# The Unitary Renormalization Group Method

## The General Idea

- Apply unitary many-body transformations to the Hamiltonian

$$\begin{tikzcd} H_j \arrow[d, "U_j"] \\ H_{j-1} \end{tikzcd}$$

"Unitary renormalisation group for correlated electrons-I: a tensor network approach" 2020a; "Unitary renormalisation group for correlated electrons-II: insights on fermionic criticality" 2020b.

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- Successively decouple high energy states
- Obtain sequence of Hamiltonians and hence scaling equations

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## Select a UV-IR Scheme

**UV shell**

$\vec{k}_N$ (zeroth RG step)

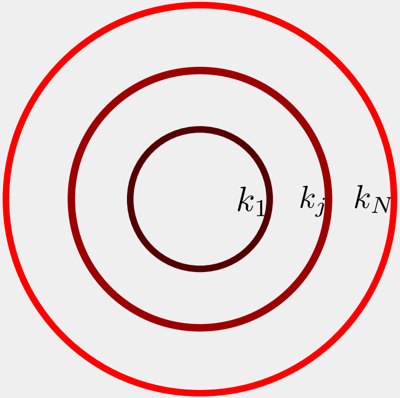
$\vdots$

$\vec{k}_j$ $\left(j^{\text{th}}\text{ RG step}\right)$

$\vdots$

$\vec{k}_1$ (Fermi surface)

**IR shell**

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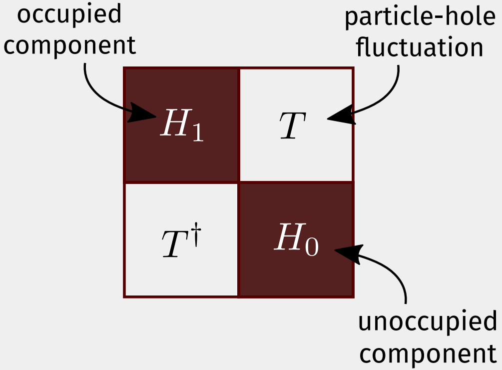
# The Unitary Renormalization Group Method

## Write Hamiltonian in the basis of $\vec{k}_j$

$$H_{(j)} = H_1 \hat{n}_j + H_0 \left(1 - \hat{n}_j\right) + c_j^\dagger T + T^\dagger c_j$$

$$2^{j-1}\text{-dim.} \longrightarrow \begin{cases} H_1, H_0 \longrightarrow \text{diagonal parts} \\ T \longrightarrow \text{off-diagonal part} \end{cases}$$

$(j)$ : $j^{\text{th}}$ RG step

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# The Unitary Renormalization Group Method

## Rotate Hamiltonian and kill off-diagonal blocks

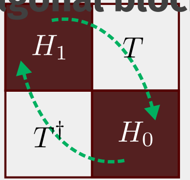
$$H_{(j-1)} = U_{(j)} H_{(j)} U^\dagger_{(j)}$$

$$U_{(j)} = \frac{1}{\sqrt{2}}\left(1 - \eta_{(j)} + \eta^\dagger_{(j)}\right), \quad \left\{\eta_{(j)}, \eta^\dagger_{(j)}\right\} = 1$$

$$\left.\eta^\dagger_{(j)} = \frac{1}{\hat{\omega}_{(j)} - H_D} c^\dagger_j T \right\} \rightarrow \text{many-particle rotation}$$

$$\hat{\omega}_{(j)} = (H_1 + H_0)_{(j-1)} + \Delta T_{(j)}$$

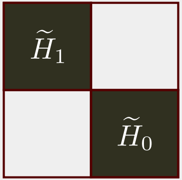
**(quantum fluctuation operator)**




$$\left[H_{(j)}, n_j\right] \neq 0$$

$$\left[H_{(j-1)}, n_j\right] = 0$$

$n_j$ becomes an **integral of motion** (IOM)

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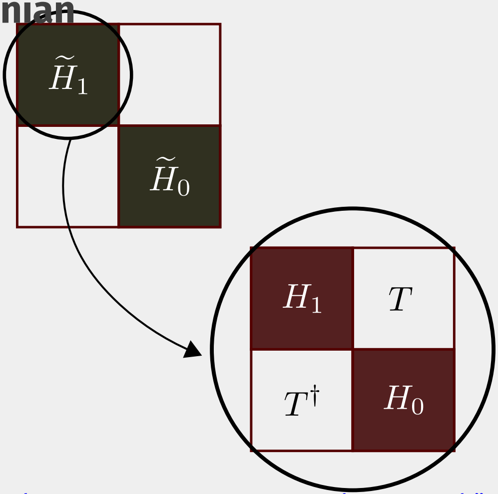
## Repeat with renormalised Hamiltonian




$$H_{(j-1)} = \widetilde{H}_1 \hat{n}_j + \widetilde{H}_0 \left(1 - \hat{n}_j\right)$$

$$\widetilde{H}_1 = H_1 \hat{n}_{j-1} + H_0 \left(1 - \hat{n}_{j-1}\right) + c_{j-1}^\dagger T + T^\dagger c_{j-1}$$

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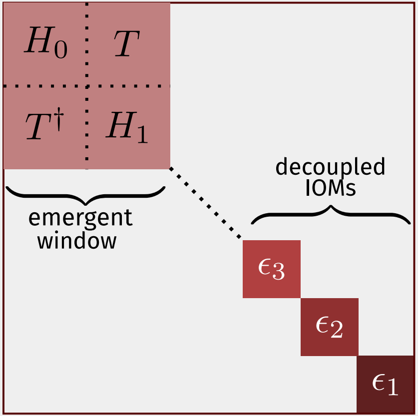
## RG Equations and Denominator Fixed Point

$$\Delta H_{(j)} = \left(\hat{n}_j - \frac{1}{2}\right)\left\{c_j^\dagger T, \eta_{(j)}\right\}$$

$$\eta_{(j)}^\dagger = \frac{1}{\hat{\omega}_{(j)} - H_D} c_j^\dagger T$$

Fixed point: $\hat{\omega}_{(j^*)} - (H_D)^* = 0$

**eigenvalue of $\hat{\omega}$ coincides with that of $H$**

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## Novel Features of the Method

- **Quantum fluctuation scale** $\hat{\omega}$ that tracks all orders of renormalisation

$$H_{(j-1)} = U_{(j)} H_{(j)} U_{(j)}^\dagger$$

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- Finite-valued fixed points for finite systems - leads to **emergent degrees of freedom**

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- **Quantum fluctuation scale** $\hat{\omega}$ that tracks all orders of renormalisation
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- **Spectrum-preserving** unitary transformations - partition function does not change
- Tractable low-energy effective Hamiltonians - allows **renormalised perturbation theory** around them

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