Unitary RG Approach to Quantum Impurity Problems

Phys. Rev. B 105, 085119

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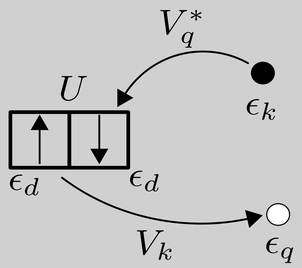
The Anderson impurity model (SIAM)

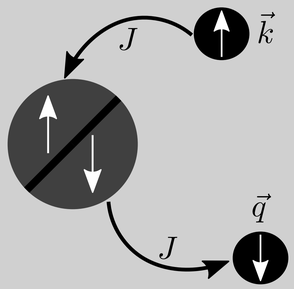


- Local impurity interacting with bath: $H_{\text{bath}} = \sum_{k\sigma} \epsilon_k \hat{n} k\sigma$
- Hubbard repulsion U on impurity and 1-particle hybridisation V with bath

$$H = H_{\text{bath}} + \epsilon_d \hat{n}_d + U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow} + V \sum_{k\sigma} \left(c^\dagger_{k\sigma} c_{d\sigma} + \text{h.c.} \right)$$

- **Microscopic origin of local moments in metals**

The Kondo model

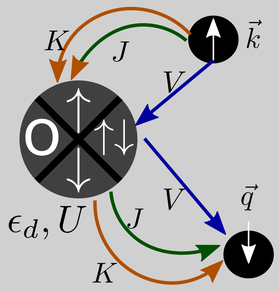


- Impurity projected to spin-half Hilbert space
- **Spin-exchange coupling** with conduction bath spin

$$H = H_{\text{bath}} + J \vec{S}_d \cdot \vec{s}$$

- Ground state is a **macroscopic singlet** formed by the impurity and the conduction bath
- Charge variant involves **isospin exchange**

Generalised Kondo-SIAM model

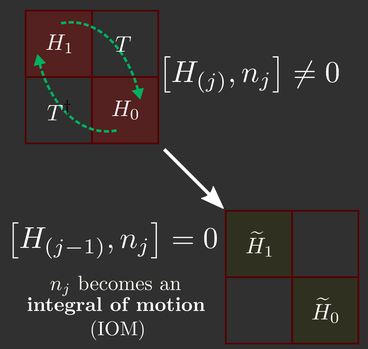


- Add spin and isospin exchange J and K to SIAM
- Exchange couplings are **dynamically generated under RG**, simpler to keep them at the start
- V renormalises at lowest order
- Describes **both spin and charge screening** in one model

Outstanding Questions

- What's the **effective Hamiltonian** for the conduction electrons that screen the impurity?
- What is the **nature of the metal** responsible for this screening?
- Quantitative insight into **many-particle entanglement** at and near the fixed point
- Does the interplay of V, J and K change the phase diagram in the generalised SIAM?
- Is there any **topological quantity** that changes in the process of screening?
- Can we track the **evolution of the impurity spectral function** along the RG flow?

The Unitary Renormalisation Group (URG) Method

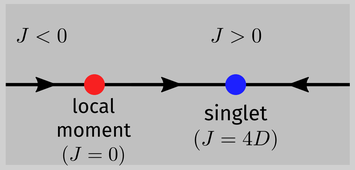


- Proceeds by **applying unitary transformations** U_j on the Hamiltonian to generate RG flow H_j

$$H_{j-1} = U_j H_j U_j^\dagger$$

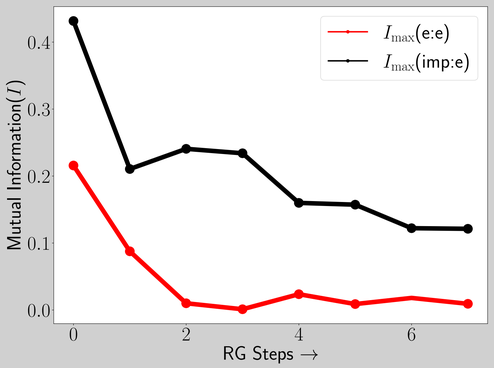
- U_j are defined so as to **remove quantum fluctuations** of high energy $k-$states
- Continues until denominator of RG equation vanishes: fixed point
- Fixed point Hamiltonian describes **emergent theory** at low energy

URG Flows of the Kondo Model: Phase Diagram



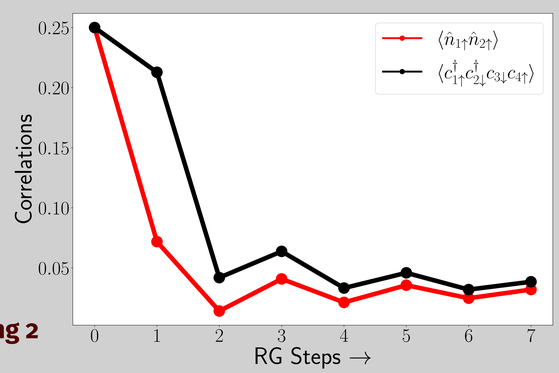
- $J = 0$ is stable for $J \leq 0$: **local moment** fixed point
- $J = 2D$ is globally stable: **strong-coupling** fixed point

RG Evolution of Entanglement in Kondo cloud



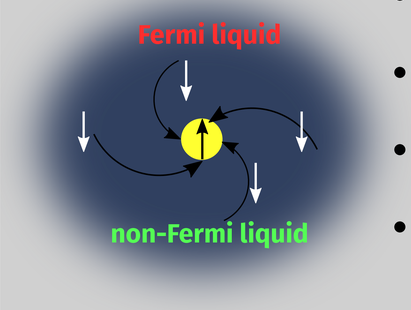
Mutual Information:
$I(1:2) = S(1) + S(2) - S(1,2)$
$S(1)$ - entanglement entropy of 1

Information obtained about 1, on measuring 2



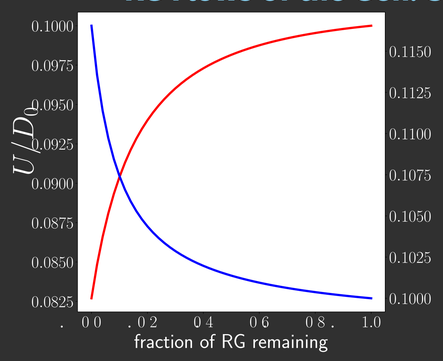
- **Mut. Info. and correlations grow** towards strong-coupling IR fixed point
- Demonstrates the screening of impurity and formation of the singlet
- Consistent with the **presence of non-FL terms** in Kondo cloud Hamiltonian

Effective Hamiltonian for Kondo Cloud



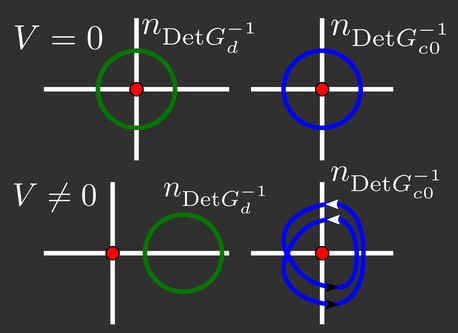
- Treat kinetic energy as perturbation above singlet ground state
- **Integrate out impurity dynamics** via Schrieffer-Wolff transformation
- Resultant effective Hamiltonian has diagonal Fermi liquid piece: $\sum \epsilon_k \hat{n}_{k\sigma} + \sum f_{kk'} \hat{n}_{k\sigma} \hat{n}_{k'\sigma'}$
- More importantly, it has **off-diagonal non-Fermi** liquid terms: $\sum_{k_1,k_2,k_3,k_4} \mathcal{F} c^\dagger_{k_1\uparrow} c^\dagger_{k_2\downarrow} c_{k_3\downarrow} c_{k_4\uparrow}$

RG Flows of the Gen. SIAM, Effective Hamiltonian



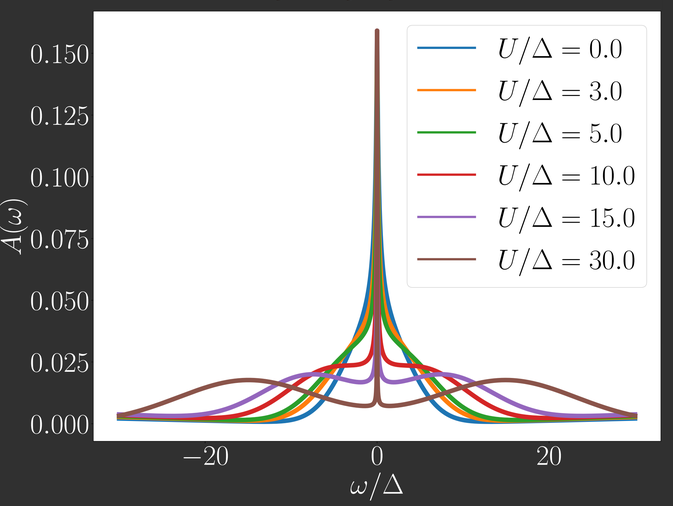
- J, V flow to strong-coupling, U may be relevant if $J > V$
- Ground state is **mixture** of spin-singlet and charge-triplet-zero
- Effective Hamiltonian of Kondo again features **non-Fermi liquid** terms
- Quantities computed from fixed point Hamiltonian **agree quantitatively** with the literature

Topological change: Increase in Luttinger's volume



- Total number of electrons is given by **Luttinger's volume** (LV)
- At strong-coupling fixed point, impurity hybridises with the impurity
- This leads to an **increase in the topological quantity** LV by 1
- **A pole is transferred** from the impurity Greens function to the bath Greens function

Evolution of Impurity Spectral Function



- Impurity spectral function obtained from effective Hamiltonian
- Single broad central peak at $J, V \gg U \rightarrow$ low-energy scattering
- Side peaks appear at $J, V \ll U$, representative of local moment

Future Directions

- Self-energy calculation of the complete cloud should reveal the type of non-Fermi liquid
- Enhance gen. SIAM to stabilise local moment - **metal-insulator transition** of DMFT
- More rich physics can be obtained from the lattice versions of the models

Acknowledgements

I thank IISER Kolkata for funding through a research fellowship. I also thank P. Majumdar, A. Mitchell, S. Sen, S. Patra, M. Mahankali, R. K. Singh for various discussions and feedback.

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