# Local metal-insulator transition in an extended Anderson impurity model

JRF-to-SRF Upgradation Presentation

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# Summary of Work

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## Completed Projects

✓ Unveiling the Kondo cloud: Unitary renormalization-group study of the Kondo model

**Phys. Rev. B 105, 085119**, arXiv:2111.10580v3

A. Mukherjee, *Abhirup Mukherjee*, N. S. Vidhyadhiraja, A. Taraphder, and S. Lal

✓ Frustration shapes multi-channel Kondo physics: A star graph perspective

**under review at PRB**, arXiv:2205.00790

S. Patra, *Abhirup Mukherjee*, A. Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, S. Lal

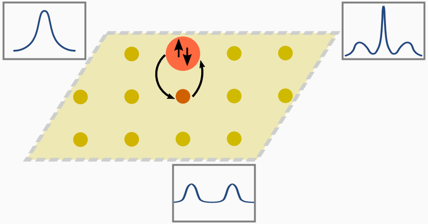
## Ongoing Projects

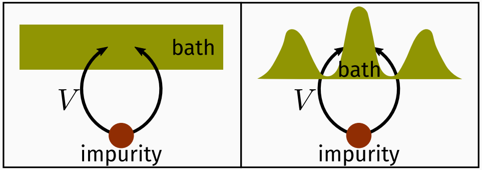
✓ Metal-insulator transition in an extended Anderson impurity model (*manuscript in preparation*)

✓ Holography and topology of entanglement scaling in free fermions (*manuscript in preparation*)

✓ URG-based auxiliary model approach to correlated systems (*ongoing*)

# Local MIT in an extended Anderson impurity model

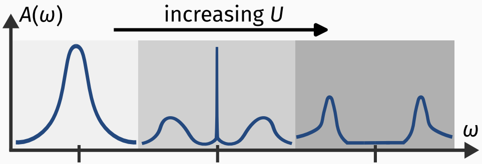




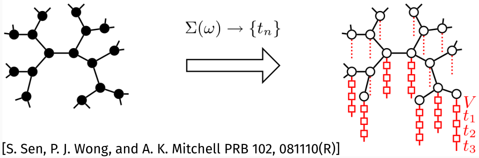
# Introducing the extended Anderson impurity model

# DMFT on the Bethe lattice: Exact in $d = \infty$

✓ shows **metal-insulator transition** on the Bethe lattice with ∞ coordination number

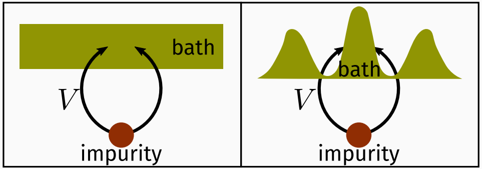



✓ Conduction bath obtained by imposing self-consistency shows **non-trivial correlations**




[S. Sen, P. J. Wong, and A. K. Mitchell PRB 102, 081110(R)]

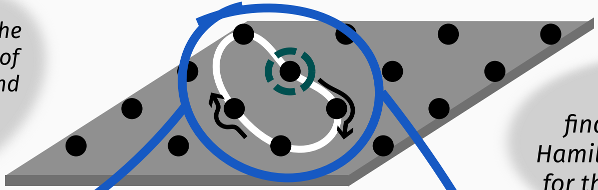
✓ Spectral function develops three peaks and then **gaps out**




Metzner et al. 1989; Georges et al. 1992; Parcollet et al. 2004; Maier et al. 2005; Kotliar et al. 2006; Ohashi et al. 2008; Held et al. 2013; Sen et al. 2020.

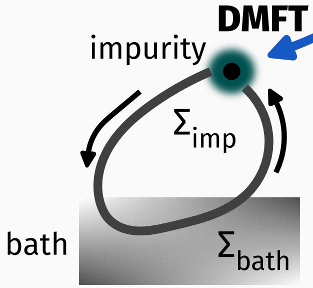
# DMFT on the Bethe lattice: Exact in $d = \infty$

**DMFT** *represents excursions from a site into the correlated bath in the form of self-energies for the bath and the impurity. This results in a three-peak DOS of the bath.*

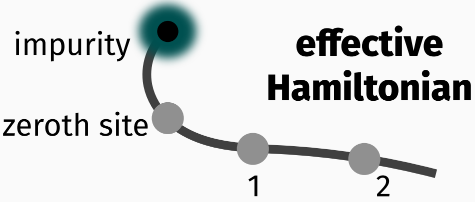


**Our Goal:**

*finding an effective Hamiltonian description for the $\Sigma$ that gives rise to the MIT.*




Similar approach adopted by Si & Kotliar for extended Hubbard model

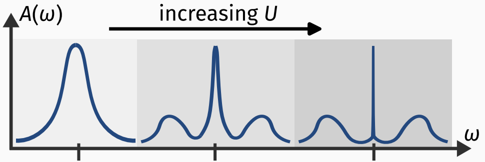



Si et al. 1993; Kotliar et al. 1993.

# Introducing the extended Anderson impurity model

**Standard Anderson impurity model**

- ✓ no local-moment phase, $A(\omega)$ gapless
- ✓ cannot explain insulating phase of DMFT



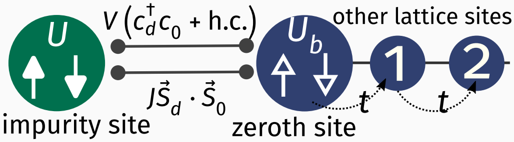

**Gap in spectral function requires additional physics!**


Anderson 1961; Anderson 1978; Wilson et al. 1974; Nozieres 1974; Krishna-murthy et al. 1980; Andrei 1980; Tsvelick et al. 1983; Hewson 1993; Costi et al. 1990; Costi 2000; Kuramoto et al. 1987; Cox et al. 1988.

# Introducing the extended Anderson impurity model

*Extended* **Anderson impurity model**

- ✓ impurity-bath spin correlation: $J$
- ✓ bath zeroth site local correlation: $U_b$




$$H = H_{\text{KE}} + V\sum_\sigma \left(c_{d\sigma}^\dagger c_{0\sigma} + \text{h.c.}\right) + \frac{U}{2}\left(\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow}\right)^2 + J\vec{S}_d \cdot \vec{S}_0 + U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2$$

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**Anderson 1961; Anderson 1978; Wilson et al. 1974; Nozieres 1974; Krishna-murthy et al. 1980; Andrei 1980; Tsvelick et al. 1983; Hewson 1993; Costi et al. 1990; Costi 2000; Kuramoto et al. 1987; Cox et al. 1988.**

# Phase Diagram & Ground-States

# Nature of RG flows

✓ URG Equations reveal **critical** point at $r = -U_b/J = 1/4$

✓ RG equation for most dominant coupling $J$:

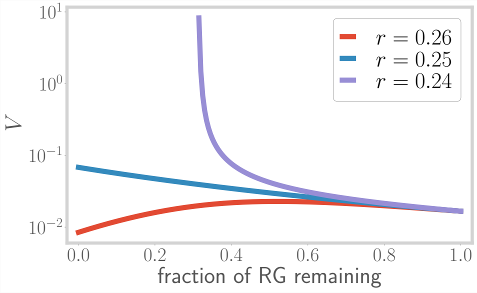
$$\Delta J = \frac{-J(J + 4U_b)n(D)}{\omega - D/2 + U_b/2 + J/4}$$

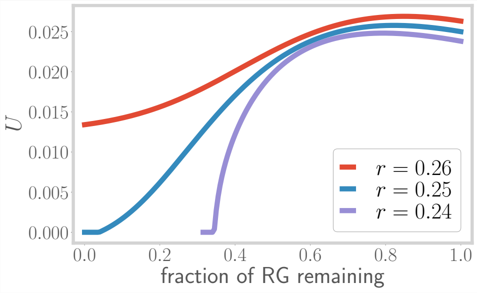
✓ Numerator structure allows **averting** strong-coupling behaviour

# Nature of RG flows

✓ $J, V, U$ reverse their behaviour across the critical point; leads to **phase transition**

✓ Bath correlation $U_b$ is always **marginal**

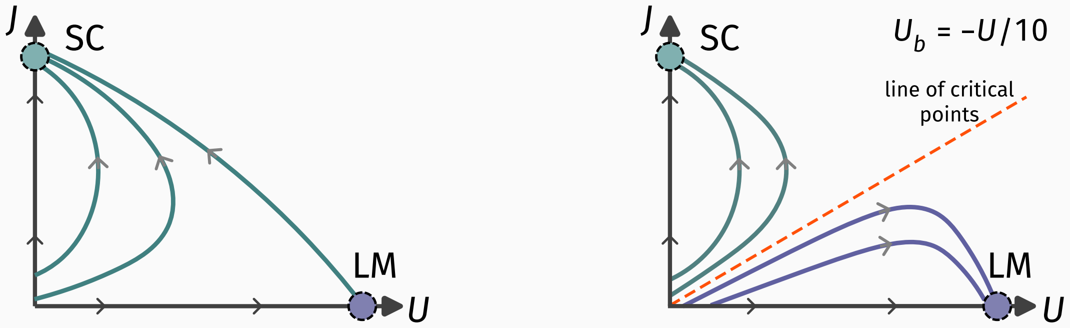


# Nature of RG flows

✓ RG equation reveals competition between Kondo flow and pairing physics

$$\Delta J = \underbrace{-\frac{(J + 2U_b)^2 n(D)}{\omega - D/2 + U_b/2 + J/4}}_{\text{usual Kondo physics}} + \underbrace{\frac{(2U_b)^2 n(D)}{\omega - D/2 + U_b/2 + J/4}}_{\text{competing pairing physics}}$$

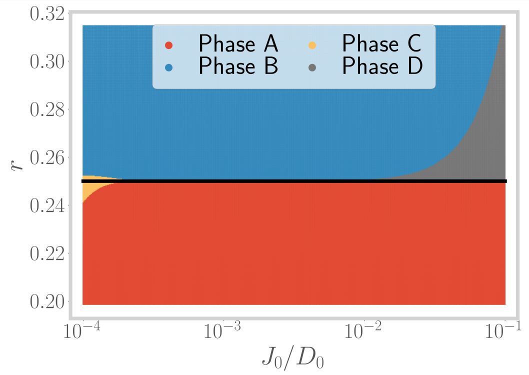


# RG Phase Diagram

✓ blue phase $\longrightarrow U_b < -J/4$ : $V, J$ are **irrelevant** $\longrightarrow$ local moment flows

✓ yellow phase: $J \ll D_0$: involves $V, U, U_b$

*vanishes for large systems*




✓ gray phase: $J \sim D_0$: **all** couplings irrelevant

*vanishes for large systems*

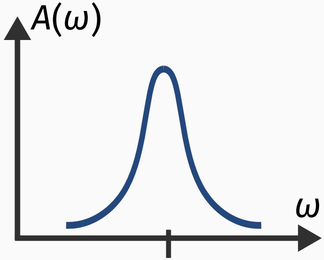
✓ red phase $\longrightarrow U_b > -J/4$: $V, J$ are **relevant** $\longrightarrow$ strong-coupling flows

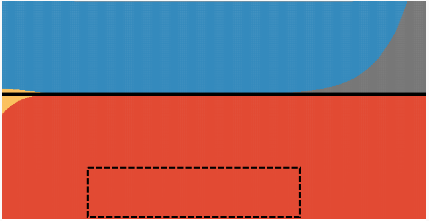
# Low-energy effective Hamiltonians and ground-states

**Regime 1:** $|U_b| < J/4$

- ✓ $J$ relevant,
- ✓ $V$ subdominant,
- ✓ $U$ irrelevant

$$H = J\vec{S}_d \cdot \vec{S}_0 - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2 + \sum_{k<\Lambda^*} \epsilon_k \hat{n}_{k\sigma}$$

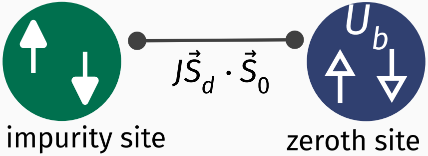


# Low-energy effective Hamiltonians and ground-states

**Regime 1:** $|U_b| < J/4$

## Zero-bandwidth limit

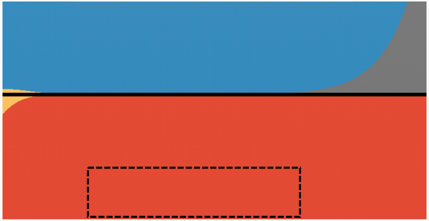



$$H = J\vec{S}_d \cdot \vec{S}_0 - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2$$

✓ two-spin Heisenberg, attractive zeroth site

✓ **singlet** ground state

$$|\Psi\rangle_{GS} = \frac{1}{\sqrt{2}}\left[|\uparrow_d\downarrow_0\rangle - |\downarrow_d\uparrow_0\rangle\right]$$



# Low-energy effective Hamiltonians and ground-states

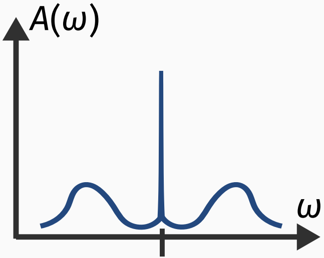
**Regime 2:** $|U_b| \sim J/4$

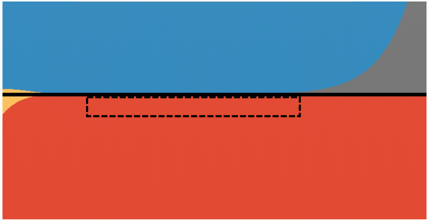
✓ $J$ relevant,

✓ $V$ relevant,

✓ $U$ irrelevant

$$H = J\vec{S}_d\cdot\vec{S}_0 + V\sum_\sigma \left(c^\dagger_{d\sigma}c_{0\sigma} + \text{h.c.}\right) - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2 + \sum_{k<\Lambda^*}\epsilon_k\hat{n}_{k\sigma}$$

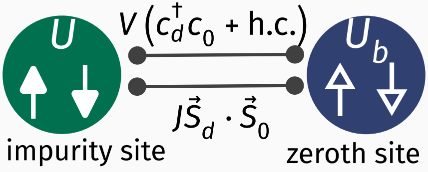


# Low-energy effective Hamiltonians and ground-states

**Regime 2:** $|U_b| \sim J/4$

**Zero-bandwidth limit**

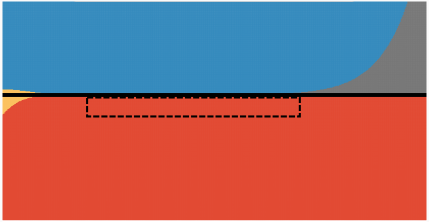



$$H = J\vec{S}_d \cdot \vec{S}_0 + V\sum_\sigma \left(c_{d\sigma}^\dagger c_{0\sigma} + \text{h.c.}\right) - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2$$

✓ **spin+charge** dimer with attractive 0th site

✓ spin-singlet + charge-triplet-zero in gr-state

$$|\Psi\rangle_{GS} = \frac{1}{2\sqrt{2}}\left[|\uparrow_d\downarrow_0\rangle - |\downarrow_d\uparrow_0\rangle\right] + \frac{1}{2\sqrt{2}}\left[|2_d, 0_0\rangle + |0_d, 2_0\rangle\right]$$



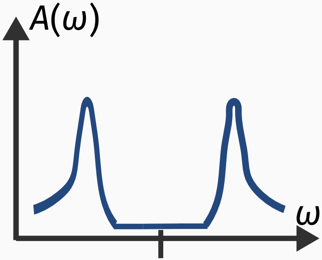
## Low-energy effective Hamiltonians and ground-states

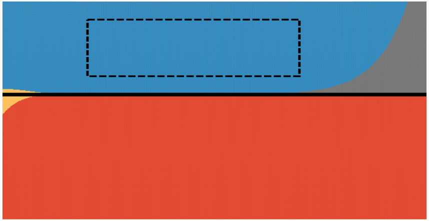
**Regime 3:** $|U_b| > J/4$

✓ $J, V$ irrelevant,

✓ $U$ relevant,

$$H = -U/2\left(\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow}\right)^2 - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2 + \sum_{k<\Lambda^*} \epsilon_k \hat{n}_{k\sigma}$$

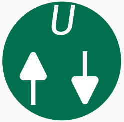


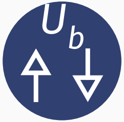
# Low-energy effective Hamiltonians and ground-states

**Regime 3:** $|U_b| > J/4$

**Zero-bandwidth limit**




impurity site



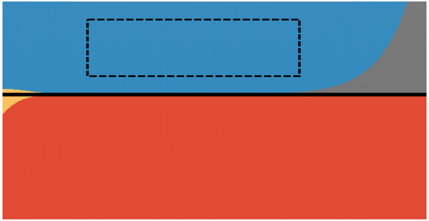

zeroth site

$$H = -U/2\left(\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow}\right)^2 - U_b\left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2$$

✓ impurity site detaches from bath

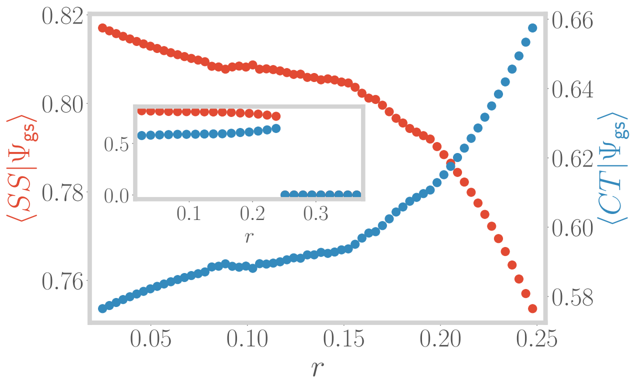
✓ **local moment** ground-state

$$|\Psi\rangle_{GS} = |\uparrow,\downarrow\rangle_d \otimes |0,2\rangle_0$$



# Low-energy effective Hamiltonians and ground-states

**Ground-state overlaps** with spin singlet and charge triplet zero

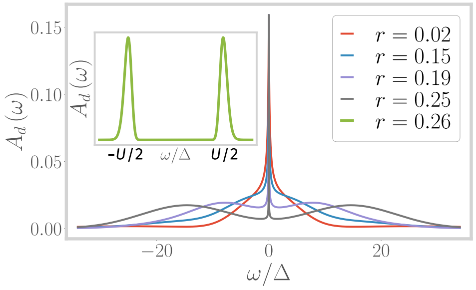


# Nature of the Transition

# Gapping of the impurity spectral function

✓ Correlated **three peak** structure at $|U_b| \lesssim J/4$

✓ Broad central peak at $|U_b| \ll J/4$



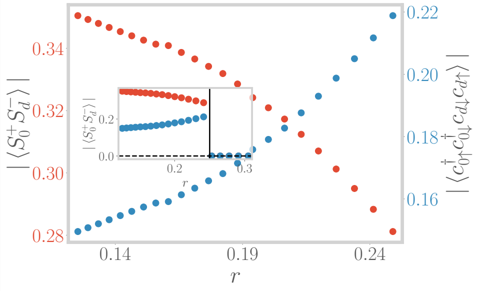

✓ hard central **gap** for $|U_b| > J/4$

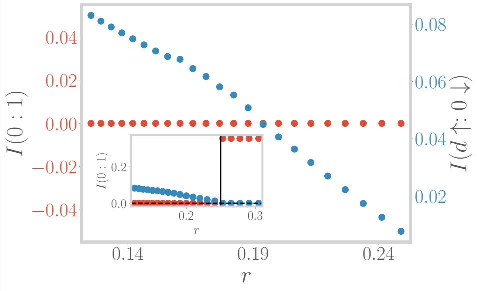
# Destruction of the Kondo cloud

**The Kondo cloud weakens, and is destroyed at the transition.**

✓ vanishing of impurity-bath correlations

✓ transfer of entanglement into the bath



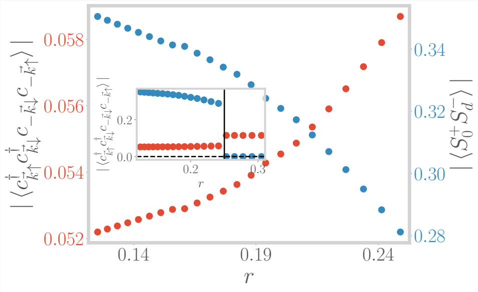



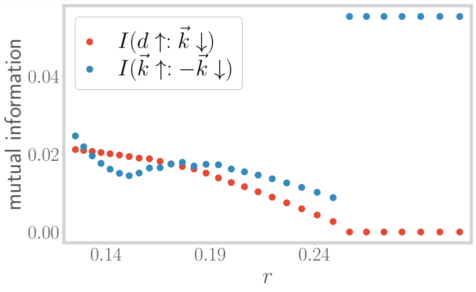
# Growth of pairing fluctuations in the bath

**Subdominant pairing fluctuations, near the transition...**

✓ growth of fluctuations in Cooper channel, at the cost of spin-flip fluctuations

✓ mutual information within the bath maximised after transition



# Universal Theory near the Transition

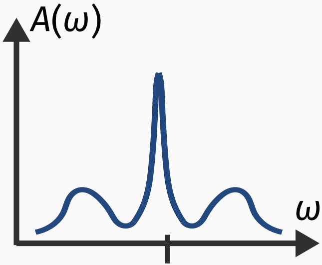
# Minimal effective model for the transition

✓ For $|U_b| \lesssim J/4$, central peak and side peaks are **well-separated**

✓ **Integrate out** charge fluctuations through Schrieffer-Wolff transformation

$$H_{\text{eff}} = \tilde{J}\vec{S}_d \cdot \vec{S}_0 - \tilde{U}_b \left(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow}\right)^2 + H_{\text{K.E.}}$$

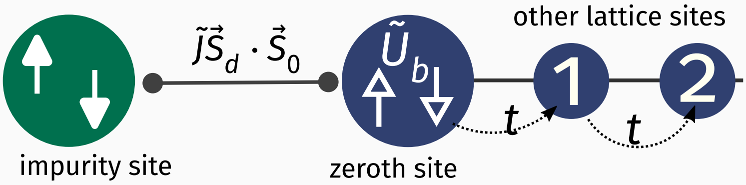
RG equation for $\tilde{J}$ : $\Delta\tilde{J} \sim \tilde{J}\left(\tilde{J} + 4\tilde{U}_b\right)$




Moeller et al. 1995; Held et al. 2013.

# Minimal effective model for the transition

✓ **captures the criticality**, and the strong-coupling and local moment phases



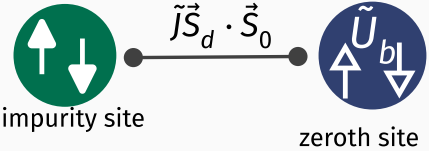

**Suggests that $J$ and $U_b$ are the minimal & universal ingredients for transition!**

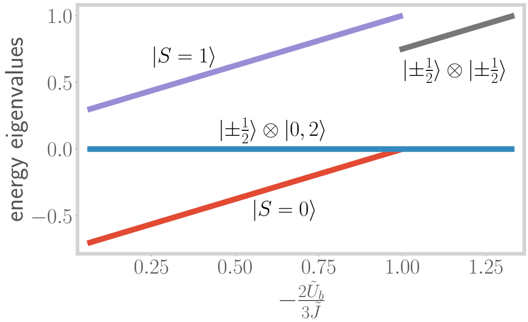
Moeller et al. 1995; Held et al. 2013.

# Capturing the level crossing at the transition from a two-site model

✓ Obtain two-site model by taking **zero bandwidth** limit

✓ spectrum shows **level crossing** between singlet and local moment states



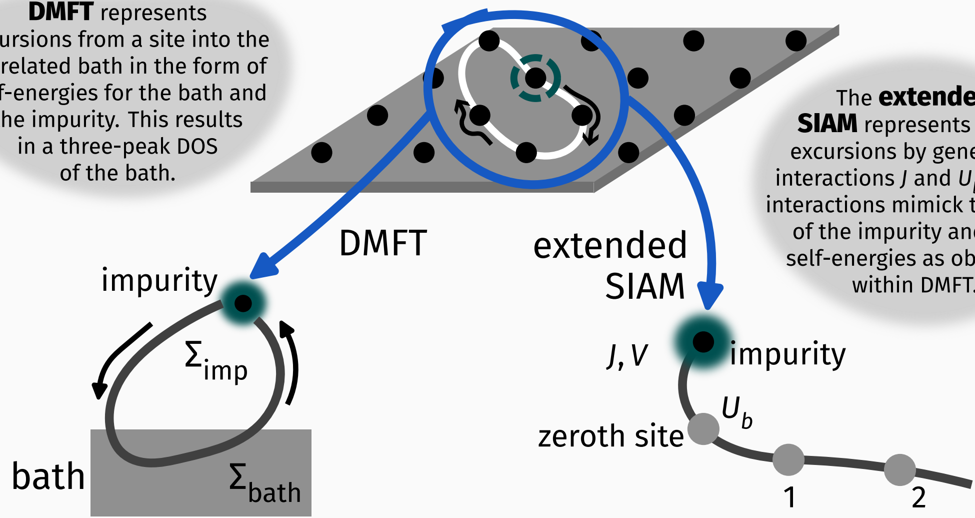



# Insights into DMFT

# Extended SIAM in the context of DMFT

**DMFT** represents excursions from a site into the correlated bath in the form of self-energies for the bath and the impurity. This results in a three-peak DOS of the bath.

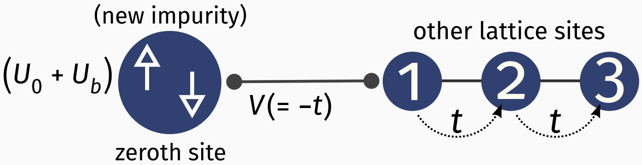
The **extended SIAM** represents these excursions by generating interactions $J$ and $U_b$. These interactions mimick the effect of the impurity and bath self-energies as observed within DMFT.



# Equivalence of the impurity site and the bath zeroth site

✓ **Integrate out impurity site** from fixed point Hamiltonian via a single URG transformation

✓ Generates additional correlation $U_0$ on zeroth site




✓ $J$ is relevant and the largest scale $\longrightarrow$ **repulsive correlation**:

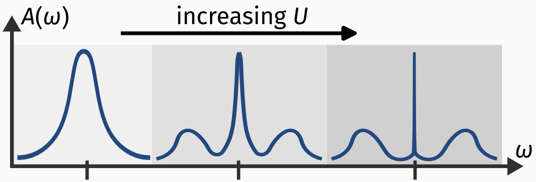
$$U_0 + U_b \simeq J > 0$$

Moeller et al. 1995.

# Equivalence of the impurity site and the bath zeroth site

✓ $J$ acts a **symmetrisation mechanism** between impurity and zeroth sites

✓ **Coherent** spin-flip scatterings ensure similarity of spectral functions

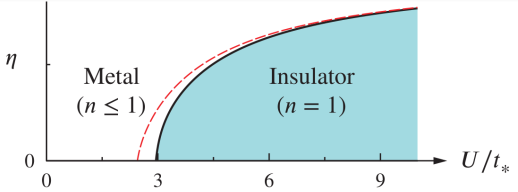



**Essence of self-consistency: Equivalence of impurity and zeroth sites!**

Moeller et al. 1995.

# Observation of a coexistence region

- ✓ DMFT observes a **coexistence region** near the critical point, for $U_{c1} < U < U_{c2}$
- ✓ Insulating when coming in from the insulator, metallic when coming in from the metal



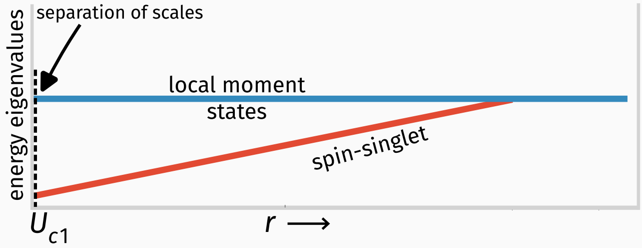

(D.E. Logan and M.R Galpin 2016 JPCM 28 025601)

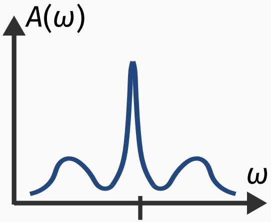
- ✓ Mott gap appears **discontinuously** after the transition, through a **continuous** sharpening of the central peak.
- ✓ **True** transition believed to occur at $U_{c2}$.

# Observation of a coexistence region

**Can be explained heuristically using the two site spectrum**

✓ Initial point is when the side peaks get separated (near-zeroes in the spectral function)

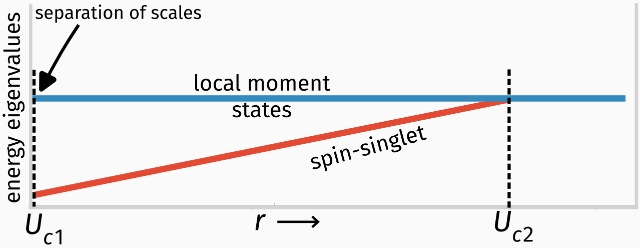


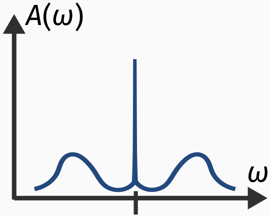



# Observation of a coexistence region

**Can be explained heuristically using the two site spectrum**

✓ $U_{c2}$ is the point where the levels cross

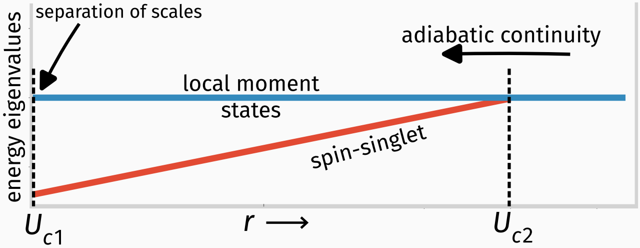


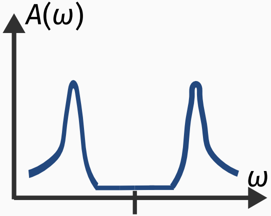



# Observation of a coexistence region

**Can be explained heuristically using the two site spectrum**

✓ Coming from $U > U_{c2}$, **adiabatic continuity** allows DMFT to stay on the local moment state...

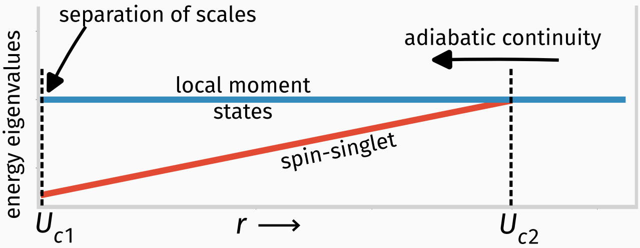


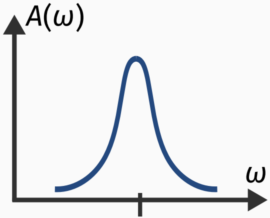



## Observation of a coexistence region

**Can be explained heuristically using the two site spectrum**

✓ For $U < U_{c1}$, local moment state is too unstable, **relaxes** to the true ground state.

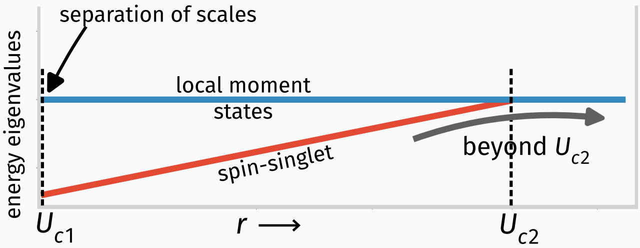


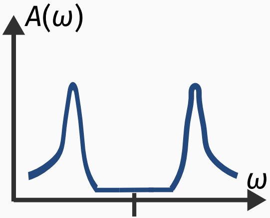



# Observation of a coexistence region

**Can be explained heuristically using the two site spectrum**

✓ For $U > U_{c1}$, charge sector separated by large $U$, leads to the **discontinuous** appearance of finite gap



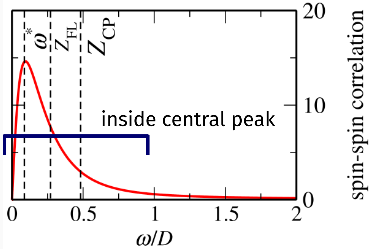
# Comparison against NRG-DMFT correlation functions

**Poor Man's scaling of the effective Kondo model**
*[K. Held, R. Peters, and A. Toschi. PRL 2013]*

✓ shows **quantitative** agreement with NRG-DMFT (crossover scale and kinks in self-energy)

✓ Suggests that the minimal model can capture **spin susceptibility**

[K. Held, R. Peters, and A. Toschi.
PRL 110, 246402 (2013)]




✓ Our $J - U_b$ model **goes further** by capturing physics beyond the transition

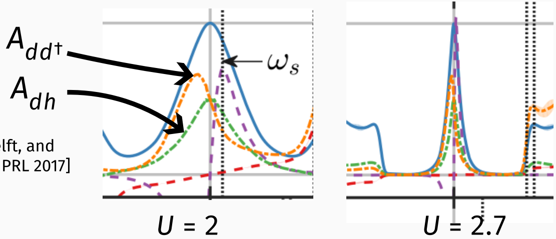
# Comparison against NRG-DMFT correlation functions

**Doublon-holon correlators of the Hubbard model**

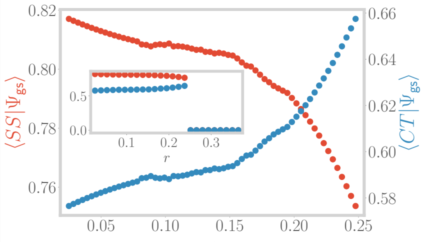
*[S. B. Lee, J. v Delft, and A. Weichselbaum. PRL 2017]*

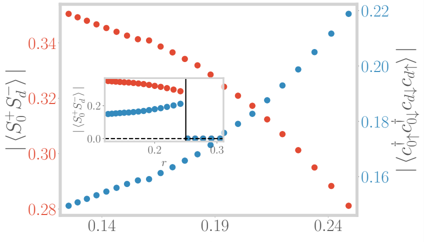
Lee et. al show **peaks** in doublon-holon correlators near zero energy within the central Kondo peak.

[S. B. Lee, J. v Delft, and A. Weichselbaum. PRL 2017]




We find support for this in the form of **increasing ground-state charge correlations and overlap**.



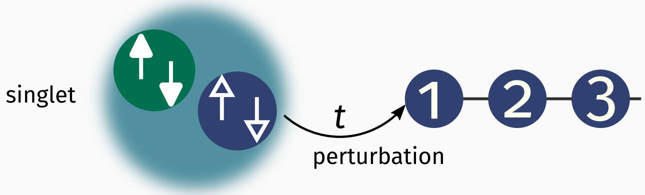



# Low-energy excitations of the bath

# Effect on the local Fermi liquid

**What about the low-energy excitations of the bath, that lie above the singlet ground state?**

✓ treat hopping between singlet and bath as perturbation




✓ Up to fourth order, charge sector becomes repulsive...

$$H_{\text{eff}} = \frac{2t^4}{\tilde{J}\left(3\tilde{J}/4 + \tilde{U}_b\right)^2}\left[\hat{n}_{1\uparrow}\hat{n}_{1\downarrow} + \left(1 - \hat{n}_{1\uparrow}\right)\left(1 - \hat{n}_{1\downarrow}\right)\right] + H_{\text{K.E.}}$$

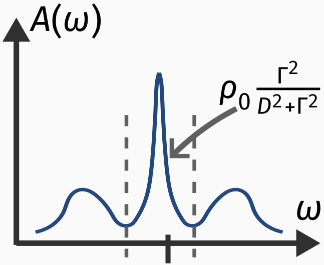
✓ FL term blows up towards transition, signaling **breakdown** of Fermi liquid theory and loss of adiabaticity.

# Effect on the local Fermi liquid

**Vanishing of the Kondo scale $T_K$ towards the transition**

✓ Consider the $J - U_b$ model, but with a **Lorentzian DOS** in the bath:

$$\rho = \rho_0 \frac{\Gamma^2}{D^2 + \Gamma^2}$$




✓ Near the transition $r = -U_b/J_0 \to \frac{1}{4}$ and $\Gamma \to 0$, the **IR energy scale** $D^*$ can be approximated as

$$D^* = D_0 \exp\left[\frac{\left(2\omega + U_b + J_0/2\right)^2}{8|U_b|\rho_0\Gamma^2} \ln|1 - 4r|\right], \quad D_0 = \text{UV cutoff}\,.$$

Held et al. 2013.

# Effect on the local Fermi liquid

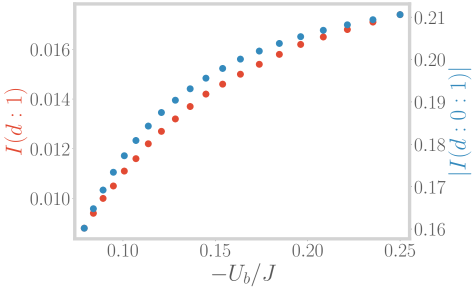
## Vanishing of the Kondo scale $T_K$ towards the transition

✓ **Kondo temperature** can be defined as $T_K = D^* / k_B$:

$$T_K = \frac{D_0}{k_B} \exp\left[\frac{\left(2\omega + U_b + J_0/2\right)^2}{8|U_b|\rho_0\Gamma^2} \ln|1 - 4r|\right]$$

✓ Kondo temperature vanishes as $r \to 1/4$:

$$T_K \sim (1 - 4r)^\alpha \to 0$$




Held et al. 2013.

# Effect on the local Fermi liquid

**How do the imaginary part of self-energy and the qp residue behave near the transition?**

✓ Following the renormalised perturbation theory approach of Hewson, $\mathrm{Im}\left[\Sigma(\omega)\right]$ is

$$\mathrm{Im}\left[\Sigma(\omega)\right] \sim u^2\omega^2, \quad u = \frac{2t^4}{\tilde{J}\left(3\tilde{J}/4 + \tilde{U}_b\right)^2}$$

✓ As $r \to 1/4$, $u \to \infty$, signalling a vanishing lifetime of the quasiparticles

✓ Quasiparticle residue $Z$ for 1-particle excitations is proportional to $T_K$:

$$Z \sim T_K$$

$$Z \to 0 \text{ as } r \to 1/4$$

Hewson 1993; Coleman 2015.

# Effective Hamiltonian at the critical point

**What is the nature of the low-energy excitations exactly at the critical point?**

✓ Singlets become **degenerate** with local moment states at the critical point:

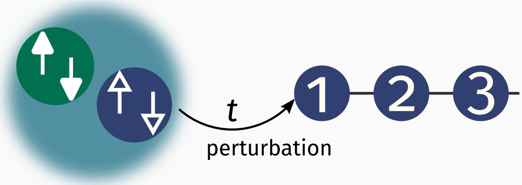
$$\frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle\right), \quad |\sigma\rangle\,|0\rangle, \quad |\sigma\rangle\,|2\rangle$$

✓ **Non-zero degeneracy** exists even in presence of hopping perturbation:

$$|N_{\text{tot}} = 2,\ S^z_{\text{tot}} = 0\rangle, \quad |N_{\text{tot}} = 3,\ S^z_{\text{tot}} = \pm 1/2\rangle,$$

$$|N_{\text{tot}} = 4,\ S^z_{\text{tot}} = 0\rangle$$

$$N_{\text{tot}} = \hat{n}_d + \hat{n}_0 + \hat{n}_1, \quad S^z_{\text{tot}} = S^z_d + S^z_0 + S^z_1$$



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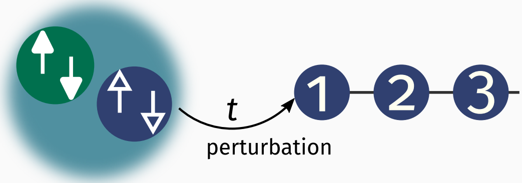



$$N_{\mathrm{tot}} = \hat{n}_d + \hat{n}_0 + \hat{n}_1,\quad S^z_{\mathrm{tot}} = S^z_d + S^z_0 + S^z_1$$

✓ Presence of magnetic states with $S^z_{\mathrm{tot}} \neq 0$ indicates **breakdown of screening**

✓ Non-vanishing degeneracy will lead to critical correlations and **non-Fermi** liquid physics

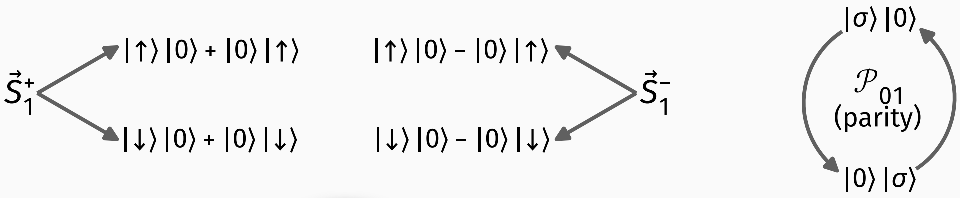
Varma et al. 2002.

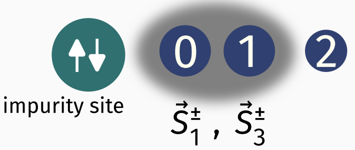
# Effective Hamiltonian at the critical point

**What is the nature of the low-energy excitations exactly at the critical point?**

✓ $S^z_{\text{tot}} = 0$ first order effective Hamiltonian acquires quasi non-local terms:

$$H_{\text{eff}} = t\vec{S}_d \cdot \left(\vec{\mathcal{S}}_1^- + \vec{\mathcal{S}}_3^- - \vec{\mathcal{S}}_1^+ - \vec{\mathcal{S}}_3^+\right) + \frac{t}{4}\mathcal{P}_{01} + H_{\text{K.E.}}$$



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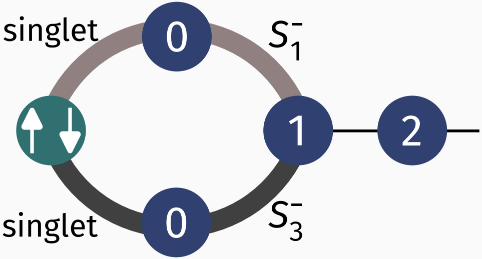
✓ Ground-state dictated by $\mathcal{S}_1^-, \mathcal{S}_3^-$: degenerate singlets

✓ Spreading of extent of bath spin indicates **stretching** of singlet in real space



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- ✓ Shows the **dispersion** of entanglement across the lattice at critical point. Precursor to decoupling of impurity spin.

- ✓ Degenerate singlets connected by hopping; reminiscent of **multichannel** Kondo physics and source of non-Fermi liquid physics

# Broad conclusions

- ✓ The extended SIAM appears to capture the phenomenology of the DMFT transition and **self-consistency**.
- ✓ The key ingredient is a **competition** between Kondo screening physics and a local attractive correlation in the bath.
- ✓ Crucial feature of the journey is the enhancement of **pairing fluctuations** in the bath: leads to destruction of Kondo cloud.

## Broad conclusions

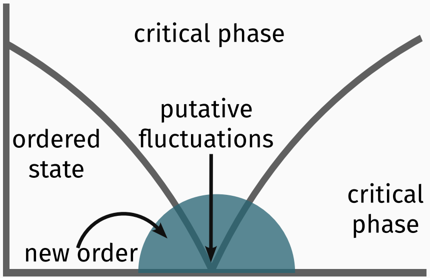
- ✓ An **emergent self-consistency** is achieved through the qualitative similarity of the spectral functions of the impurity and zeroth sites.
- ✓ Approach towards criticality is marked by **vanishing** metallic energy scale and quasiparticle residue.
- ✓ **Non-Fermi liquid** physics arises at critical point, through the expansion of the singlet into the next lattice site.

# Future Prospects

# Future Prospects

✓ The extended SIAM can be improved by considering **multiple impurities** and general impurity **filling**.

✓ We are developing a new **tiling-based auxiliary model method** can used for studying other models of strong-correlations as well as topologically active or flat band systems.




✓ The URG can be applied to **heavy-fermion materials** towards a study of phase diagram and unconventional superconductivity, as well as Kondo insulators.

✓ Interacting systems in a magnetic field is also a potential area of study, specifically **fractional Chern insulators** (e.g. the fractional quantum hall effects).

# Acknowledgements

Gracious thanks to

- ✓ my collaborators **S. Patra, A. Mukherjee, Prof. A. Taraphder** and **Prof. N. S. Vidhyadhiraja**,
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- ✓ **Prof. H. Casini, Prof. N. Banerjee** and **Shibendu G. Chowdhury** for very fruitful discussions, and
- ✓ IISER Kolkata for funding.

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# More insight into self-consistency

## More insight into self-consistency

**Meaning of exact self-consistency**

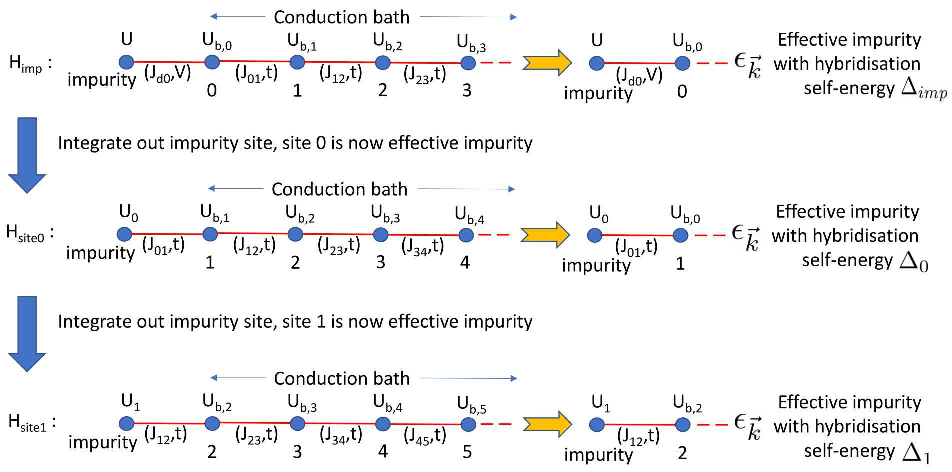
✓ Minimal impurity model $\implies$ qualitative self-consistency $\implies$ impurity site = zeroth site

✓ Full self-consistency $\implies$ equivalence of self-energy on all sites

# More insight into self-consistency

## Meaning of exact self-consistency

✓ Can be achieved through an extended impurity model with $U_b$ and $J$ on all sites in the bath



# Other Projects

# Theory for the single-channel Kondo cloud

**Phys. Rev. B 105, 085119**

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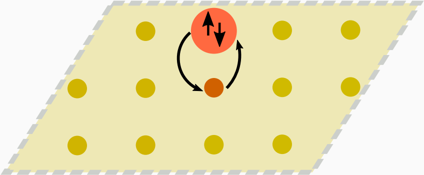
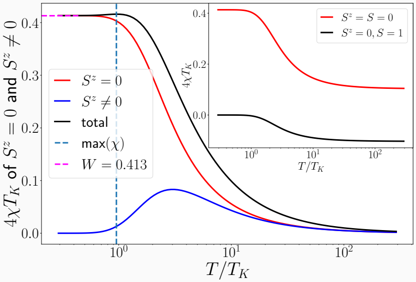
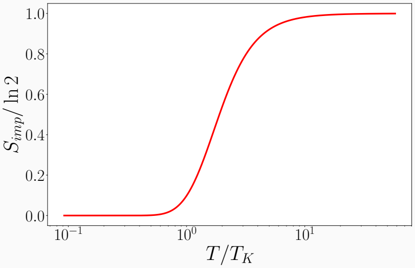


# Theory for the single-channel Kondo cloud

✓ spectral function & magnetic susceptibility



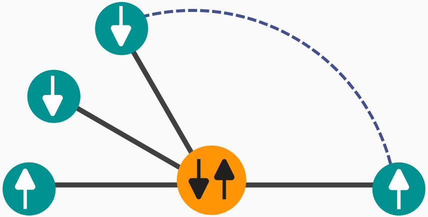




✓ local Fermi liq. & orthogonality catastrophe

✓ thermal entropy

Kondo 1964; Wilson 1975; Andrei et al. 1983; Hewson 1993; Nozieres 1974; Anderson 1970; Tsvelick et al. 1983; Affleck et al. 1993; Goldhaber-Gordon et al. 1998; V. Borzenets et al. 2020; Sakai et al. 1989; Costi et al. 1990; Nozaki et al. 2012; Affleck 1995.

# Role of degeneracy in the multi-channel Kondo problem



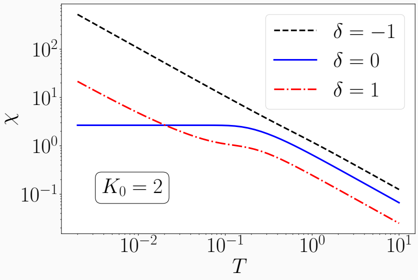
**arXiv:2205.00790**

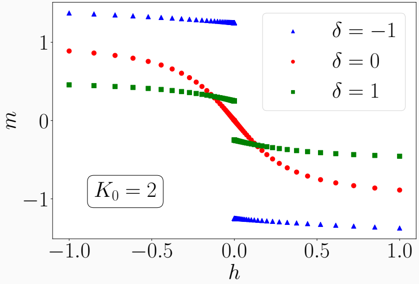
Siddhartha Patra, **Abhirup Mukherjee**, Anirban Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, Siddhartha Lal

# Role of degeneracy in the multi-channel Kondo problem

✓ Intermediate-coupling RG fixed point Hamiltonian and **degenerate** ground states

✓ Degree of compensation, magnetization and susceptibility show **incomplete screening**



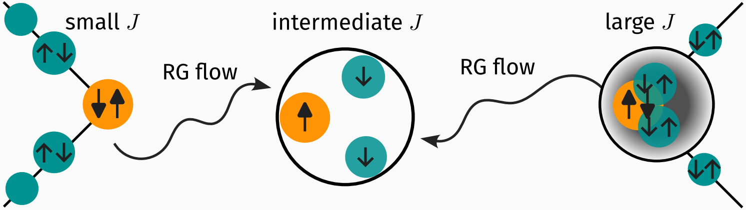




Nozières, Ph. et al. 1980; Tsvelick et al. 1985; Affleck et al. 1993; Gan 1994; Affleck et al. 1991; Emery et al. 1992; Bulla et al. 1998.

# Role of degeneracy in the multi-channel Kondo problem

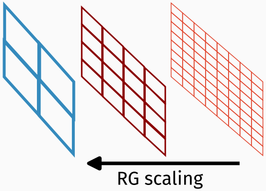
✓ Local **marginal Fermi liquid** within the low-energy excitations of the bath

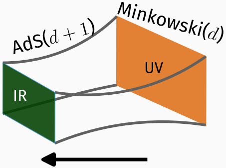
✓ **Duality** relations constrain the RG flows of the MCK model




Nozières, Ph. et al. 1980; Tsvelick et al. 1985; Affleck et al. 1993; Gan 1994; Affleck et al. 1991; Emery et al. 1992; Bulla et al. 1998.

# Entanglement scaling in free fermions: holography & topology



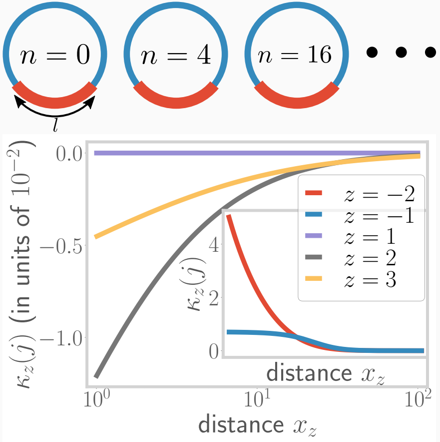
# Entanglement scaling in free fermions: holography & topology - Summary

✓ Under coarse-graining or fine-graining in $k$–space, entanglement of free Dirac fermions shows hierarchical onion-like structure.

✓ Entanglement scaling can be used to define distances, leads to additional spatial dimension $\longrightarrow$ holography.

✓ Emergent distances and curvature can be related to RG beta function; the sign of the curvature is topological.

✓ Pole structure of the entanglement tracks the Luttinger volume - invariant under the scaling transformations.



# Creating subsystems

Free Dirac fermions on torus: $k_x^n = \frac{2\pi}{L_x} n, \quad n \in \mathbb{Z};$ define **sparsity** $= \Delta n = 1$

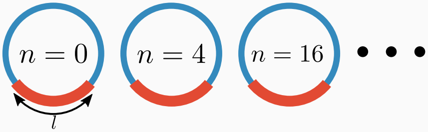
**Simplest** choice: the entire set

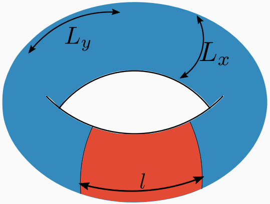
sparsity = 1 $\longrightarrow n \in \{-N, -(N-1), -(N-2), \dots, -1, 0, 1, \dots, N-2, N-1, N\}$

**Coarser** choices: increase sparsity

sparsity = 2 $\longrightarrow n \in \{-N, -(N-2), -(N-4), \dots, -2, 0, 2, \dots, N-4, N-2, N\}$

sparsity = 4 $\longrightarrow n \in \{-N, -(N-4), -(N-8), \dots, -4, 0, 4, \dots, N-8, N-4, N\}$



# Subsystem entanglement entropy: Entanglement hierarchy

$$S_{\mathcal{A}_z(j)} = f_z(j) c \alpha L_x - c \log \left| 2 \sin \left( \pi f_z(j) \phi \right) \right|$$

$$i < j, \quad S_{i \cup j} = \begin{cases} S_i, & z > 0 \\ S_j, & z < 0 \end{cases}$$

✓ presents a **hierarchy** of entanglement ⟶ EE distributed across RG steps
RG transformation ⟶ reveals entanglement

✓ distribution of entanglement also present in **multipartite** entanglement

**Calabrese et al. 2004; Casini et al. 2005; Arias et al. 2015; Chen et al. 2017; Murciano et al. 2020.**

# Mutual information = distance

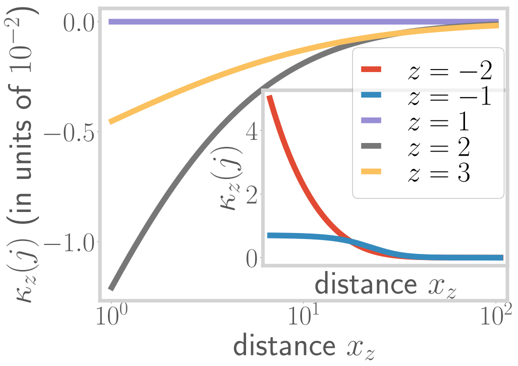
**Mutual information:** $I^2(A:B) \equiv S(A) + S(B) - S(A \cup B)$ (non-negative)

Define distances using mut. info.

$$x_z(j) = \log t_z(j), \quad y_z(j) = \log t_z(j \pm 1)$$

$$v_z(j) \equiv \Delta y_z(j)/\Delta x_z(j), \quad v' = \Delta v_z(j)/\Delta x_z(j)$$

Curvature as well: $\kappa_z(j) = \dfrac{v_z'(j)}{\left[1 + v_z(j)^2\right]^{\frac{3}{2}}}$




Van Raamsdonk 2010; Lee et al. 2016; Mukherjee et al. 2022; Lee 2010; Lee 2014; Qi 2013; Lee et al. 2016; Mukherjee et al. 2020a; Mukherjee et al. 2020b; Ryu et al. 2006b; Ryu et al. 2006a; Nozaki et al. 2012.

# RG evolution = emergent distance

✓ Distances and curvature can be related to an RG **beta function**

✓ Amounts to an **explicit demonstration** of the holographic principle

✓ Sign of curvature is **topological**, can be written in terms of winding numbers

Maldacena 1999; Ryu et al. 2006a; Holzhey et al. 1994.

# Topological nature of geometry-independent term

$$S_{\mathcal{A}_z(j)} = f_z(j)c\alpha L_x - \underbrace{c \log \left|2 \sin\left(\pi f_z(j)\phi\right)\right|}_{=Q(\phi),\text{geometry-independent term}}$$

✓ $Q(\phi)$ is periodic in the flux $\phi$, $\phi = 1$ transports a charge across Fermi surface

✓ pole structure of $\left(\sin\frac{\pi}{4} - |\sin\left(\pi f_z(j)\right)\phi|\right)^{-1}$ counts number of states $\longrightarrow$ tracks Luttinger volume

✓ Luttinger volume is topological, so is $Q(\phi)$; $Q(\phi)$ can be expressed in terms of winding numbers

**Luttinger et al. 1960; Luttinger 1960; Oshikawa 2000; Seki et al. 2017; Mukherjee et al. 2020a; Heath et al. 2020.**