# Pseudogapped Mott criticality: Stretching Kondo Screening to Breaking Point

## How a Fermi liquid gives way to Mott insulator in 2D

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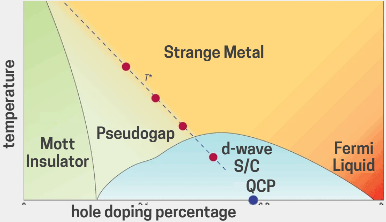
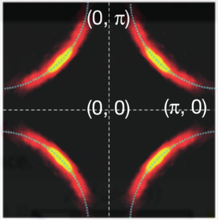
# Some Questions

The anomalous **pseudogap** (PG) phase exhibits nodal-antinodal dichotomy.

No general consensus yet regarding

- nature of $T = 0$ ground states of the cuprates
- **relation** of PG to Mott insulating and superconducting phases proximate to it
- how pseudogap **evolves** from weak- to strong-coupling
- nature of correlations and entanglement within the transition to the model

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Keimer et al. 2015; Proust and Taillefer 2019; Loeser et al. 1996; Norman et al. 1998; Hashimoto et al. 2014; Kyung et al. 2006; Macridin et al. 2006; Wu et al. 2018; Mukherjee and Lal 2020; Hille et al. 2020.

# New Auxiliary Model Approach To Interacting Fermions

1. Solve an appropriate **impurity model**, $H_{\text{imp}}$
   - Lattice symmetry
   - Impurity phase transition

2. **Construct lattice** model by applying manybody translation operators:

$$H_{\text{latt}} = \sum_{\mathbf{r}} T^{\dagger}(\mathbf{r}) H_{\text{imp}}(\mathbf{r_0}) T(\mathbf{r})$$

3. Relate computables across the models, using manybody Bloch's theorem

**Greens functions**:

$$\tilde{G}(\mathbf{K}\sigma;\omega) = G^{>}(T^{\dagger}_{\mathbf{K}\sigma}, \omega - \varepsilon_{\mathbf{K}}) + G^{<}(T^{\dagger}_{\mathbf{K}\sigma}, \omega + \varepsilon_{\mathbf{K}})$$

Equal-time **correlation** functions:

$$C_O(\mathbf{k}_1, \mathbf{k}_2) = \sum_{\Delta} \langle \mathbf{r}_c + \Delta | \tilde{O}(\mathbf{k}_2) | \mathbf{r}_c \rangle \langle \mathbf{r}_c | \tilde{O}^{\dagger}(\mathbf{k}_1) | \mathbf{r}_c \rangle$$

where

$$G^{>}(O^{\dagger}, t) = -i \langle O(t) O^{\dagger} \rangle$$

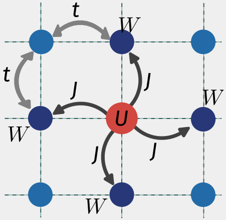
$$T_{\mathbf{K}\sigma} = c_{\mathbf{K}\sigma} \left( \textstyle\sum_{\sigma'} c^{\dagger}_{d\sigma} + \text{h.c.} \right) + c_{\mathbf{K}\sigma} \left( S^{+}_{d} + \text{h.c.} \right)$$

$$\tilde{O}(\mathbf{r}) = O(\mathbf{r}) O^{\dagger}(d)$$

Stoyanova 2006; Maier et al. 2005; Sakai 2023.

# The Core Ingredient: A Lattice-Embedded Impurity Model

$$H_{\text{imp}} = H_{\text{2D-TB-KE}} + V\sum_{Z,\sigma}\left(c^{\dagger}_{d\sigma}c_{Z\sigma} + \text{h.c.}\right) + J\sum_{Z}\mathbf{S}_d\cdot\mathbf{S}_Z - \frac{W}{2}\sum_{Z}\left(n_{Z\uparrow} - n_{Z\downarrow}\right)^2$$




- $J_{\mathbf{k},\mathbf{k}'}$ has $C_4$**-symmetry** instead of $s$-wave symmetry

$$J_{\mathbf{k},\mathbf{k}'} = \frac{J}{2}\left[\cos\left(\mathbf{k}_x - \mathbf{k}'_x\right) + \cos\left(\mathbf{k}_y - \mathbf{k}'_y\right)\right]$$

Kai-Hua et al. 2002.

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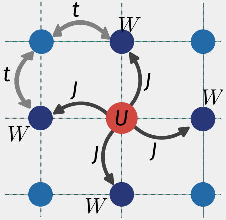


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## Map to Hubbard-Heisenberg Model

$$H_{\text{latt}} = \sum_{\mathbf{r}} T^{\dagger}(\mathbf{r})H_{\text{imp}}(\mathbf{r_0})T(\mathbf{r})$$

$$= -\frac{\tilde{t}}{\sqrt{Z}}\sum_{\langle\mathbf{r}_i,\mathbf{r}_j\rangle;\sigma}\left(c^{\dagger}_{\mathbf{r}_i,\sigma}c_{\mathbf{r}_j,\sigma} + \text{h.c.}\right) - \tilde{\mu}\sum_{\mathbf{r}}\hat{n}_{\mathbf{r},\sigma}$$

$$+ \frac{\tilde{J}}{Z}\sum_{\langle\mathbf{r}_i,\mathbf{r}_j\rangle}\mathbf{S}_{\mathbf{r}_i}\cdot\mathbf{S}_{\mathbf{r}_j} - \frac{1}{2}\tilde{U}\sum_{\mathbf{r}}\left(\hat{n}_{\mathbf{r},\uparrow} - \hat{n}_{\mathbf{r},\downarrow}\right)^2$$

$$\tilde{t} = t + 2V$$

$$\tilde{U} = U + W$$

$$\tilde{\mu} = 2\mu + \eta, \quad \tilde{J} = 2J$$

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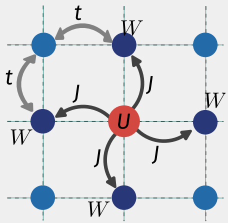


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- We work in large $U$ limit
- SW transformation $\rightarrow$ $J - W$ **model**

Kai-Hua et al. 2002.

# Pseudogapping Transition from Kondo Breakdown

Unitary RG analysis - integrate out high-energy states in the conduction bath:

$$\Delta J^{(j)}_{\mathbf{k}_1,\mathbf{k}_2} = -\sum_{\mathbf{q}\in \mathrm{PS}} \frac{J^{(j)}_{\mathbf{k}_2,\mathbf{q}} J^{(j)}_{\mathbf{q},\mathbf{k}_1} + 4 J^{(j)}_{\mathbf{q},\bar{\mathbf{q}}} W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}}}{\omega - \frac{1}{2}|\varepsilon_j| + J^{(j)}_{\mathbf{q}}/4 + W_{\mathbf{q}}/2}$$

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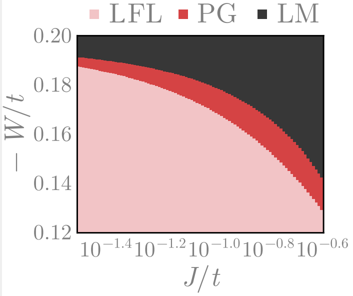


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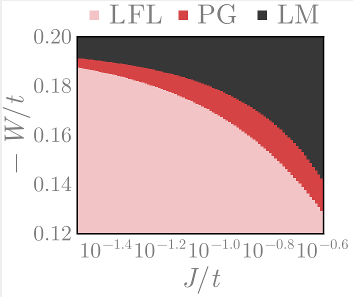
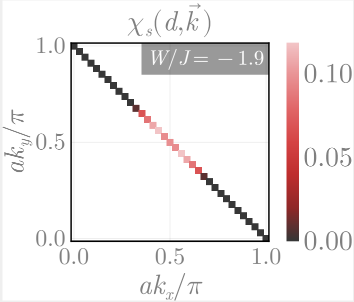
- Impurity-bath spin correlations show $k$-differentiation



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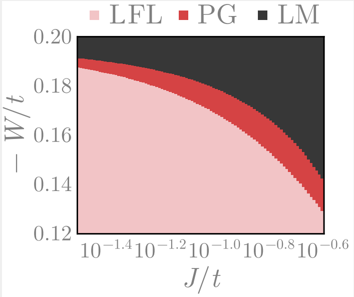
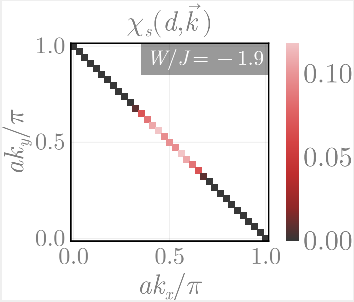
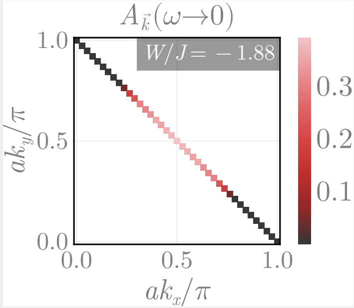
- Impurity-bath spin correlations show $k$–differentiation
- Lattice model DOS shows **P-gap**



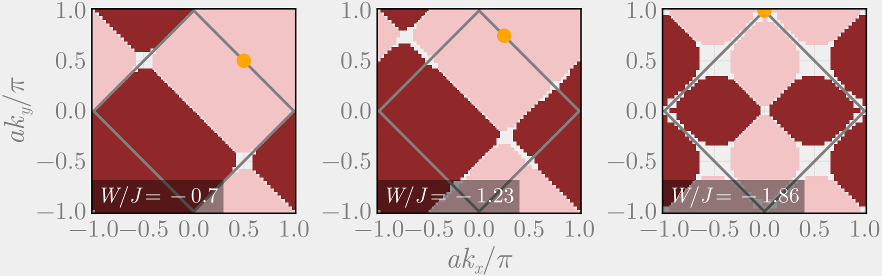



# Unravelling of Kondo screening

The Kondo breakdown process can be visualised in terms of **zeros** of $J_{\mathbf{k}_N,\mathbf{k}}$.

- $J_{\mathbf{k}_N,\mathbf{k}}$ for $\mathbf{k}$ close to the **adjacent nodes** turn RG irrelevant first, and a patch of zeros subsequently appears in $J_{\mathbf{k}_N,\mathbf{k}}$ around this point.



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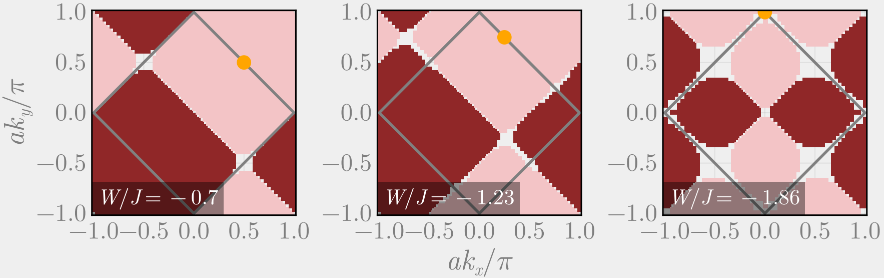

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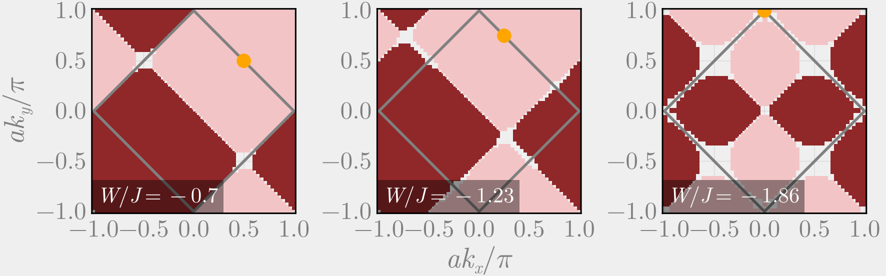
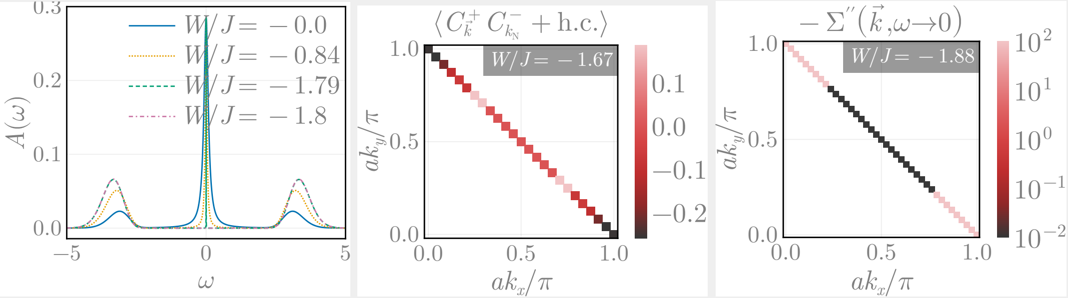
- At $W = W_{PG}$, the **antinode** joins this connected region of zeros in $J_{\mathbf{k}_1,\mathbf{k}_2}$, marking the onset of the PG.



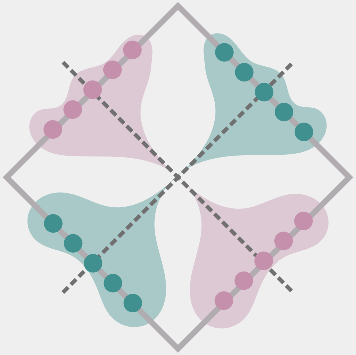
# Dynamical Spectral Weight Transfer

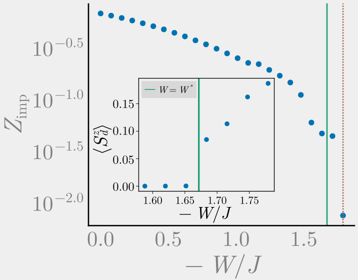
- strong fluctuations observed in **charge correlations** between the gapless nodal and gapped antinodal regions in PG regime
- PG formation results from the **transfer of spectral weight** from low to high energies
- PG coincides with the appearance of poles of the lattice model self-energy $\Sigma(\mathbf{k}, \omega = 0)$ near the antinodes

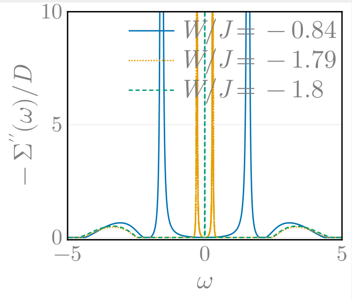


# Non-Fermi liquid nature of the pseudogap

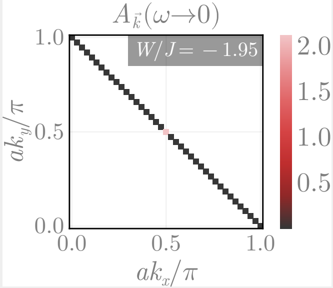
- In PG phase, Kondo processes between adjacent $k$–space quadrants are removed at low-energies
- Effective **two-channel Kondo** description - each pair of opposite quadrants forms a channel
- **Non-Fermi liquid** physics - vanishing quasiparticle residue, and Σ poles near $\omega = 0$





# Singular Nodal Metal




Close to the transition, Kondo cloud consists of concentrated **nodal regions**

Low-energy excitations

- Integrate out impurity spin-flips ( $J^2/W$)
- SW transformation → effective Hamiltonian

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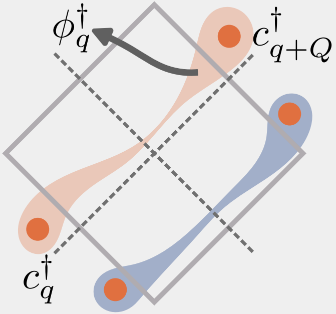


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Emergent modes: $\phi_{\mathbf{q},\sigma} = \frac{1}{\sqrt{2}}\left(c_{\mathbf{N}_1+\mathbf{q},\sigma} - c_{\mathbf{N}_1+\mathbf{Q}_1-\mathbf{q},\sigma}\right)$, $r = \phi^\dagger\phi$

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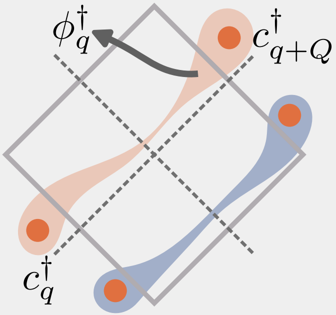


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$$\Delta\tilde{H} = \underbrace{\sum_{\mathbf{q},\sigma} \frac{|\varepsilon_{\mathbf{N}_1+\mathbf{q}}|\varepsilon_{\mathbf{N}_1+\mathbf{q}}}{-W} r_{\mathbf{q},\sigma}}_{\text{dispersion}} + \sum_{\mathbf{q}_1,\mathbf{q}_2,\sigma} \frac{J^{*2}}{-4W}\left[\underbrace{r_{\mathbf{q}_1\sigma}\left(1 - r_{\mathbf{q}_2\bar{\sigma}}\right)}_{\text{density interaction}} -(1 - \delta_{\mathbf{q}_1,\mathbf{q}_2})\underbrace{\phi^\dagger_{\mathbf{q}_1,\bar{\sigma}}\phi^\dagger_{\mathbf{q}_1,\sigma}\phi_{\mathbf{q}_2,\sigma}\phi_{\mathbf{q}_2,\bar{\sigma}}}_{\text{fwd/tang. pair transfer}}\right]$$

# Singular Nodal Metal: $q_1 = q_2$

We focus on the simplified case of zero momentum transfer $q_1 = q_2$:

$$\Delta\tilde{H} = \sum_{\mathbf{q},\sigma} \epsilon_{\mathbf{q}} r_{\mathbf{q},\sigma} + u \sum_{\mathbf{q},\sigma} r_{\mathbf{q}\sigma} r_{\mathbf{q}\bar{\sigma}}, \quad \phi_{\mathbf{q},\sigma} = \frac{1}{\sqrt{2}}\left(c_{\mathbf{N}_1+\mathbf{q},\sigma} - c_{\mathbf{N}_1+\mathbf{Q}_1-\mathbf{q},\sigma}\right), \; r = \phi^\dagger \phi$$

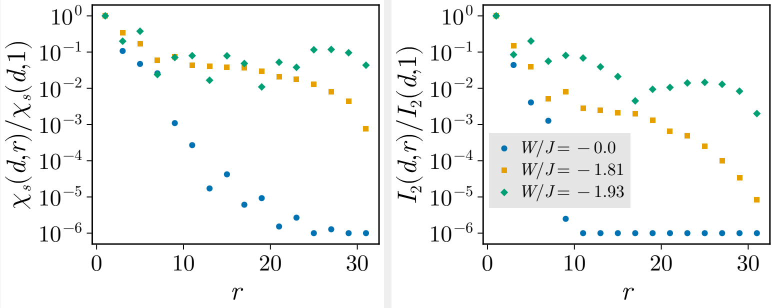
$$\epsilon_{\mathbf{q}} = \frac{|\varepsilon_{\mathbf{N}_1+\mathbf{q}}|\varepsilon_{\mathbf{N}_1+\mathbf{q}}}{-W} + \frac{J^{*2}}{-4W}, \; u = \frac{J^{*2}}{4W}$$

- Nodal metal is described by a **Hatsugai-Kohmoto model**.
- Non-Fermi liquid excitations.

$$\Sigma \sim \frac{u^2}{\omega}, Z \sim \omega^2$$

# Non-local nature of the pseudogap

- real-space correlations and entanglement undergo a crossover within the pseudogap from short-ranged to **long-ranged** behaviour
- This is further evidence of the **breakdown of local Kondo screening**, and resulting Landau quasiparticle excitations
- the Mott transition observed by us for the Hubbard-Heisenberg model on the square lattice lies well beyond the paradigm of **local quantum criticality**



# Conclusions

- On a 2D square lattice, a Fermi liquid must morph into a **non-Fermi liquid pseudogap phase** in order to give rise to a Mott insulator
- $k$-space differentiated **Kondo breakdown** lies at the heart of this physics
- the pseudogap features increasingly **non-local correlations** as the system is driven towards the transition

## Future Directions

- Heavy fermions?
- Doping the pseudogap phase?
- Other impurity model geometries - spin liquids?