Research Progress Report: 2024 - 2025

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Acknowledgements

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Publications and Ongoing Projects

Published

- 2023 New J. Phys. 25 113011
- 2024 J. Phys. A: Math. Theor. 57 275401
- 2022 Phys. Rev. B 105, 085119
- 2023 J. Phys.: Condens. Matter 35 315601

Currently in Progress

- Development of auxiliary model-based method for interacting electrons [**arXiv**:2507.17201].
- Studies of plateau-to-plateau transition in integer quantum hall systems.

Ongoing Collaborations

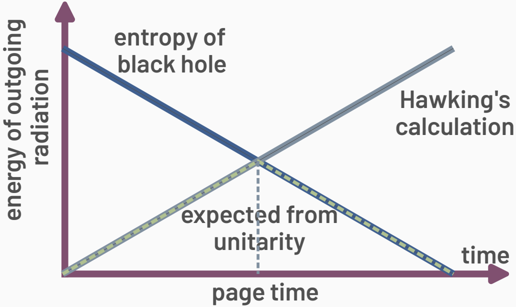
- Breakdown of Kondo screening in presence of magnetic field [DD, **AM**, SL]
- Quantum critical Mott MIT in 3-orbital impurity model [AK, DD, **AM**, NSV, SL]
- Universal features of Kondo breakdown in impurity models [DD, **AM**, SL]
- Search for non-Fermi liquid physics in mixed-valence eSIAM [AS, **AM**, SL]
- Spin-liquid and non-Fermi liquid phases in frustrated systems [SD, **AM**, SL]
- Hawking-Page entanglement transition in a Kondo system [DD, **AM**, SL]

Ongoing Collaborations

Hawking-Page Entanglement Transition in a Kondo System

Hawking radiation + black hole evaporation appears to **violate unitarity**

- Initially system is in **pure state**.
- Hawking's calculation: increasing entanglement between radiation and interior
- This leads to **mixed state** at $t = \infty$, but mixedness cannot change under **unitary evolution**.



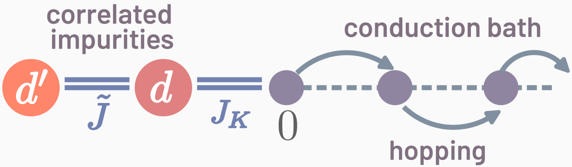
- Unitarity means entanglement of radiation **must decrease** after page time.
- Can we replicate this behaviour in a **quantum system**?

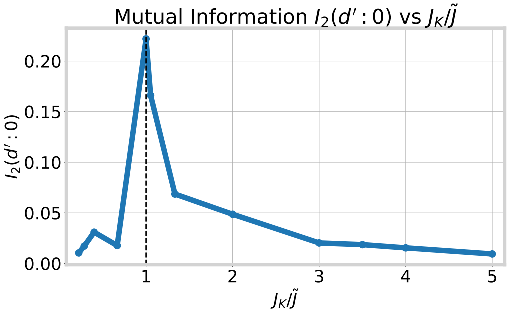
Hawking 1974; Hawking 1975; Page 1993.

(D Debata, **A Mukherjee**, S Lal)

Consider a **two-impurity model** (d, d'), with spin-exchange interaction between the impurities

- $d'(d) \rightarrow$ black hole interior (boundary)
- $0 \rightarrow$ radiation outside the black hole

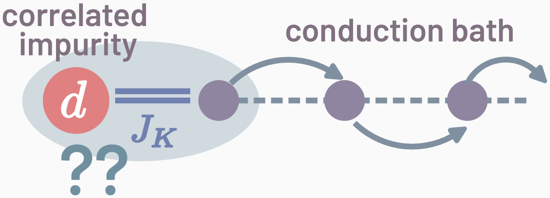




Mutual-information between d' and 0 shows behaviour similar to Page curve.

Universal Features of Kondo Breakdown in Impurity Models

Kondo screening (or its breakdown) lies at the heart of emergence in interacting electrons.



What is screening?
Whether the impurity spin forms a correlated **non-magnetic state** with local spin density of conduction electrons, at low temperature

- SCREENING → gapless **1-particle** excitations
- ABSENCE of screening → excitations are **gapped**
- PARTIAL breakdown → excitations are **exotic**

Obtaining a broader understanding of features of partial screening is important, particularly since such states appear near **critical points**!

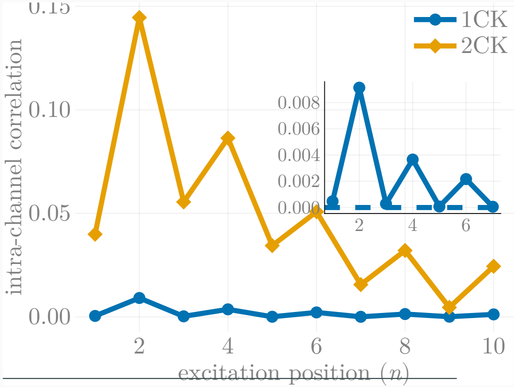
Fabrizio 2017; Rademaker 2025.

Universal Features of Kondo Breakdown in Impurity Models

(D Debata, **A Mukherjee**, S Lal)

We are studying two models that display partial breakdown of Kondo screening:

- eSIAM: Kondo breakdown through charge fluctuations in the bath
- two-channel Kondo model: Kondo breakdown due to competing screening tendencies



- Universal signatures of Kondo breakdown include **partial magnetisation**, partial phase shift.
- More recently, we are looking into **long-range entanglement** and correlations near Kondo breakdown.

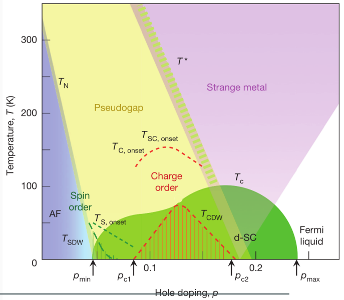
Mukherjee et al. 2023; Patra et al. 2023.

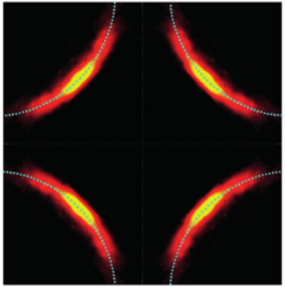
A New Auxiliary Model Approach to Systems of Interacting Electrons

(*A Mukherjee*, S R Hassan, Anamitra Mukherjee, N S Vidhyadhiraja, A Taraphder, S Lal. arXiv:2507.17201)

Broad Objectives

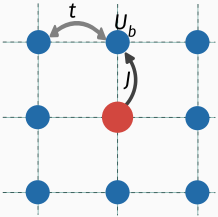
- ✓ Designing a **new method** by which to leverage quantum impurity models towards understanding phase transitions in correlated electrons in low dimensions
- ✓ Study the highly non-trivial Mott MIT through the **pseudogap** in the context of the cuprates
- ✓ Characterise the **parent metal** that gives way to an interacting Mott insulator.





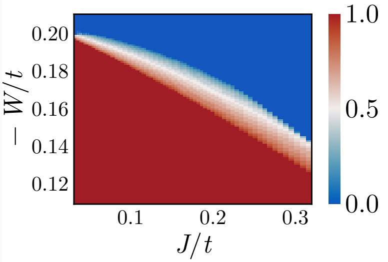
Keimer et al. 2015; Sebastian et al. 2014; Norman et al. 1998.

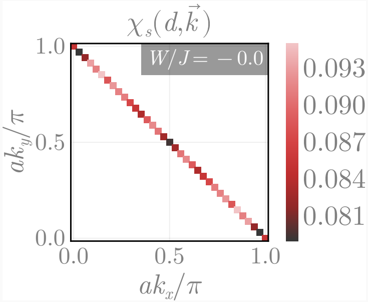
Momentum-Resolved Renormalisation Group Flows

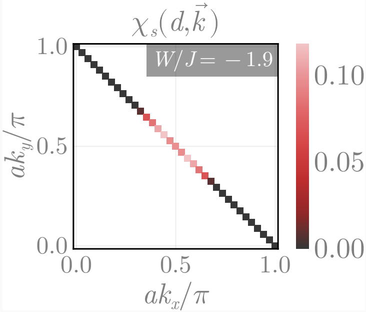


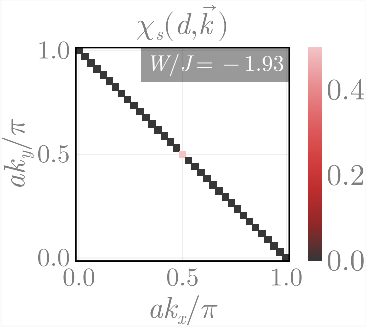
Hamiltonian RG equations of **embedded e-SIAM**

$$\Delta J^{(j)}_{\mathbf{k}_1,\mathbf{k}_2} = -\sum_{\mathbf{q}\in \mathrm{PS}} \frac{J^{(j)}_{\mathbf{k}_2,\mathbf{q}}J^{(j)}_{\mathbf{q},\mathbf{k}_1} + 4J^{(j)}_{\mathbf{q},\bar{\mathbf{q}}}W_{\bar{\mathbf{q}},\mathbf{k}_2,\mathbf{k}_1,\mathbf{q}}}{\omega - \frac{1}{2}|\varepsilon_j| + J^{(j)}_{\mathbf{q}}/4 + W_{\mathbf{q}}/2}$$









‘Periodising’ the Hamiltonian and Eigenstates

Periodising the Hamiltonian creates an **extended-Hubbard** model:

$$H_{\mathrm{tiled}} = \sum_{\mathbf{r}} T^{\dagger}(\mathbf{r} - \mathbf{r}_d) H_{\mathrm{aux}}(\mathbf{r}_d) T(\mathbf{r} - \mathbf{r}_d)$$

Wavefunctions can be related using a many-body **Bloch’s theorem**:

$$|\Psi_{\mathrm{gs}}\rangle = \frac{1}{\sqrt{N}} \sum_{\mathbf{r}_d} e^{i\mathbf{k}\cdot\mathbf{r}_d} \, |\psi_{\mathrm{gs}}(\mathbf{r}_d)\rangle$$

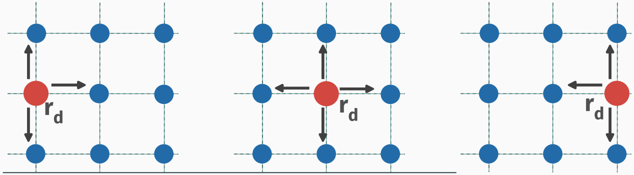
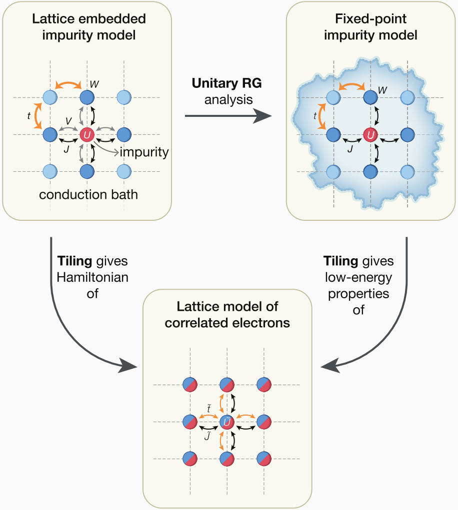


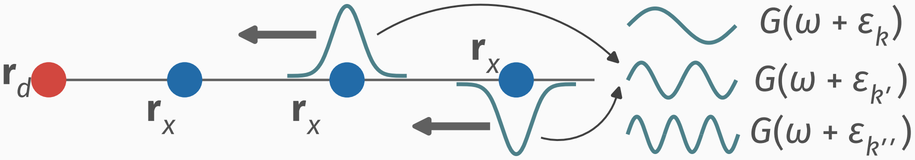
Image: Mayank Shreshtha



Periodising the Greens Functions

Greens function = sum of 1-particle *k*-space Greens functions starting from all sites in impurity model.

$$\tilde{G}(\mathbf{r};\tilde{\omega}) = \frac{1}{N}\sum_{\mathbf{k},\mathbf{r}_x}\Big[e^{i(\mathbf{k}-\mathbf{k}_0)\cdot(\mathbf{r}-\mathbf{r}_x)}G_p\left(\mathbf{r}_x;\omega+\varepsilon_{\mathbf{k}}\right) + e^{-i(\mathbf{k}-\mathbf{k}_0)\cdot(\mathbf{r}-\mathbf{r}_x)}G_h\left(\mathbf{r}_x;\omega-\varepsilon_{\mathbf{k}}\right)\Big]$$



Subsequently allows periodising spectral functions and self-energies

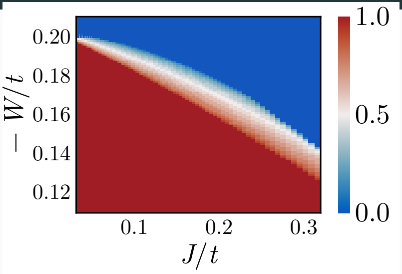
$$\tilde{A}(\mathrm{K};\omega) = -\frac{1}{\pi}\mathrm{Im}\left[\tilde{G}(\mathrm{K};\tilde{\omega})\right]$$

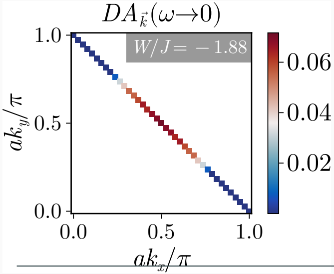
$$\tilde{\Sigma}(\mathrm{K};\omega) = \left(\tilde{G}^{(0)}(\mathrm{K};\tilde{\omega})\right)^{-1} - \left(\tilde{G}(\mathrm{K};\tilde{\omega})\right)^{-1}$$

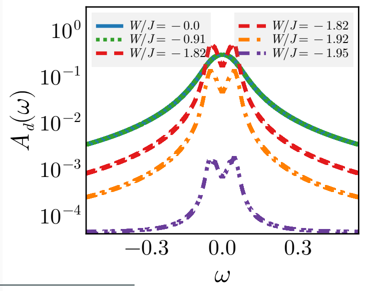
Kotliar et al. 2001; Verret et al. 2022.

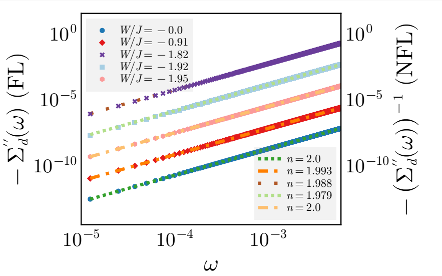
Characterising Low-energy Excitations of the Pseudogap

- Impurity spectral function shows **pseudogap** (loss of central peak)
- Tiled k-space DOS shows **partial gapping** around antinodes
- Self-energy shows **anomalous exponent** throughout the phase



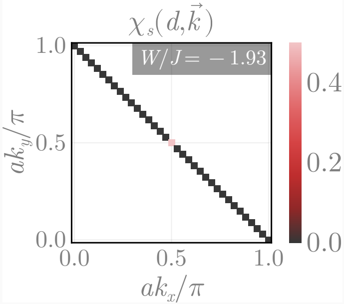






Baskaran 1991; Hatsugai and Kohmoto 1992; Nave, Zhao, and Phillips 2025.

Theory for the Nodal Non-Fermi Liquid



Exactly solvable model emerges at the **critical point**.

$$\Delta\tilde{H}_{q_1=q_2} = \sum_{q,\sigma} \epsilon_q n_{q,\sigma} + u \sum_{q,\sigma} n_{q\sigma} n_{q\bar{\sigma}}$$

- Diverging self-energy at zero energy: $\Sigma'' \sim 1/\omega$
- Holon-doublon deconfining excitations

Greens function shows bifurcation:

$$G = \frac{1}{2}\left[\frac{1}{\omega - u} + \frac{1}{\omega + u}\right]$$

→ $\mathbb{Z}_2$-symmetry breaking
→ change in analytic structure of self-energy

degenerate poles | **split poles**

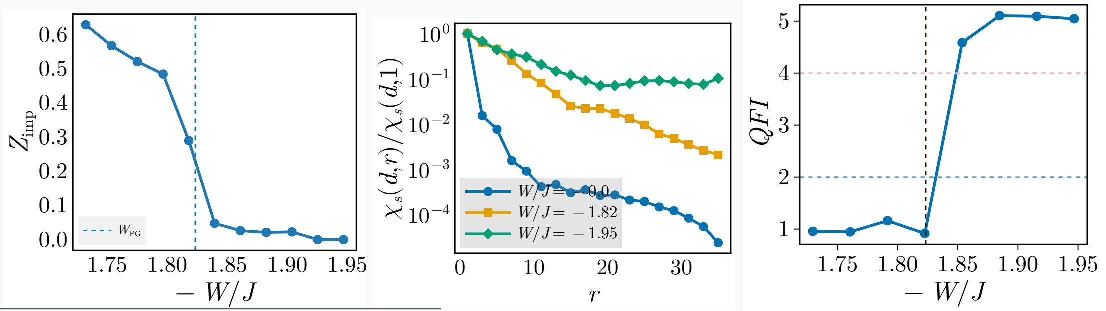
$$G = \frac{1}{\omega}$$

$$G = \frac{1/2}{\omega - u} + \frac{1/2}{\omega - u}$$

'Un-Fermi' Liquid Nature of the Pseudogap

Characteristics of the '**Mott metal**'

- low-energy excitations **cannot be described** by effectively one-particle excitations (vanishing Z)
- **long-range** correlations present in the pseudogap (quantum critical matter)
- entanglement is **multipartite** (Quantum Fisher information > 4)



Georgi 2007; Phillips 2014; Hauke et al. 2016.

The Big Picture

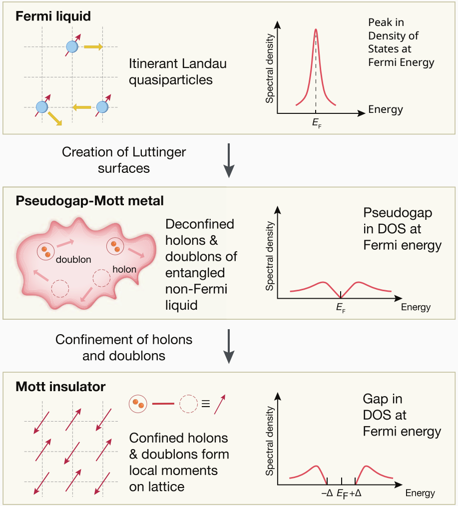


Image: Mayank Shreshtha

Paradigm of **spontaneous symmetry breaking**

- by non-zero local **order parameter**
- **short-range** entanglement and correlations
- excitations are **one-particle** in nature (single electrons or single bosons)
- superconductivity, antiferromagnetism

Paradigm of **fermionic criticality**

- no SSB, **analytic properties** of self-energy dictate phases
- excitations can be **many-particle** in nature
- non-trivial entanglement (**long-range**/topological)
- Examples: Mott metal-to-Mott insulator

Future Plans

Future Plans

- Study heavy-fermion physics using auxiliary model approach.
- Start consolidating results in the form of a thesis.

Thank You!