HW1

Due date: 04/05/2018

1. Durrett excercise 5.2.2

2. Complete the steps of Durrett example 5.2.4 (if you use the the Borel-Cantelli lemma, explain why you can use it)

3. Durrett excercise 5.2.13

4. Let τ be a stopping time with respect to a filtration $\{\mathcal{F}_n\}$. Show that

   (a) $\mathcal{F}_\tau = \{A \in \mathcal{F} \mid A \cap \{\tau \le n\} \in \mathcal{F}_n \text{ for all } n\}$ is a σ-field

   (b) σ is another stopping time such that $\sigma \le \tau$ a.s. then $\mathcal{F}_\sigma \subset \mathcal{F}_\tau$

   (c) If X_n is adapted to $\{\mathcal{F}_n\}$, then X_τ is $\mathcal{F}_\tau$-measurable

5. Let $\{X_n\}$ be a martingale with $\sup_n |X_n| < \infty$. Show that there is a representation $X_n = Y_n - Z_n$ where $\{Y_n\}$ and $\{Z_n\}$ are both non-negative martingales with $\sup_n |Y_n| < \infty$ and $\sup_n |Z_n| < \infty$.

6. Let $\{X_n\}$ and $\{Y_n\}$ be positive integrable processes adapted to a filtration $\{\mathcal{F}_n\}$ such that $\sum_{n\ge 1} Y_n < \infty$ a.s. and $E(X_{n+1}|\mathcal{F}_n) \le (1 + Y_n)X_n + Y_n$. Prove that X_n converges a.s. to a finite limit as $n \to \infty$.