

# **EMERGENCE IN CORRELATED FERMIONS: FROM IMPURITY MODELS TO THE BULK**

**ABHIRUP MUKHERJEE**

**RPC PRESENTATION 2022-2023**

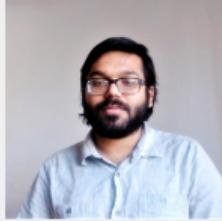
**EMERGENT PHENOMENA IN QUANTUM MATTER GROUP  
DEPARTMENT OF PHYSICAL SCIENCES, IISER KOLKATA**

**JULY 11, 2023**

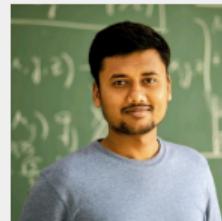




Siddhartha Lal



Anirban Mukherjee



Siddhartha Patra



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**A huge thanks to all my collaborators!  
Thanks to IISER K and SERB for financial support.**  
~~~~~



Arghya Taraphder  
IIT Kharagpur



N. S. Vidhyadhiraja  
JNCASR Bangalore



# **LIST OF COMPLETED AND ONGOING PROJECTS**

## LIST OF PUBLICATIONS, PREPRINTS AND ONGOING PROJECTS

- ✓ Unveiling the Kondo cloud: Unitary RG study of the Kondo model.  
2022 **Phys. Rev. B** 105, 085119.  
A Mukherjee, **Abhirup Mukherjee**, N. S. Vidhyadhiraja, A. Taraphder, S Lal
  
- ✓ Frustration shapes multi-channel Kondo physics: a star graph perspective.  
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- Kondo frustration via charge fluctuations: a route to Mott localisation.  
2023 arXiv:2302.02328. **under review** at New Journal of Physics.  
**Abhirup Mukherjee**, N. S. Vidhyadhiraja, A. Taraphder, S Lal
- Holographic entanglement renormalisation for fermionic quantum matter.  
2023 arXiv:2302.10590. **under review** at Journal of HEP.  
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## Currently in progress

- Development of auxiliary model-based method for studying bulk correlated systems.
- Studies of the plateau-to-plateau transition in integer quantum hall systems.

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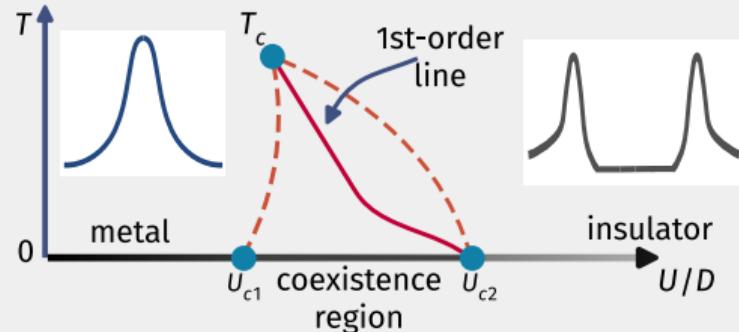
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# **QUANTUM PHASE TRANSITION IN AN EXTENDED-SIAM**

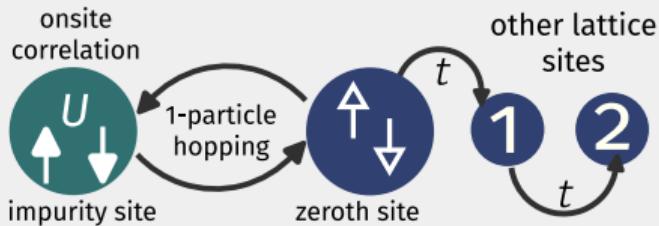
**ABHIRUP MUKHERJEE, N. S. VIDHYADHIRAJA, A. TARAPHDER, SID-DHARTHA LAL**  
**ARXIV:2302.02328. (2023)**

# SOME BROAD QUESTIONS

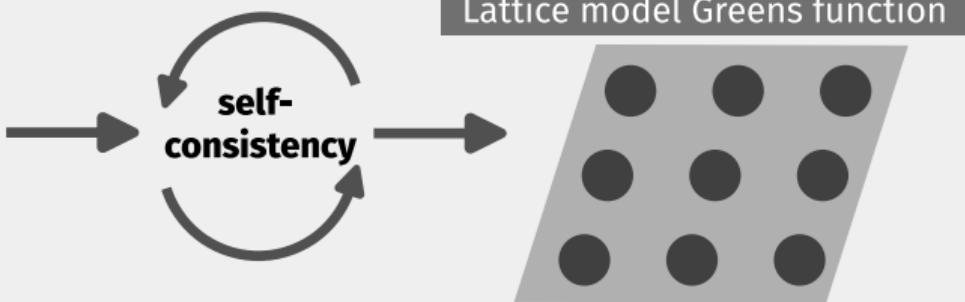
Dynamical mean-field theory shows **metal-insulator transition** for the Hubbard model in  $\infty$  dimensions.



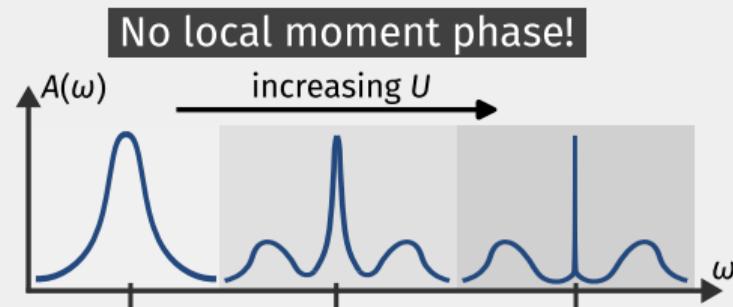
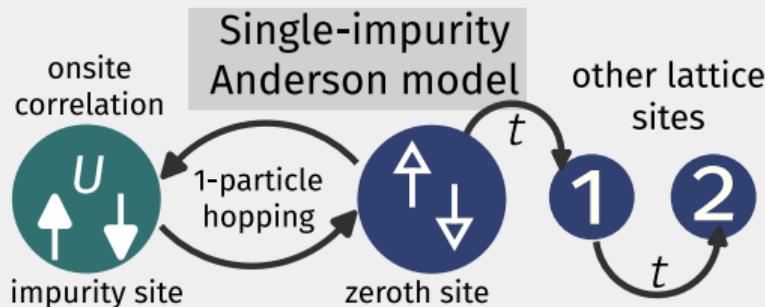
## Single-impurity Anderson model



## Lattice model Greens function



# SOME BROAD QUESTIONS



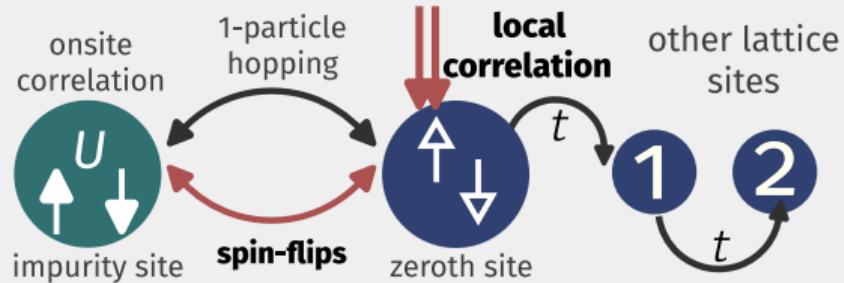
Standard Anderson model shows **no transition**, can't explain DMFT phase diagram.

- Which impurity model is realised through **self-consistency**?
- What physics leads to  $U_{c1}$  and  $U_{c2}$ ?
- What is the state precisely at the  $T = 0$  **transition**?

# RESULTS

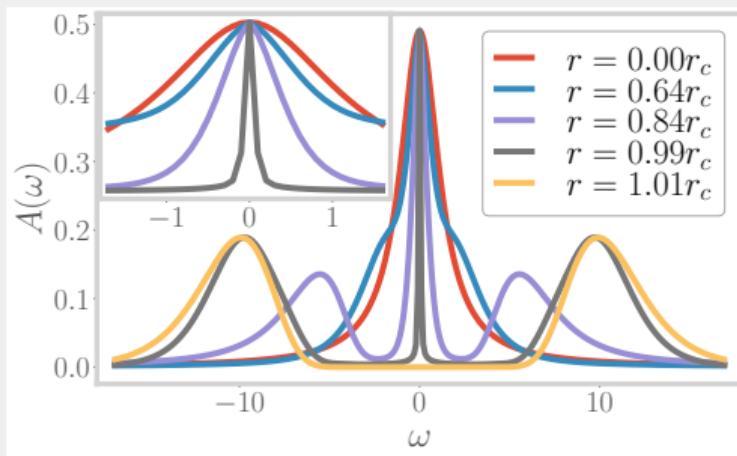
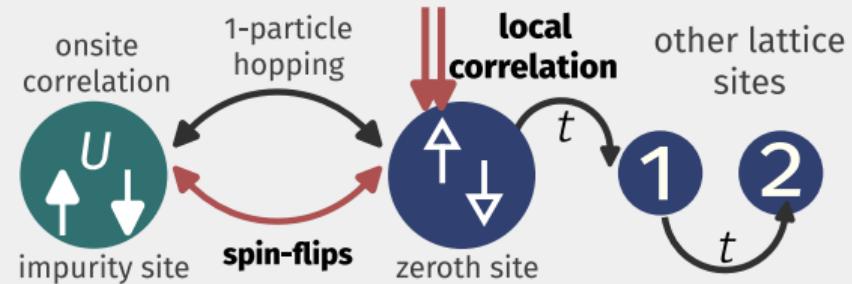
Phase **transition** occurs upon adding

- spin-flip correlation between impurity and bath
- **local correlation** on bath zeroth site



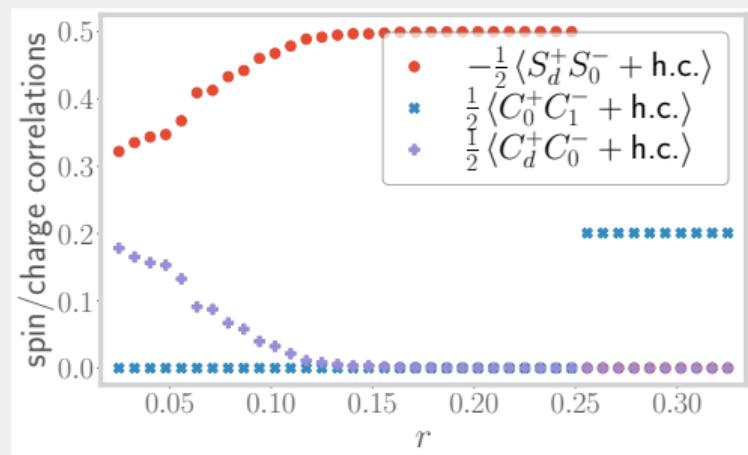
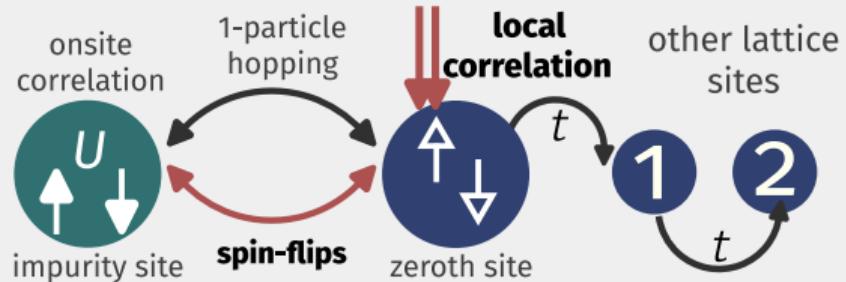
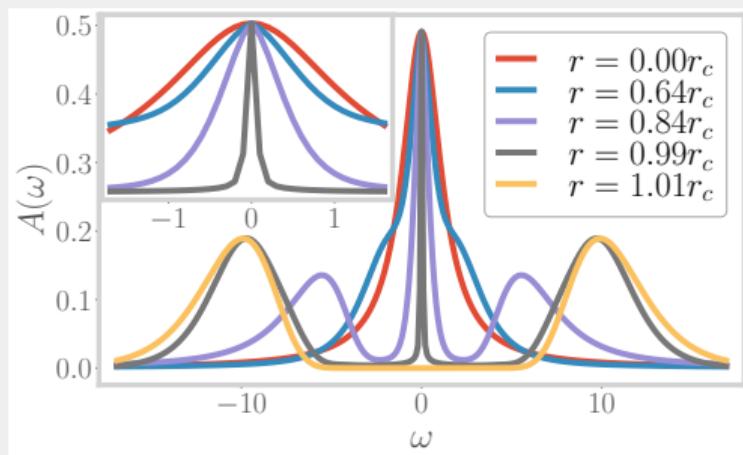
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- Local spectral function shows gap.



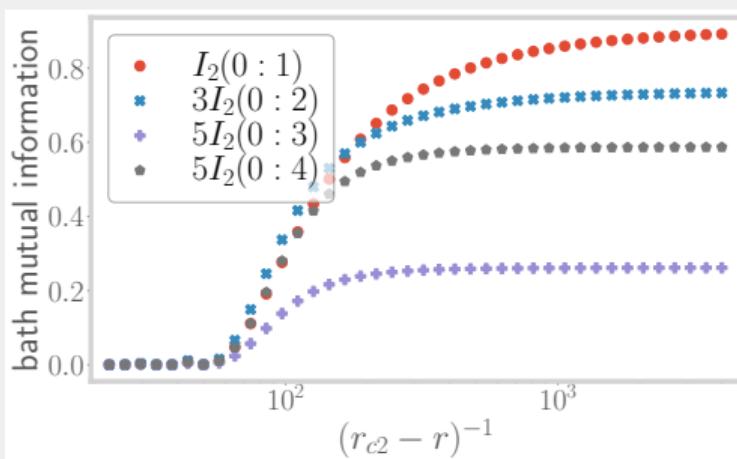
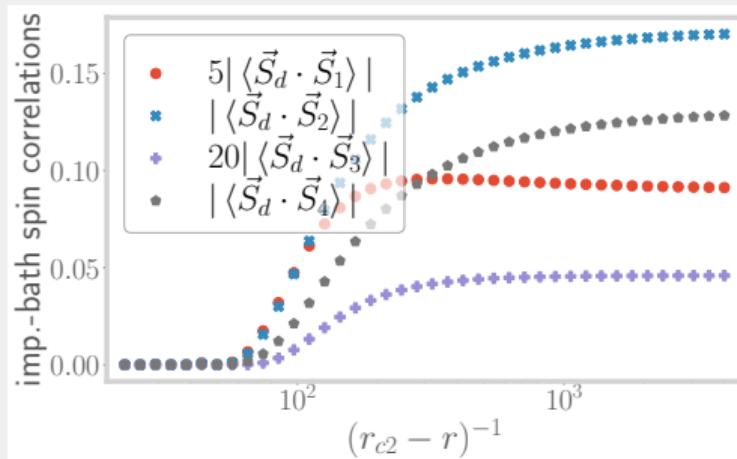
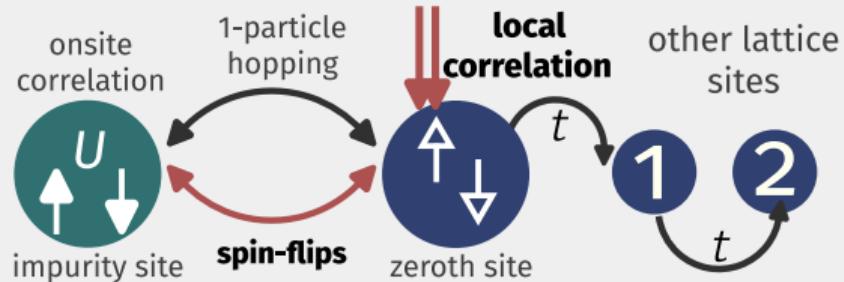
# RESULTS

- Local spectral function shows gap.
- Spin correlations vanish, **pairing correlations** grow at the transition



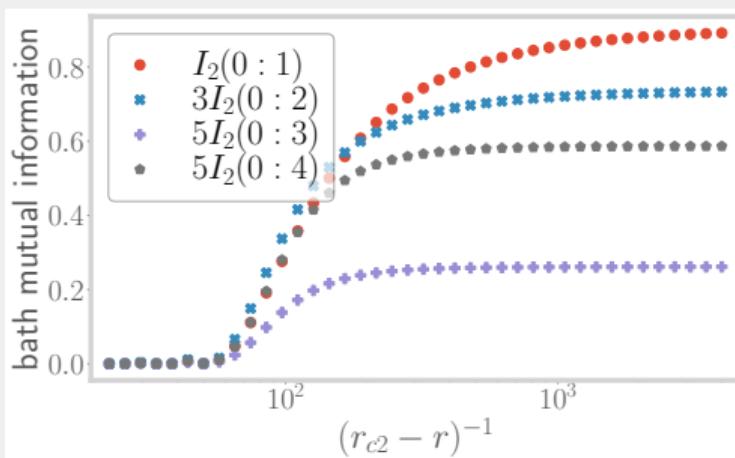
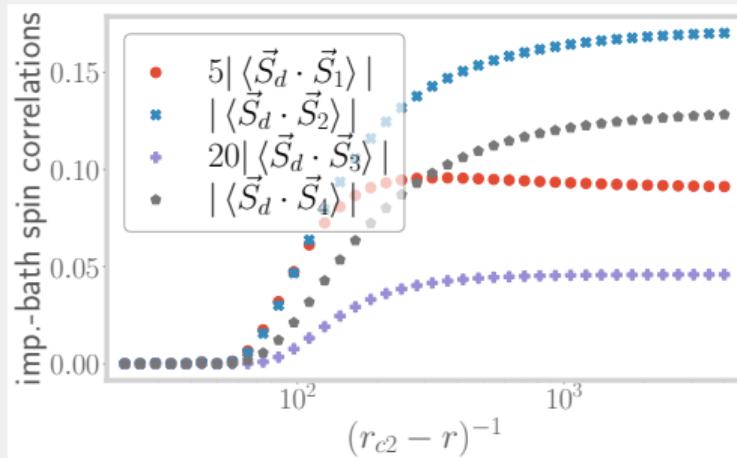
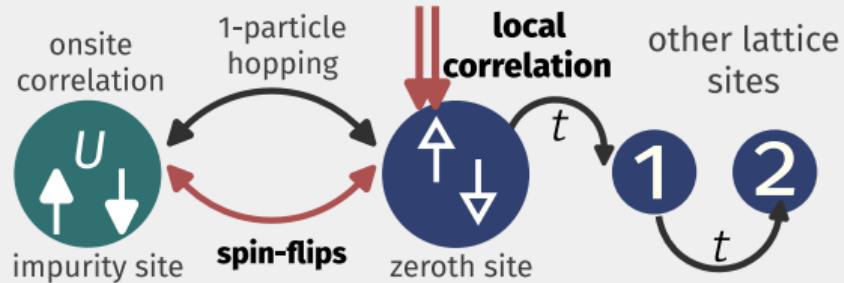
# RESULTS

- Quantum critical point shows **long-ranged correlations**.



# RESULTS

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- Excitations at the QCP are non-Fermi liquid in nature.



# **HOLOGRAPHY OF ENTANGLEMENT IN 2D FREE FERMIONS**

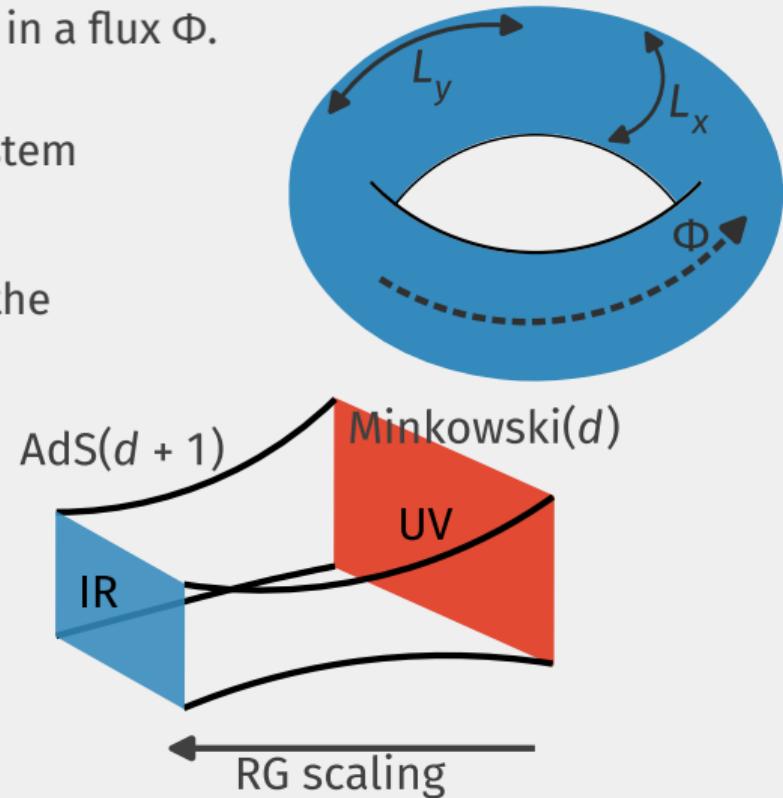
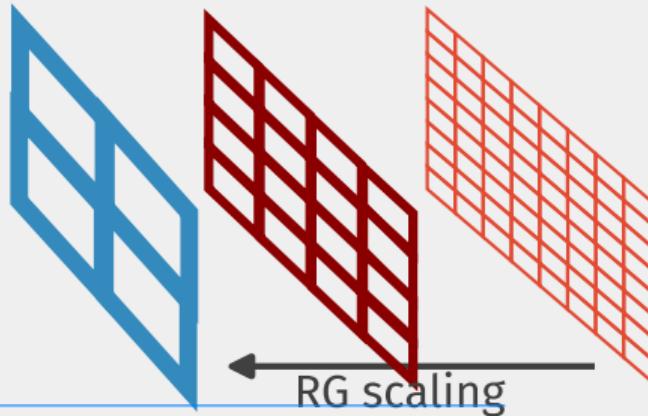
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**ABHIRUP MUKHERJEE, SIDDHARTHA PATRA, SIDDHARTHA LAL**  
**ARXIV:2302.10590. (2023)**

# SOME BROAD QUESTIONS

We consider 2D electrons placed on a torus in a flux  $\Phi$ .

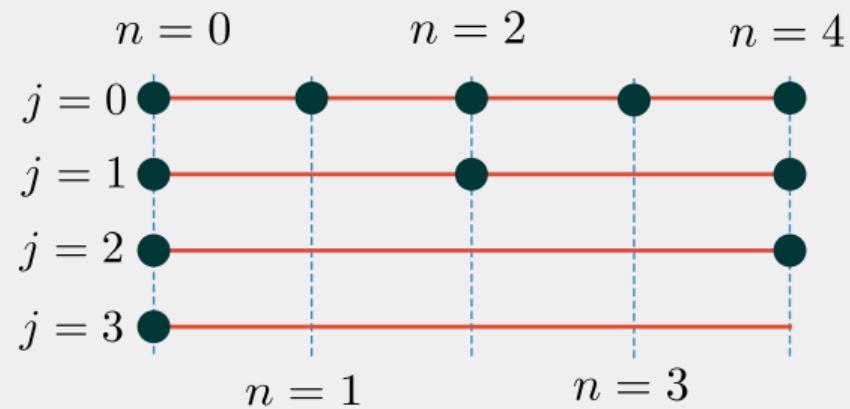
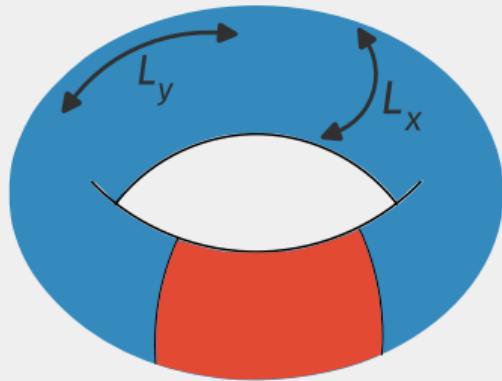
- Is the entanglement content of this system **holographic**?
- Is there any **topological** notion within the entanglement measures?



# RESULTS

- Choose subsystem in real space (red region)
- Apply **coarse-graining transformations** in  $k$ -space

Evolution of subspace entanglement shows interesting properties.

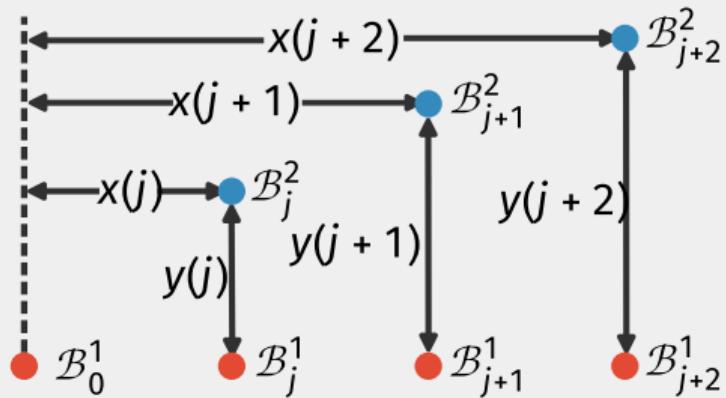


# RESULTS

Use mutual information  $I_2$  to define **distance**.

- Larger  $I_2 \implies$  smaller distance
- Allows notion of **curvature** as well.

Coarse-graining transformations lead to **emergent** spatial dimension

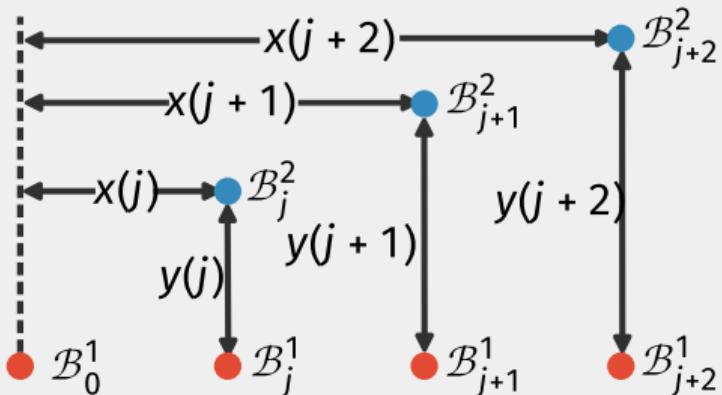


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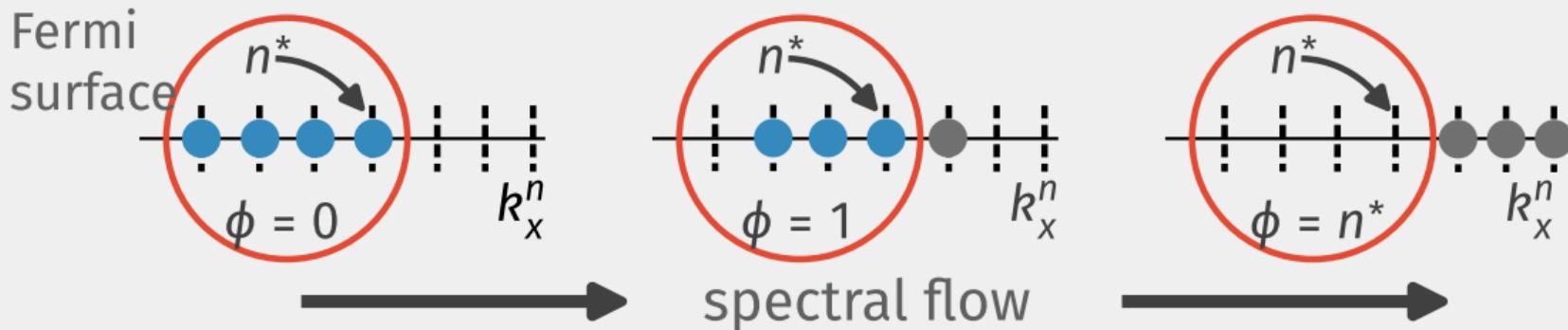
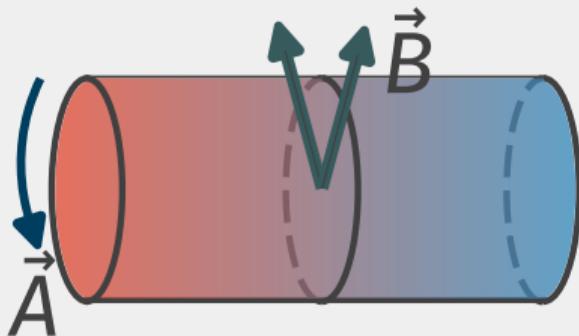


Other consequences:

- **hierarchy** of entanglement exists along the RG
- hierarchy also present in **multipartite entanglement**

# RESULTS

- By tuning flux, we relate Luttinger's volume to functions of entanglement
- Entanglement spectral flow is also related to Chern numbers in presence of magnetic field

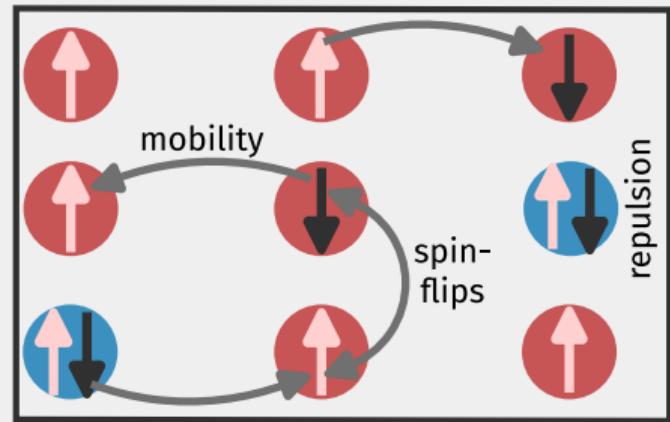


# **TILING A LATTICE WITH THE EXTENDED SIAM**

**ONGOING PROJECT**

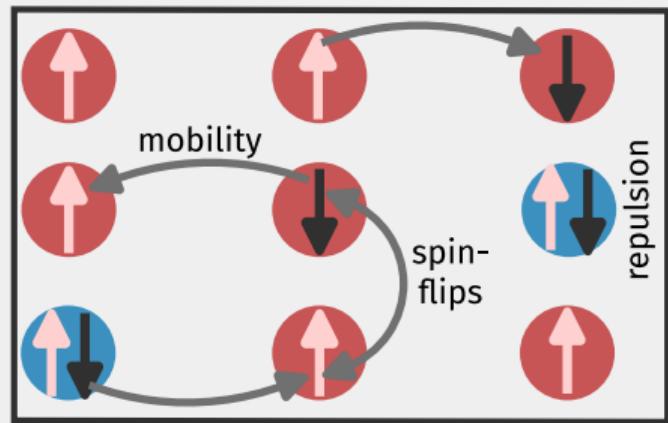
# BROAD QUESTIONS

- Studying the **Mott MIT** on the 2D Hubbard-Heisenberg model



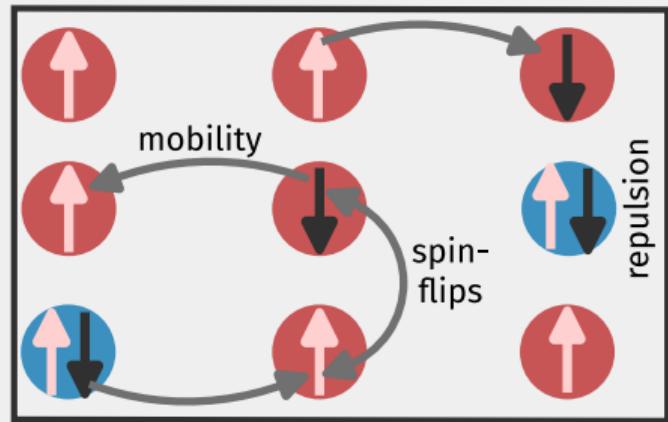
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- Obtaining a **momentum-space** picture of the transition.



# BROAD QUESTIONS

- Studying the **Mott MIT** on the 2D Hubbard-Heisenberg model
- Obtaining a **momentum-space** picture of the transition.
- Devising a method to tackle lattice models **using impurity models**



## RESULTS

We use impurity model eigenstates (Hamiltonians) and **Bloch's theorem** to reconstruct full eigenstates (Hamiltonian):

$$|\Psi_{\vec{k}}\rangle \sim \sum_{\vec{R}_i} e^{i\vec{k}\cdot\vec{R}_i} |\psi_{\text{aux}}(\vec{R}_i)\rangle, \quad H \sim \sum_{\vec{R}_i} H_{\text{aux}}(\vec{R}_i)$$



## RESULTS

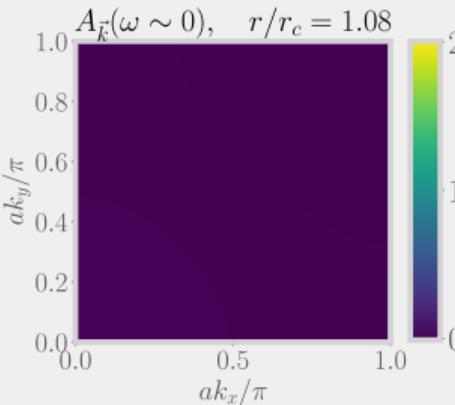
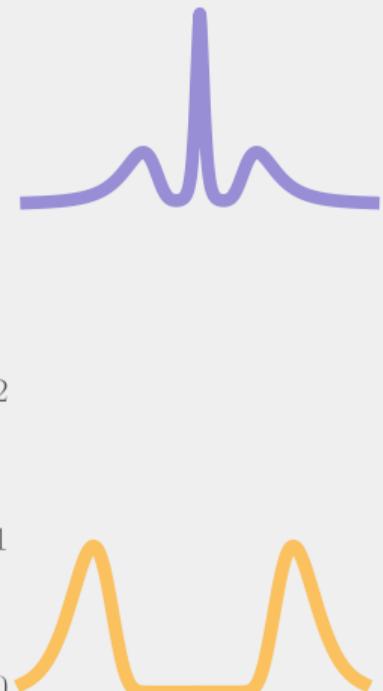
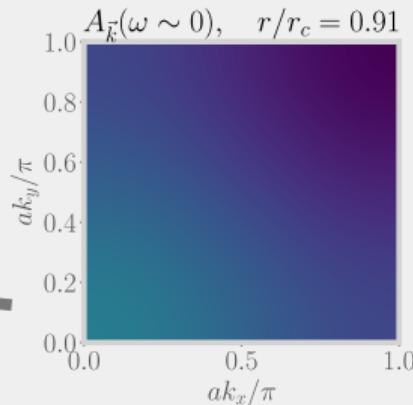
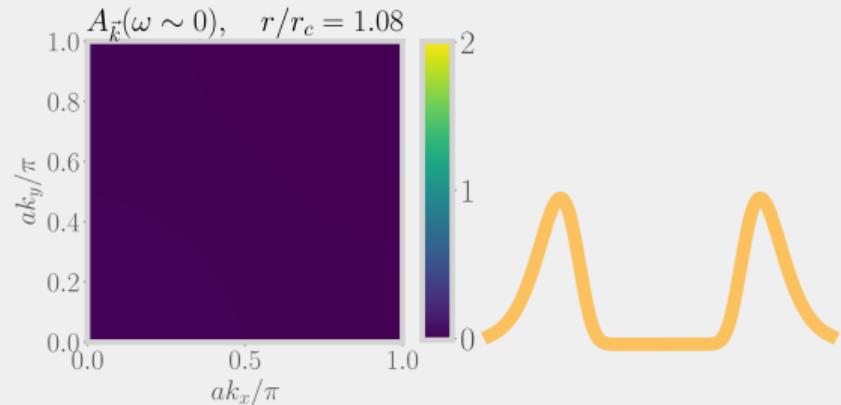
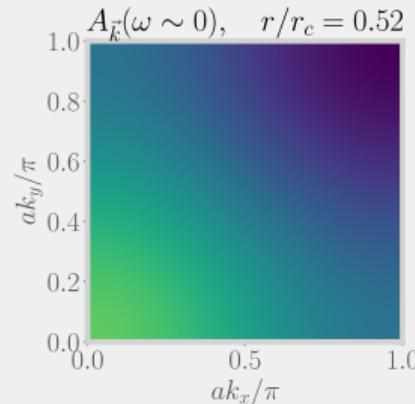
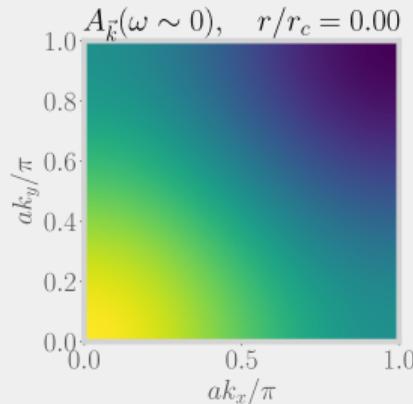
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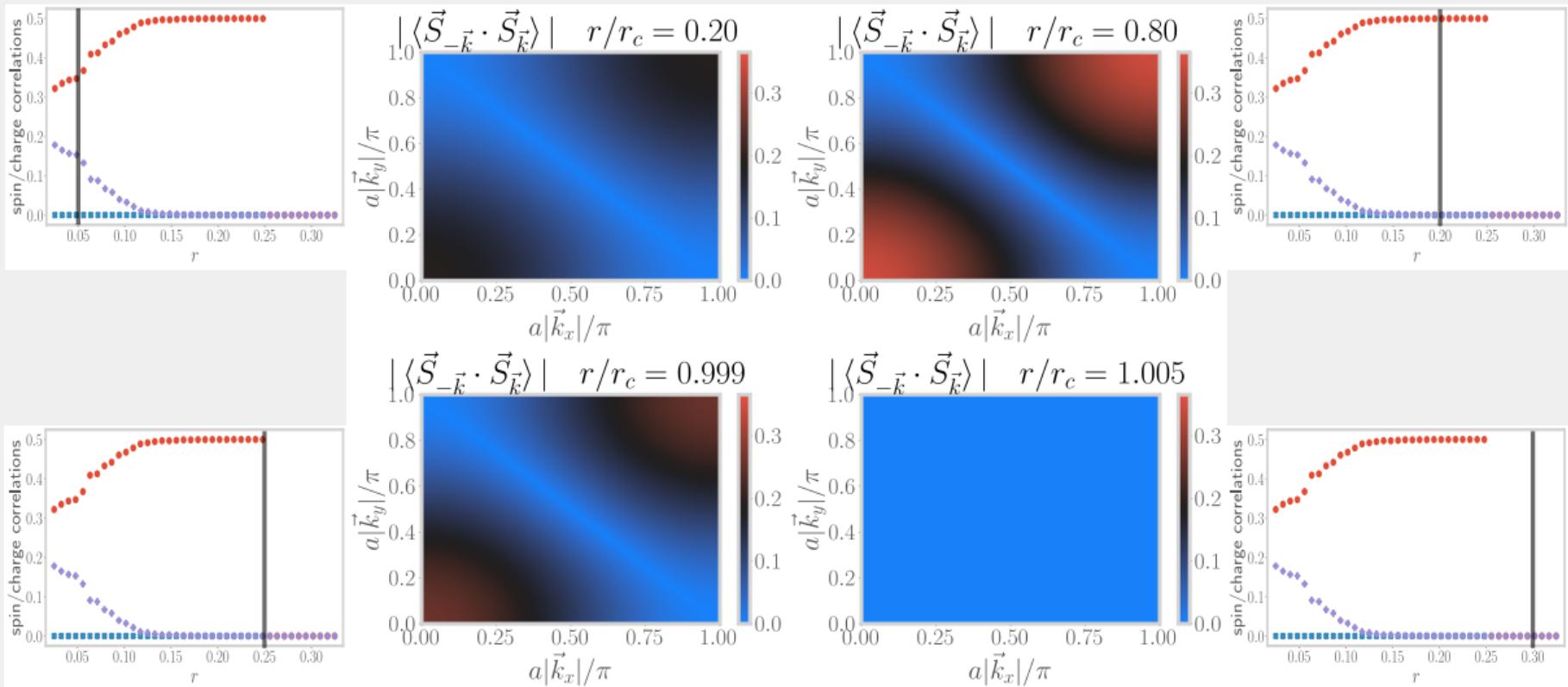
Allow us to **relate corresponding objects** between the impurity model and the lattice model

- Greens functions and self-energies
- Two-particle correlation functions
- entanglement measures

# RESULTS: MOMENTUM SPACE SPECTRAL FUNCTION



# RESULTS: MOMENTUM SPACE SPIN CORRELATIONS

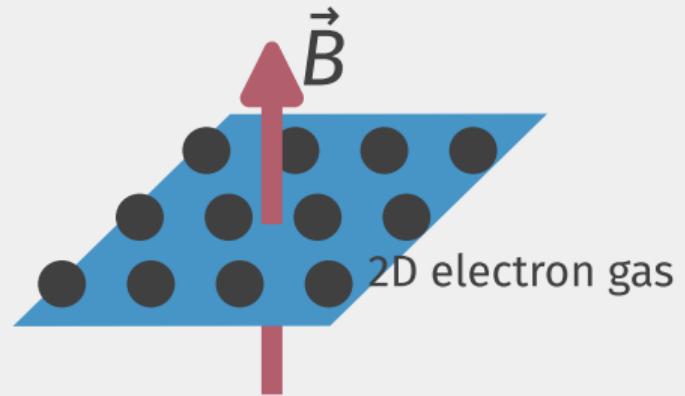


# **SEARCH FOR PUNCTURED-CHERN TOPOLOGY AT IQHE TRANSITIONS**

**ONGOING PROJECT**

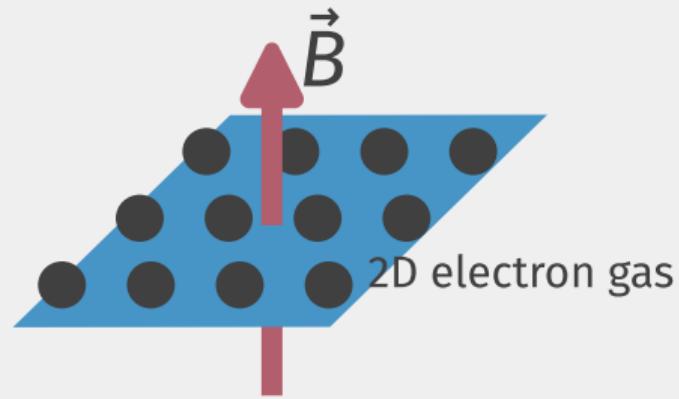
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- Obtaining the **IQHE phase diagram** from a model of 2D lattice electrons



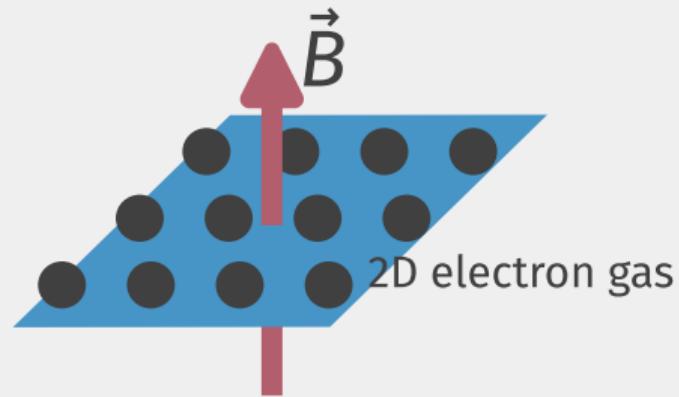
# BROAD QUESTIONS

- Obtaining the **IQHE phase diagram** from a model of 2D lattice electrons
- Understanding the **topology** of the ground state precisely at a transition



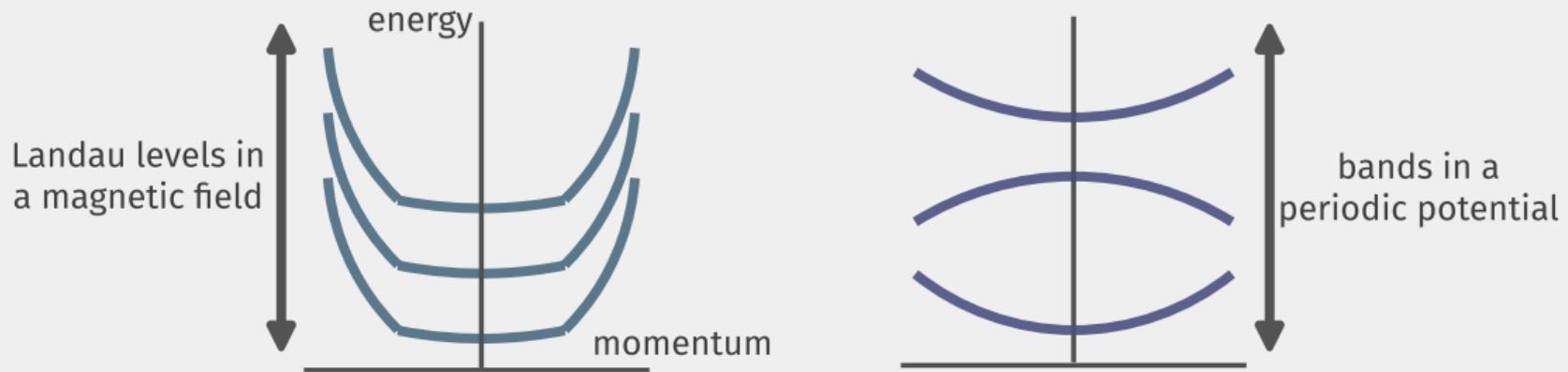
# BROAD QUESTIONS

- Obtaining the **IQHE phase diagram** from a model of 2D lattice electrons
- Understanding the **topology** of the ground state precisely at a transition
- Extending this to systems with **disorder**.



## PRELIMINARY RESULTS

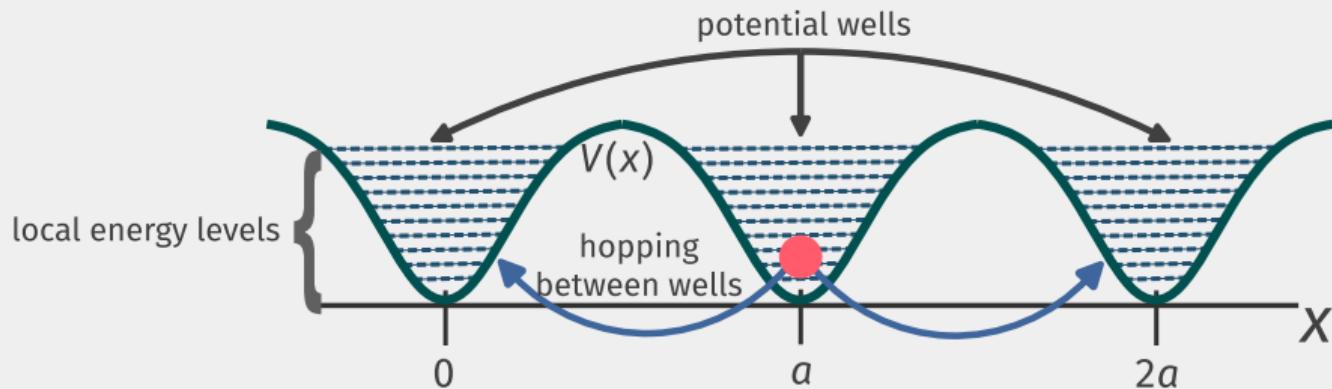
Emergence of **Landau levels** in a magnetic field is similar to the formation of **bands** in a periodic potential.



We first studied the simpler problem of **particle in a periodic potential**.

## PRELIMINARY RESULTS

We first studied the simpler problem of **particle in a periodic potential**.



- Can understand the formation of bands under RG
- Obtained insights regarding the **effective center of mass** degrees of freedom
- Needs to be extended by incorporating a **magnetic field**

# SUMMARY

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### Currently in progress

- Development of auxiliary model-based method for studying bulk correlated systems.
  - Studies of the plateau-to-plateau transition in integer quantum hall systems.
- 

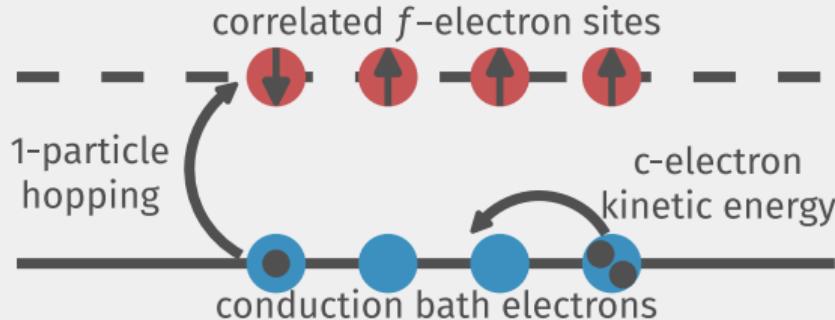
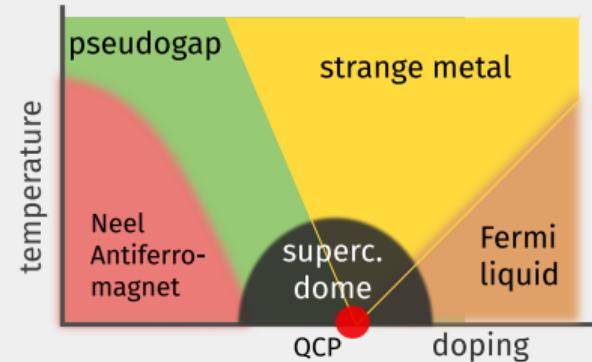
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# FUTURE PLANS

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## Lattice models of impurities

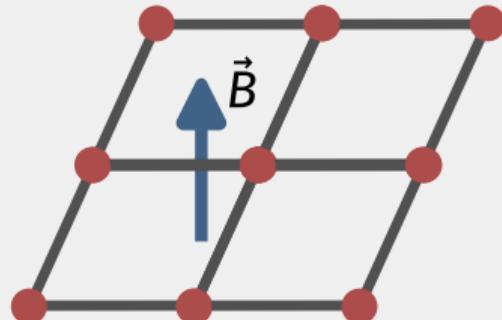
- either directly or through the auxiliary model approach
- phase diagrams: strange metals and QCPs
- unconventional superconductivity



## FUTURE PLANS

### Fractional Chern insulators

- microscopic understanding of the FQHE ground states
- emergence of composite degrees of freedom and topological theories



## FUTURE PLANS

### **Classification of RG flows in fermionic models**

- growth of multipartite entanglement towards stable fixed points
- extending this to impurity models
- connections with the URG noise operator

THANK YOU.