

# LOCAL METAL-INSULATOR TRANSITION IN AN EXTENDED ANDERSON IMPURITY MODEL

JRF-to-SRF Upgradation Presentation

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## **Summary of Work**

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# Summary of Work

## Completed Projects

- ✓ Unveiling the Kondo cloud: Unitary renormalization-group study of the Kondo model

**Phys. Rev. B 105, 085119**, arXiv:2111.10580v3

A. Mukherjee, Abhirup Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, and S. Lal

- ✓ Frustration shapes multi-channel Kondo physics: A star graph perspective

**under review at PRB**, arXiv:2205.00790

S. Patra, Abhirup Mukherjee, A. Mukherjee, N. S. Vidhyadhiraja, A. Taraphder, S. Lal

## Ongoing Projects

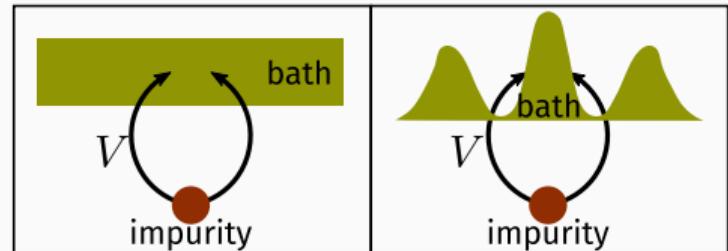
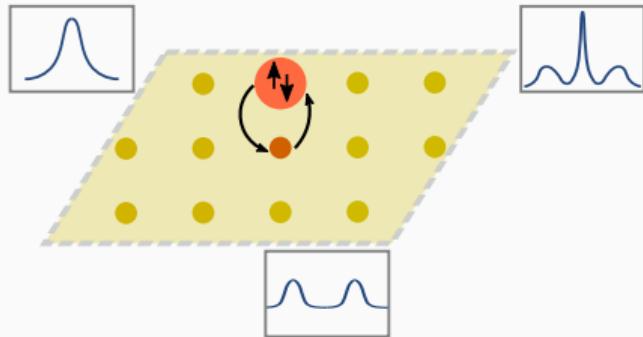
- ✓ Metal-insulator transition in an extended Anderson impurity model (*manuscript in preparation*)
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- ✓ Holography and topology of entanglement scaling in free fermions (*manuscript in preparation*)

- ✓ URG-based auxiliary model approach to correlated systems (*ongoing*)

# Local MIT in an extended Anderson impurity model

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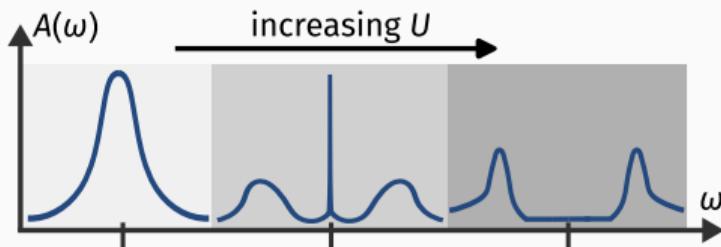


## **Introducing the extended Anderson impurity model**

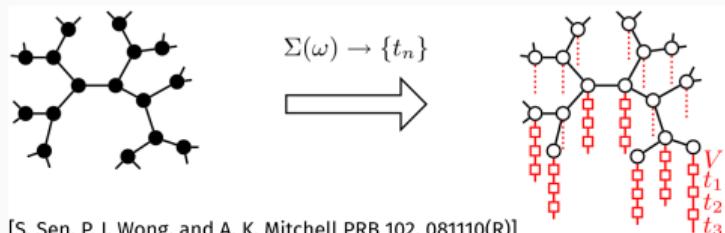
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## DMFT on the Bethe lattice: Exact in $d = \infty$

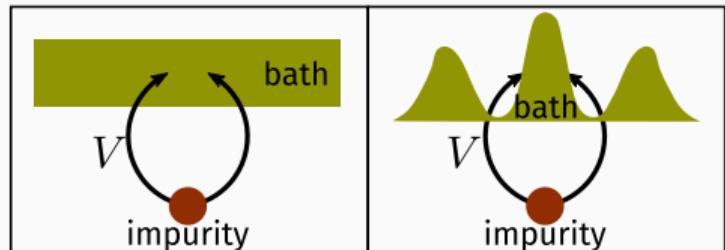
- ✓ shows **metal-insulator transition** on the Bethe lattice with  $\infty$  coordination number



- ✓ Conduction bath obtained by imposing self-consistency shows **non-trivial correlations**



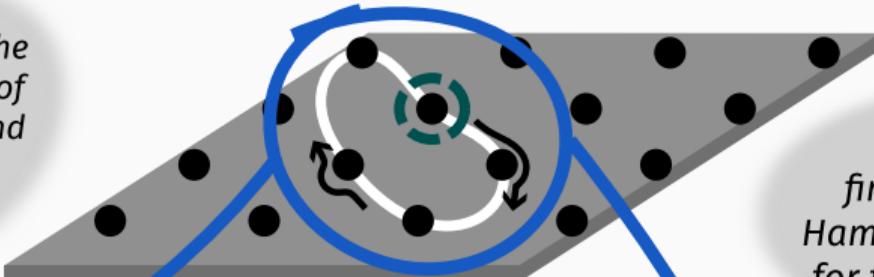
- ✓ Spectral function develops three peaks and then **gaps out**



Metzner et al. 1989; Georges et al. 1992; Parcollet et al. 2004; Maier et al. 2005; Kotliar et al. 2006; Ohashi et al. 2008; Held et al. 2013; Sen et al. 2020.

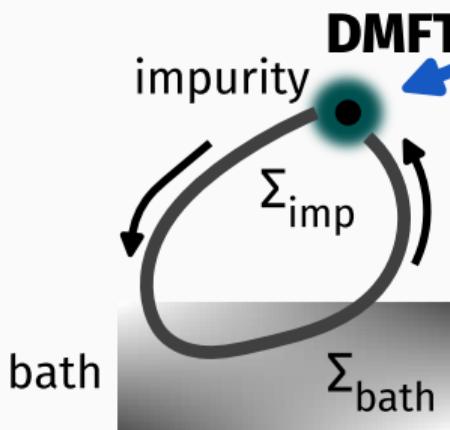
# DMFT on the Bethe lattice: Exact in $d = \infty$

**DMFT** represents excursions from a site into the correlated bath in the form of self-energies for the bath and the impurity. This results in a three-peak DOS of the bath.

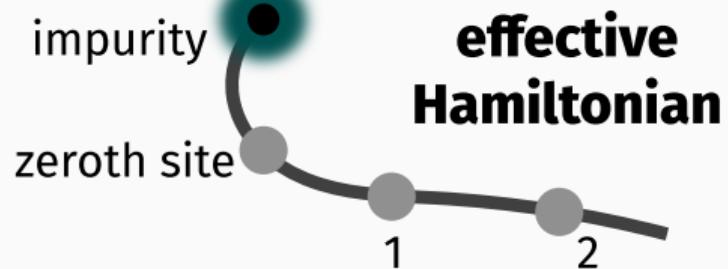


**Our Goal:**

finding an effective Hamiltonian description for the  $\Sigma$  that gives rise to the MIT.



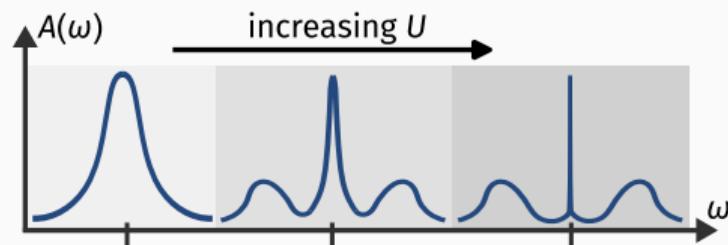
Similar approach adopted by Si & Kotliar for extended Hubbard model



# Introducing the extended Anderson impurity model

## Standard Anderson impurity model

- ✓ no local-moment phase,  $A(\omega)$  gapless
- ✓ cannot explain insulating phase of DMFT

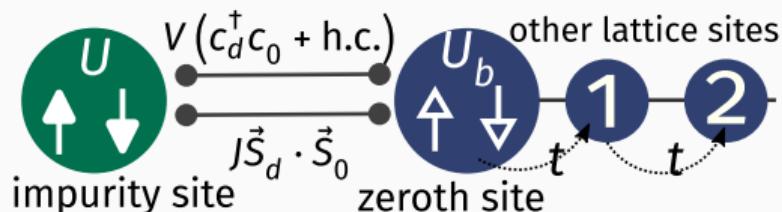


**Gap in spectral function requires additional physics!**

# Introducing the extended Anderson impurity model

## Extended Anderson impurity model

- ✓ impurity-bath spin correlation:  $J$
- ✓ bath zeroth site local correlation:  $U_b$



$$H = H_{\text{KE}} + V \sum_{\sigma} (c_{d\sigma}^\dagger c_{0\sigma} + \text{h.c.}) + \frac{U}{2} (\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow})^2 + J \vec{S}_d \cdot \vec{S}_0 + U_b (\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})^2$$

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Anderson 1961; Anderson 1978; Wilson et al. 1974; Nozieres 1974; Krishna-murthy et al. 1980; Andrei 1980; Tsvelick et al. 1983; Hewson 1993; Costi et al. 1990; Costi 2000; Kuramoto et al. 1987; Cox et al. 1988.

## **Phase Diagram & Ground-States**

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## Nature of RG flows

- ✓ URG Equations reveal **critical** point at  $r = -U_b/J = 1/4$

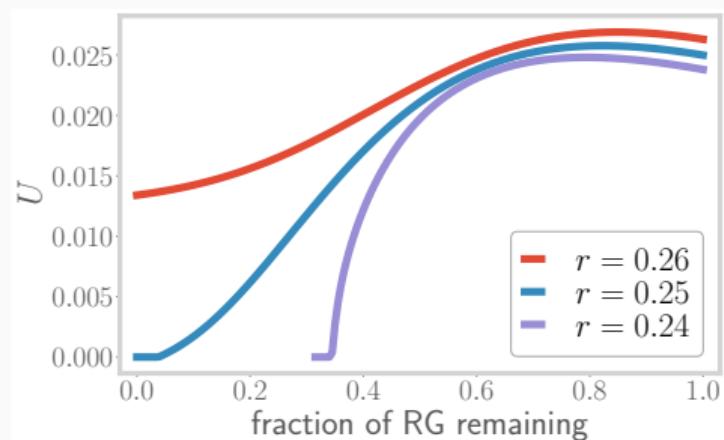
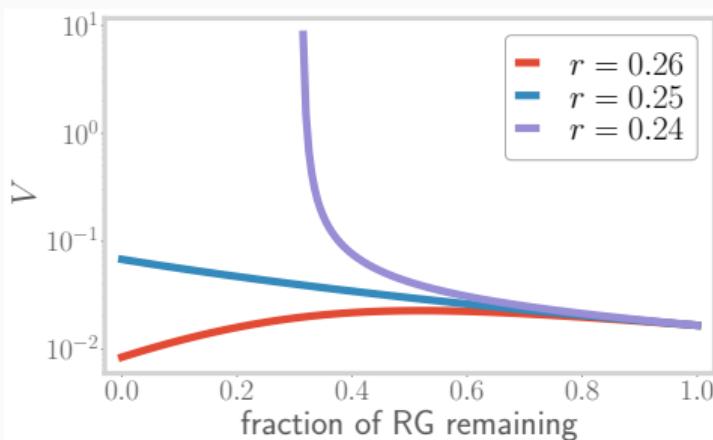
- ✓ RG equation for most dominant coupling  $J$ :

$$\Delta J = \frac{-J(J + 4U_b)n(D)}{\omega - D/2 + U_b/2 + J/4}$$

- ✓ Numerator structure allows **averting** strong-coupling behaviour

## Nature of RG flows

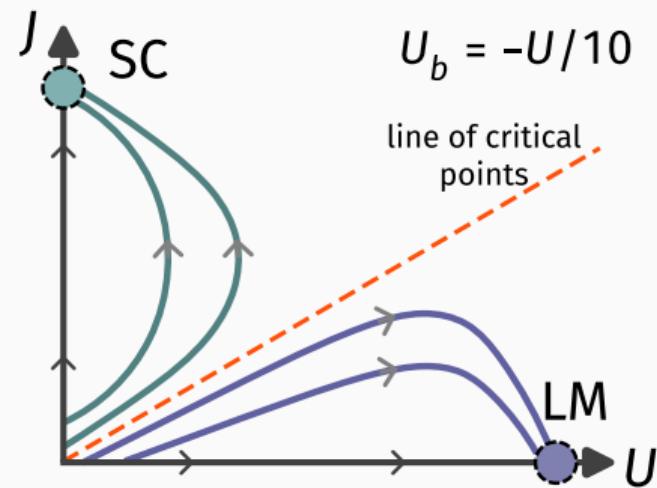
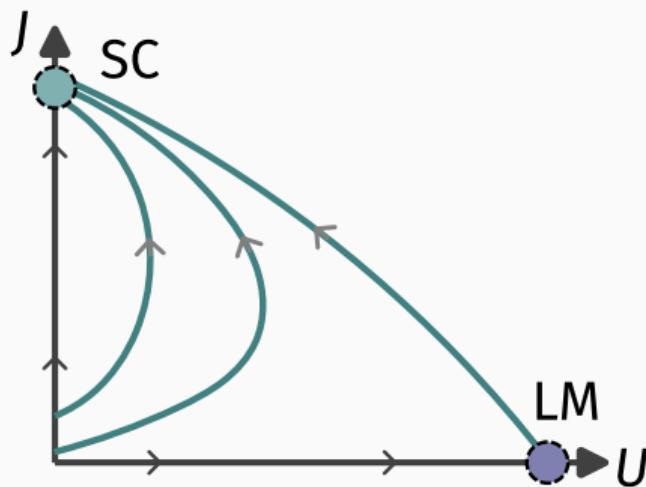
- ✓  $J, V, U$  reverse their behaviour across the critical point; leads to **phase transition**
- ✓ Bath correlation  $U_b$  is always **marginal**



## Nature of RG flows

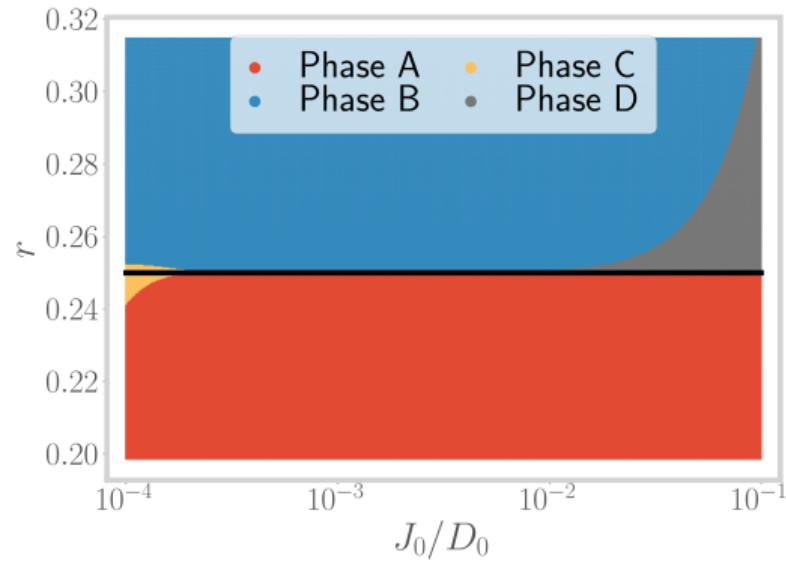
- ✓ RG equation reveals competition between Kondo flow and pairing physics

$$\Delta J = - \underbrace{\frac{(J + 2U_b)^2 n(D)}{\omega - D/2 + U_b/2 + J/4}}_{\text{usual Kondo physics}} + \underbrace{\frac{(2U_b)^2 n(D)}{\omega - D/2 + U_b/2 + J/4}}_{\text{competing pairing physics}}$$



## RG Phase Diagram

- ✓ blue phase  $\rightarrow U_b < -J/4$ :  $V, J$  are **irrelevant**  $\rightarrow$  local moment flows
- ✓ yellow phase:  $J \ll D_0$ : involves  **$V, U, U_b$**   
*vanishes for large systems*
- ✓ gray phase:  $J \sim D_0$ : **all** couplings irrelevant  
*vanishes for large systems*
- ✓ red phase  $\rightarrow U_b > -J/4$ :  $V, J$  are **relevant**  $\rightarrow$  strong-coupling flows

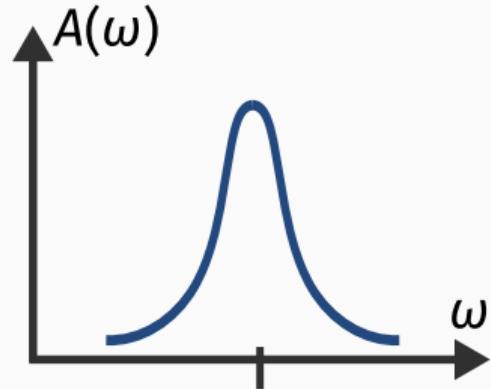


# Low-energy effective Hamiltonians and ground-states

**Regime 1:**  $|U_b| < J/4$

- ✓  $J$  relevant,
- ✓  $V$  subdominant,
- ✓  $U$  irrelevant

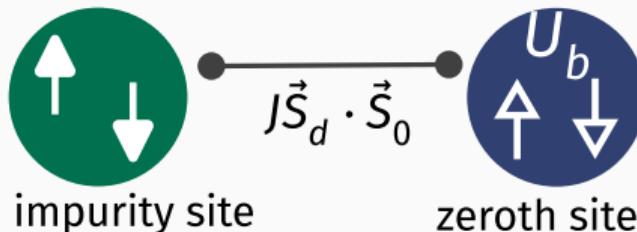
$$H = J \vec{S}_d \cdot \vec{S}_0 - U_b (\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})^2 + \sum_{k < \Lambda^*} \epsilon_k \hat{n}_{k\sigma}$$



# Low-energy effective Hamiltonians and ground-states

**Regime 1:**  $|U_b| < J/4$

**Zero-bandwidth limit**



$$H = J\vec{S}_d \cdot \vec{S}_0 - U_b (\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})^2$$

- ✓ two-spin Heisenberg, attractive zeroth site
- ✓ **singlet** ground state

$$|\Psi\rangle_{GS} = \frac{1}{\sqrt{2}} [|\uparrow_d \downarrow_0\rangle - |\downarrow_d \uparrow_0\rangle]$$

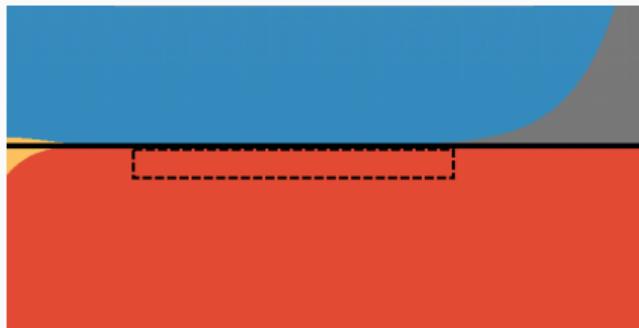
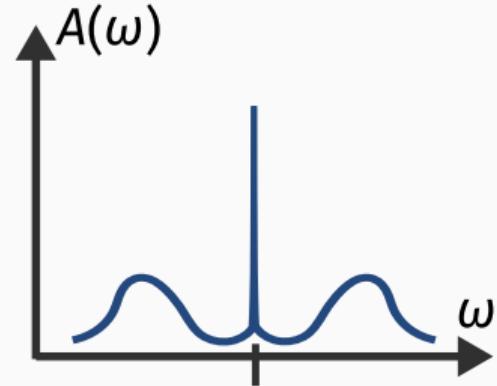


# Low-energy effective Hamiltonians and ground-states

**Regime 2:**  $|U_b| \sim J/4$

- ✓  $J$  relevant,
- ✓  $V$  relevant,
- ✓  $U$  irrelevant

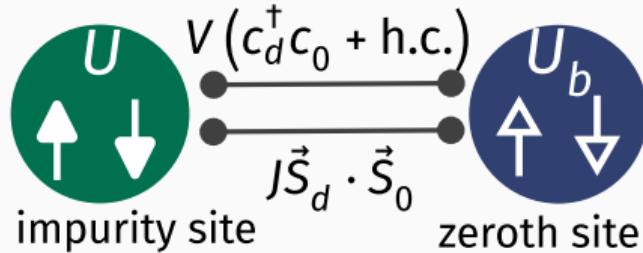
$$H = J\vec{S}_d \cdot \vec{S}_0 + V \sum_{\sigma} (c_{d\sigma}^\dagger c_{0\sigma} + \text{h.c.}) - U_b (\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})^2 + \sum_{k < \Lambda^*} \epsilon_k \hat{n}_{k\sigma}$$



## Low-energy effective Hamiltonians and ground-states

**Regime 2:**  $|U_b| \sim J/4$

**Zero-bandwidth limit**

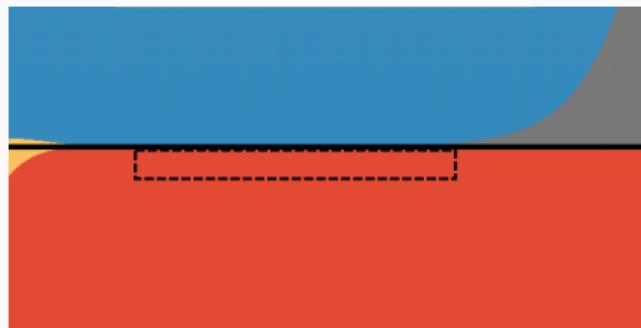


$$H = JS_d \cdot S_0 + V \sum_{\sigma} (c_{d\sigma}^\dagger c_{0\sigma} + \text{h.c.}) - U_b (\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})^2$$

✓ **spin+charge** dimer with attractive 0<sup>th</sup> site

✓ spin-singlet + charge-triplet-zero in gr-state

$$|\Psi\rangle_{GS} = \frac{1}{2\sqrt{2}} [|\uparrow_d \downarrow_0\rangle - |\downarrow_d \uparrow_0\rangle] + \frac{1}{2\sqrt{2}} [|\downarrow_d, 0_0\rangle + |\uparrow_d, 2_0\rangle] \quad (1)$$

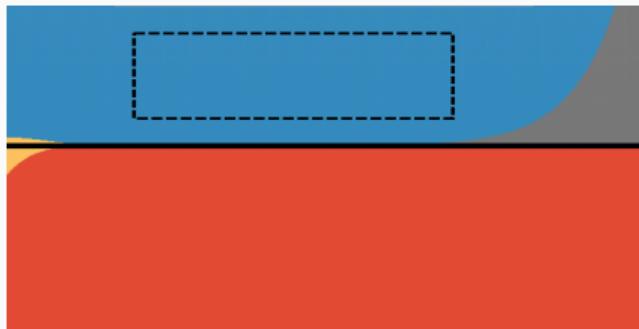
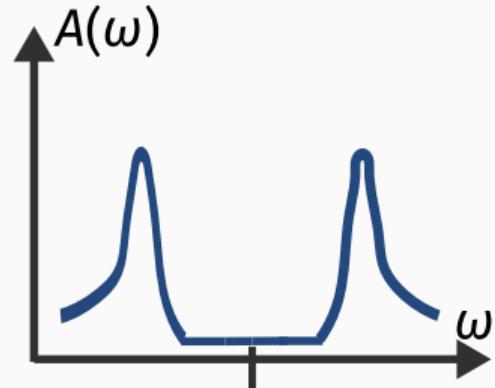


## Low-energy effective Hamiltonians and ground-states

**Regime 3:**  $|U_b| > J/4$

- ✓  $J, V$  irrelevant,
- ✓  $U$  relevant,

$$H = -U/2 (\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow})^2 - U_b (\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})^2 + \sum_{k < \Lambda^*} \epsilon_k \hat{n}_{k\sigma}$$



# Low-energy effective Hamiltonians and ground-states

**Regime 3:**  $|U_b| > J/4$

**Zero-bandwidth limit**



impurity site

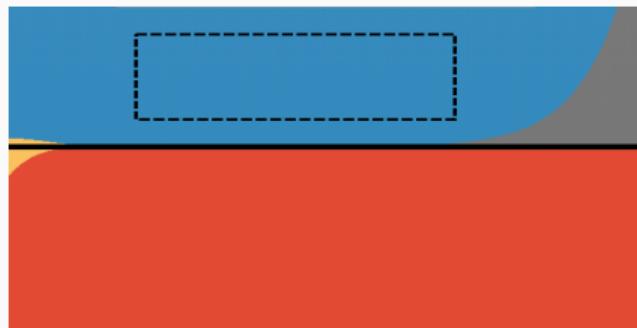


zeroth site

- ✓ impurity site detaches from bath
- ✓ **local moment** ground-state

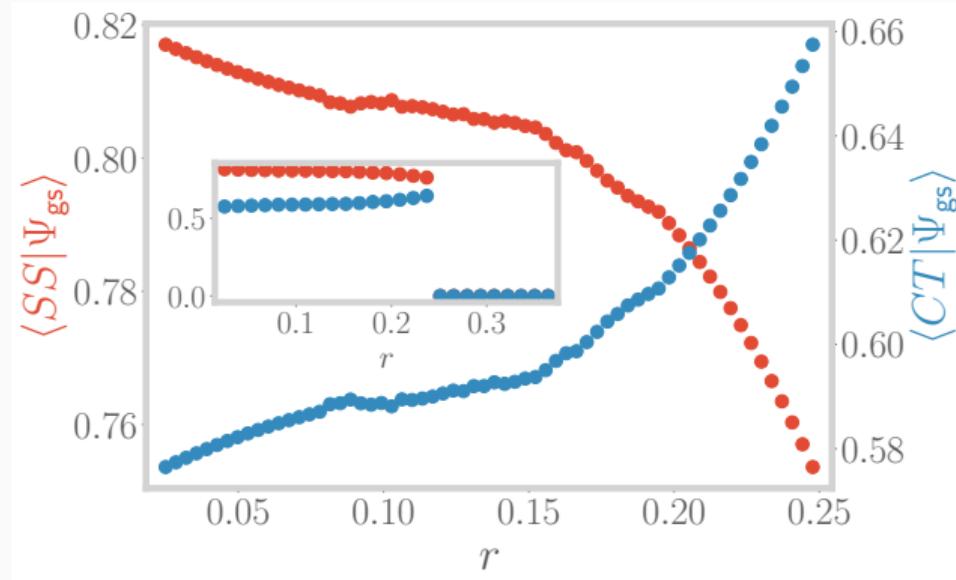
$$|\Psi\rangle_{GS} = |\uparrow, \downarrow\rangle_d \otimes |0, 2\rangle_0$$

$$H = -U/2(\hat{n}_{d\uparrow} - \hat{n}_{d\downarrow})^2 - U_b(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})^2$$



# Low-energy effective Hamiltonians and ground-states

Ground-state overlaps with spin singlet and charge triplet zero



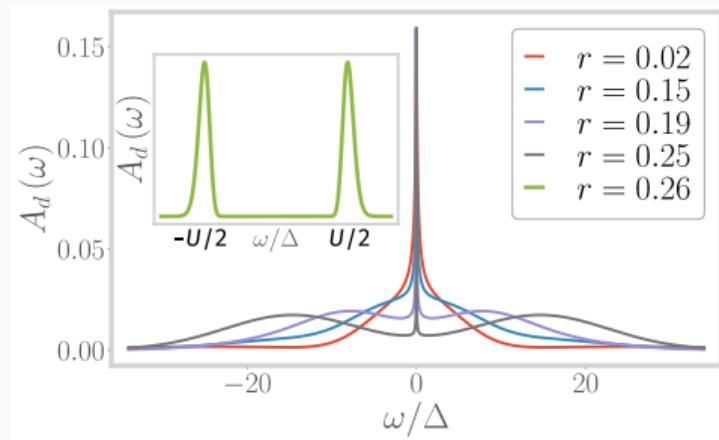
## **Nature of the Transition**

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# Gapping of the impurity spectral function

- ✓ Broad central peak at  $|U_b| \ll J/4$

- ✓ Correlated **three peak** structure at  $|U_b| \lesssim J/4$

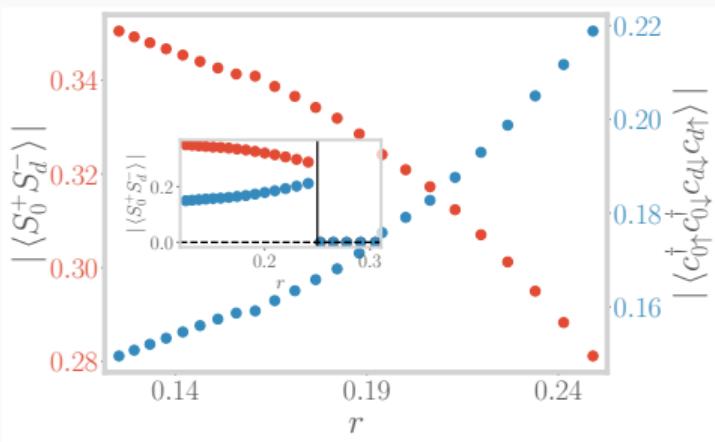


- ✓ hard central **gap** for  $|U_b| > J/4$

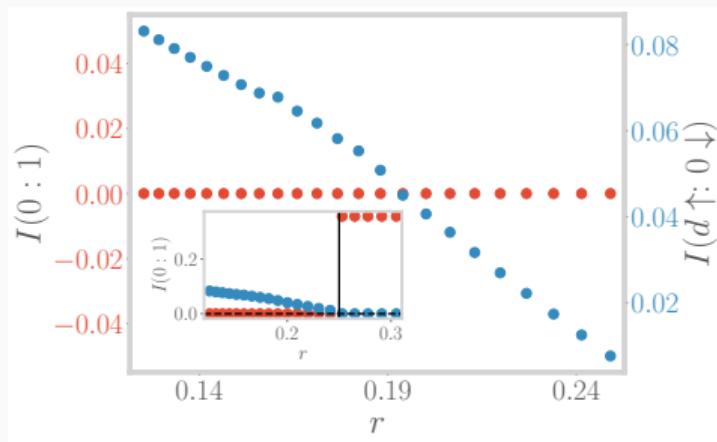
# Destruction of the Kondo cloud

The Kondo cloud **weakens, and is destroyed at the transition.**

- ✓ vanishing of impurity-bath correlations



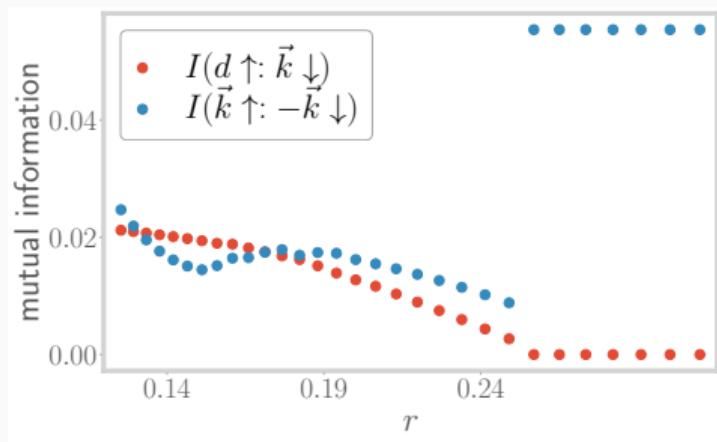
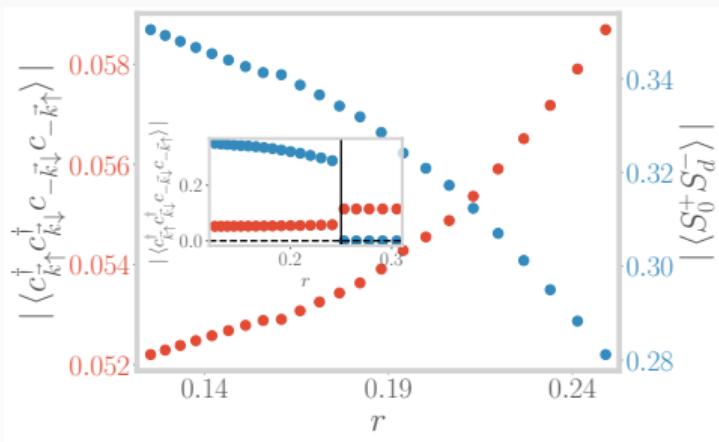
- ✓ transfer of entanglement into the bath



# Growth of pairing fluctuations in the bath

## Subdominant pairing fluctuations, near the transition...

- ✓ growth of fluctuations in Cooper channel, at the cost of spin-flip fluctuations
- ✓ mutual information within the bath maximised after transition



## **Universal Theory near the Transition**

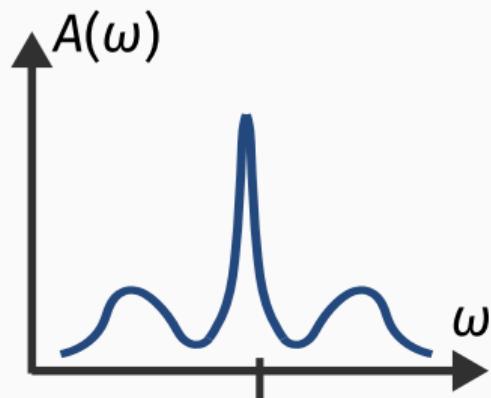
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## Minimal effective model for the transition

- ✓ For  $|U_b| \lesssim J/4$ , central peak and side peaks are **well-separated**
- ✓ **Integrate out** charge fluctuations through Schrieffer-Wolff transformation

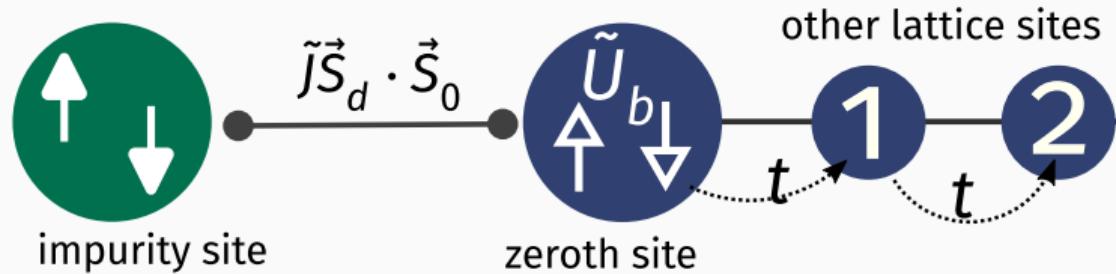
$$H_{\text{eff}} = \tilde{J} \vec{S}_d \cdot \vec{S}_0 - \tilde{U}_b (\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})^2 + H_{\text{K.E.}}$$

$$\text{RG equation for } \tilde{J} : \Delta \tilde{J} \sim \tilde{J} (\tilde{J} + 4 \tilde{U}_b)$$



## Minimal effective model for the transition

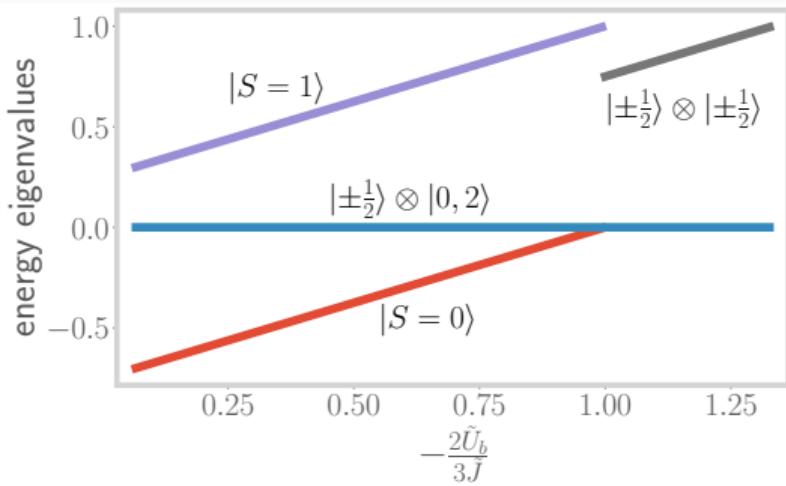
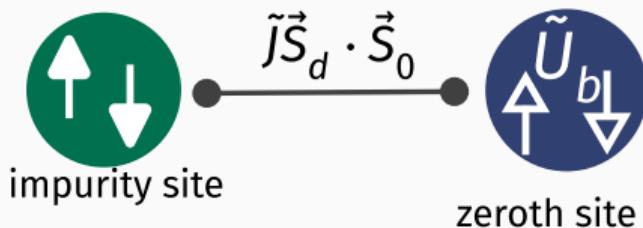
- ✓ captures the criticality, and the strong-coupling and local moment phases



Suggests that  $J$  and  $U_b$  are the minimal & universal ingredients for transition!

## Capturing the level crossing at the transition from a two-site model

- ✓ Obtain two-site model by taking **zero bandwidth** limit
- ✓ spectrum shows **level crossing** between singlet and local moment states

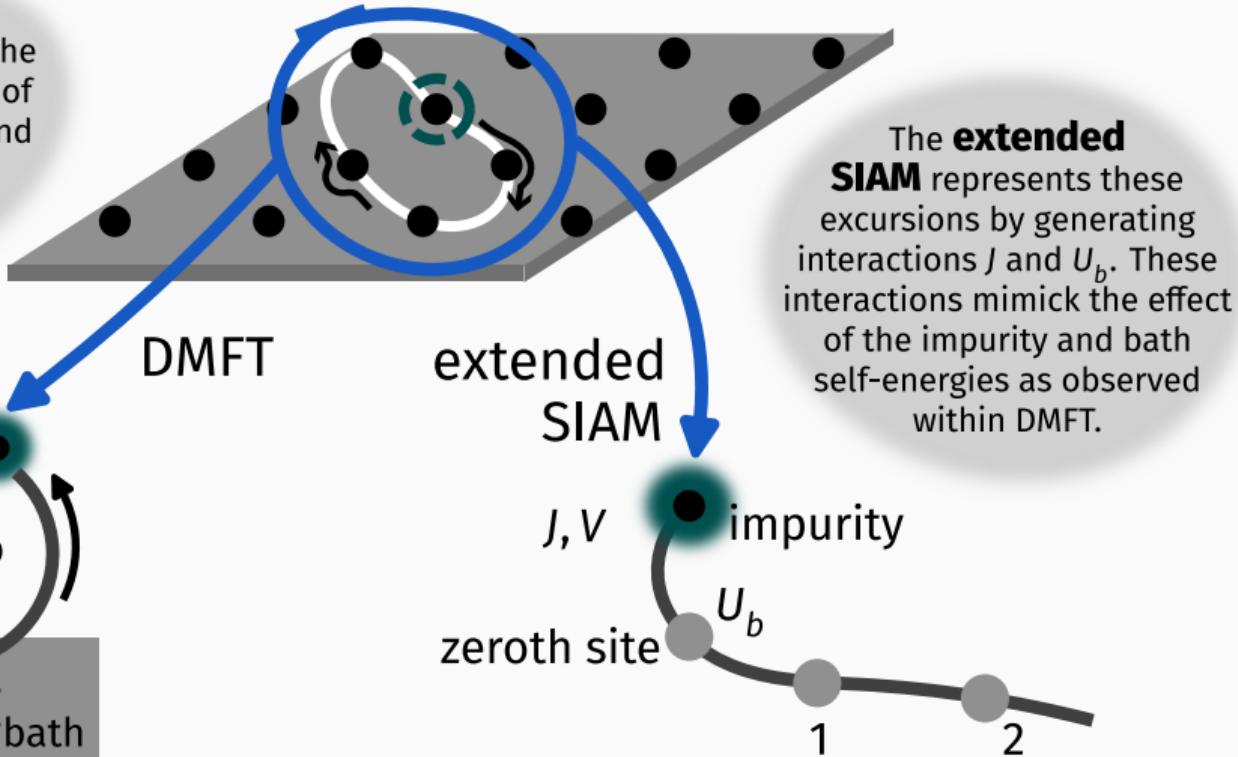
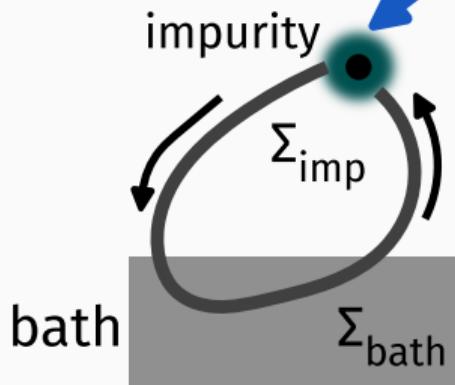


## **Insights into DMFT**

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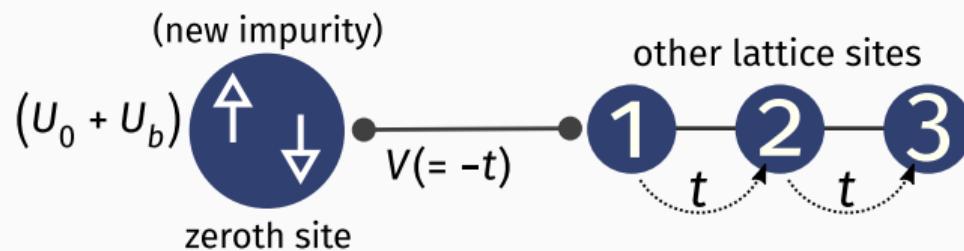
## Extended SIAM in the context of DMFT

**DMFT** represents excursions from a site into the correlated bath in the form of self-energies for the bath and the impurity. This results in a three-peak DOS of the bath.



## Equivalence of the impurity site and the bath zeroth site

- ✓ Integrate out impurity site from fixed point Hamiltonian via a single URG transformation
- ✓ Generates additional correlation  $U_0$  on zeroth site

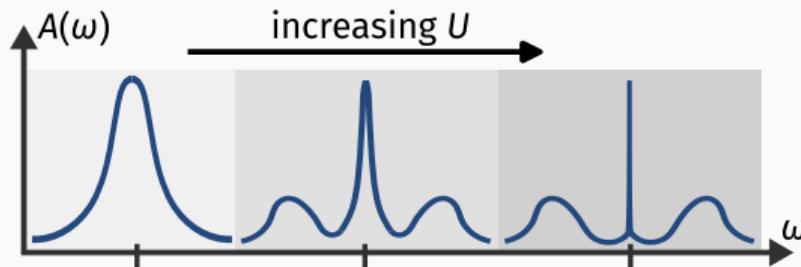


- ✓  $J$  is relevant and the largest scale → **repulsive correlation:**

$$U_0 + U_b \approx J > 0$$

## Equivalence of the impurity site and the bath zeroth site

- ✓  $J$  acts a **symmetrisation mechanism** between impurity and zeroth sites
- ✓ **Coherent** spin-flip scatterings ensure similarity of spectral functions

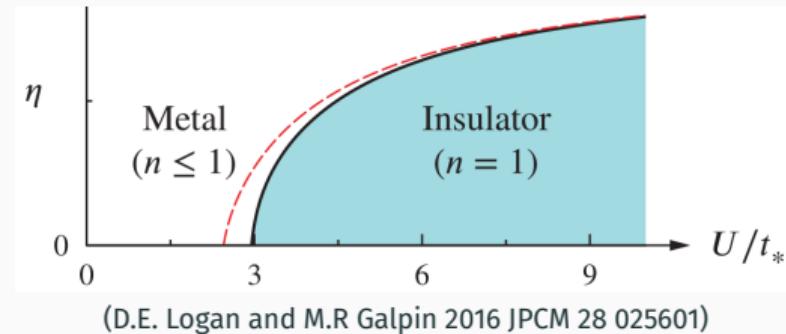


**Essence of self-consistency:** Equivalence of impurity and zeroth sites!

## Observation of a coexistence region

- ✓ DMFT observes a **coexistence region** near the critical point, for  $U_{c1} < U < U_{c2}$

- ✓ Insulating when coming in from the insulator, metallic when coming in from the metal

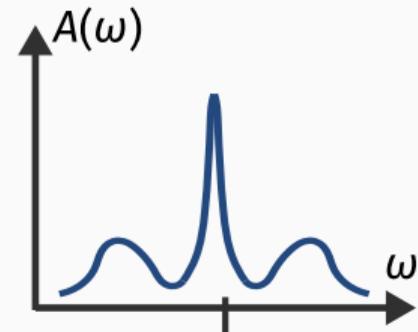
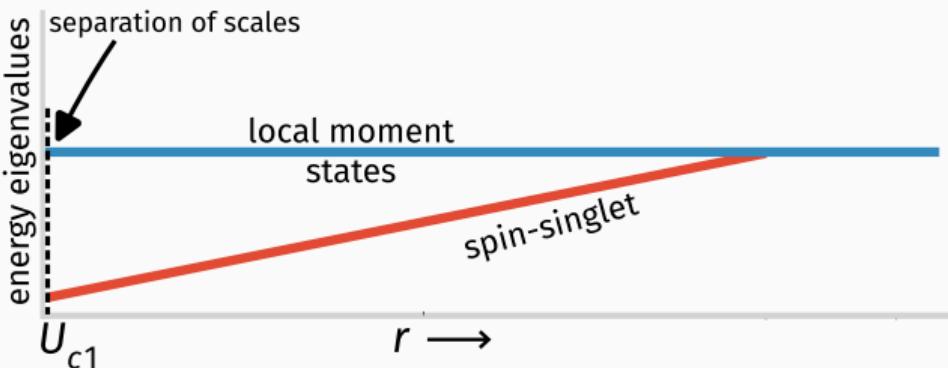


- ✓ Mott gap appears **discontinuously** after the transition, through a **continuous** sharpening of the central peak.
- ✓ **True** transition believed to occur at  $U_{c2}$ .

# Observation of a coexistence region

Can be explained heuristically using the two site spectrum

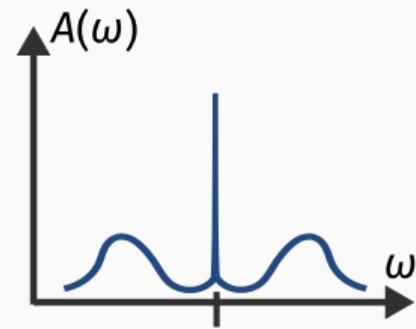
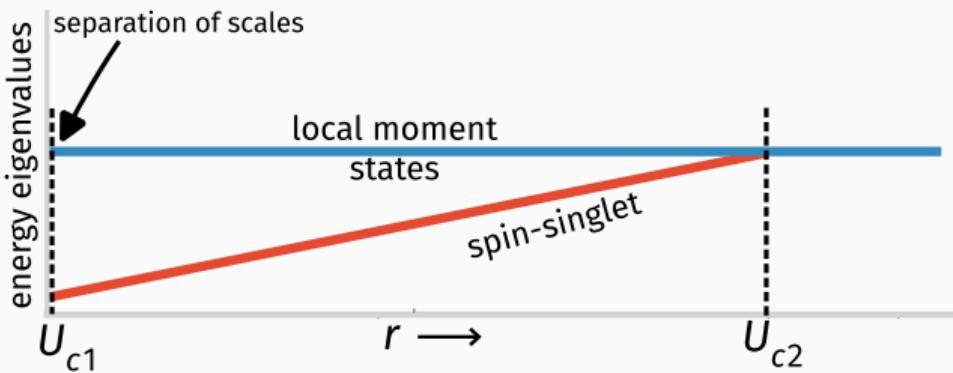
- ✓ Initial point is when the side peaks get separated (near-zeroes in the spectral function)



# Observation of a coexistence region

Can be explained heuristically using the two site spectrum

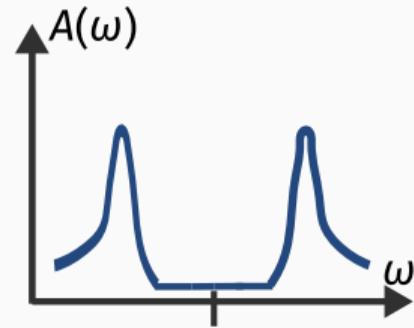
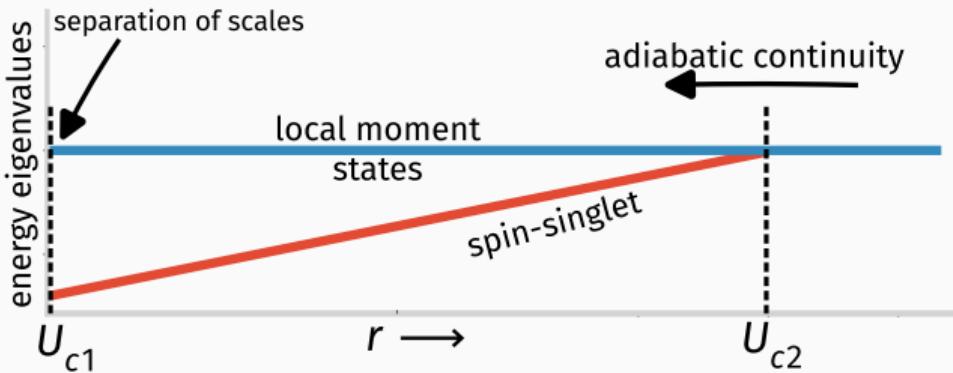
- ✓  $U_{c2}$  is the point where the levels cross



# Observation of a coexistence region

Can be explained heuristically using the two site spectrum

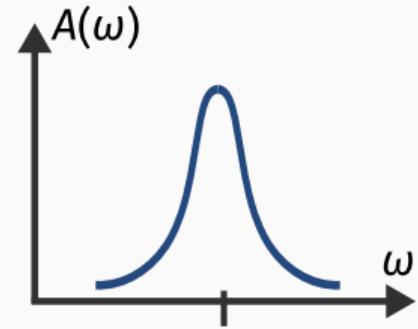
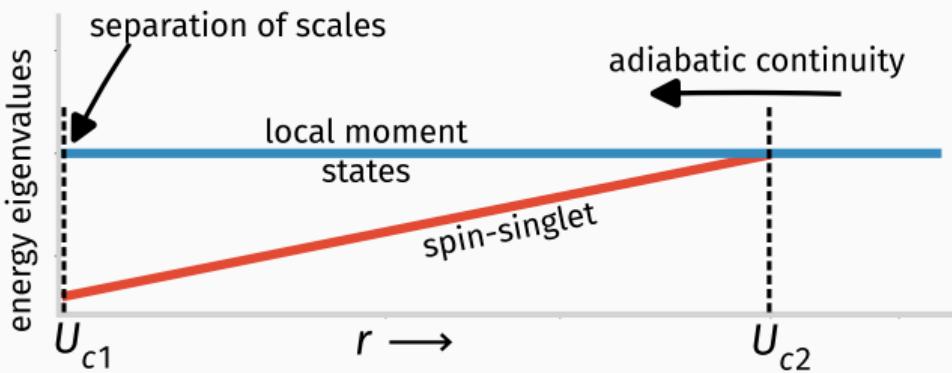
- ✓ Coming from  $U > U_{c2}$ , **adiabatic continuity** allows DMFT to stay on the local moment state...



# Observation of a coexistence region

Can be explained heuristically using the two site spectrum

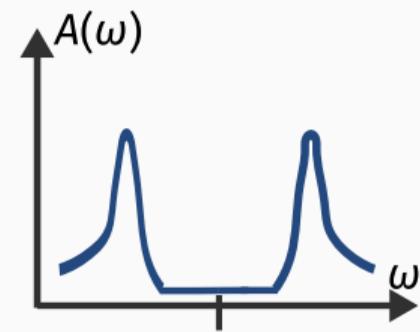
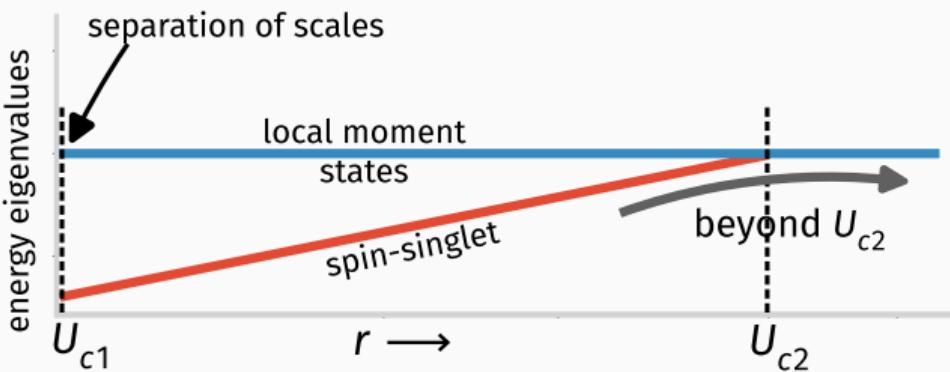
- ✓ For  $U < U_{c1}$ , local moment state is too unstable, relaxes to the true ground state.



# Observation of a coexistence region

Can be explained heuristically using the two site spectrum

- ✓ For  $U > U_{c1}$ , charge sector separated by large  $U$ , leads to the **discontinuous** appearance of finite gap



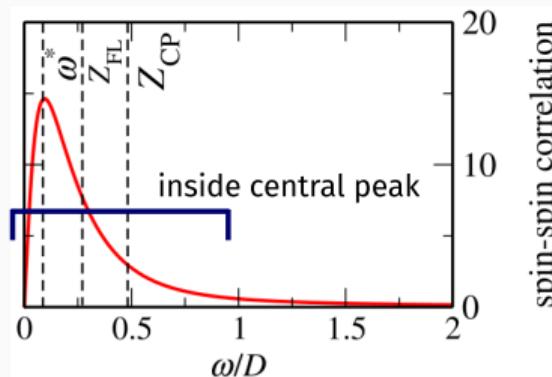
# Comparison against NRG-DMFT correlation functions

## Poor Man's scaling of the effective Kondo model

[K. Held, R. Peters, and A. Toschi. PRL 2013]

- ✓ shows **quantitative** agreement with NRG-DMFT (crossover scale and kinks in self-energy)
- ✓ Suggests that the minimal model can capture **spin susceptibility**

[K. Held, R. Peters, and A. Toschi.  
PRL 110, 246402 (2013)]



- ✓ Our  $J - U_b$  model **goes further** by capturing physics beyond the transition

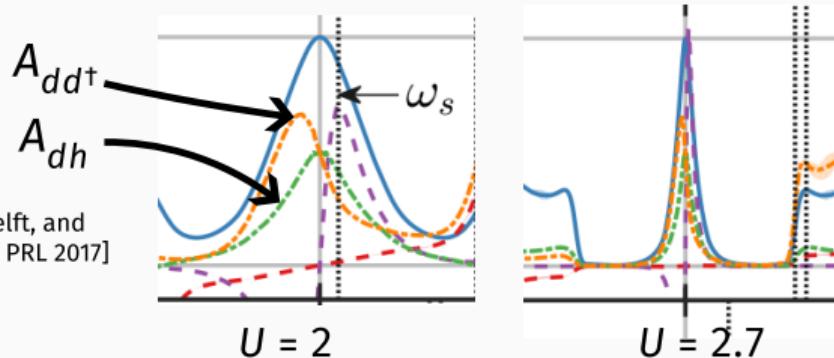
# Comparison against NRG-DMFT correlation functions

## Doublon-holon correlators of the Hubbard model

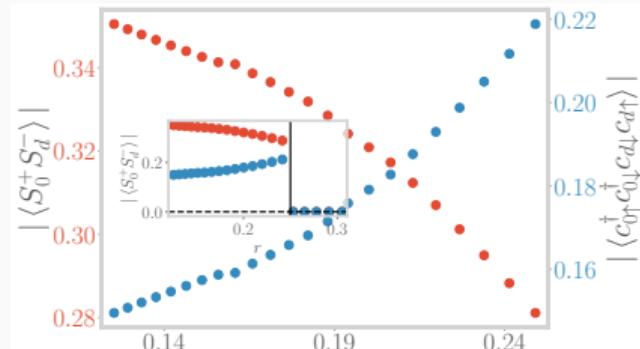
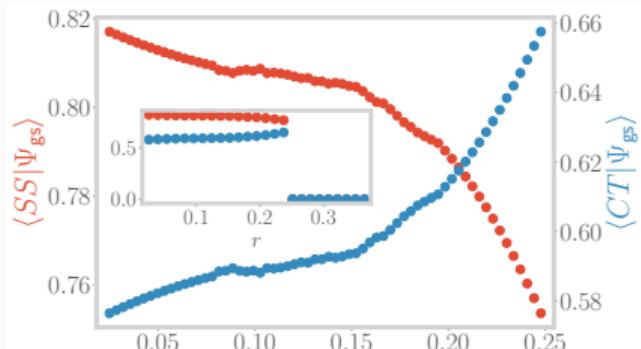
[S. B. Lee, J. v Delft, and A. Weichselbaum. PRL 2017]

Lee et. al show **peaks** in doublon-holon correlators near zero energy within the central Kondo peak.

[S. B. Lee, J. v Delft, and A. Weichselbaum. PRL 2017]



We find support for this in the form of **increasing ground-state charge correlations and overlap**.



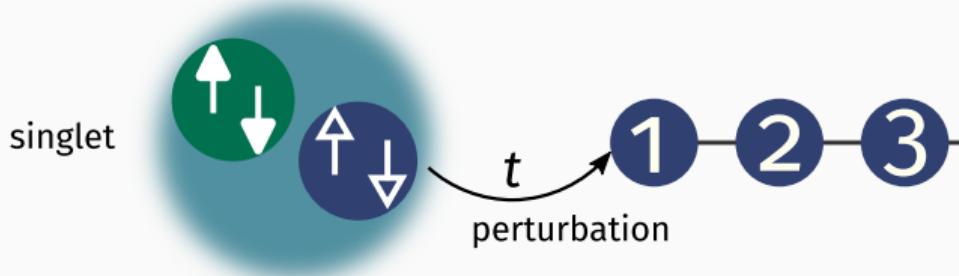
## **Low-energy excitations of the bath**

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## Effect on the local Fermi liquid

What about the **low-energy excitations** of the bath, that lie above the singlet ground state?

- ✓ treat hopping between singlet and bath as perturbation



- ✓ Up to fourth order, charge sector becomes repulsive...

$$H_{\text{eff}} = \frac{2t^4}{\tilde{J}(3\tilde{J}/4 + \tilde{U}_b)^2} [\hat{n}_{1\uparrow}\hat{n}_{1\downarrow} + (1 - \hat{n}_{1\uparrow})(1 - \hat{n}_{1\downarrow})] + H_{\text{K.E.}}$$

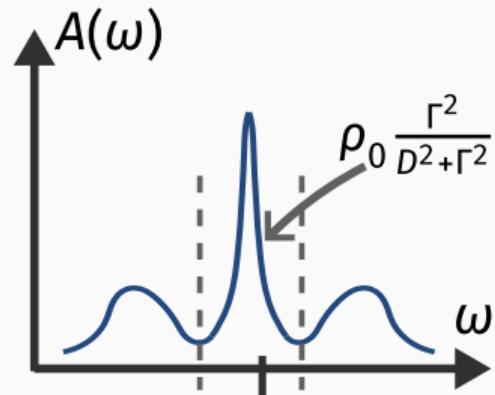
- ✓ FL term blows up towards transition, signaling **breakdown** of Fermi liquid theory and loss of adiabaticity.

## Effect on the local Fermi liquid

Vanishing of the **Kondo scale**  $T_K$  towards the transition

- ✓ Consider the  $J - U_b$  model, but with a **Lorentzian DOS** in the bath:

$$\rho = \rho_0 \frac{\Gamma^2}{D^2 + \Gamma^2}$$



- ✓ Near the transition  $r = -U_b/J_0 \rightarrow \frac{1}{4}$  and  $\Gamma \rightarrow 0$ , the **IR energy scale**  $D^*$  can be approximated as

$$D^* = D_0 \exp \left[ \frac{(2\omega + U_b + J_0/2)^2}{8|U_b|\rho_0\Gamma^2} \ln |1 - 4r| \right], \quad D_0 = \text{UV cutoff}.$$

## Effect on the local Fermi liquid

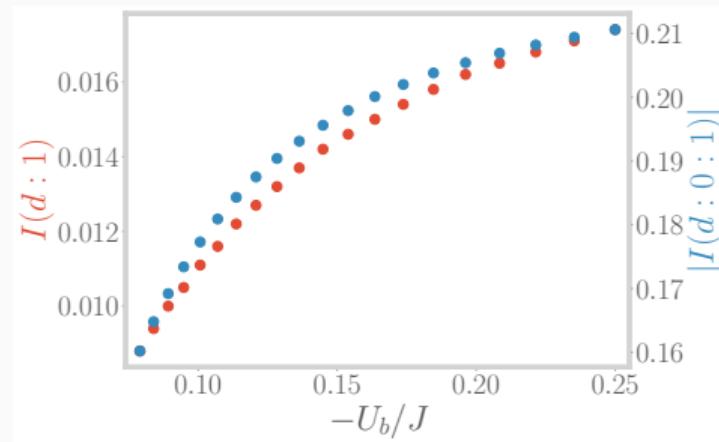
**Vanishing of the Kondo scale  $T_K$  towards the transition**

- ✓ **Kondo temperature** can be defined as  $T_K = D^*/k_B$ :

$$T_K = \frac{D_0}{k_B} \exp \left[ \frac{(2\omega + U_b + J_0/2)^2}{8|U_b|\rho_0\Gamma^2} \ln |1 - 4r| \right]$$

- ✓ Kondo temperature vanishes as  $r \rightarrow 1/4$ :

$$T_K \sim (1 - 4r)^\alpha \rightarrow 0$$



## Effect on the local Fermi liquid

**How do the imaginary part of self-energy and the qp residue behave near the transition?**

- ✓ Following the renormalised perturbation theory approach of Hewson,  $\text{Im} [\Sigma(\omega)]$  is

$$\text{Im} [\Sigma(\omega)] \sim u^2 \omega^2, \quad u = \frac{2t^4}{\tilde{J}(3\tilde{J}/4 + \tilde{U}_b)^2}$$

- ✓ As  $r \rightarrow 1/4$ ,  $u \rightarrow \infty$ , signalling a vanishing lifetime of the quasiparticles
- ✓ Quasiparticle residue  $Z$  for 1-particle excitations is proportional to  $T_K$ :

$$Z \sim T_K$$

$$Z \rightarrow 0 \text{ as } r \rightarrow 1/4$$

## Effective Hamiltonian at the critical point

What is the nature of the low-energy excitations exactly at the critical point?

- ✓ Singlets become **degenerate** with local moment states at the critical point:

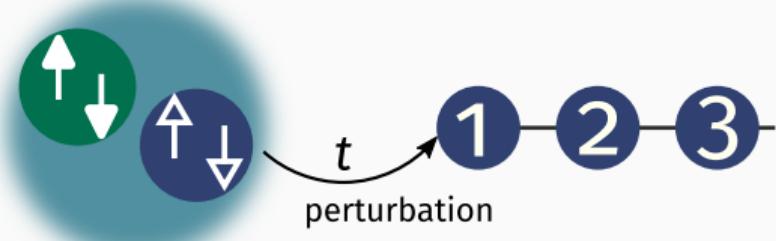
$$\frac{1}{\sqrt{2}} (|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle), \quad |\sigma\rangle |0\rangle, \quad |\sigma\rangle |2\rangle$$

- ✓ **Non-zero degeneracy** exists even in presence of hopping perturbation:

$$|N_{\text{tot}} = 2, S_{\text{tot}}^z = 0\rangle, \quad |N_{\text{tot}} = 3, S_{\text{tot}}^z = \pm 1/2\rangle,$$

$$|N_{\text{tot}} = 4, S_{\text{tot}}^z = 0\rangle$$

$$N_{\text{tot}} = \hat{n}_d + \hat{n}_0 + \hat{n}_1, \quad S_{\text{tot}}^z = S_d^z + S_0^z + S_1^z$$



## Effective Hamiltonian at the critical point

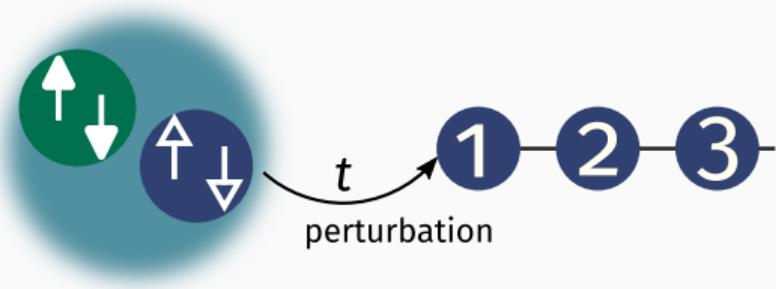
What is the nature of the low-energy excitations exactly at the critical point?

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$$|N_{\text{tot}} = 2, S_{\text{tot}}^z = 0\rangle, |N_{\text{tot}} = 3, S_{\text{tot}}^z = \pm 1/2\rangle,$$

$$|N_{\text{tot}} = 4, S_{\text{tot}}^z = 0\rangle$$

$$N_{\text{tot}} = \hat{n}_d + \hat{n}_0 + \hat{n}_1, \quad S_{\text{tot}}^z = S_d^z + S_0^z + S_1^z$$



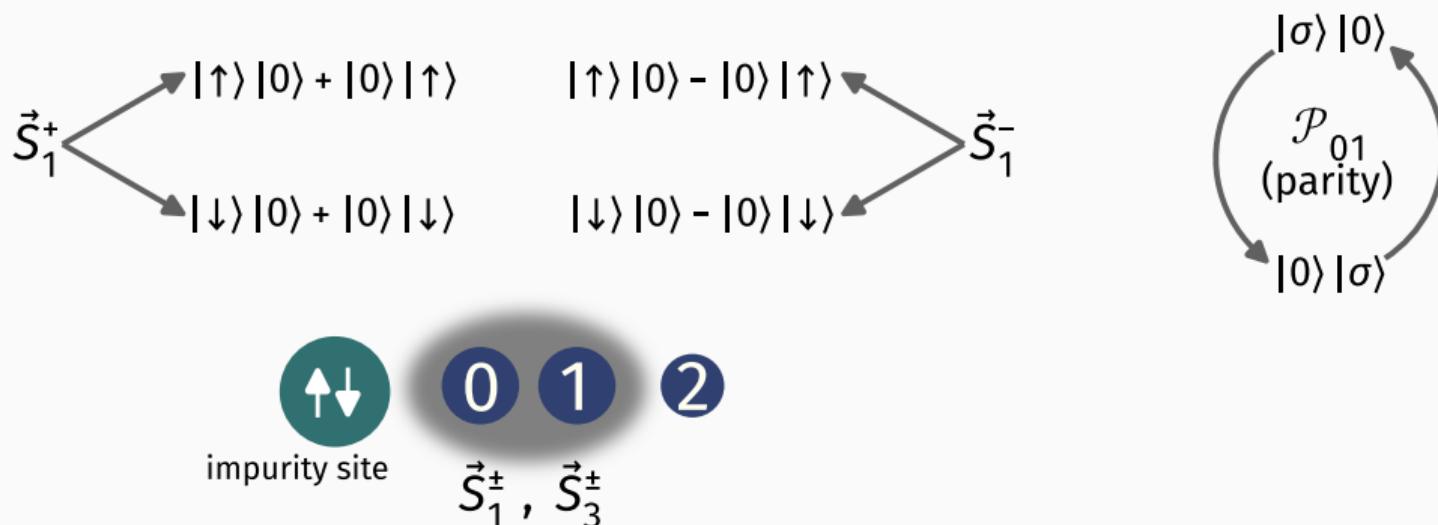
- ✓ Presence of magnetic states with  $S_{\text{tot}}^z \neq 0$  indicates **breakdown of screening**
- ✓ Non-vanishing degeneracy will lead to critical correlations and **non-Fermi** liquid physics

## Effective Hamiltonian at the critical point

What is the nature of the low-energy excitations exactly at the critical point?

- ✓  $S_{\text{tot}}^z = 0$  first order effective Hamiltonian acquires quasi non-local terms:

$$H_{\text{eff}} = t \vec{S}_d \cdot (\vec{S}_1^- + \vec{S}_3^- - \vec{S}_1^+ - \vec{S}_3^+) + \frac{t}{4} \mathcal{P}_{01} + H_{\text{K.E.}}$$



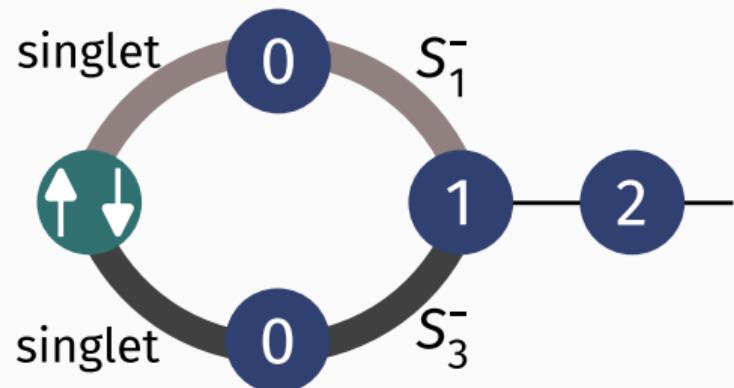
## Effective Hamiltonian at the critical point

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- ✓ Ground-state dictated by  $\mathcal{S}_1^-, \mathcal{S}_3^-$ : degenerate singlets
- ✓ Spreading of extent of bath spin indicates **stretching** of singlet in real space



## Effective Hamiltonian at the critical point

What is the nature of the low-energy excitations exactly at the critical point?

- ✓  $S_{\text{tot}}^z = 0$  first order effective Hamiltonian acquires quasi non-local terms:

$$H_{\text{eff}} = t \vec{S}_d \cdot (\vec{\mathcal{S}}_1^- + \vec{\mathcal{S}}_3^- - \vec{\mathcal{S}}_1^+ - \vec{\mathcal{S}}_3^+) + \frac{t}{4} \mathcal{P}_{01} + H_{\text{K.E.}}$$

- ✓ Shows the dispersion of entanglement across the lattice at critical point. Precursor to decoupling of impurity spin.
- ✓ Degenerate singlets connected by hopping; reminiscent of multichannel Kondo physics and source of non-Fermi liquid physics

## Broad conclusions

- ✓ The extended SIAM appears to capture the phenomenology of the DMFT transition and **self-consistency**.
- ✓ The key ingredient is a **competition** between Kondo screening physics and a local attractive correlation in the bath.
- ✓ Crucial feature of the journey is the enhancement of **pairing fluctuations** in the bath: leads to destruction of Kondo cloud.

## Broad conclusions

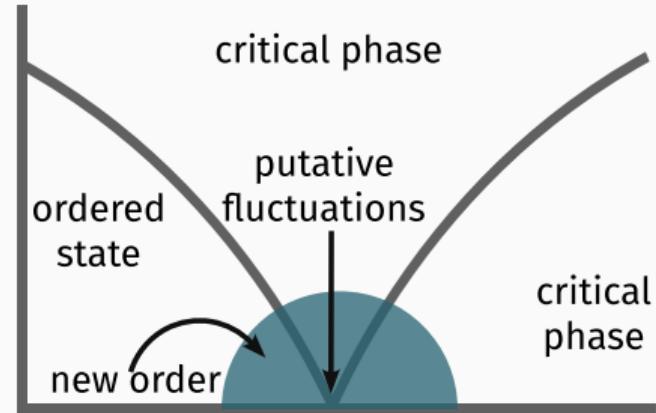
- ✓ An **emergent self-consistency** is achieved through the qualitative similarity of the spectral functions of the impurity and zeroth sites.
- ✓ Approach towards criticality is marked by **vanishing** metallic energy scale and quasiparticle residue.
- ✓ **Non-Fermi liquid** physics arises at critical point, through the expansion of the singlet into the next lattice site.

## **Future Prospects**

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## Future Prospects

- ✓ The extended SIAM can be improved by considering **multiple impurities** and general impurity **filling**.
- ✓ We are developing a new **tiling-based auxiliary model method** can used for studying other models of strong-correlations as well as topologically active or flat band systems.
- ✓ The URG can be applied to **heavy-fermion materials** towards a study of phase diagram and unconventional superconductivity, as well as Kondo insulators.
- ✓ Interacting systems in a magnetic field is also a potential area of study, specifically **fractional Chern insulators** (e.g. the fractional quantum hall effects).



## Acknowledgements

Gracious thanks to

- ✓ my collaborators **S. Patra, A. Mukherjee, Prof. A. Taraphder** and **Prof. N. S. Vidhyadhiraja**,
- ✓ my RPC members **Prof. Ritesh Singh** and **Prof. Anandamohan Ghosh** for instructive feedback,
- ✓ **Prof. H. Casini, Prof. N. Banerjee** and **Shibendu G. Chowdhury** for very fruitful discussions, and
- ✓ IISER Kolkata for funding.

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## **More insight into self-consistency**

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## More insight into self-consistency

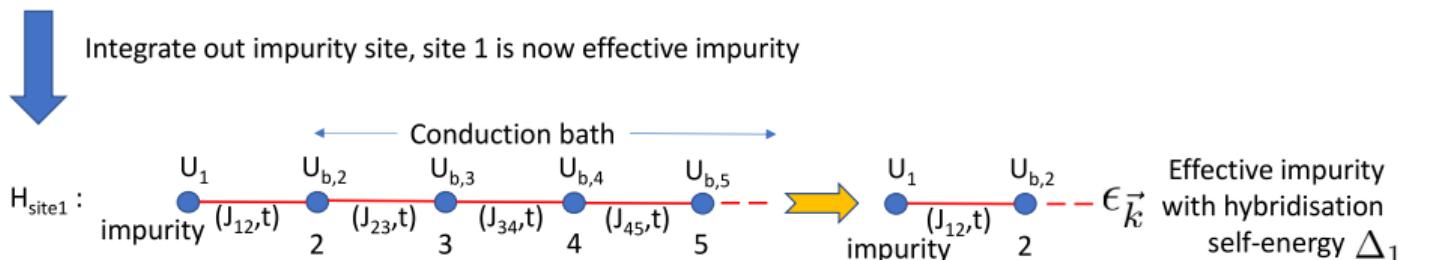
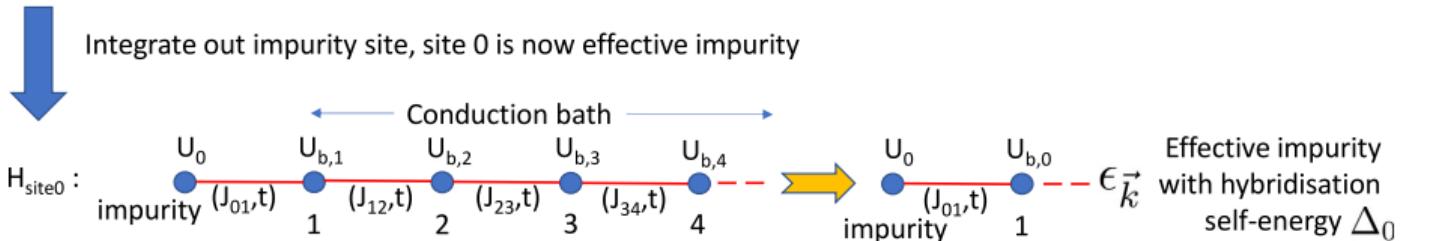
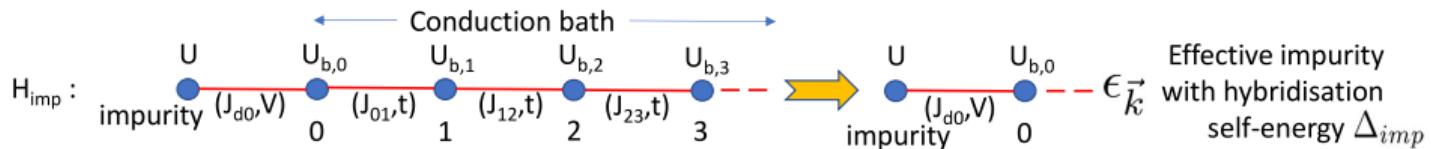
### Meaning of exact self-consistency

- ✓ Minimal impurity model  $\Rightarrow$  qualitative self-consistency  $\Rightarrow$  impurity site = zeroth site
- ✓ Full self-consistency  $\Rightarrow$  equivalence of self-energy on all sites

## More insight into self-consistency

### Meaning of exact self-consistency

- ✓ Can be achieved through an extended impurity model with  $U_b$  and  $J$  on all sites in the bath



## **Other Projects**

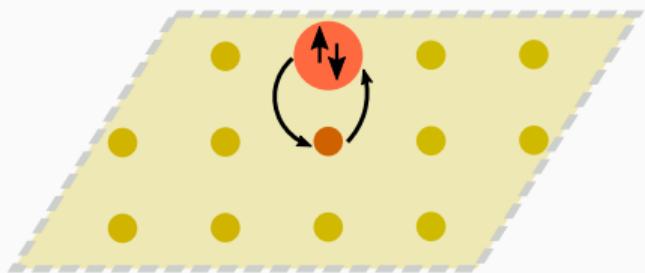
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# Theory for the single-channel Kondo cloud

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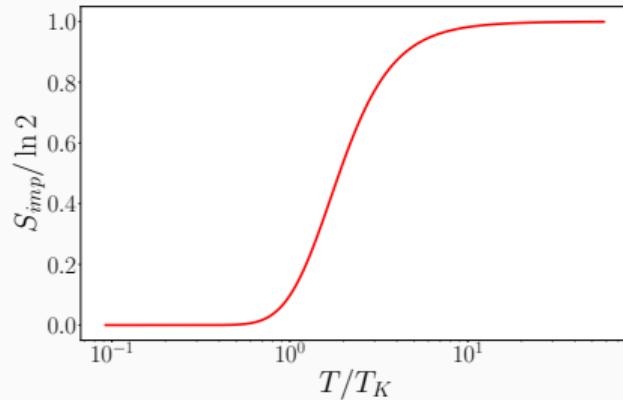
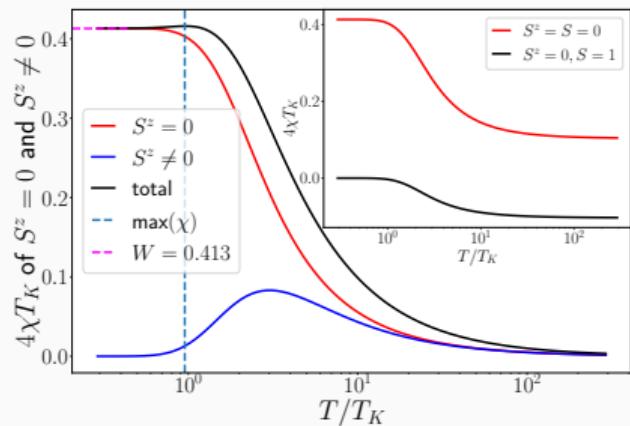
Phys. Rev. B 105, 085119

Anirban Mukherjee, **Abhirup Mukherjee**, N. S. Vidhyadhiraja,  
A. Taraphder, and Siddhartha Lal



# Theory for the single-channel Kondo cloud

- ✓ spectral function & magnetic susceptibility



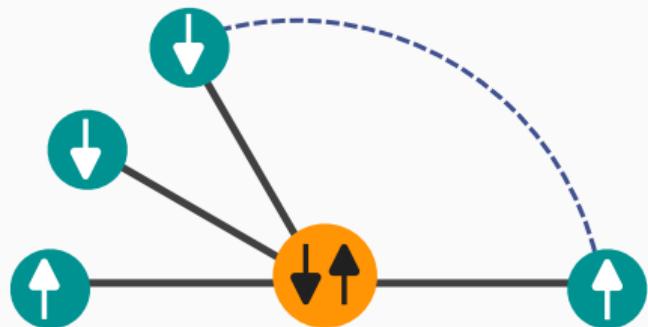
- ✓ local Fermi liq. & orthogonality catastrophe
- ✓ thermal entropy

# Role of degeneracy in the multi-channel Kondo problem

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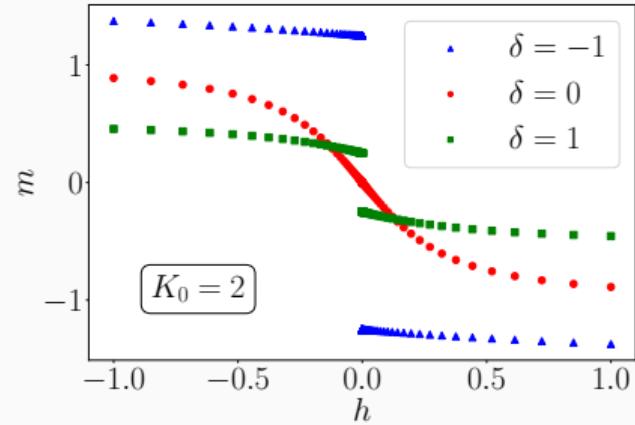
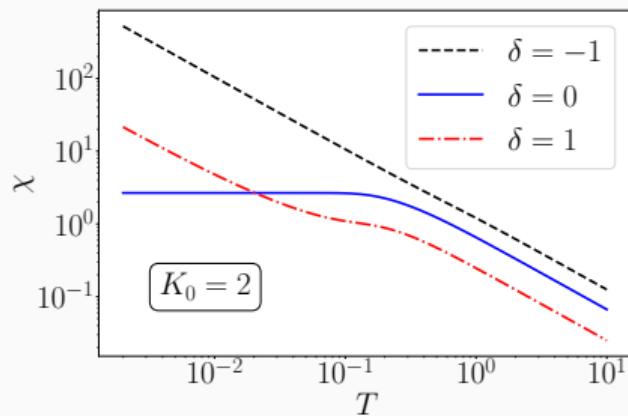
arXiv:2205.00790

Siddhartha Patra, **Abhirup Mukherjee**, Anirban Mukherjee, N.  
S. Vidhyadhiraja, A. Taraphder, Siddhartha Lal



# Role of degeneracy in the multi-channel Kondo problem

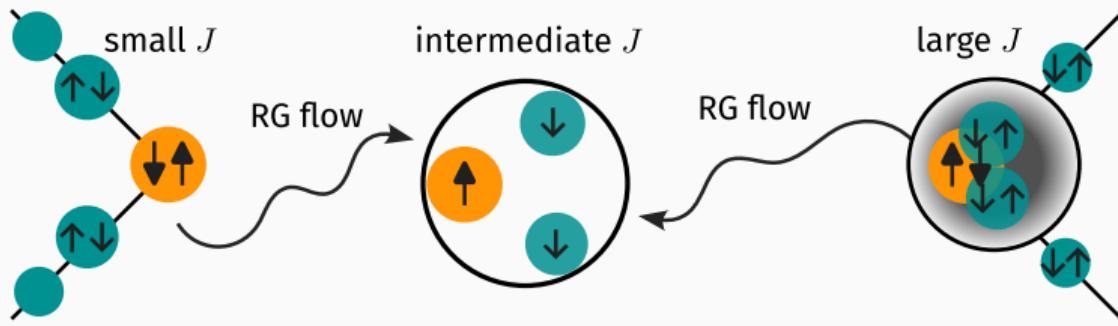
- ✓ Intermediate-coupling RG fixed point Hamiltonian and **degenerate** ground states
- ✓ Degree of compensation, magnetization and susceptibility show **incomplete screening**



Nozières, Ph. et al. 1980; Tsvelick et al. 1985; Affleck et al. 1993; Gan 1994; Affleck et al. 1991; Emery et al. 1992; Bulla et al. 1998.

## Role of degeneracy in the multi-channel Kondo problem

- ✓ Local **marginal Fermi liquid** within the low-energy excitations of the bath
- ✓ **Duality** relations constrain the RG flows of the MCK model

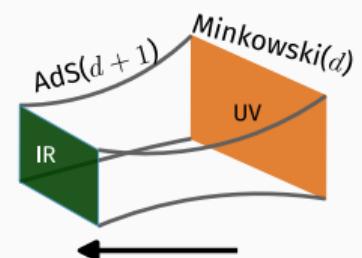
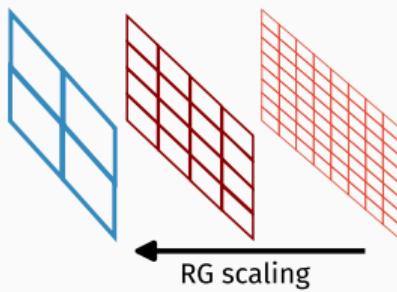


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Nozières, Ph. et al. 1980; Tsvelick et al. 1985; Affleck et al. 1993; Gan 1994; Affleck et al. 1991; Emery et al. 1992; Bulla et al. 1998.

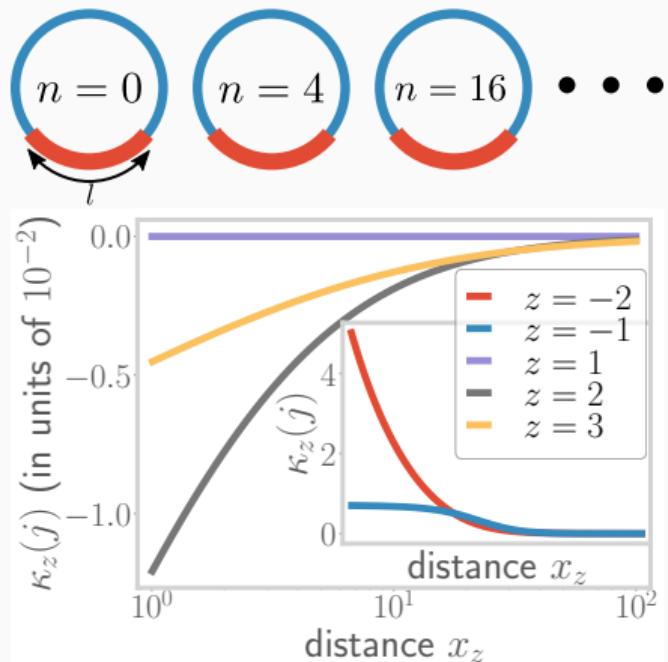
# Entanglement scaling in free fermions: holography & topology

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## Entanglement scaling in free fermions: holography & topology - Summary

- ✓ Under coarse-graining or fine-graining in  $k$ -space, entanglement of free Dirac fermions shows hierarchical onion-like structure.
- ✓ Entanglement scaling can be used to define distances, leads to additional spatial dimension  $\rightarrow$  holography.
- ✓ Emergent distances and curvature can be related to RG beta function; the sign of the curvature is topological.
- ✓ Pole structure of the entanglement tracks the Luttinger volume - invariant under the scaling transformations.



## Creating subsystems

Free Dirac fermions on torus:  $k_x^n = \frac{2\pi}{L_x} n, \quad n \in \mathbb{Z}; \quad$  define **sparsity** =  $\Delta n = 1$

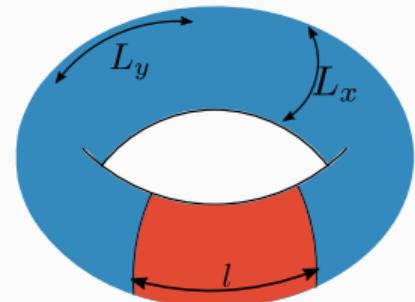
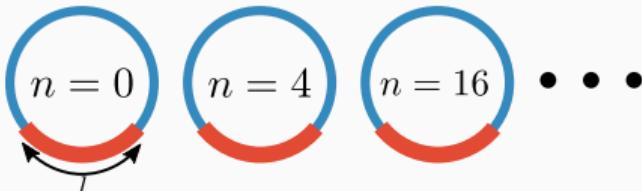
**Simplest** choice: the entire set

sparsity = 1  $\longrightarrow n \in \{-N, -(N - 1), -(N - 2), \dots, -1, 0, 1, \dots, N - 2, N - 1, N\}$

**Coarser** choices: increase sparsity

sparsity = 2  $\longrightarrow n \in \{-N, -(N - 2), -(N - 4), \dots, -2, 0, 2, \dots, N - 4, N - 2, N\}$

sparsity = 4  $\longrightarrow n \in \{-N, -(N - 4), -(N - 8), \dots, -4, 0, 4, \dots, N - 8, N - 4, N\}$



## Subsystem entanglement entropy: Entanglement hierarchy

$$S_{\mathcal{A}_z(j)} = f_z(j) c \alpha L_x - c \log |2 \sin(\pi f_z(j)\phi)|$$

$$i < j, \quad S_{i \cup j} = \begin{cases} S_i, & z > 0 \\ S_j, & z < 0 \end{cases}$$



- ✓ presents a **hierarchy** of entanglement → EE distributed across RG steps  
RG transformation → reveals entanglement
- ✓ distribution of entanglement also present in **multipartite** entanglement

## Mutual information = distance

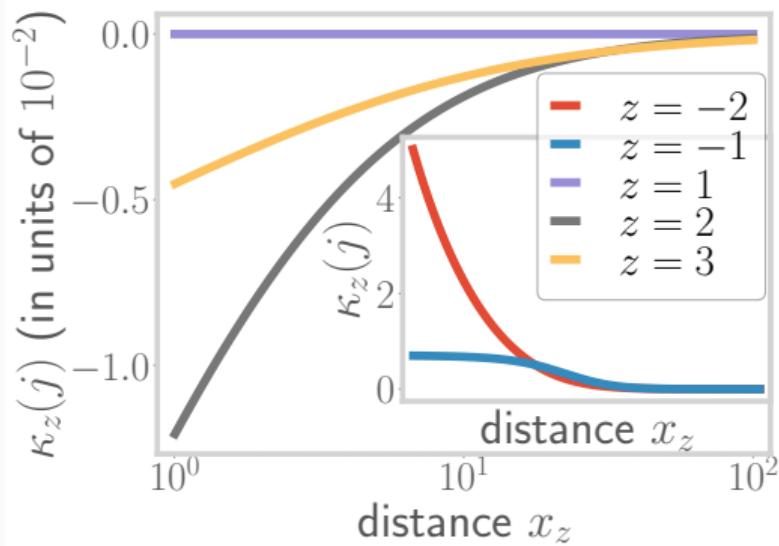
**Mutual information:**  $I^2(A : B) \equiv S(A) + S(B) - S(A \cup B)$  (non-negative)

Define distances using mut. info.

$$x_z(j) = \log t_z(j), \quad y_z(j) = \log t_z(j \pm 1)$$

$$v_z(j) \equiv \Delta y_z(j)/\Delta x_z(j), \quad v' = \Delta v_z(j)/\Delta x_z(j)$$

Curvature as well:  $\kappa_z(j) = \frac{v'_z(j)}{[1 + v_z(j)^2]^{\frac{3}{2}}}$



## RG evolution = emergent distance

- ✓ Distances and curvature can be related to an RG **beta function**
- ✓ Amounts to an **explicit demonstration** of the holographic principle
- ✓ Sign of curvature is **topological**, can be written in terms of winding numbers

## Topological nature of geometry-independent term

$$S_{\mathcal{A}_z(j)} = f_z(j)caL_x - \underbrace{c \log |2 \sin(\pi f_z(j)\phi)|}_{=Q(\phi), \text{geometry-independent term}}$$

- ✓  $Q(\phi)$  is periodic in the flux  $\phi$ ,  $\phi = 1$  transports a charge across Fermi surface
- ✓ pole structure of  $(\sin \frac{\pi}{4} - |\sin(\pi f_z(j))\phi|)^{-1}$  counts number of states → tracks Luttinger volume
- ✓ Luttinger volume is topological, so is  $Q(\phi)$ ;  $Q(\phi)$  can be expressed in terms of winding numbers