

URG analysis of the extended Kondo model with d-wave interaction

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I. HAMILTONIAN

We consider an impurity spin \vec{S}_d interacting with a two-dimensional tight-binding conduction bath through the d-wave channel :

$$J\vec{S}_d \cdot \vec{S}_f, \quad \vec{S}_f = \frac{1}{2} \sum_{\sigma, \sigma'} \vec{\tau}_{\sigma, \sigma'} f_{\sigma}^{\dagger} f_{\sigma'}. \quad (1)$$

where $\vec{\tau}$ is the vector of sigma matrices. The spin \vec{S}_f is constructed in terms of the d-wave electron $f_{\sigma} = \frac{1}{2} (c_{L, \sigma}^{\dagger} + c_{R, \sigma}^{\dagger} - c_{U, \sigma}^{\dagger} - c_{D, \sigma}^{\dagger})$, where L, R, U and D indicate electrons at the positions $(x, y) = (-a, 0), (a, 0), (0, a)$ and $(0, -a)$ respectively, a being the lattice spacing of the conduction bath lattice. We also consider local bath correlations in the d -wave channel:

$$-\frac{W}{2} (f_{\uparrow}^{\dagger} f_{\uparrow} - f_{\downarrow}^{\dagger} f_{\downarrow})^2. \quad (2)$$

In order to facilitate a momentum-space decoupling RG scheme, we Fourier transform the d -wave operator to momentum space:

$$f_{\sigma} = \frac{1}{2} (c_{L, \sigma}^{\dagger} + c_{R, \sigma}^{\dagger} - c_{U, \sigma}^{\dagger} - c_{D, \sigma}^{\dagger}) = \frac{1}{2} \sum_{\vec{k}} c_{\vec{k}, \sigma}^{\dagger} [e^{-ik^x a} + e^{ik^x a} - e^{-ik^y a} - e^{ik^y a}] = \sum_{\vec{k}} [\cos(ak^x) - \cos(ak^y)] c_{\vec{k}, \sigma}^{\dagger}. \quad (3)$$

The Kondo interaction term takes the following form in momentum space:

$$J\vec{S}_d \cdot \vec{S}_f = \frac{1}{2} J \sum_{\vec{k}_1, \vec{k}_2, \sigma, \sigma'} \vec{S}_d \cdot \vec{\tau}_{\sigma, \sigma'} c_{\vec{k}_1, \sigma}^{\dagger} c_{\vec{k}_2, \sigma'} \prod_{i=1,2} [\cos(ak_i^x) - \cos(ak_i^y)]. \quad (4)$$

The local correlation term can be similarly written in momentum space:

$$-\frac{W}{2} (f_{\uparrow}^{\dagger} f_{\uparrow} - f_{\downarrow}^{\dagger} f_{\downarrow})^2 = -\frac{W}{2} \sum_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4, \sigma} (c_{\vec{k}_1, \sigma}^{\dagger} c_{\vec{k}_2, \sigma} c_{\vec{k}_3, \sigma}^{\dagger} c_{\vec{k}_4, \sigma} - c_{\vec{k}_1, \sigma}^{\dagger} c_{\vec{k}_2, \sigma} c_{\vec{k}_3, \bar{\sigma}}^{\dagger} c_{\vec{k}_4, \bar{\sigma}}) \prod_{i=1,2,3,4} [\cos(ak_i^x) - \cos(ak_i^y)]. \quad (5)$$

Combining all the terms, the Hamiltonian can be formally written as

$$H = \sum_{\vec{k}, \sigma} \varepsilon_{\vec{k}} c_{\vec{k}, \sigma}^{\dagger} c_{\vec{k}, \sigma} + \frac{1}{2} \sum_{\vec{k}_1, \vec{k}_2, \sigma, \sigma'} J_{\vec{k}_1, \vec{k}_2} \vec{S}_d \cdot \vec{\tau}_{\sigma, \sigma'} c_{\vec{k}_1, \sigma}^{\dagger} c_{\vec{k}_2, \sigma'} - \frac{1}{2} \sum_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4, \sigma, \sigma'} W_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4} \sigma \sigma' c_{\vec{k}_1, \sigma}^{\dagger} c_{\vec{k}_2, \sigma} c_{\vec{k}_3, \sigma'}^{\dagger} c_{\vec{k}_4, \sigma'}. \quad (6)$$

with

$$\begin{aligned} \varepsilon_{\vec{k}} &= -2t [\cos(ak^x) + \cos(ak^y)], \\ J_{\vec{k}_1, \vec{k}_2} &= J \prod_{i=1,2} [\cos(ak_i^x) - \cos(ak_i^y)], \\ W_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4} &= W \prod_{i=1,2,3,4} [\cos(ak_i^x) - \cos(ak_i^y)]. \end{aligned} \quad (7)$$

The coupling $W_{\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4}$ has certain symmetries. It is independent of the sequence of momentum indices. It remains invariant if any number of the momenta undergo rotation by an integer multiple of $\pi/2$. It also remains invariant if an even number of momenta are reflected about the nodal point in the corresponding quadrant.

II. RG SCHEME

At any given step j of the RG procedure, we decouple the states $\{\vec{q}\}$ on the isoenergetic surface of energy ε_j . The diagonal Hamiltonian H_D for this step consists of all terms that do not change the occupancy of the states $\{\vec{q}\}$:

$$H_D^{(j)} = \varepsilon_j \sum_{q,\sigma} \tau_{q,\sigma} + \sum_{\vec{q}} J_{\vec{q},\vec{q}} S_d^z (\hat{n}_{\vec{q},\uparrow} - \hat{n}_{\vec{q},\downarrow}) - \frac{1}{2} \sum_{\vec{q}_1} W_{\vec{q}_1,\vec{q}_1,\vec{q}_1,\vec{q}_1} (\hat{n}_{\vec{q}_1,\uparrow} - \hat{n}_{\vec{q}_1,\downarrow})^2 . \quad (8)$$

where $\tau = \hat{n} - 1/2$. The three terms, respectively, are the kinetic energy of the momentum states on the isoenergetic shell that we are decoupling, the Ising interaction energy between the impurity spin and the spins formed by these momentum states and, finally, the local correlation energy associated with these states arising from the W term.

The off-diagonal part of the Hamiltonian on the other hand leads to scattering in the states $\{\vec{q}\}$:

$$\begin{aligned} H_X^{(j)} = & \underbrace{\sum_{\vec{k},\vec{q},\sigma,\sigma'} J_{\vec{k},\vec{q}} \vec{S}_d \cdot \vec{\tau}_{\sigma,\sigma'} \left[c_{\vec{q}\sigma}^\dagger c_{\vec{k},\sigma} + \text{h.c.} \right]}_{T_1^\dagger + T_1} \\ & - \frac{1}{2} \underbrace{\sum_{\vec{q}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4,\sigma,\sigma'} W_{\vec{q}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4} \sigma \sigma' \left[c_{\vec{q}_1,\sigma}^\dagger c_{\vec{k}_2,\sigma} c_{\vec{k}_3,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} + \text{h.c.} \right]}_{T_2^\dagger + T_2} \\ & - \frac{1}{2} \underbrace{\sum_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{k}_4,\sigma,\sigma'} W_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{k}_4} \sigma \sigma' \left[c_{\vec{q}_1,\sigma}^\dagger c_{\vec{k}_2,\sigma} c_{\vec{q}_2,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} + \text{h.c.} \right]}_{T_3^\dagger + T_3} \\ & - \frac{1}{2} \underbrace{\sum_{\vec{q}_1,\vec{k}_2,\vec{k}_3,\vec{q}_2,\sigma,\sigma'} W_{\vec{q}_1,\vec{k}_2,\vec{k}_3,\vec{q}_2} \sigma \sigma' c_{\vec{q}_1,\sigma}^\dagger c_{\vec{k}_2,\sigma} c_{\vec{k}_3,\sigma'}^\dagger c_{\vec{q}_2,\sigma'}}_{T_4} \\ & - \frac{1}{2} \underbrace{\sum_{\vec{q}_1,\vec{k}_2,\vec{k}_3,\vec{q}_2,\sigma,\sigma'} W_{\vec{q}_1,\vec{k}_2,\vec{k}_3,\vec{q}_2} \sigma \sigma' c_{\vec{k}_3,\sigma'}^\dagger c_{\vec{q}_2,\sigma'} c_{\vec{q}_1,\sigma}^\dagger c_{\vec{k}_2,\sigma}}_{T_5} \\ & - \frac{1}{2} \underbrace{\sum_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{q}_3,\sigma,\sigma'} W_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{q}_3} \sigma \sigma' \left[c_{\vec{q}_1,\sigma}^\dagger c_{\vec{k}_2,\sigma} c_{\vec{q}_2,\sigma'}^\dagger c_{\vec{q}_3,\sigma'} + \text{h.c.} \right]}_{T_6 + T_6^\dagger} . \end{aligned} \quad (9)$$

The first term T_1 is an impurity-mediated scattering between the states \vec{q} at energy ε_j and the states \vec{k} at energies below ε_j . Terms T_2 through T_6 involve two-particle scattering between these momentum states involving an increasing number of participating states from the isoenergetic shell ε_j , through the Hubbard-like local term W . The renormalisation of the Hamiltonian is constructed from the general expression

$$\Delta H^{(j)} = H_X \frac{1}{\omega - H_D} H_X . \quad (10)$$

III. RENORMALISATION OF THE BATH CORRELATION TERM W

In order to lead to a renormalisation of the W -term, there must be a total of four uncontracted momentum indices k_i and two contracted indices q_1, q_2 . The following combinations of scattering processes are compatible: (i) $T_2^\dagger GT_6 + T_6^\dagger GT_2$, (ii) $T_3^\dagger GT_3 + T_3 GT_3^\dagger$, (iii) $T_4 GT_4$, (iv) $T_5 GT_5$ and (v) $T_4 GT_5 + T_5 GT_4$.

The first term is of the form

$$T_2^\dagger GT_6 = \sum \sigma_1 \sigma_1' W_{\vec{q}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4} c_{\vec{q}_1,\sigma_1}^\dagger c_{\vec{k}_2,\sigma_1} c_{\vec{k}_3,\sigma_1'}^\dagger c_{\vec{k}_4,\sigma_1'} \frac{1}{\omega - H_D} \sigma_1 \sigma_2' W_{\vec{q}_1,\vec{q}_2,\vec{q}_2,\vec{k}_2'} \hat{n}_{\vec{q}_2,\sigma_2'} c_{\vec{k}_2',\sigma_1'}^\dagger c_{\vec{q}_1,\sigma_1} . \quad (11)$$

The change in occupancy of the state $\vec{q}_1\sigma_1$ from 1 to 0 leads to an excited state energy $H_D = \varepsilon(q_1)\tau_{q_1\sigma_1} - \frac{1}{2}W_{\vec{q}_1}(\hat{n}_{\vec{q}_1,\uparrow} - \hat{n}_{\vec{q}_1,\downarrow})^2 = -\frac{1}{2}\varepsilon(q_1) - \frac{1}{2}W_{\vec{q}_1}$, where $W_{\vec{q}_1}$ is a shorthand for $W_{\vec{q}_1,\vec{q}_1,\vec{q}_1,\vec{q}_1}$. The number operator $\hat{n}_{\vec{q}_2,\sigma'_2}$ projects the initial state to that in which \vec{q}_2 is occupied. The operator $c_{\vec{q}_1,\sigma_1}$ can be combined with its hermitian conjugate on the other side to give another number operator, leading to another projection. Both the number operators reduce to unity upon acting on the projected states:

$$T_2^\dagger GT_6 = \sum \sigma_1^2 \sigma'_1 W_{\vec{q}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4} W_{\vec{q}_1,\vec{q}_2,\vec{q}_2,\vec{k}'_2} c_{\vec{k}_2,\sigma_1} c_{\vec{k}_3,\sigma'_1}^\dagger c_{\vec{k}_4,\sigma'_1} c_{\vec{k}'_2,\sigma_1}^\dagger \frac{1}{\omega + \frac{1}{2}\varepsilon(q_1) + \frac{1}{2}W_{\vec{q}_1}} \sum_{\sigma'_2} \sigma'_2. \quad (12)$$

Since there is a sum over the spin factor $\sigma'_2 = \pm 1$, this term vanishes identically. The particle-hole transformed term $T_6^\dagger GT_2$ vanishes for the same reason.

We now consider the second term $T_3^\dagger GT_3$:

$$\begin{aligned} T_3^\dagger GT_3 &= \frac{1}{4} \sum W_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{k}_4} \sigma \sigma' c_{\vec{q}_1,\sigma}^\dagger c_{\vec{k}_2,\sigma} c_{\vec{q}_2,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} \frac{1}{\omega - H_D} W_{\vec{q}_1,\vec{k}'_2,\vec{q}_2,\vec{k}'_4} \sigma \sigma' c_{\vec{k}'_4,\sigma'}^\dagger c_{\vec{q}_2,\sigma'} c_{\vec{k}'_2,\sigma}^\dagger c_{\vec{q}_1,\sigma} \\ &= \frac{1}{4} \sum \hat{n}_{\vec{q}_1,\sigma} \hat{n}_{\vec{q}_2,\sigma'} c_{\vec{k}_2,\sigma} c_{\vec{k}_4,\sigma'}^\dagger c_{\vec{k}'_4,\sigma'}^\dagger c_{\vec{k}'_2,\sigma} \frac{W_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{k}_4} W_{\vec{q}_1,\vec{k}'_2,\vec{q}_2,\vec{k}'_4}}{\omega + \frac{1}{2}[\varepsilon(q_1) + \varepsilon(q_2)] + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})} \\ &= \frac{1}{4} \sum_{\substack{4',4,2',2, \\ \sigma,\sigma'}} c_{\vec{k}'_4,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} c_{\vec{k}'_2,\sigma}^\dagger c_{\vec{k}_2,\sigma} \sum_{\vec{q}_1,\vec{q}_2 \in \text{PS}} \frac{W_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{k}_4} W_{\vec{q}_1,\vec{k}'_2,\vec{q}_2,\vec{k}'_4}}{\omega + \frac{1}{2}[\varepsilon(q_1) + \varepsilon(q_2)] + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})}, \end{aligned} \quad (13)$$

where \vec{q}_1, \vec{q}_2 are summed over all momentum states in the isoenergetic shell and in the particle sector (PS) (states are occupied in the ground state).

The particle-hole transformed term is $T_3 GT_3^\dagger$:

$$\begin{aligned} T_3 GT_3^\dagger &= \frac{1}{4} \sum W_{\vec{q}_1,\vec{k}'_2,\vec{q}_2,\vec{k}'_4} \sigma \sigma' c_{\vec{k}'_4,\sigma'}^\dagger c_{\vec{q}_2,\sigma'} c_{\vec{k}'_2,\sigma}^\dagger c_{\vec{q}_1,\sigma} \frac{1}{\omega - H_D} W_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{k}_4} \sigma \sigma' c_{\vec{q}_1,\sigma}^\dagger c_{\vec{k}_2,\sigma} c_{\vec{q}_2,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} \\ &= \frac{1}{4} \sum (1 - \hat{n}_{\vec{q}_1,\sigma}) (1 - \hat{n}_{\vec{q}_2,\sigma'}) c_{\vec{k}'_4,\sigma'}^\dagger c_{\vec{k}'_2,\sigma}^\dagger c_{\vec{k}_2,\sigma} c_{\vec{k}_4,\sigma'} \frac{W_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{k}_4} W_{\vec{q}_1,\vec{k}'_2,\vec{q}_2,\vec{k}'_4}}{\omega - \frac{1}{2}[\varepsilon(q_1) + \varepsilon(q_2)] + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})} \\ &= \frac{1}{4} \sum_{\substack{4',4,2',2, \\ \sigma,\sigma'}} c_{\vec{k}'_4,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} c_{\vec{k}'_2,\sigma}^\dagger c_{\vec{k}_2,\sigma} \sum_{\vec{q}_1,\vec{q}_2 \in \text{HS}} \frac{W_{\vec{q}_1,\vec{k}_2,\vec{q}_2,\vec{k}_4} W_{\vec{q}_1,\vec{k}'_2,\vec{q}_2,\vec{k}'_4}}{\omega - \frac{1}{2}[\varepsilon(q_1) + \varepsilon(q_2)] + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})}, \end{aligned} \quad (14)$$

where the hole projectors $(1 - \hat{n}_{\vec{q}_1,\sigma}) (1 - \hat{n}_{\vec{q}_2,\sigma'})$ force the momenta \vec{q}_1, \vec{q}_2 to extend over all states in the hole sector (states that are unoccupied in the ground state). The change in the sign of $[\varepsilon(q_1) + \varepsilon(q_2)]$ in the denominator compared to the denominator in $T_3^\dagger GT_3$ is for the same reason.

The remaining sets of terms that we need to consider are:

$$\begin{aligned} T_4 GT_4 &= \frac{1}{4} \sum W_{\vec{q}_1,\vec{k}_1,\vec{q}_2,\vec{k}_2} \sigma \sigma' c_{\vec{q}_1,\sigma}^\dagger c_{\vec{k}_2,\sigma} c_{\vec{k}_1,\sigma'}^\dagger c_{\vec{q}_2,\sigma'} \frac{1}{\omega - H_D} W_{\vec{q}_1,\vec{k}_3,\vec{q}_2,\vec{k}_4} \sigma \sigma' c_{\vec{q}_2,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} c_{\vec{k}_3,\sigma}^\dagger c_{\vec{q}_1,\sigma} \\ &= \frac{1}{4} \sum \hat{n}_{\vec{q}_1,\sigma} (1 - \hat{n}_{\vec{q}_2,\sigma'}) c_{\vec{k}_2,\sigma} c_{\vec{k}_1,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} c_{\vec{k}_3,\sigma}^\dagger \frac{W_{\vec{q}_1,\vec{k}_1,\vec{q}_2,\vec{k}_2} W_{\vec{q}_1,\vec{k}_3,\vec{q}_2,\vec{k}_4}}{\omega + \frac{1}{2}\varepsilon(q_1) - \frac{1}{2}\varepsilon(q_2) + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})} \\ &= -\frac{1}{4} \sum_{\substack{1,2,3,4, \\ \sigma,\sigma'}} c_{\vec{k}_1,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} c_{\vec{k}_3,\sigma}^\dagger c_{\vec{k}_2,\sigma} \sum_{\substack{\vec{q}_1 \in \text{PS}, \\ \vec{q}_2 \in \text{HS}}} \frac{W_{\vec{q}_1,\vec{k}_1,\vec{q}_2,\vec{k}_2} W_{\vec{q}_1,\vec{k}_3,\vec{q}_2,\vec{k}_4}}{\omega + \frac{1}{2}\varepsilon(q_1) - \frac{1}{2}\varepsilon(q_2) + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})}. \end{aligned} \quad (15)$$

$$\begin{aligned} T_5 GT_5 &= \frac{1}{4} \sum W_{\vec{q}_1,\vec{k}_1,\vec{q}_2,\vec{k}_2} \sigma \sigma' c_{\vec{k}_1,\sigma'}^\dagger c_{\vec{q}_2,\sigma'} c_{\vec{q}_1,\sigma}^\dagger c_{\vec{k}_2,\sigma} \frac{1}{\omega - H_D} W_{\vec{q}_1,\vec{k}_3,\vec{q}_2,\vec{k}_4} \sigma \sigma' c_{\vec{k}_3,\sigma}^\dagger c_{\vec{q}_1,\sigma} c_{\vec{q}_2,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} \\ &= \frac{1}{4} \sum \hat{n}_{\vec{q}_1,\sigma} (1 - \hat{n}_{\vec{q}_2,\sigma'}) c_{\vec{k}_1,\sigma'}^\dagger c_{\vec{k}_2,\sigma} c_{\vec{k}_3,\sigma}^\dagger c_{\vec{k}_4,\sigma'} \frac{W_{\vec{q}_1,\vec{k}_1,\vec{q}_2,\vec{k}_2} W_{\vec{q}_1,\vec{k}_3,\vec{q}_2,\vec{k}_4}}{\omega + \frac{1}{2}\varepsilon(q_1) - \frac{1}{2}\varepsilon(q_2) + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})} \\ &= -\frac{1}{4} \sum_{\substack{1,2,3,4, \\ \sigma,\sigma'}} c_{\vec{k}_1,\sigma'}^\dagger c_{\vec{k}_4,\sigma'} c_{\vec{k}_3,\sigma}^\dagger c_{\vec{k}_2,\sigma} \sum_{\substack{\vec{q}_1 \in \text{PS}, \\ \vec{q}_2 \in \text{HS}}} \frac{W_{\vec{q}_1,\vec{k}_1,\vec{q}_2,\vec{k}_2} W_{\vec{q}_1,\vec{k}_3,\vec{q}_2,\vec{k}_4}}{\omega + \frac{1}{2}\varepsilon(q_1) - \frac{1}{2}\varepsilon(q_2) + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})}. \end{aligned} \quad (16)$$

$$\begin{aligned}
T_4GT_5 &= \frac{1}{4} \sum W_{\vec{q}_1, \vec{k}_1, \vec{q}_2, \vec{k}_2} \sigma \sigma' c_{\vec{q}_1, \sigma}^\dagger c_{\vec{k}_2, \sigma} c_{\vec{k}_1, \sigma'}^\dagger c_{\vec{q}_2, \sigma'} \frac{1}{\omega - H_D} W_{\vec{q}_1, \vec{k}_3, \vec{q}_2, \vec{k}_4} \sigma \sigma' c_{\vec{k}_3, \sigma}^\dagger c_{\vec{q}_1, \sigma} c_{\vec{q}_2, \sigma'}^\dagger c_{\vec{k}_4, \sigma'} \\
&= -\frac{1}{4} \sum \hat{n}_{\vec{q}_1, \sigma} (1 - \hat{n}_{\vec{q}_2, \sigma'}) c_{\vec{k}_2, \sigma} c_{\vec{k}_1, \sigma'}^\dagger c_{\vec{k}_3, \sigma}^\dagger c_{\vec{k}_4, \sigma'} \frac{W_{\vec{q}_1, \vec{k}_1, \vec{q}_2, \vec{k}_2} W_{\vec{q}_1, \vec{k}_3, \vec{q}_2, \vec{k}_4}}{\omega + \frac{1}{2}\varepsilon(q_1) - \frac{1}{2}\varepsilon(q_2) + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})} \\
&= -\frac{1}{4} \sum_{\substack{1,2,3,4, \\ \sigma, \sigma'}} c_{\vec{k}_1, \sigma'}^\dagger c_{\vec{k}_4, \sigma'} c_{\vec{k}_3, \sigma}^\dagger c_{\vec{k}_2, \sigma} \sum_{\substack{\vec{q}_1 \in \text{PS}, \\ \vec{q}_2 \in \text{HS}}} \frac{W_{\vec{q}_1, \vec{k}_1, \vec{q}_2, \vec{k}_2} W_{\vec{q}_1, \vec{k}_3, \vec{q}_2, \vec{k}_4}}{\omega + \frac{1}{2}\varepsilon(q_1) - \frac{1}{2}\varepsilon(q_2) + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})} .
\end{aligned} \tag{17}$$

$$\begin{aligned}
T_5GT_4 &= \frac{1}{4} \sum W_{\vec{q}_1, \vec{k}_3, \vec{q}_2, \vec{k}_4} \sigma \sigma' c_{\vec{k}_3, \sigma}^\dagger c_{\vec{q}_2, \sigma} c_{\vec{q}_1, \sigma'}^\dagger c_{\vec{k}_4, \sigma'} \frac{1}{\omega - H_D} W_{\vec{q}_1, \vec{k}_1, \vec{q}_2, \vec{k}_2} \sigma \sigma' c_{\vec{q}_2, \sigma}^\dagger c_{\vec{k}_2, \sigma} c_{\vec{k}_1, \sigma'}^\dagger c_{\vec{q}_1, \sigma'} \\
&= -\frac{1}{4} \sum \hat{n}_{\vec{q}_1, \sigma} (1 - \hat{n}_{\vec{q}_2, \sigma'}) c_{\vec{k}_3, \sigma}^\dagger c_{\vec{k}_4, \sigma'} c_{\vec{k}_2, \sigma} c_{\vec{k}_1, \sigma'}^\dagger \frac{W_{\vec{q}_1, \vec{k}_1, \vec{q}_2, \vec{k}_2} W_{\vec{q}_1, \vec{k}_3, \vec{q}_2, \vec{k}_4}}{\omega + \frac{1}{2}\varepsilon(q_1) - \frac{1}{2}\varepsilon(q_2) + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})} \\
&= -\frac{1}{4} \sum_{\substack{1,2,3,4, \\ \sigma, \sigma'}} c_{\vec{k}_3, \sigma}^\dagger c_{\vec{k}_2, \sigma} c_{\vec{k}_1, \sigma'}^\dagger c_{\vec{k}_4, \sigma'} \sum_{\substack{\vec{q}_1 \in \text{PS}, \\ \vec{q}_2 \in \text{HS}}} \frac{W_{\vec{q}_1, \vec{k}_1, \vec{q}_2, \vec{k}_2} W_{\vec{q}_1, \vec{k}_3, \vec{q}_2, \vec{k}_4}}{\omega + \frac{1}{2}\varepsilon(q_1) - \frac{1}{2}\varepsilon(q_2) + \frac{1}{2}(W_{\vec{q}_1} + W_{\vec{q}_2})} .
\end{aligned} \tag{18}$$