# Local metal-insulator transition in an extended Anderson impurity model

JRF-to-SRF Upgradation Presentation

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August 24, 2022

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# Summary of Work

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## Completed Projects

- ✓ Unveiling the Kondo cloud: Unitary renormalization-group study of the Kondo model
  **Phys. Rev. B 105, 085119**, arXiv:2111.10580v3
  A. Mukherjee, *Abhirup Mukherjee*, N. S. Vidhyadhiraja, A. Taraphder, and S. Lal

- ✓ Frustration shapes multi-channel Kondo physics: A star graph perspective
  **under review at PRB**, arXiv:2205.00790
  S. Patra, *Abhirup Mukherjee*, A. Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, S. Lal

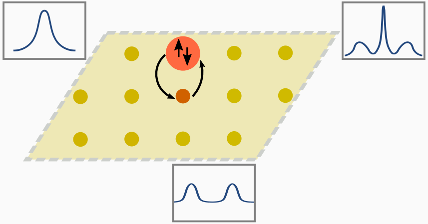
## Ongoing Projects

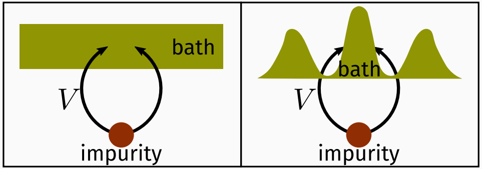
- ✓ Metal-insulator transition in an extended Anderson impurity model (*manuscript in preparation*)

- ✓ Holography and topology of entanglement scaling in free fermions (*manuscript in preparation*)

- ✓ URG-based auxiliary model approach to correlated systems (*ongoing*)

# Local MIT in an extended Anderson impurity model

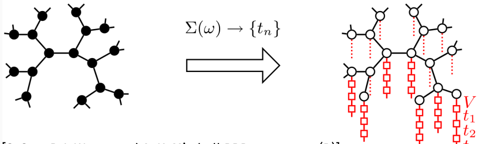




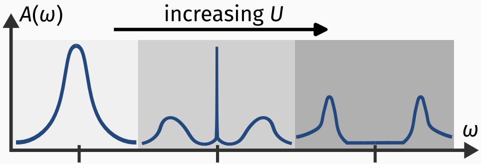
# Introducing the extended Anderson impurity model

# DMFT on the Bethe lattice: Exact in $d = \infty$

✓ shows **metal-insulator transition** on the Bethe lattice with ∞ coordination number

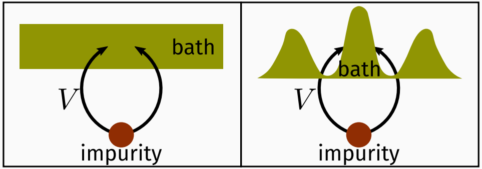



[S. Sen, P. J. Wong, and A. K. Mitchell PRB 102, 081110(R)]




✓ Spectral function develops three peaks and then **gaps out**

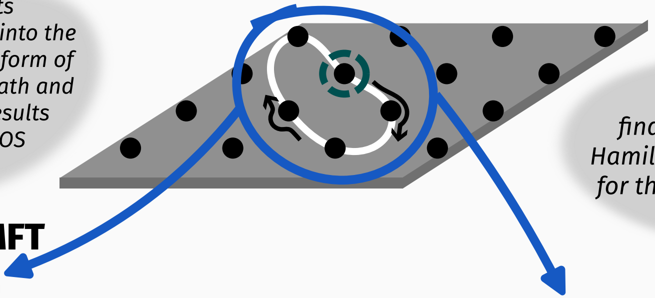
✓ Conduction bath obtained by imposing self-consistency shows **non-trivial correlations**

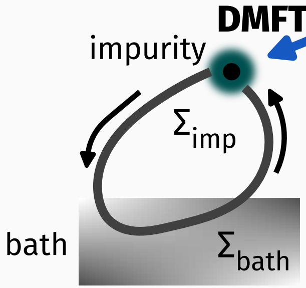



Metzner et al. 1989; Georges et al. 1992; Parcollet et al. 2004; Maier et al. 2005; Kotliar et al. 2006; Ohashi et al. 2008; Held et al. 2013; Sen et al. 2020.

# DMFT on the Bethe lattice: Exact in $d = \infty$

**DMFT** *represents excursions from a site into the correlated bath in the form of self-energies for the bath and the impurity. This results in a three-peak DOS of the bath.*

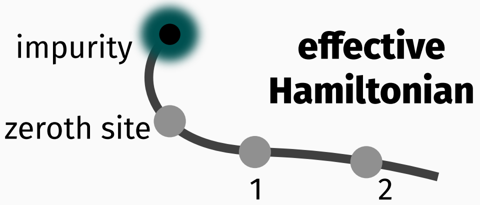





**Our Goal:**

*finding an effective Hamiltonian description for the Σ that gives rise to the MIT.*

Similar approach adopted by Si & Kotliar for extended Hubbard model

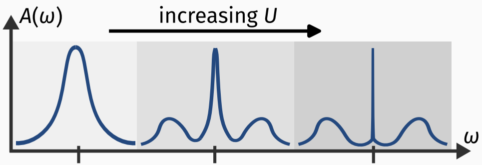



Si et al. 1993; Kotliar et al. 1993.

# Introducing the extended Anderson impurity model

**Standard Anderson impurity model**

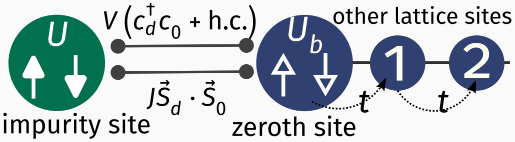
- ✓ no local-moment phase, $A(\omega)$ gapless
- ✓ cannot explain insulating phase of DMFT




**Gap in spectral function requires additional physics!**

*Extended* **Anderson impurity model**

- ✓ impurity-bath spin correlation: $J$
- ✓ bath zeroth site local correlation: $U_b$





Anderson 1961; Anderson 1978; Wilson et al. 1974; Nozieres 1974; Krishna-murthy et al. 1980; Andrei 1980; Tsvelick et al. 1983; Hewson 1993; Costi et al. 1990; Costi 2000; Kuramoto et al. 1987; Cox et al. 1988.

# Phase Diagram & Ground-States

# Nature of RG flows

✓ URG Equations reveal **critical** point at $r = -U_b/J = 1/4$

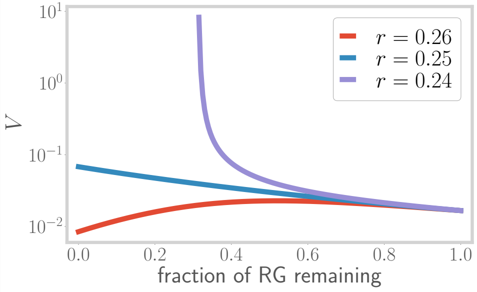
✓ RG equation for most dominant coupling $J$:

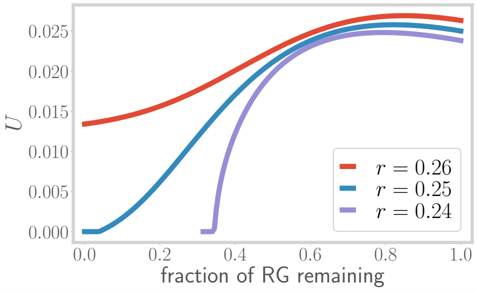
$$\Delta J = -J(J + 4U_b)n(D)\frac{1}{\omega - D/2 + U_b/2 + J/4}$$

✓ Numerator structure allows **averting** strong-coupling behaviour

# Nature of RG flows

✓ Bath correlation $U_b$ is always **marginal**

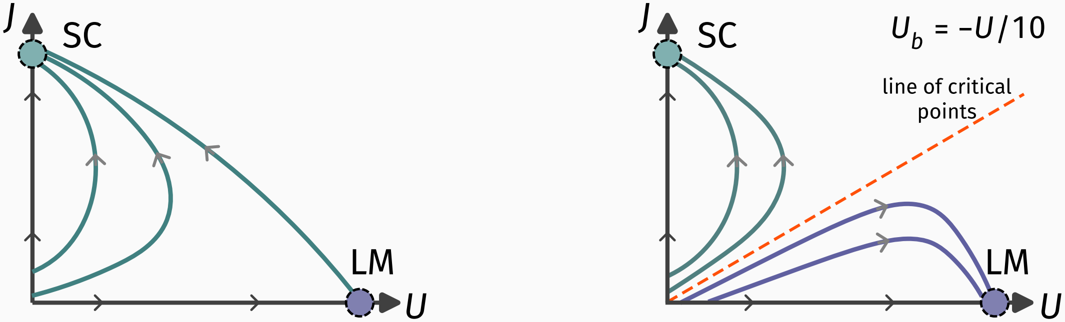


# Nature of RG flows

✓ RG equation reveals competition between Kondo flow and pairing physics

$$\Delta J = -\frac{(J + 2U_b)^2 n(D)}{\omega - D/2 + U_b/2 + J/4} + \frac{(2U_b)^2 n(D)}{\omega - D/2 + U_b/2 + J/4}$$



# RG Phase Diagram

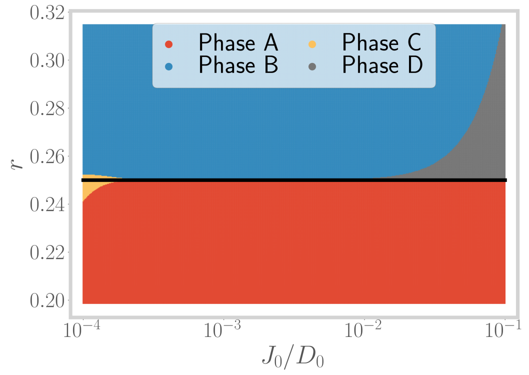
✓ blue phase ⟶ $U_b < -J/4$ : $V, J$ are **irrelevant** ⟶ local moment flows

✓ yellow phase: $J \ll D_0$: involves $V, U, U_b$

*vanishes for large systems*

✓ gray phase: $J \sim D_0$: **all** couplings irrelevant

*vanishes for large systems*



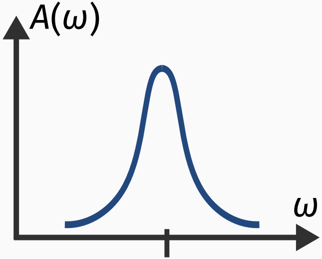

✓ red phase ⟶ $U_b > -J/4$: $V, J$ are **relevant** ⟶ strong-coupling flows

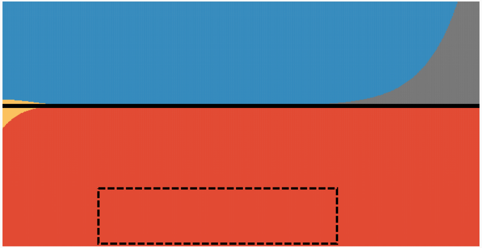
# Low-energy effective Hamiltonians and ground-states

**Regime 1:** $|U_b| < J/4$

- ✓ $J$ relevant,
- ✓ $V$ subdominant,
- ✓ $U$ irrelevant

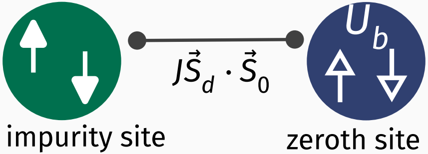
$$H = J\vec{S}_d \cdot \vec{S}_0 - U_b \left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2 + \sum_{k<\Lambda^*} \epsilon_k \hat{n}_{k\sigma}$$



**Regime 1:** $|U_b| < J/4$

## Zero-bandwidth limit

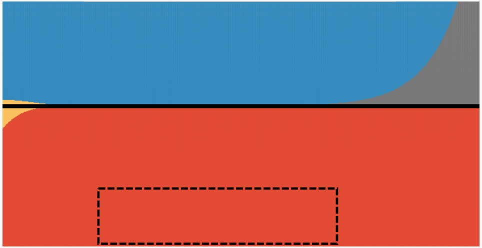



$$H = J\vec{S}_d \cdot \vec{S}_0 - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2$$

✓ two-spin Heisenberg, attractive zeroth site

✓ **singlet** ground state

$$|\Psi\rangle_{GS} = \frac{1}{\sqrt{2}}\left[|\uparrow_d\downarrow_0\rangle - |\downarrow_d\uparrow_0\rangle\right]$$



# Low-energy effective Hamiltonians and ground-states

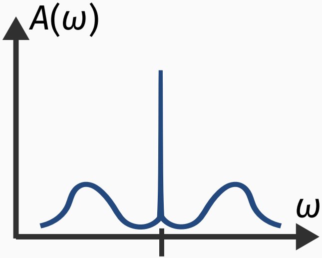
**Regime 2:** $|U_b| \sim J/4$

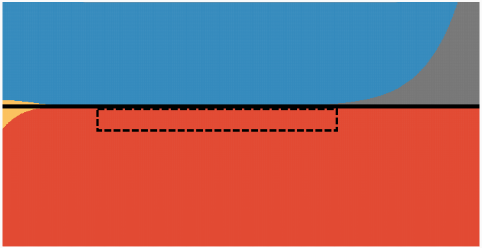
✓ $J$ relevant,

✓ $V$ relevant,

✓ $U$ irrelevant

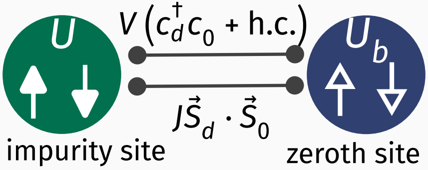
$$H = J\vec{S}_d\cdot\vec{S}_0 + V\sum_\sigma\left(c^\dagger_{d\sigma}c_{0\sigma} + \text{h.c.}\right) - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2 + \sum_{k<\Lambda^*}\epsilon_k\hat{n}_{k\sigma}$$



**Regime 2:** $|U_b| \sim J/4$

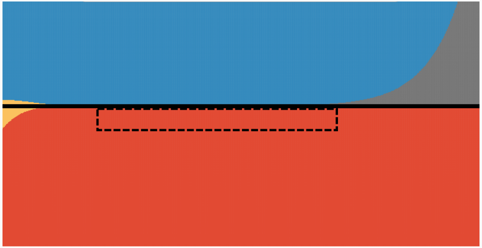
**Zero-bandwidth limit**




$$H = J\vec{S}_d\cdot\vec{S}_0 + V\sum_\sigma\left(c_{d\sigma}^\dagger c_{0\sigma} + \text{h.c.}\right) - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2$$

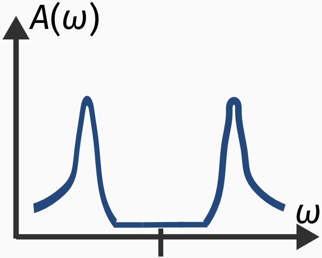
- ✓ **spin+charge** dimer with attractive zeroth site
- ✓ spin-singlet + charge-triplet-zero in gr-state

$$|\Psi\rangle_{GS} = \frac{1}{2\sqrt{2}}\left[|\uparrow_d\downarrow_0\rangle - |\downarrow_d\uparrow_0\rangle\right] + \frac{1}{2\sqrt{2}}\left[|2_d, 0_0\rangle - |0_d, 2_0\rangle\right]$$



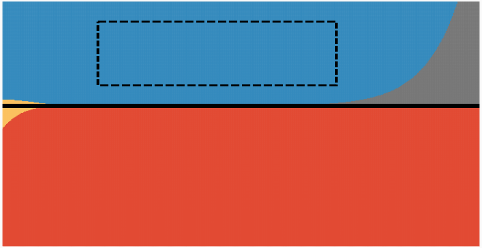
# Low-energy effective Hamiltonians and ground-states

**Regime 3:** $|U_b| > J/4$



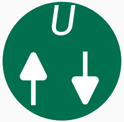

- ✓ $J, V$ irrelevant,
- ✓ $U$ relevant,

$$H = -U/2\left(\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow}\right)^2 - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2 + \sum_{k<\Lambda^*} \epsilon_k \hat{n}_{k\sigma}$$

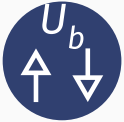


### Regime 3: $|U_b| > J/4$

### Zero-bandwidth limit




impurity site



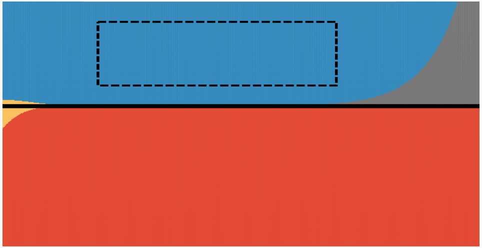

zeroth site

$$H = -U/2\left(\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow}\right)^2 - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2$$

✓ impurity site detaches from bath

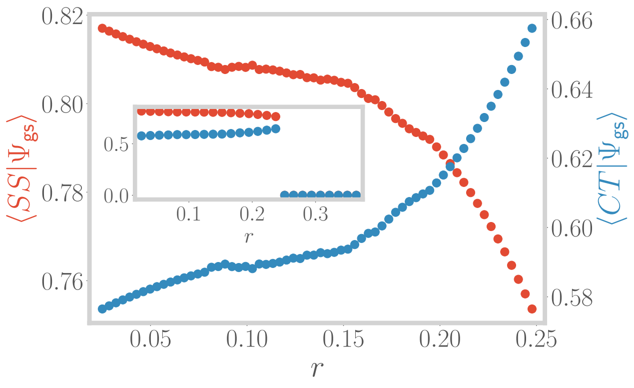
✓ **local moment** ground-state

$$|\Psi\rangle_{GS} = |\uparrow,\downarrow\rangle_d \otimes |0,2\rangle_0$$



# Low-energy effective Hamiltonians and ground-states

**Ground-state overlaps** with spin singlet and charge triplet zero

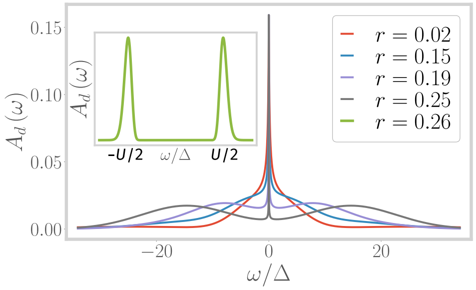


# Nature of the Transition

# Gapping of the impurity spectral function

✓ Correlated **three peak** structure at $|U_b| \lesssim J/4$

✓ Broad central peak at $|U_b| \ll J/4$

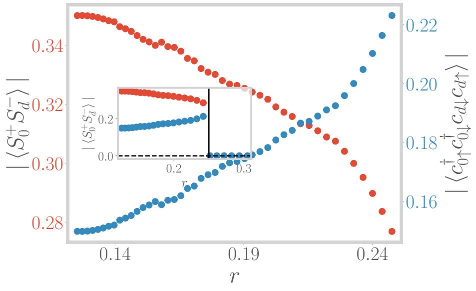



✓ hard central **gap** for $|U_b| > J/4$

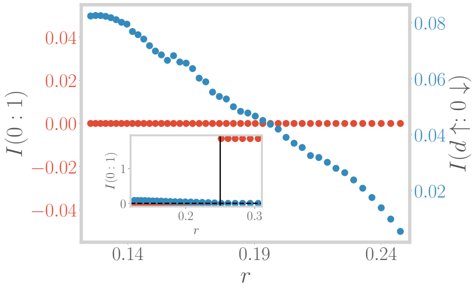
# Destruction of the Kondo cloud

The Kondo cloud **weakens, and is destroyed** at the transition.

✓ vanishing of impurity-bath correlations




✓ transfer of entanglement into the bath

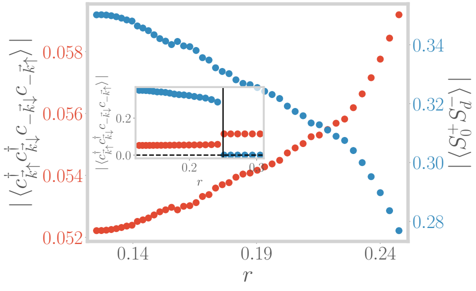


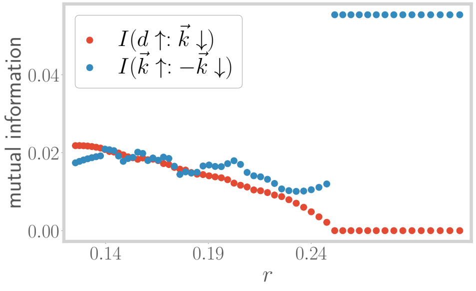
# Growth of pairing fluctuations in the bath

**Subdominant pairing fluctuations**, near the transition...

✓ growth of fluctuations in Cooper channel, at the cost of spin-flip fluctuations

✓ mutual information within the bath maximised after transition



# Universal Theory near the Transition

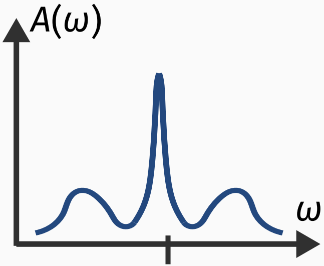
## Minimal effective model for the transition

✓ For $|U_b| \lesssim J/4$, central peak and side peaks are **well-separated**

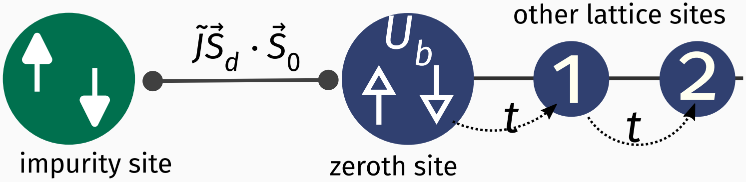
✓ **Integrate out** charge fluctuations through Schrieffer-Wolff transformation

$$H_{\text{eff}} = \tilde{J}\vec{S}_d \cdot \vec{S}_0 - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2 + H_{\text{K.E.}}$$

$$\text{RG equation for } \tilde{J}: \quad \Delta\tilde{J} \sim \tilde{J}\left(\tilde{J} + 4U_b\right)$$




✓ **captures** the criticality, and the strong-coupling and local moment phases

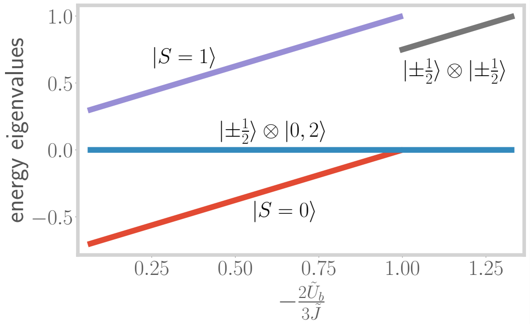



Suggests that $J$ **and** $U_b$ **are the minimal & universal ingredients** for transition!

# Capturing the level crossing at the transition from a two-site model

✓ Obtain two-site model by taking **zero bandwidth** limit

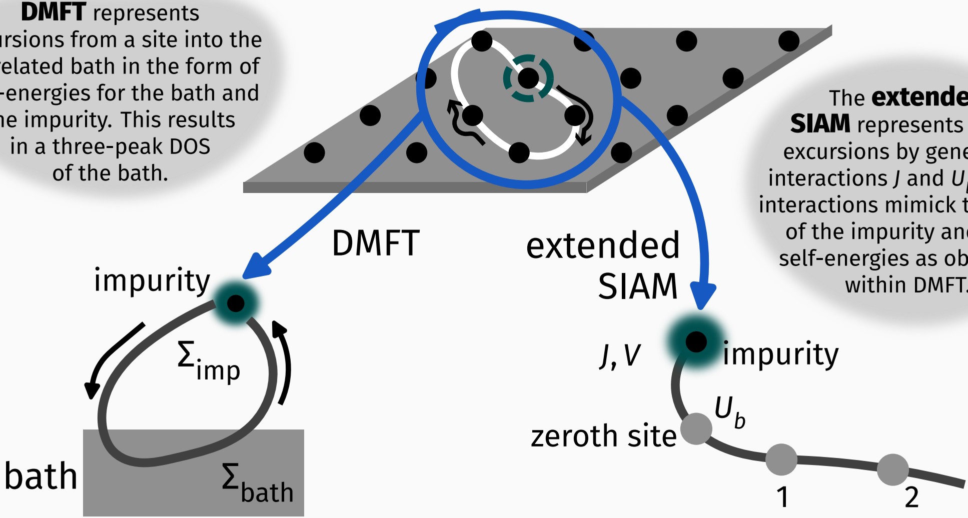
✓ spectrum shows **level crossing** between singlet and local moment states



# Insights into DMFT

# Extended SIAM in the context of DMFT

**DMFT** represents excursions from a site into the correlated bath in the form of self-energies for the bath and the impurity. This results in a three-peak DOS of the bath.

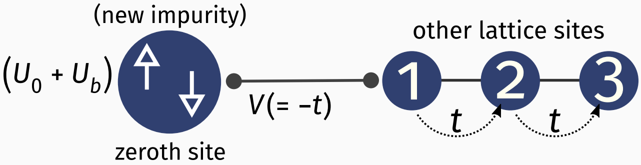



The **extended SIAM** represents these excursions by generating interactions $J$ and $U_b$. These interactions mimick the effect of the impurity and bath self-energies as observed within DMFT.

# Equivalence of the impurity site and the bath zeroth site

✓ **Integrate out impurity site** from fixed point Hamiltonian via a single URG transformation

✓ Generates additional correlation $U_0$ on zeroth site




✓ $J$ is relevant and the largest scale $\longrightarrow$ **repulsive correlation**:

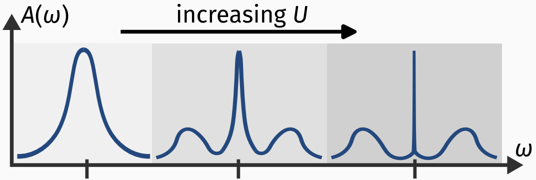
$$U_0 + U_b \simeq J > 0$$

Moeller et al. 1995.

# Equivalence of the impurity site and the bath zeroth site

✓ $J$ acts a **symmetrisation mechanism** between impurity and zeroth sites

✓ **Coherent** spin-flip scatterings ensure similarity of spectral functions



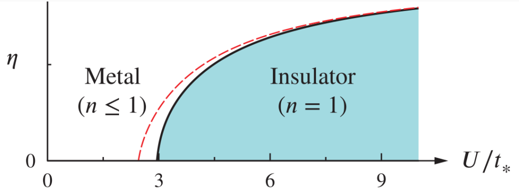

Essence of **self-consistency**: Equivalence of impurity and zeroth sites!

Moeller et al. 1995.

# Observation of a coexistence region

✓ DMFT observes a **coexistence region** near the critical point, for $U_{c1} < U < U_{c2}$

✓ Insulating when coming in from the insulator, metallic when coming in from the metal




(D.E. Logan and M.R Galpin 2016 JPCM 28 025601)

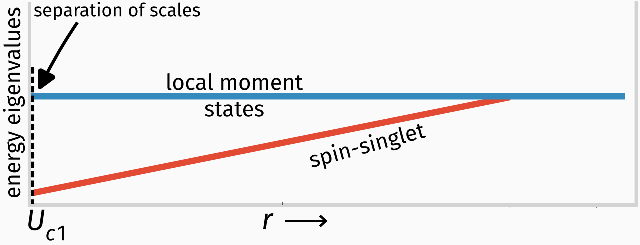
✓ Mott gap appears **discontinuously** after the transition, through a **continuous** sharpening of the central peak.

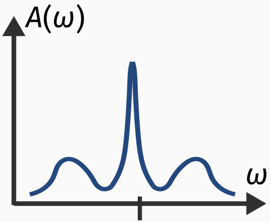
✓ **True** transition believed to occur at $U_{c2}$.

# Observation of a coexistence region

**Can be explained** heuristically using the two site spectrum

✓ Initial point is when the side peaks get separated (near-zeroes in the spectral function)

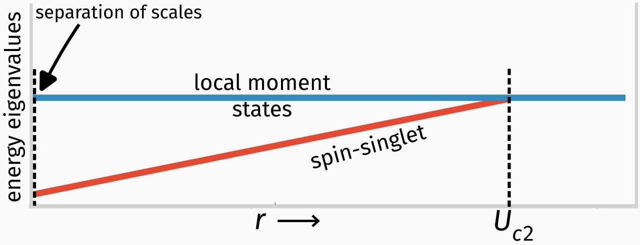


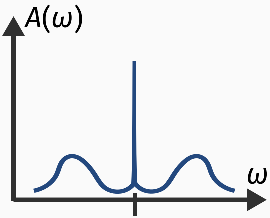



# Observation of a coexistence region

**Can be explained** heuristically using the two site spectrum

✓ $U_{c2}$ is the point where the levels cross

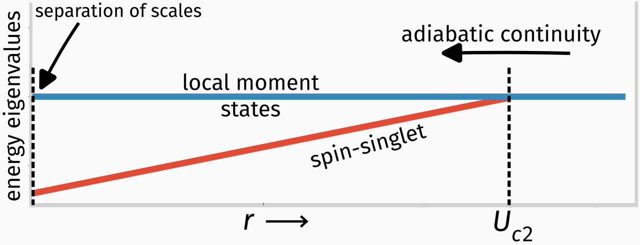


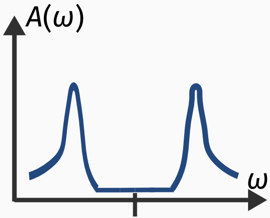



# Observation of a coexistence region

**Can be explained** heuristically using the two site spectrum

✓ Coming from $U > U_{c2}$, **adiabatic continuity** allows DMFT to stay on the local moment state...

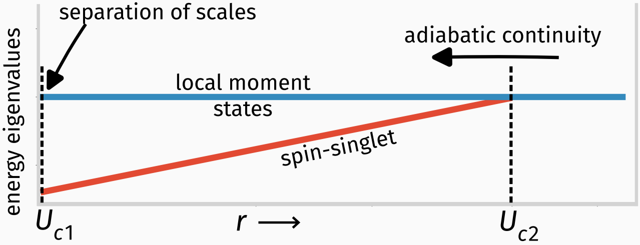


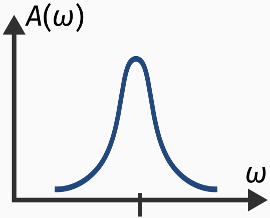



# Observation of a coexistence region

**Can be explained** heuristically using the two site spectrum

✓ For $U < U_{c1}$, local moment state is too unstable, **relaxes** to the true ground state.

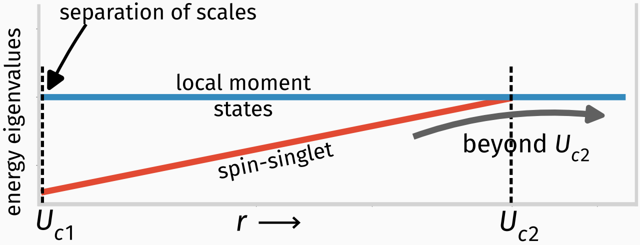


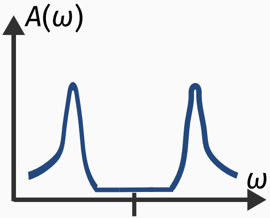



# Observation of a coexistence region

**Can be explained** heuristically using the two site spectrum

✓ For $U > U_{c1}$, charge sector separated by large $U$, leads to the **discontinuous** appearance of finite gap



# Comparison against NRG-DMFT correlation functions

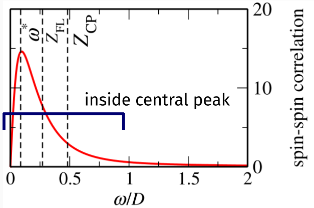
**Poor Man's scaling** of the effective Kondo model

*[K. Held, R. Peters, and A. Toschi. PRL 2013]*

✓ shows **quantitative** agreement with NRG-DMFT (crossover scale and kinks in self-energy)

✓ Suggests that the minimal model can capture **spin susc.**

[K. Held, R. Peters, and A. Toschi.
PRL 110, 246402 (2013)]




✓ Our $J - U_b$ model **goes further** by capturing physics beyond the transition

# Comparison against NRG-DMFT correlation functions

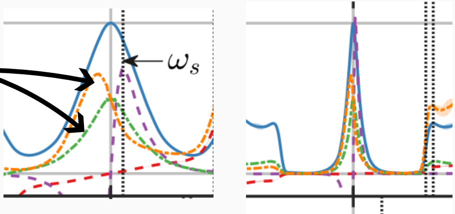
**Doublon-holon correlators** of the Hubbard model

*[S. B. Lee, J. v Delft, and A. Weichselbaum. PRL 2017]*

Lee et. al show **peaks** in doublon-holon correlators near zero energy within the central peak.

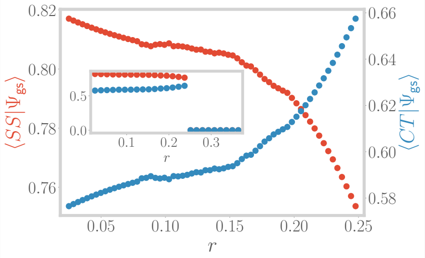
doublon-holon correlators

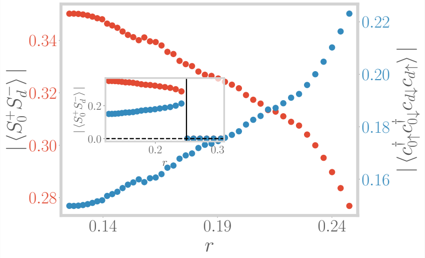
[S. B. Lee, J. v Delft, and A. Weichselbaum. PRL 2017]




$U$ = 2 $U$ = 2.7

We find support for this in the form of **increasing ground-state charge correlations and overlap**.



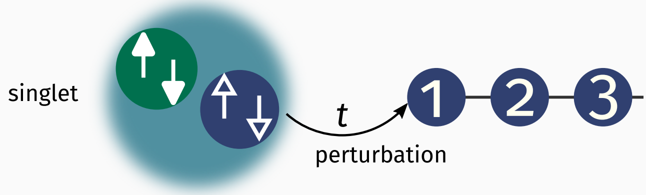



# Low-energy excitations of the bath

# Effect on the local Fermi liquid

What about the **low-energy excitations** of the bath, that lie above the singlet ground state?

✓ treat hopping between singlet and bath as perturbation




✓ Up to fourth order, charge sector becomes repulsive...

$$H_{\text{eff}} = \frac{2t^4}{\tilde{J}\left(3\tilde{J}/4 + \tilde{U}_b\right)^2}\left[\hat{n}_{1\uparrow}\hat{n}_{1\downarrow} + \text{p-h}\right] + H_{\text{K.E.}}$$

✓ FL term blows up towards transition, signaling **breakdown** of Fermi liquid theory and loss of adiabaticity.

# Effect on the local Fermi liquid

Vanishing of the **Kondo scale** $T_K$ towards the transition

✓ Kondo temperature scale $T_K$ can be obtained from the $J - U_b$ model, but with a Lorentzian DOS in the bath, $\rho = \rho_0 \Gamma^2 / (D^2 + \Gamma^2)$.

✓ Near the transition $r = -U_b / J_0 \to \frac{1}{4}$ and $\Gamma \to 0$, the fixed-point momentum energy scale $D^*$ can be approximated as

$$D^* = D_0 \exp\left[ \frac{\left(2\omega + U_b + J_0/2\right)^2}{8 U_b \rho_0 \Gamma^2} \ln |1 - 4r| \right], \quad D_0 = \text{UV cutoff} .$$

✓ Kondo temperature can be defined as $T_K = D^* / k_B$, vanishes towards the critical point

$$T_K = \frac{D_0}{k_B} \exp\left[ \frac{\left(2\omega + U_b + J_0/2\right)^2}{8 U_b \rho_0 \Gamma^2} \ln |1 - 4r| \right]$$

Held et al. 2013.

# Effect on the local Fermi liquid

How do the imaginary part of **self-energy** $\Sigma$ and the **quasiparticle residue** behave near the transition?

✓ Following the renormalised perturbation theory approach of Hewson, $\mathrm{Im}\left[\Sigma(\omega)\right]$ is

$$\mathrm{Im}\left[\Sigma(\omega)\right] \sim u^2\omega^2, \quad u = \frac{2t^4}{\tilde{J}\left(3\tilde{J}/4 + \tilde{U}_b\right)^2}$$

✓ As $r \to 0$, $u \to \infty$, signalling a vanishing lifetime of the quasiparticles

✓ Quasiparticle residue $Z$ for 1-particle excitations is proportional to $T_K$:

$$Z \sim T_K$$

$$Z \to 0 \text{ as } r \to 0$$

Hewson 1993; Coleman 2015.

# Broad conclusions

- ✓ The extended SIAM appears to capture the DMFT transition and **self-consistency**.
- ✓ The key ingredient is a **competition** between Kondo screening physics and a local attractive correlation in the bath.
- ✓ Crucial feature of the journey is the enhancement of **pairing fluctuations** in the bath - leads to destruction of Kondo cloud.
- ✓ An emergent self-consistency is achieved through the qualitative similarity of the spectral functions of the impurity and zeroth sites.
- ✓ SOMETHING ABOUT THE FINAL CALCULATIONS (MAYBE NON-FERMI LIQUID PHYSICS)

# Future Prospects

# Future Prospects

- ✓ The extended SIAM can be improved by considering **multiple impurities** and general impurity **filling**.
- ✓ We are developing a new **tiling-based auxiliary model method** can used for studying other models of strong-correlations as well as topologically active or flat band systems.
- ✓ The URG can be applied to **heavy-fermion materials** towards a study of phase diagram and unconventional superconductivity, as well as Kondo insulators.
- ✓ Interacting systems in a magnetic field is also a potential area of study, specifically **fractional Chern insulators** (e.g. the fractional quantum hall effects).

# Acknowledgements

Gracious thanks to

- ✓ my collaborators **S. Patra, A. Mukherjee, Prof. A. Taraphder** and **Prof. N. S. Vidhyadhiraja**,
- ✓ my RPC members **Prof. Ritesh Singh** and **Prof. Anandamohan Ghosh** for instructive feedback,
- ✓ **Prof. H. Casini, Prof. N. Banerjee** and **Shibendu G. Chowdhury** for very fruitful discussions, and
- ✓ IISER Kolkata for funding.

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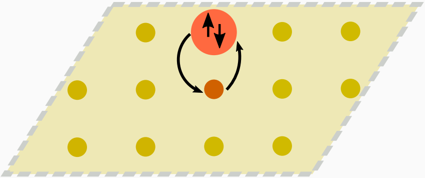
# Other Projects

# Theory for the single-channel Kondo cloud

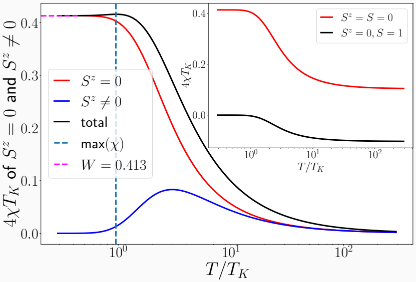
**Phys. Rev. B 105, 085119**

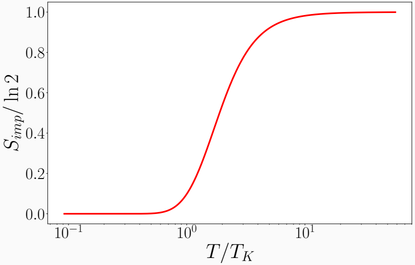
Anirban Mukherjee, **Abhirup Mukherjee**, N. S. Vidhyadhiraja, A. Taraphder, and Siddhartha Lal



# Theory for the single-channel Kondo cloud

✓ spectral function & magnetic susceptibility




✓ local Fermi liq. & orthogonality catastrophe

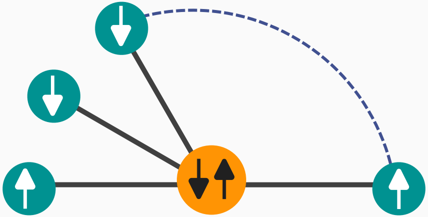
✓ thermal entropy

Kondo 1964; Wilson 1975; Andrei et al. 1983; Hewson 1993; Nozieres 1974; Anderson 1970; Tsvelick et al. 1983; Affleck et al. 1993; Goldhaber-Gordon et al. 1998; V. Borzenets et al. 2020; Sakai et al. 1989; Costi et al. 1990; Nozaki et al. 2012; Affleck 1995.

# Role of degeneracy in the multi-channel Kondo problem

**arXiv:2205.00790**

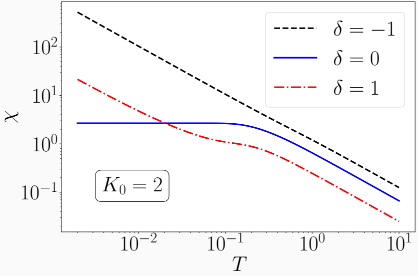
Siddhartha Patra, **Abhirup Mukherjee**, Anirban Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, Siddhartha Lal

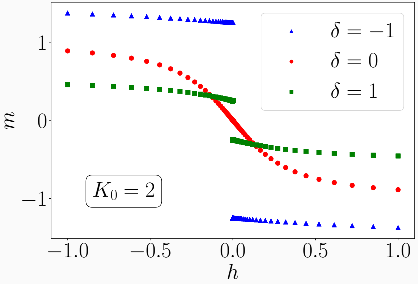


# Role of degeneracy in the multi-channel Kondo problem

✓ Intermediate-coupling RG fixed point Hamiltonian and **degenerate** ground states

✓ Degree of compensation, magnetization and susceptibility show **incomplete screening**



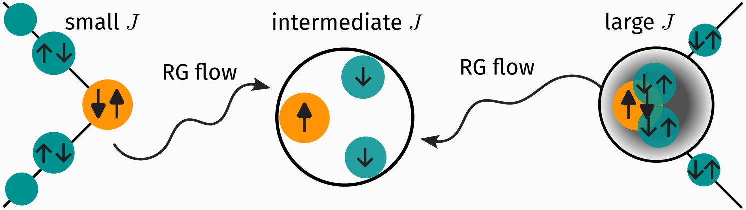




Nozières, Ph. et al. 1980; Tsvelick et al. 1985; Affleck et al. 1993; Gan 1994; Affleck et al. 1991; Emery et al. 1992; Bulla et al. 1998.

# Role of degeneracy in the multi-channel Kondo problem

✓ Local **marginal Fermi liquid** within the low-energy excitations of the bath

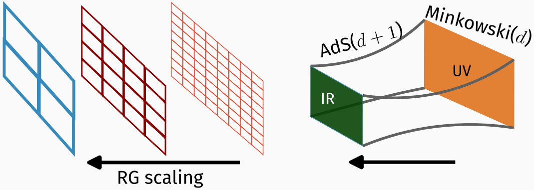
✓ **Duality** relations constrain the RG flows of the MCK model





Nozières, Ph. et al. 1980; Tsvelick et al. 1985; Affleck et al. 1993; Gan 1994; Affleck et al. 1991; Emery et al. 1992; Bulla et al. 1998.

# Entanglement scaling in free fermions: holography & topology



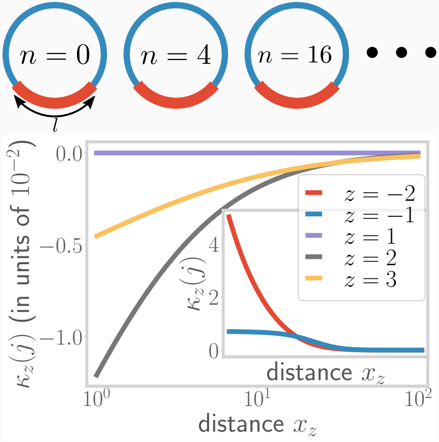
# Entanglement scaling in free fermions: holography & topology - Summary

✓ Under coarse-graining or fine-graining in $k$–space, entanglement of free Dirac fermions shows hierarchical onion-like structure.

✓ Entanglement scaling can be used to define distances, leads to additional spatial dimension $\longrightarrow$ holography.

✓ Emergent distances and curvature can be related to RG beta function; the sign of the curvature is topological.

✓ Pole structure of the entanglement tracks the Luttinger volume - invariant under the scaling transformations.



# Creating subsystems

Free Dirac fermions on torus: $k_x^n = \dfrac{2\pi}{L_x} n, \quad n \in \mathbb{Z}$; define **sparsity** = $\Delta n = 1$

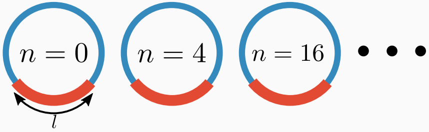
**Simplest** choice: the entire set

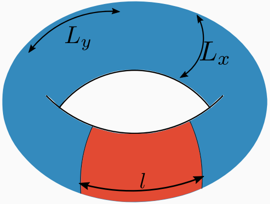
sparsity = 1 $\longrightarrow n \in \{-N, -(N-1), -(N-2), \ldots, -1, 0, 1, \ldots, N-2, N-1, N\}$

**Coarser** choices: increase sparsity

sparsity = 2 $\longrightarrow n \in \{-N, -(N-2), -(N-4), \ldots, -2, 0, 2, \ldots, N-4, N-2, N\}$

sparsity = 4 $\longrightarrow n \in \{-N, -(N-4), -(N-8), \ldots, -4, 0, 4, \ldots, N-8, N-4, N\}$



# Subsystem entanglement entropy: Entanglement hierarchy

$$S_{A_z(j)} = f_z(j) c \alpha L_x - c \log \left| 2 \sin\left(\pi f_z(j) \phi\right) \right|$$

$$i < j, \quad S_{i \cup j} = \begin{cases} S_i, & z > 0 \\ S_j, & z < 0 \end{cases}$$

✓ presents a **hierarchy** of entanglement ⟶ EE distributed across RG steps
RG transformation ⟶ reveals entanglement

✓ distribution of entanglement also present in **multipartite** entanglement

Calabrese et al. 2004; Casini et al. 2005; Arias et al. 2015; Chen et al. 2017; Murciano et al. 2020.

# Mutual information = distance

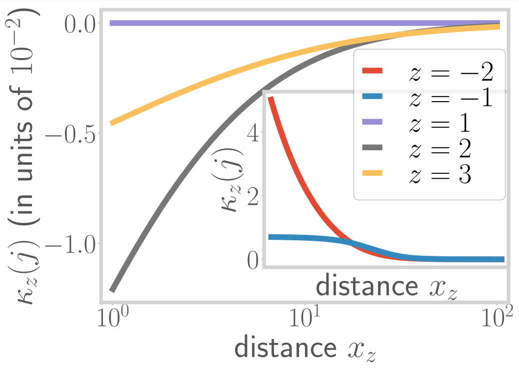
**Mutual information**: $I^2(A:B) \equiv S(A) + S(B) - S(A \cup B)$ (non-negative)

Define distances using mut. info.

$$x_z(j) = \log t_z(j), \quad y_z(j) = \log t_z(j \pm 1)$$

$$v_z(j) \equiv \Delta y_z(j)/\Delta x_z(j), \quad v' = \Delta v_z(j)/\Delta x_z(j)$$

Curvature as well: $\kappa_z(j) = \dfrac{v_z'(j)}{[1 + v_z(j)^2]^{\frac{3}{2}}}$




Van Raamsdonk 2010; Lee et al. 2016; Mukherjee et al. 2022; Lee 2010; Lee 2014; Qi 2013; Lee et al. 2016; Mukherjee et al. 2020a; Mukherjee et al. 2020b; Ryu et al. 2006b; Ryu et al. 2006a; Nozaki et al. 2012.

# RG evolution = emergent distance

✓ Distances and curvature can be related to an RG **beta function**

✓ Amounts to an **explicit demonstration** of the holographic principle

✓ Sign of curvature is **topological**, can be written in terms of winding numbers

Maldacena 1999; Ryu et al. 2006a; Holzhey et al. 1994.

# Topological nature of geometry-independent term

$$S_{A_z(j)} = f_z(j) c \alpha L_x - \underbrace{c \log \left| 2 \sin\left(\pi f_z(j) \phi\right) \right|}_{=Q(\phi),\text{geometry-independent term}}$$

- ✓ $Q(\phi)$ is periodic in the flux $\phi$, $\phi = 1$ transports a charge across Fermi surface
- ✓ pole structure of $\left(\sin \frac{\pi}{4} - |\sin\left(\pi f_z(j)\right)\phi|\right)^{-1}$ counts number of states $\longrightarrow$ tracks Luttinger volume
- ✓ Luttinger volume is topological, so is $Q(\phi)$; $Q(\phi)$ can be expressed in terms of winding numbers

**Luttinger et al. 1960; Luttinger 1960; Oshikawa 2000; Seki et al. 2017; Mukherjee et al. 2020a; Heath et al. 2020.**