugates of the two contributions considered above, and therefore also cancel each other.

B. Scattering arising from spin-flip processes

We now come to the processes that involve spin-flips. Considering T_{F1} and T_{F4} first, we get

$$\begin{aligned} T_{F1}^{\dagger}G_W T_{F4} &= \sum_{\sigma}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}\sum_{\mathbf{q}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\bar{\sigma}}c_{\mathbf{k}_4,\bar{\sigma}} G_W W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}} c^{\dagger}_{\mathbf{k}_1\sigma}c_{\mathbf{q}\sigma}n_{\mathbf{q}\bar{\sigma}} \\ &= -\sum_{1,2,3,4}\sum_{\sigma} c^{\dagger}_{\mathbf{k}_1\sigma}c_{\mathbf{k}_2\sigma}c^{\dagger}_{\mathbf{k}_3\bar{\sigma}}c_{\mathbf{k}_4\bar{\sigma}}\sum_{\mathbf{q}\in \mathrm{PS}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_4,\mathbf{k}_4}G_W W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}}\ , \\ T_{F4}G_W T_{F1}^{\dagger} &= \sum_{\sigma}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}\sum_{\mathbf{q}} W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}} c^{\dagger}_{\mathbf{k}_1\sigma}c_{\mathbf{q}\sigma}n_{\mathbf{q}\bar{\sigma}} G_W W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\bar{\sigma}}c_{\mathbf{k}_4,\bar{\sigma}} \\ &= \sum_{1,2,3,4}\sum_{\sigma} c^{\dagger}_{\mathbf{k}_1\sigma}c_{\mathbf{k}_2\sigma}c^{\dagger}_{\mathbf{k}_3\bar{\sigma}}c_{\mathbf{k}_4\bar{\sigma}}\sum_{\mathbf{q}\in \mathrm{HS}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_4,\mathbf{k}_4}G_W W_{\mathbf{k}_1,\mathbf{q},\mathbf{q},\mathbf{q}}\ . \end{aligned} \tag{18}$$

By the same arguments as in the previous subsection, these terms cancel each other out. Their hermitian conjugate contributions $T_{F1}G_W T_{F4}^{\dagger}$ and $T_{F4}^{\dagger}G_W T_{F1}$ also cancel out. The other two terms are T_{F2} and T_{F3}, and their contributions also cancel out for the same reason:

$$\begin{aligned} T_{F2}G_W T_{F2} &= \sum_{\sigma}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}\sum_{\mathbf{q}} W_{\mathbf{q},\bar{\mathbf{q}},\mathbf{k}_3,\mathbf{k}_4} c^{\dagger}_{\mathbf{q},\sigma}c_{\bar{\mathbf{q}},\sigma}c^{\dagger}_{\mathbf{k}_3,\bar{\sigma}}c_{\mathbf{k}_4,\bar{\sigma}} G_W W_{\bar{\mathbf{q}},\mathbf{q},\mathbf{k}_1,\mathbf{k}_2} c^{\dagger}_{\bar{\mathbf{q}},\sigma}c_{\mathbf{q},\sigma}c^{\dagger}_{\mathbf{k}_1,\bar{\sigma}}c_{\mathbf{k}_2,\bar{\sigma}} \\ &= \sum_{1,2,3,4}\sum_{\sigma} c^{\dagger}_{\mathbf{k}_1\sigma}c_{\mathbf{k}_2\sigma}c^{\dagger}_{\mathbf{k}_3\bar{\sigma}}c_{\mathbf{k}_4\bar{\sigma}}\sum_{\mathbf{q}\in \mathrm{PS}} W_{\mathbf{q},\bar{\mathbf{q}},\mathbf{k}_3,\mathbf{k}_4}G_W W_{\bar{\mathbf{q}},\mathbf{q},\mathbf{k}_1,\mathbf{k}_2}\ , \\ T_{F3}G_W T_{F3} &= \sum_{\sigma}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4}\sum_{\mathbf{q}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\bar{\mathbf{q}}} c^{\dagger}_{\mathbf{q},\sigma}c_{\mathbf{k}_2,\sigma}c^{\dagger}_{\mathbf{k}_3,\bar{\sigma}}c_{\bar{\mathbf{q}},\bar{\sigma}} G_W W_{\bar{\mathbf{q}},\mathbf{k}_4,\mathbf{k}_1,\mathbf{q}} c^{\dagger}_{\bar{\mathbf{q}},\bar{\sigma}}c_{\mathbf{k}_4,\bar{\sigma}}c^{\dagger}_{\mathbf{k}_1,\sigma}c_{\mathbf{q},\sigma} \\ &= -\sum_{1,2,3,4}\sum_{\sigma} c^{\dagger}_{\mathbf{k}_1\sigma}c_{\mathbf{k}_2\sigma}c^{\dagger}_{\mathbf{k}_3\bar{\sigma}}c_{\mathbf{k}_4\bar{\sigma}}\sum_{\mathbf{q}\in \mathrm{PS}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_3,\bar{\mathbf{q}}}G_W W_{\bar{\mathbf{q}},\mathbf{k}_4,\mathbf{k}_1,\mathbf{q}}\ , \end{aligned} \tag{19}$$

C. Scattering involving both spin-flip and spin-preserving processes

These processes involve the combination of terms like T_{P1} with T_{F4}, and T_{P2} with T_{F1}. These again cancel each other out for the same reasons as outline above.

D. Net renormalisation for the bath correlation term

Since all the contributions cancel out in pairs, the bath correlation term W is *marginal.*

IV. RENORMALISATION OF THE KONDO SCATTERING TERM J

We focus on the renormalisation of the spin-flip part of the Kondo interaction. For these processes, the intermediate many-body state always involves the impurity spin being anti-correlated with the conduction electron spin, such that the propagator for that state is $G_J = 1/\left(\omega - |\varepsilon_j|/2 + J_{\mathbf{q}}/4 + W_{\mathbf{q}}/2\right)$).

A. Impurity-mediated spin-flip scattering purely through Kondo-like processes

The following processes arising from the Kondo term renormalise the spin-flip interaction:

$$\begin{aligned}
T^\dagger_{KT1} G_J \left(T_{KZ1} + T^\dagger_{KZ1}\right) &= \frac{1}{4}\sum_{\mathbf{k}_1,\mathbf{k}_1,\mathbf{q}} J_{\mathbf{q},\mathbf{k}_2} S_d^+ \left[-c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{k}_2\uparrow} G_J c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{q}\downarrow} + c^\dagger_{\mathbf{k}_2\downarrow} c_{\mathbf{q}\uparrow} G_J c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}_1\uparrow}\right] J_{\mathbf{k}_1,\mathbf{q}} S_d^z \\
&= -\frac{1}{8}\sum_{\mathbf{k}_1,\mathbf{k}_1,\mathbf{q}} J_{\mathbf{q},\mathbf{k}_2} S_d^+ \left[c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{k}_2\uparrow} G_J n_{\mathbf{q}\downarrow} + c^\dagger_{\mathbf{k}_2\downarrow} c_{\mathbf{k}_1\uparrow} (1 - n_{\mathbf{q}\uparrow}) G_J\right] J_{\mathbf{k}_1,\mathbf{q}} \\
&= -\frac{1}{8}\sum_{\mathbf{k}_1,\mathbf{k}_1} S_d^+ c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{k}_2\uparrow} \sum_{\mathbf{q}\in\mathrm{PS}} \left[J_{\mathbf{q},\mathbf{k}_2} J_{\mathbf{k}_1,\mathbf{q}} + J_{\bar{\mathbf{q}},\mathbf{k}_1} J_{\mathbf{k}_2,\bar{\mathbf{q}}}\right] G_J \ .
\end{aligned} \tag{20}$$

In getting the final expression, we used the sigma matrix relation $S_d^+ S_d^z = -\frac{1}{2} S_d^+$, and absorbed the projector $1 - n_{\mathbf{q}\uparrow}$ into the sum over the particle sector by replacing q with its particle-hole transformed counterpart $\bar{q}$. An identical contribution is obtained by switching the sequence of processes:

$$\begin{aligned}
\left(T_{KZ1} + T^\dagger_{KZ1}\right) G_J T^\dagger_{KT1} &= \frac{1}{4}\sum_{\mathbf{k}_1,\mathbf{k}_1,\mathbf{q}} J_{\mathbf{k}_1,\mathbf{q}} S_d^z \left[-c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{q}\downarrow} G_J c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{k}_2\uparrow} + c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}_1\uparrow} G_J c^\dagger_{\mathbf{k}_2\downarrow} c_{\mathbf{q}\uparrow}\right] J_{\mathbf{q},\mathbf{k}_2} S_d^+ \\
&= -\frac{1}{8}\sum_{\mathbf{k}_1,\mathbf{k}_1} S_d^+ c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{k}_2\uparrow} \sum_{\mathbf{q}\in\mathrm{PS}} \left[J_{\bar{\mathbf{q}},\mathbf{k}_2} J_{\mathbf{k}_1,\bar{\mathbf{q}}} + J_{\mathbf{q},\mathbf{k}_1} J_{\mathbf{k}_2,\mathbf{q}}\right] G_J \ .
\end{aligned} \tag{21}$$

B. Scattering processes involving interplay between the Kondo interaction and conduction bath interaction

Looking at $T^\dagger_{KT1}$ first, we have

$$T^\dagger_{KT1} G_J \left(T_{F4} + T^\dagger_{F4}\right) = \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} J_{\mathbf{k}_2,\mathbf{q}} S_d^+ \left(c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{k}_2\uparrow} G_J W_{\mathbf{q},\mathbf{q},\mathbf{k}_1,\mathbf{q}} n_{q\uparrow} c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{q}\downarrow} + c^\dagger_{\mathbf{k}_2\downarrow} c_{\mathbf{q}\uparrow} G_J W_{\bar{\mathbf{q}},\bar{\mathbf{q}},\mathbf{q},\mathbf{k}_1} n_{\bar{q}\downarrow} c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}_1\uparrow}\right) \ . \tag{22}$$

For either of the two choices of the functional form of W, it is easy to show that $W_{\mathbf{q},\mathbf{q},\mathbf{k}_1,\mathbf{q}} = W_{\bar{\mathbf{q}},\bar{\mathbf{q}},\mathbf{q},\mathbf{k}_1}$.

$$T^\dagger_{KT1} G_J \left(T_{F4} + T^\dagger_{F4}\right) = \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} J_{\mathbf{k}_2,\mathbf{q}} W_{\mathbf{q},\mathbf{q},\mathbf{k}_1,\mathbf{q}} G_J S_d^+ \left[-c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{k}_2\uparrow} n_{\mathbf{q}\downarrow} n_{q\uparrow} + c^\dagger_{\mathbf{k}_2\downarrow} c_{\mathbf{k}_1\uparrow} (1 - n_{\mathbf{q}\uparrow}) n_{\bar{q}\downarrow}\right] \ . \tag{23}$$

Another contribution is obtained by switching the sequence of the scattering processes:

$$\begin{aligned}
\left(T_{F4} + T^\dagger_{F4}\right) G_J T^\dagger_{KT1} &= \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} \left(W_{\mathbf{q},\mathbf{q},\mathbf{k}_1,\mathbf{q}} n_{\bar{q}\uparrow} c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{q}\downarrow} G_J c^\dagger_{\mathbf{q}\downarrow} c_{\mathbf{k}_2\uparrow} + W_{\bar{\mathbf{q}},\bar{\mathbf{q}},\mathbf{q},\mathbf{k}_1} n_{q\downarrow} c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}_1\uparrow} G_J c^\dagger_{\mathbf{k}_2\downarrow} c_{\mathbf{q}\uparrow}\right) J_{\mathbf{k}_2,\mathbf{q}} S_d^+ \\
&= \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} \left(c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{k}_2\uparrow} n_{\bar{q}\uparrow} (1 - n_{\mathbf{q}\downarrow}) - c^\dagger_{\mathbf{k}_2\downarrow} c_{\mathbf{k}_1\uparrow} n_{q\downarrow} n_{\mathbf{q}\uparrow}\right) W_{\mathbf{q},\mathbf{q},\mathbf{k}_1,\mathbf{q}} G_J J_{\mathbf{k}_2,\mathbf{q}} S_d^+
\end{aligned} \tag{24}$$

The two contributions (eqs. 23 and 24) arising from T_{KT1} cancel each other.

We now consider the other spin-exchange process $T^\dagger_{KT2}$. One such contribution is

$$
\begin{aligned}
T^\dagger_{KT2} G_J T_{F3} &= \frac{1}{2} \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} J_{\mathbf{q},\bar{\mathbf{q}}} S_d^+ \left(c^\dagger_{\mathbf{q}\downarrow} c_{\bar{\mathbf{q}}\uparrow} G_J c^\dagger_{\bar{\mathbf{q}}\uparrow} c_{\mathbf{k}_2\uparrow} c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{q}\downarrow} + c^\dagger_{\bar{\mathbf{q}}\downarrow} c_{\mathbf{q}\uparrow} G_J c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}_2\uparrow} c^\dagger_{\mathbf{k}_1\downarrow} c_{\bar{\mathbf{q}}\downarrow} \right) W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}} \\
&= -\frac{1}{2} \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} S_d^+ c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{k}_2\uparrow} \left[n_{\mathbf{q}\downarrow}(1-n_{\bar{\mathbf{q}}\uparrow}) + n_{\bar{\mathbf{q}}\downarrow}(1-n_{\mathbf{q}\uparrow}) \right] J_{\mathbf{q},\bar{\mathbf{q}}} G_J W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}} \\
&= -\frac{1}{2} \sum_{\mathbf{k}_1,\mathbf{k}_2} S_d^+ c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{k}_2\uparrow} \sum_{\mathbf{q}\in \mathrm{PS}} \left(J_{\mathbf{q},\bar{\mathbf{q}}} W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}} + J_{\bar{\mathbf{q}},\mathbf{q}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_1,\bar{\mathbf{q}}} \right) G_J \ .
\end{aligned} \tag{25}
$$

An identical contribution is obtained from the reversed term:

$$
\begin{aligned}
T_{F3} G_J T^\dagger_{KT2} &= \frac{1}{2} \sum_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}} W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}} \left(c^\dagger_{\bar{\mathbf{q}}\uparrow} c_{\mathbf{k}_2\uparrow} c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{q}\downarrow} G_J c^\dagger_{\mathbf{q}\downarrow} c_{\bar{\mathbf{q}}\uparrow} + c^\dagger_{\mathbf{q}\uparrow} c_{\mathbf{k}_2\uparrow} c^\dagger_{\mathbf{k}_1\downarrow} c_{\bar{\mathbf{q}}\downarrow} G_J c^\dagger_{\bar{\mathbf{q}}\downarrow} c_{\mathbf{q}\uparrow} \right) J_{\mathbf{q},\bar{\mathbf{q}}} S_d^+ \\
&= -\frac{1}{2} \sum_{\mathbf{k}_1,\mathbf{k}_2} S_d^+ c^\dagger_{\mathbf{k}_1\downarrow} c_{\mathbf{k}_2\uparrow} \sum_{\mathbf{q}\in \mathrm{PS}} \left(J_{\mathbf{q},\bar{\mathbf{q}}} W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}} + J_{\bar{\mathbf{q}},\mathbf{q}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_1,\bar{\mathbf{q}}} \right) G_J \ .
\end{aligned} \tag{26}
$$

C. Net renormalisation to the Kondo interaction

Combining the results from eqs. 20, 21, 25 and 26, as well as using the properties $J_{\bar{\mathbf{q}},\mathbf{k}_1} J_{\mathbf{k}_2,\bar{\mathbf{q}}} = J_{\mathbf{q},\mathbf{k}_2} J_{\mathbf{k}_1,\mathbf{q}}$ and $J_{\mathbf{q},\bar{\mathbf{q}}} W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}} = J_{\bar{\mathbf{q}},\mathbf{q}} W_{\mathbf{q},\mathbf{k}_2,\mathbf{k}_1,\bar{\mathbf{q}}}$, the total renormalisation in the momentum-resolved Kondo coupling $J^{(j)}$ at the j^{th} step amounts to

$$
\Delta J^{(j)}_{\mathbf{k}_1,\mathbf{k}_2} = -\sum_{\mathbf{q}\in \mathrm{PS}} \frac{\frac{1}{2}\left(J^{(j)}_{\mathbf{k}_2,\mathbf{q}} J^{(j)}_{\mathbf{q},\mathbf{k}_1} + J^{(j)}_{\mathbf{k}_1,\mathbf{q}} J^{(j)}_{\mathbf{q},\mathbf{k}_2} \right) + 4 J^{(j)}_{\mathbf{q},\bar{\mathbf{q}}} W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}}}{\omega - \frac{1}{2}|\varepsilon_j| + J^{(j)}_{\mathbf{q}}/4 + W_{\mathbf{q}}/2} \tag{27}
$$