Consensus Problem & Paxos Protocol

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Distributed System

Set of processes that communicate with each other via message

- e.g., sending messages via https. No bound on communication ; no
- processes can fail, e.g., can crash / be slow

Consensus Problem

Example: Consensus

Agreeing on a primary process:

- Set of processes p, q, r, …
- Each process can propose a value, (in this case, the next **primary**)
- All processes need to agree on a common value (e.g., primary)

Consensus Problem

Requirements:

- **Termination**:
  - All **correct** processes will **decide**
- **Agreement**:
  - **All** processes that decide, agree on the same value
- **Validity**:
  - Decision value was **proposed** by one process

Specification

- What if we drop validity requirement?
- What if we drop termination?
- What if we change agreement to:
  - „*All correct processes agree on the same value*“?

More Examples

Why do we need consensus?

Consensus

Agreement on

- **key/value stores**
- ordering of messages
- the members of a set
- state updates (to keep replicas in sync)
- ...

How can one implement consensus?

System Model

Set of N (unique) processes

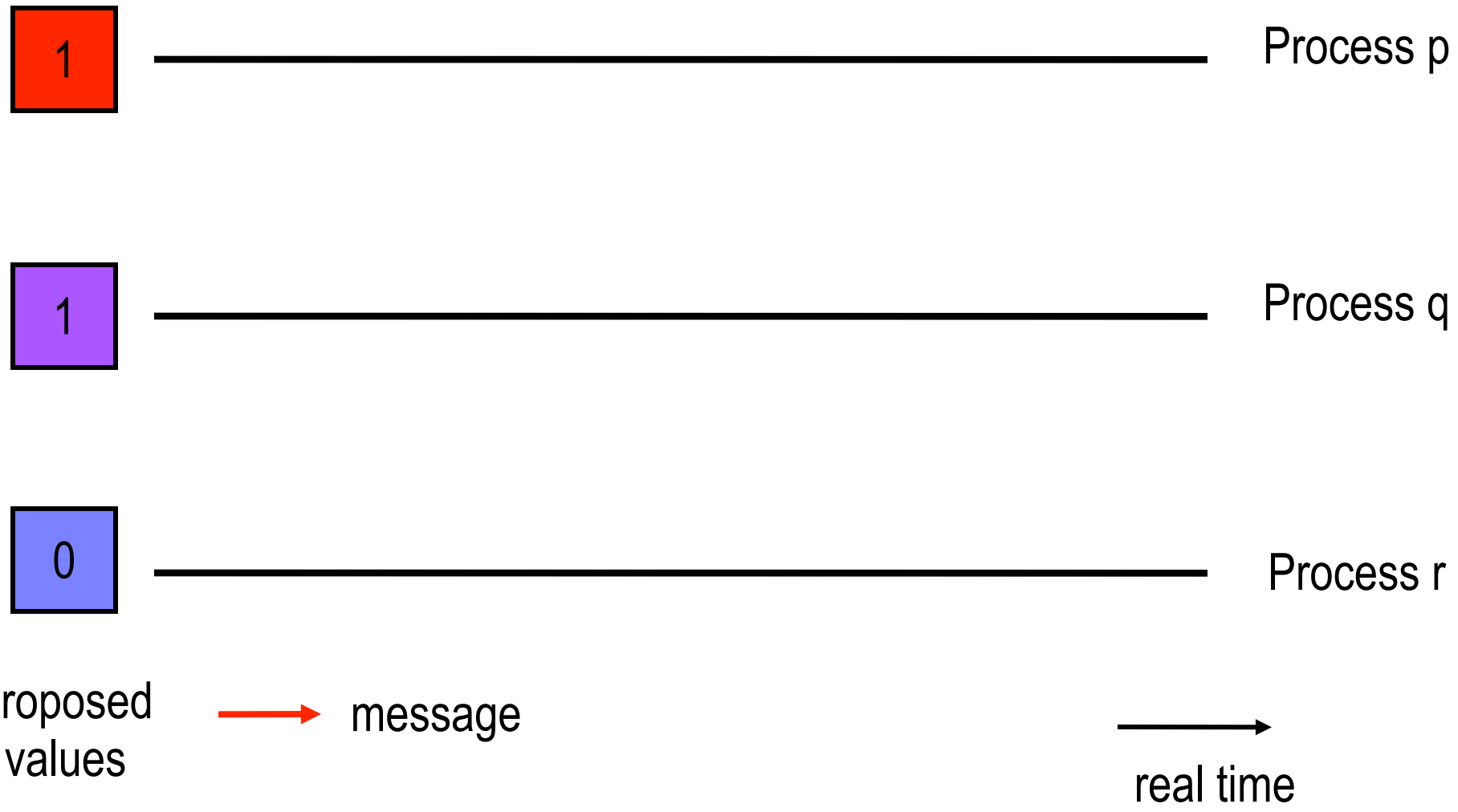
- each executed by some computer
- we know a priori all processes

Processes can exchange messages

- e.g., can use TCP to exchange info

An execution of a program is called a **run**

Example run 1: decision = min {values}



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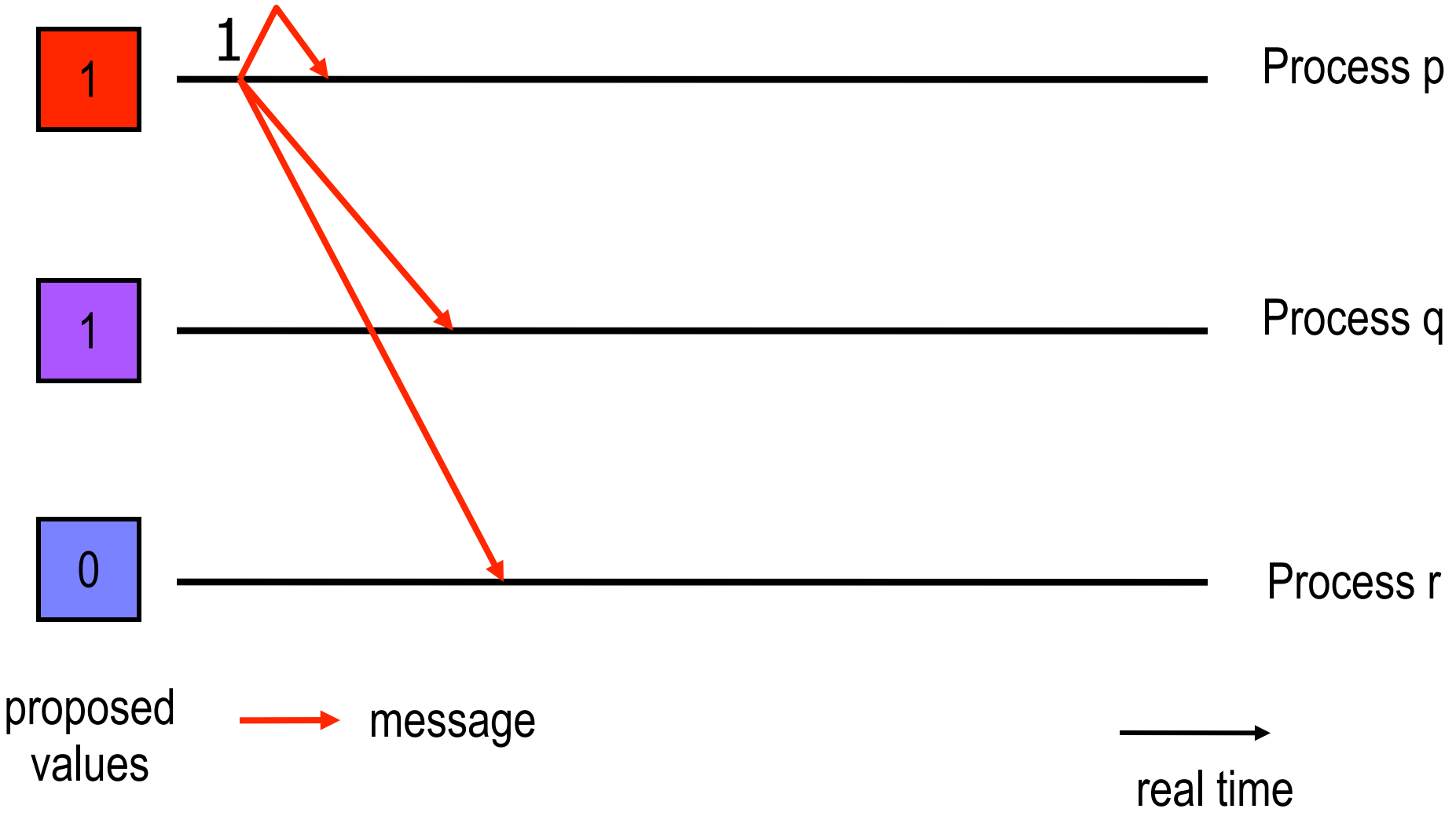

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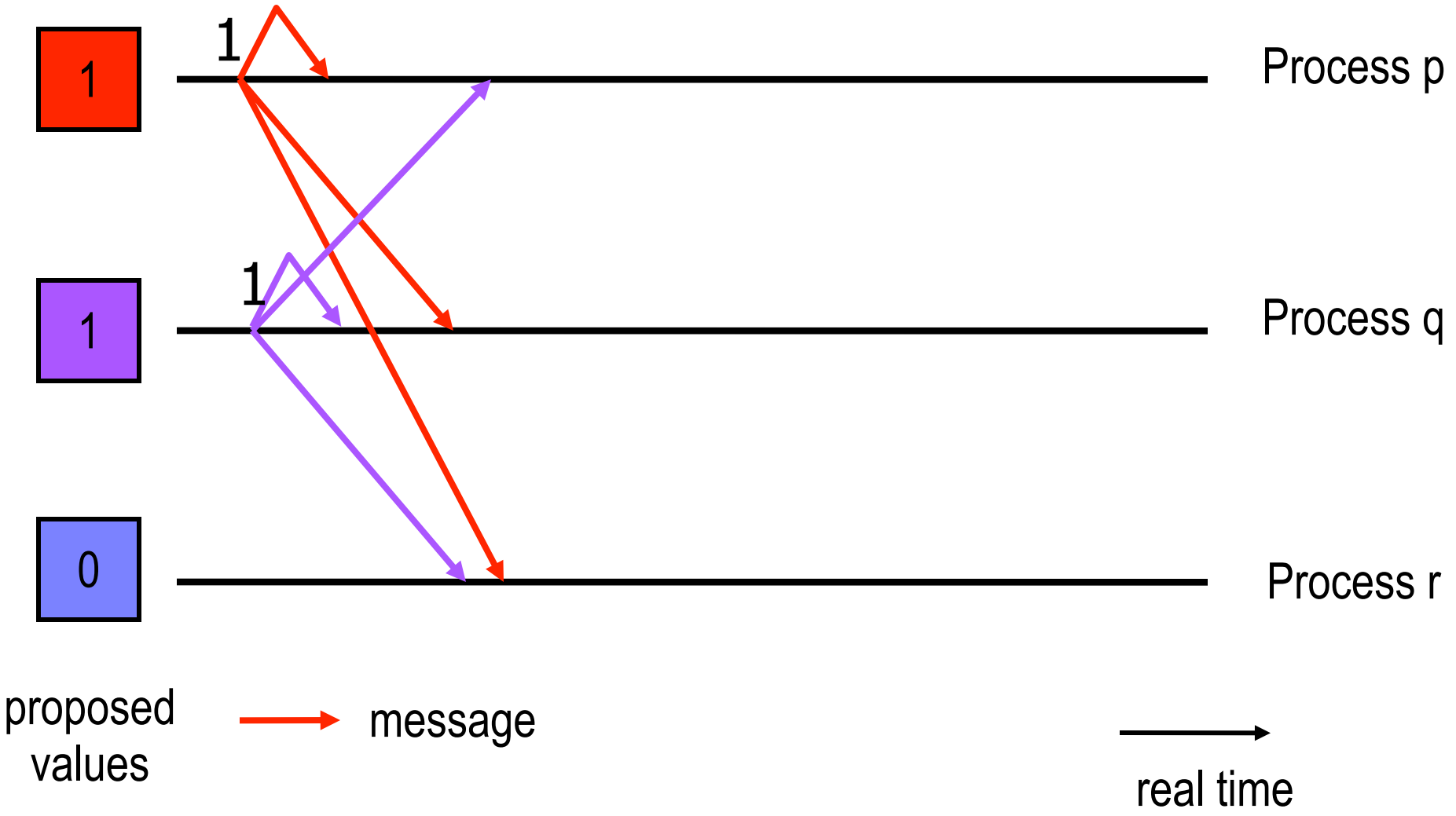

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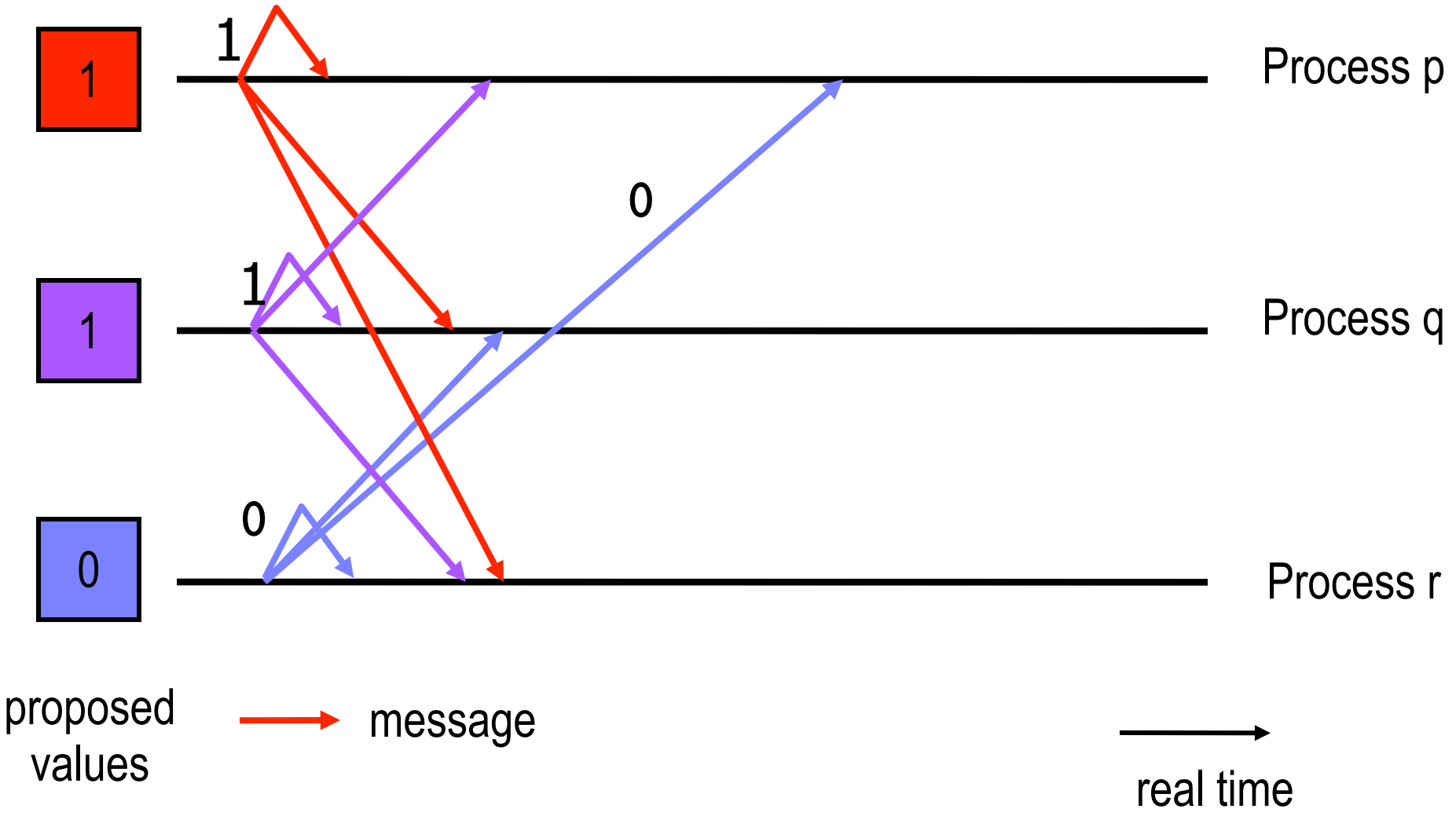

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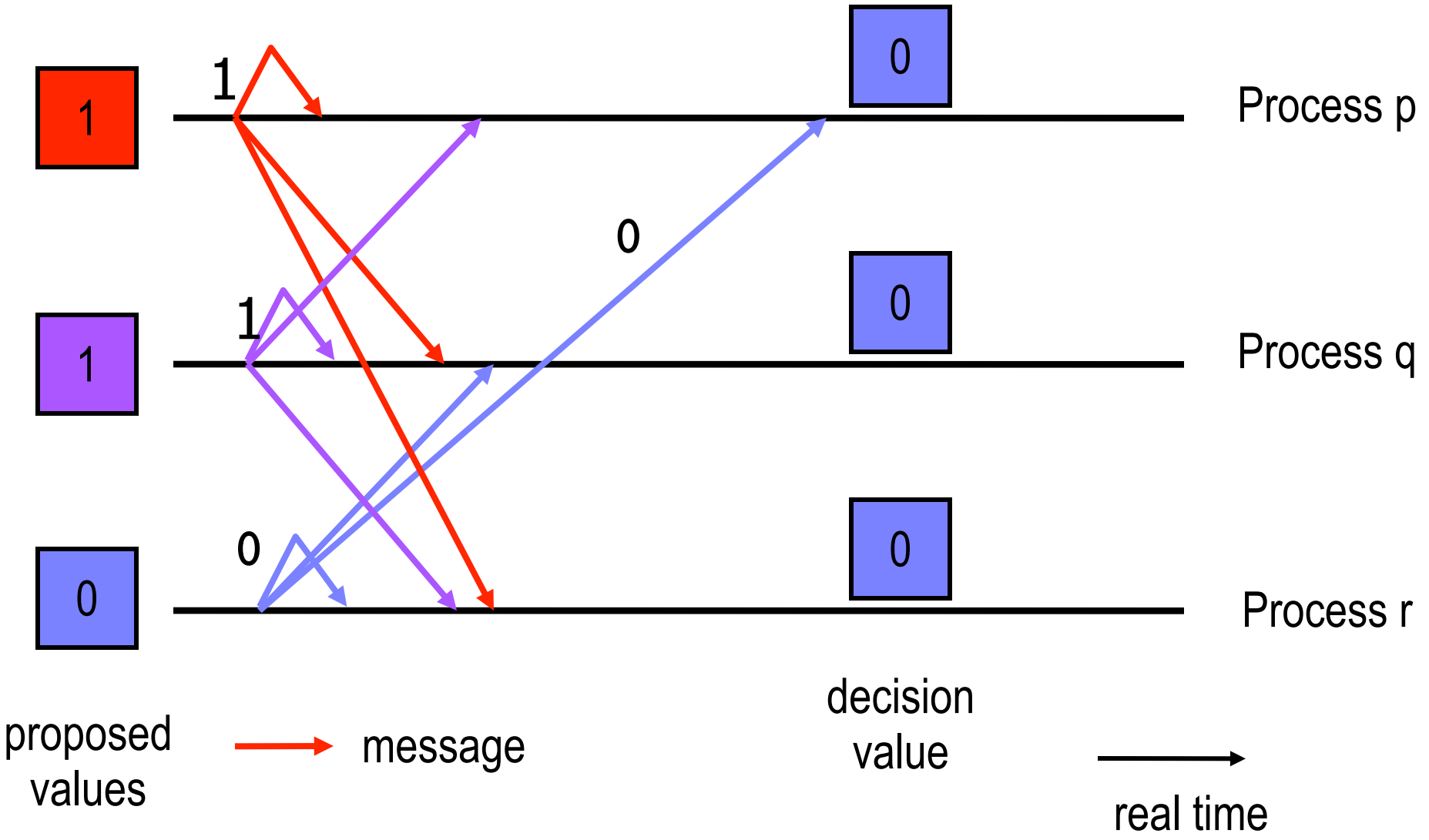

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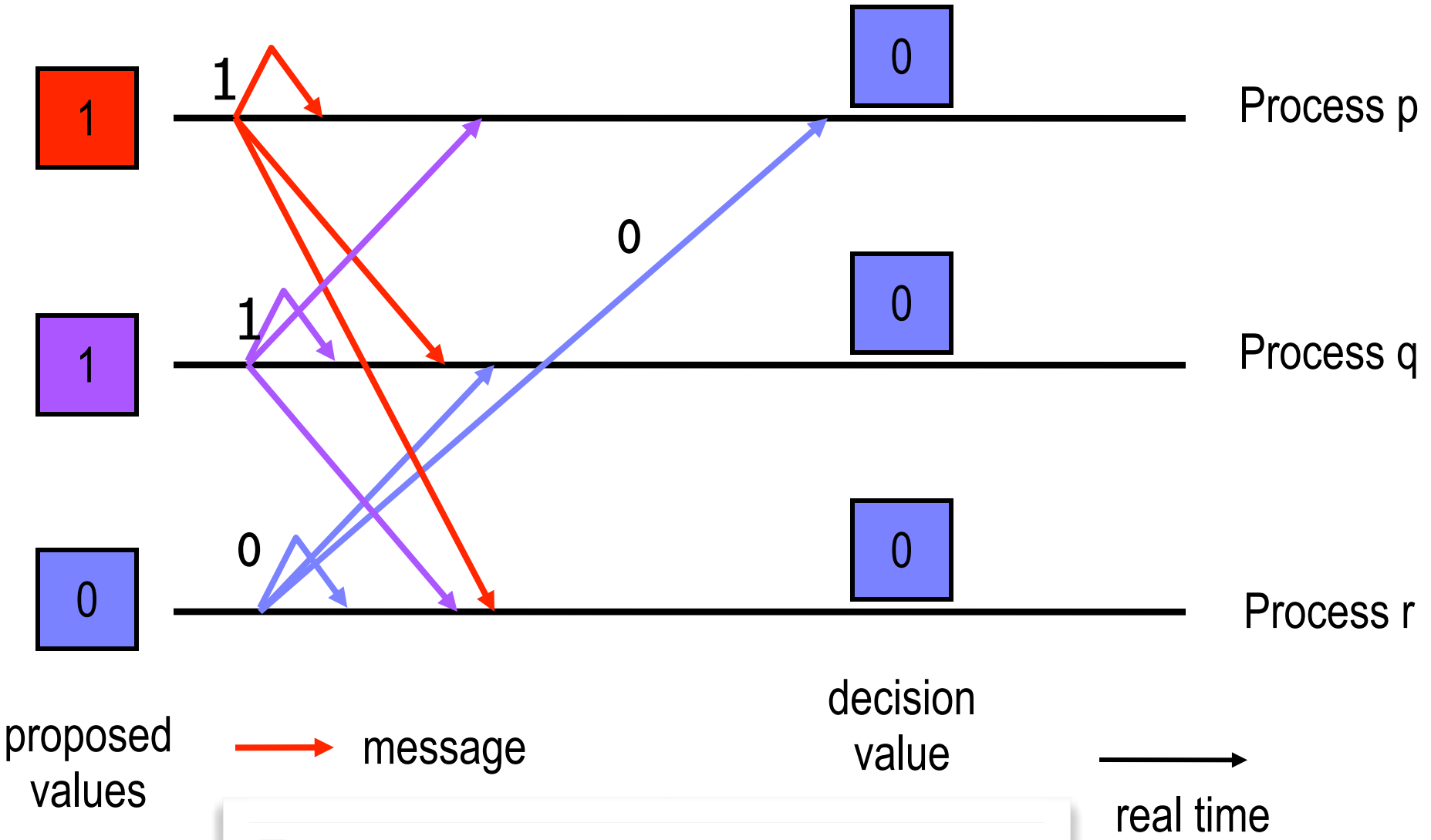

This is not a correct consensus protocol

Protocol

```
Set(int) Values = {};  // proposed values received
int num_msgs = 0;   // number of messages received

void propose(int proposed_value) {
    broadcast(proposed_value);
}

on receive(int value) from p {
    Values.insert(value);
    if (++num_msgs == N) {
        decide(min(Values));
    }
}
```

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set of all proposed values gotten so far

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called to initiate consensus - broadcasts value to all processes

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receive a broadcast msg:
add value to set of values

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```

if we receive messages from all N processes:
decide on minimum value

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```

upcall to say that we reached a decision

Broadcast

```
(1..N) Π;  // set of process ids

void broadcast(value) {
    foreach p in Π {
        send(value) to p;
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```

broadcast value to all processes in set

Broadcast

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void broadcast(value) {
    foreach p in Π {
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    }
}
```

iterate over all processes and send point-to-point message

Note

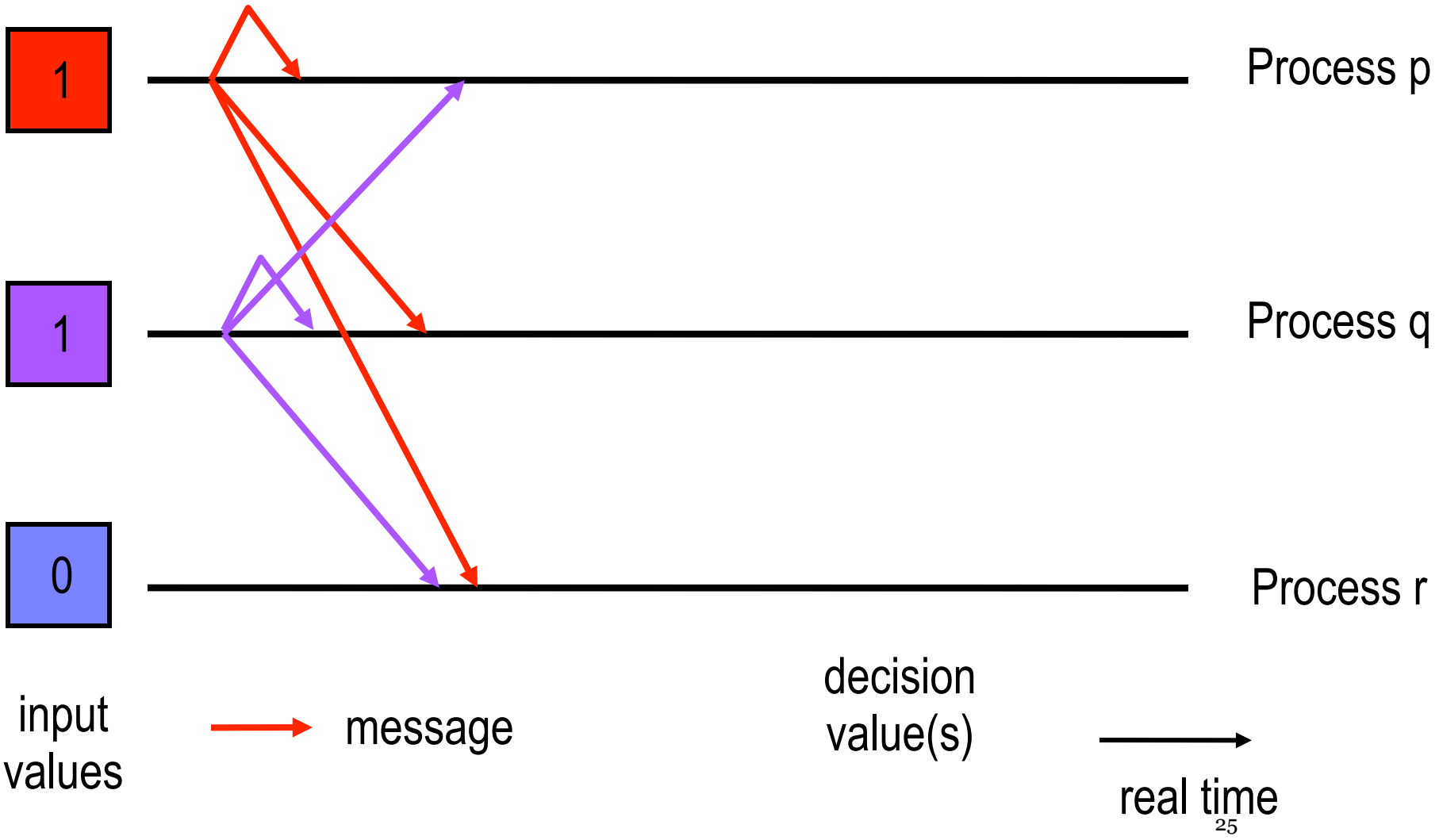
```
(1..N) Π;  // set of process ids

void broadcast(value) {
    foreach p in Π {
        send(value) to p;
    }
```

Broadcast is not **atomic**:
- crash of process could result in only a subset of processes receiving the value

What about failures?

Example: Omission Failure



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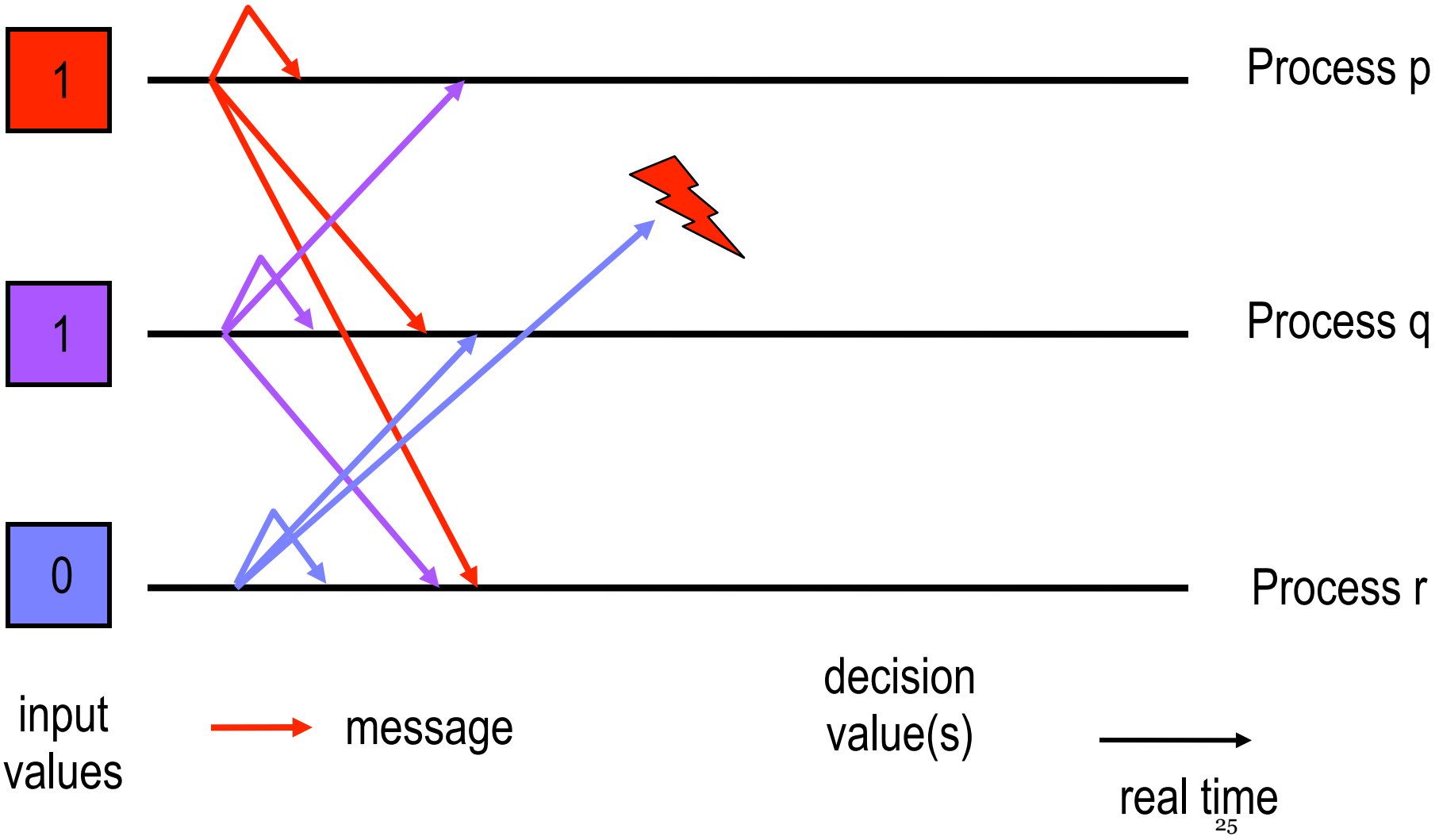

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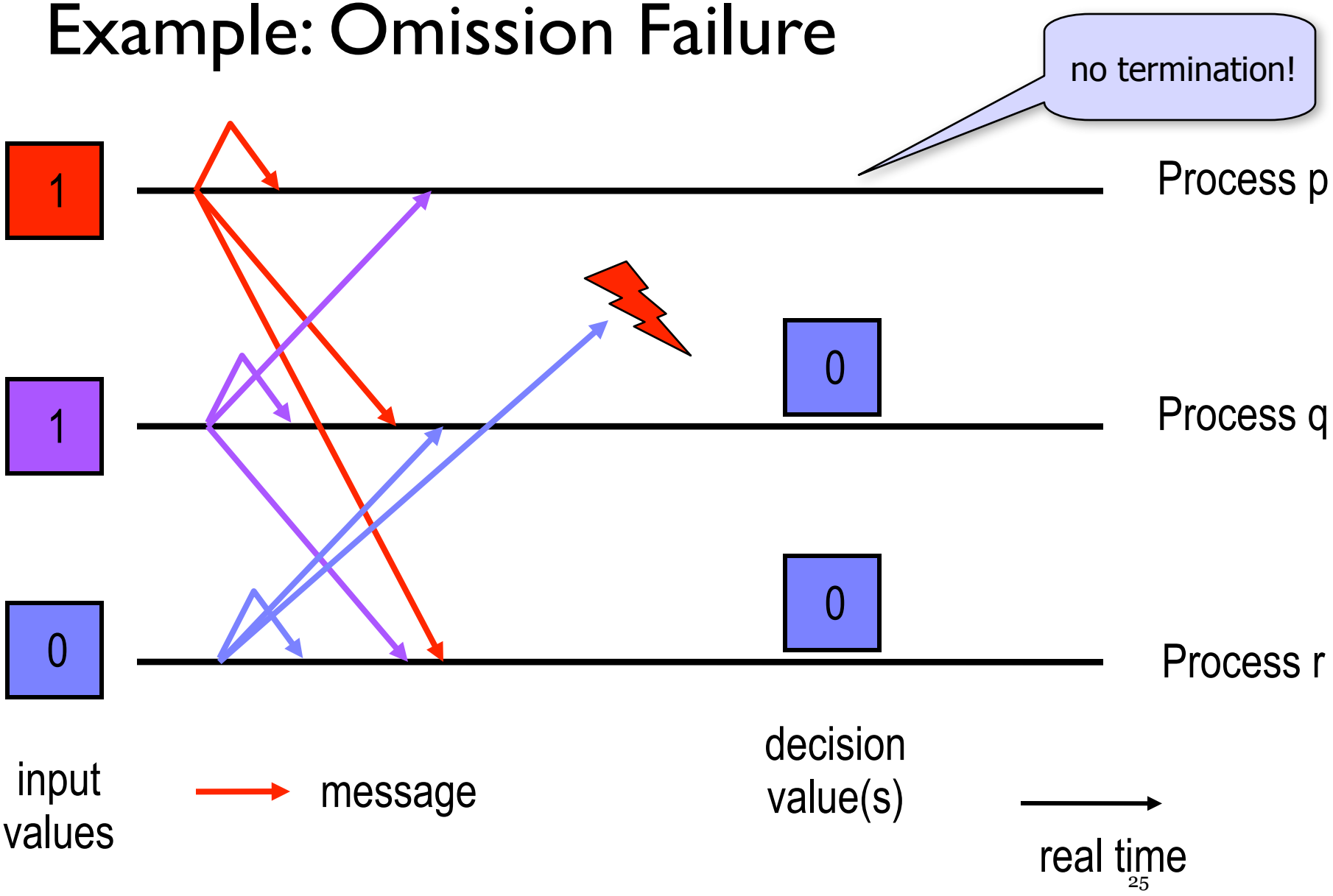
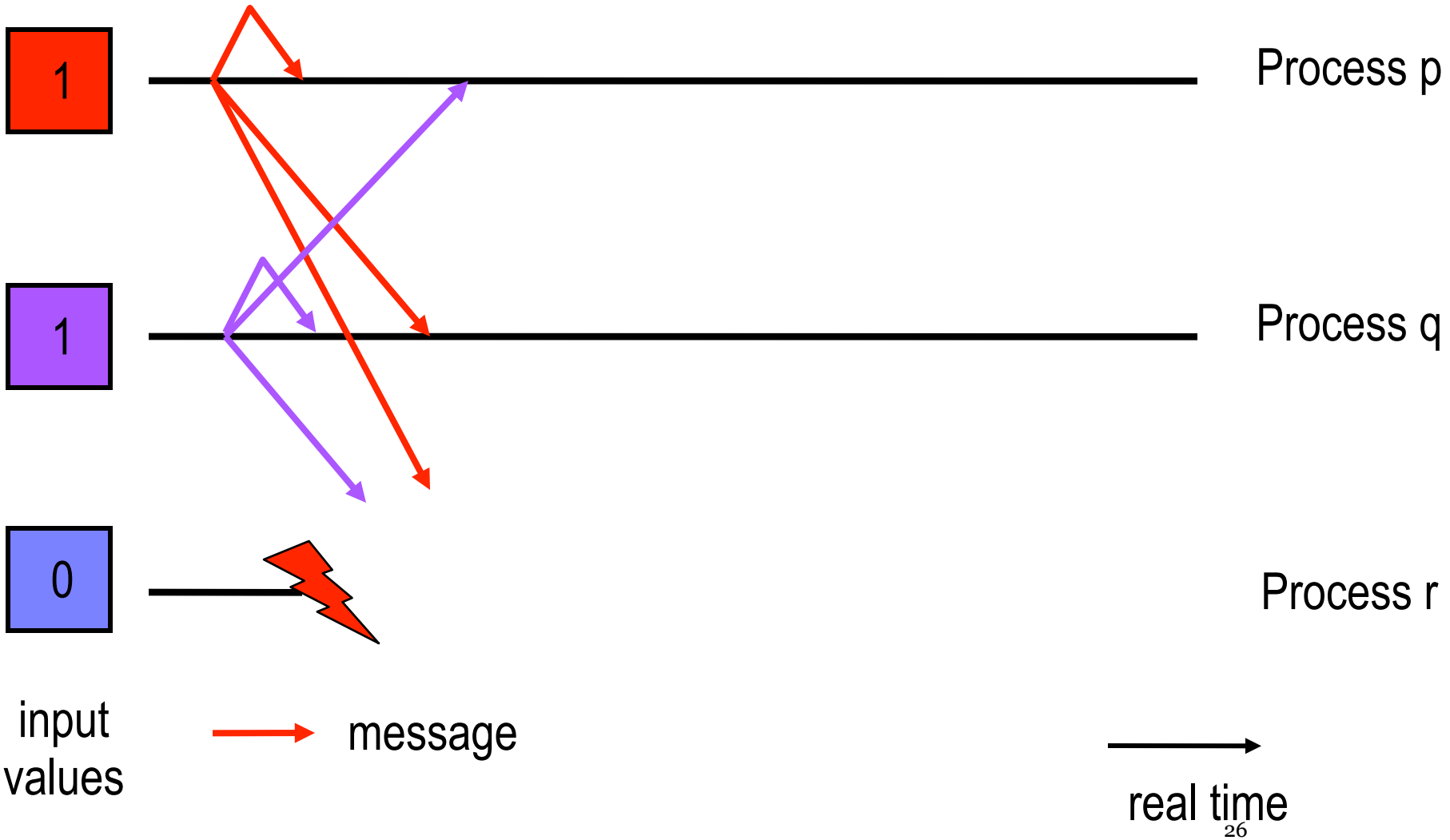


Example: Crash Failure



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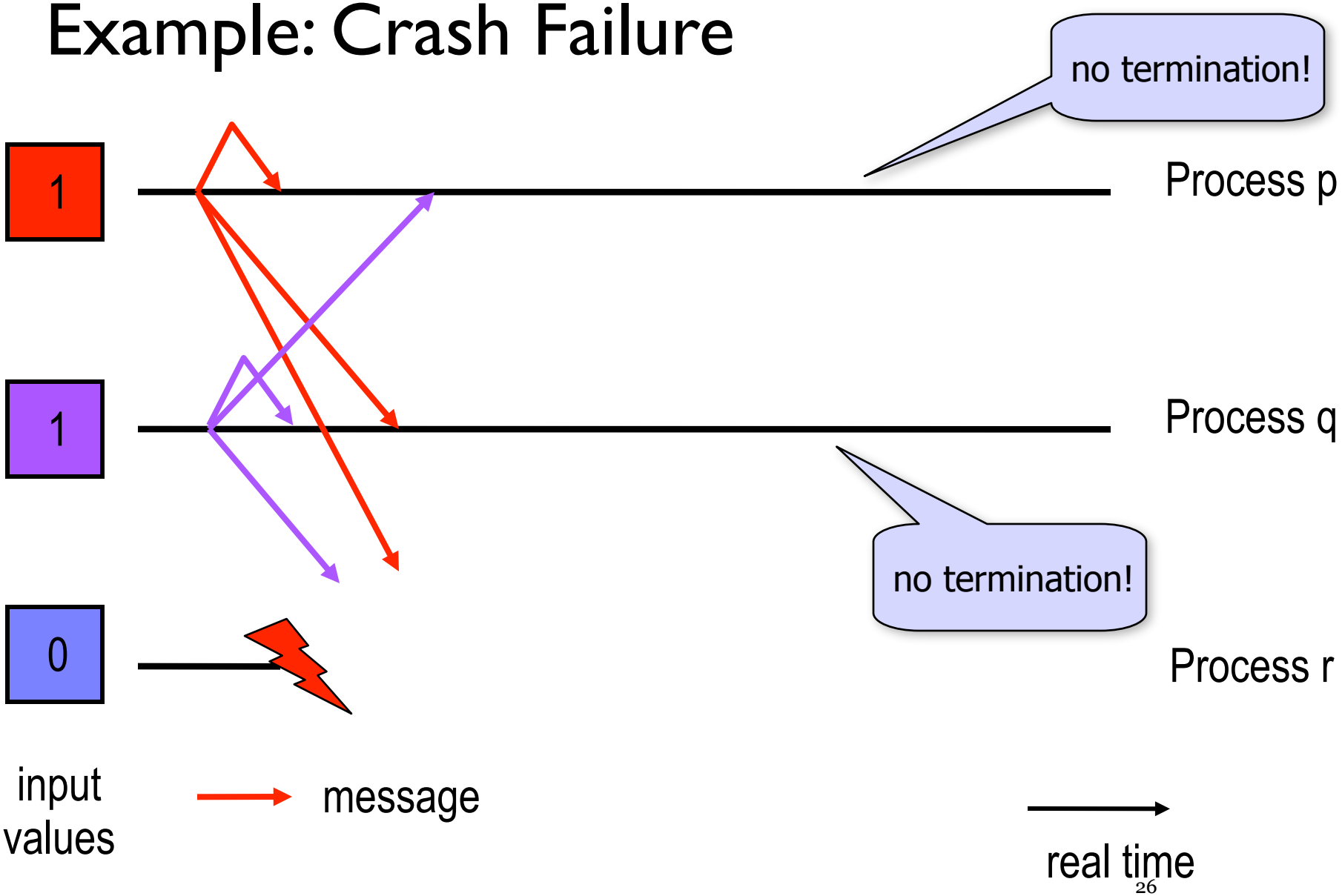
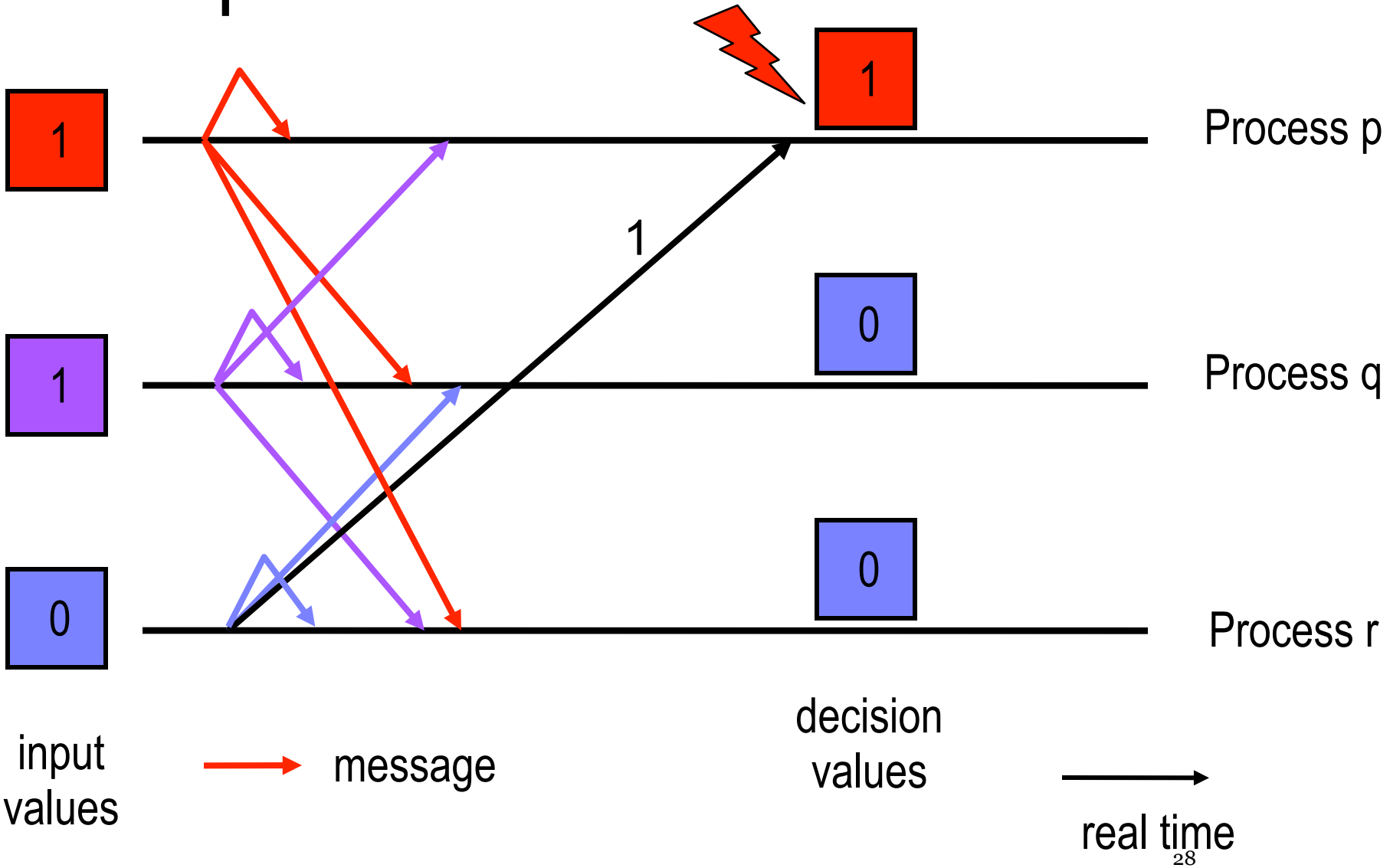


What about other failures?

Example: Value Failures



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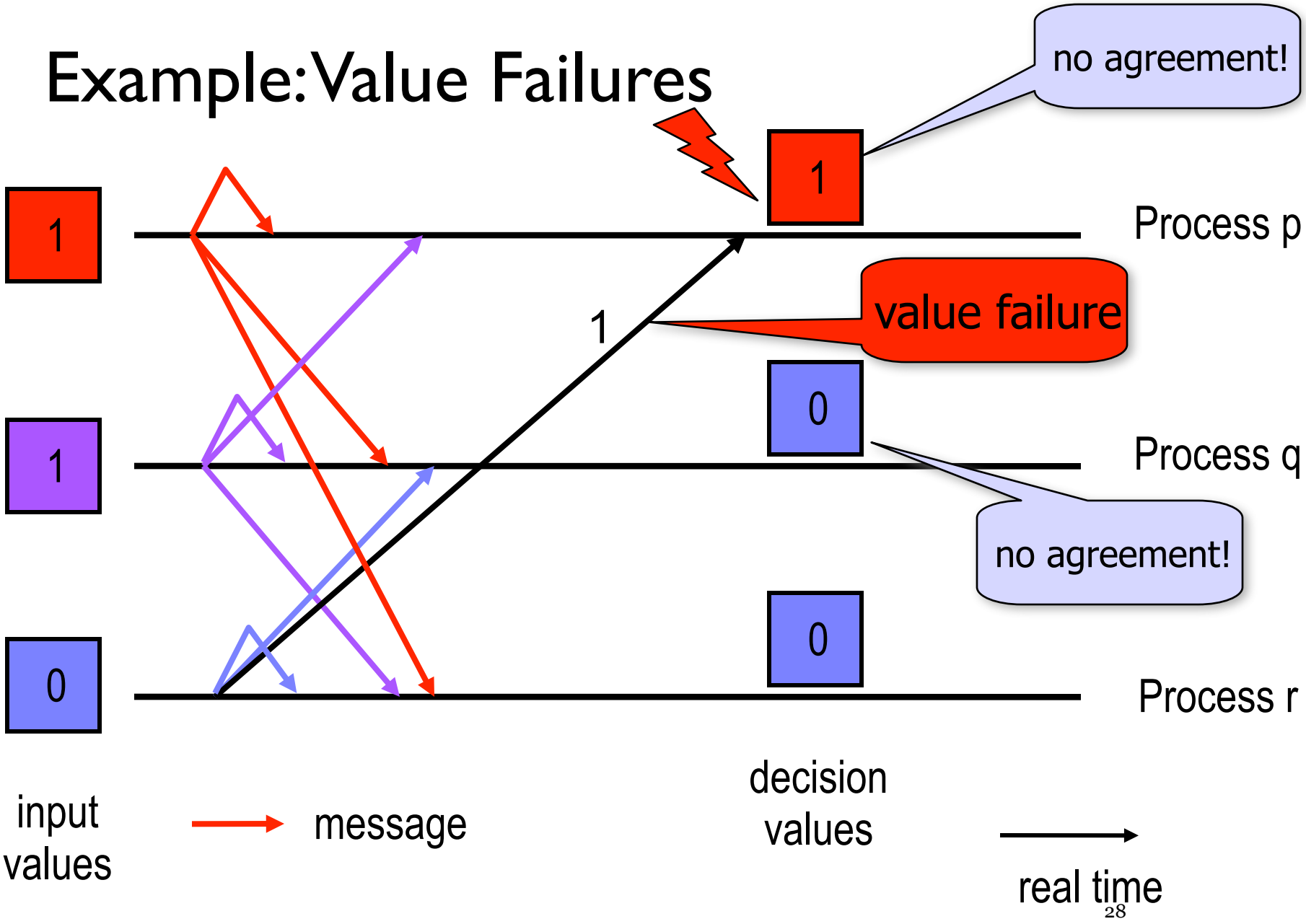

Simplify The Problem!

Failure Assumption

Processes can only fail by crashing!

Crash failure:

- process stops executing the protocol

For now, simplify:

- we assume crashes failures only
- i.e., **no other failures:** no value failures, no message omission, …!

Is the problem
now easier to solve?

Bivalent Configuration

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- Assume proposed values are in {0,1}

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  - configuration 1,1,1 result in 1 (1-deciding)
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- **Bivalent configurations**
  - permit decision values of either 0 or 1.

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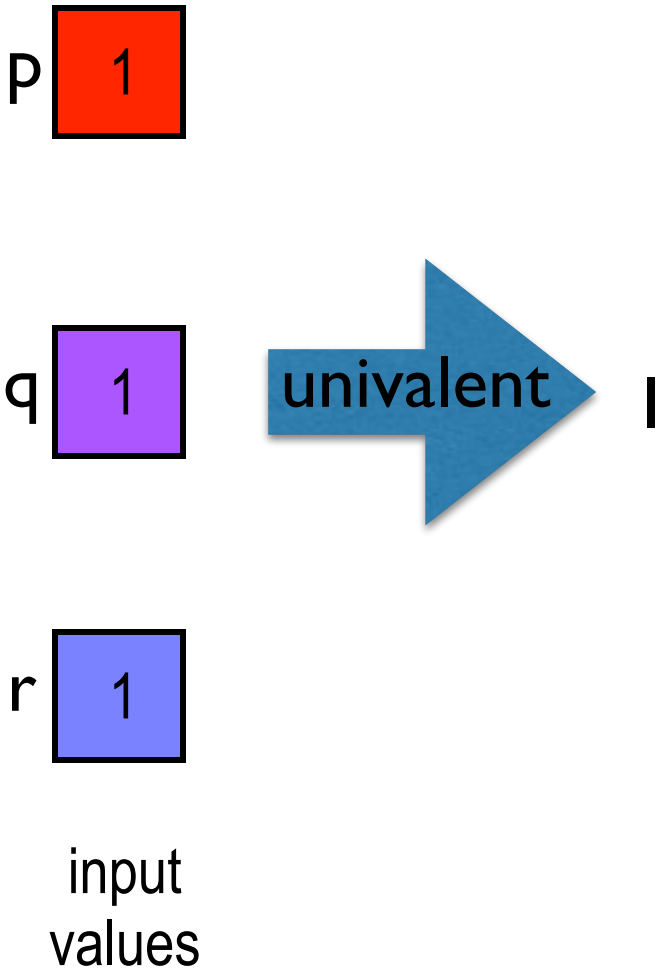

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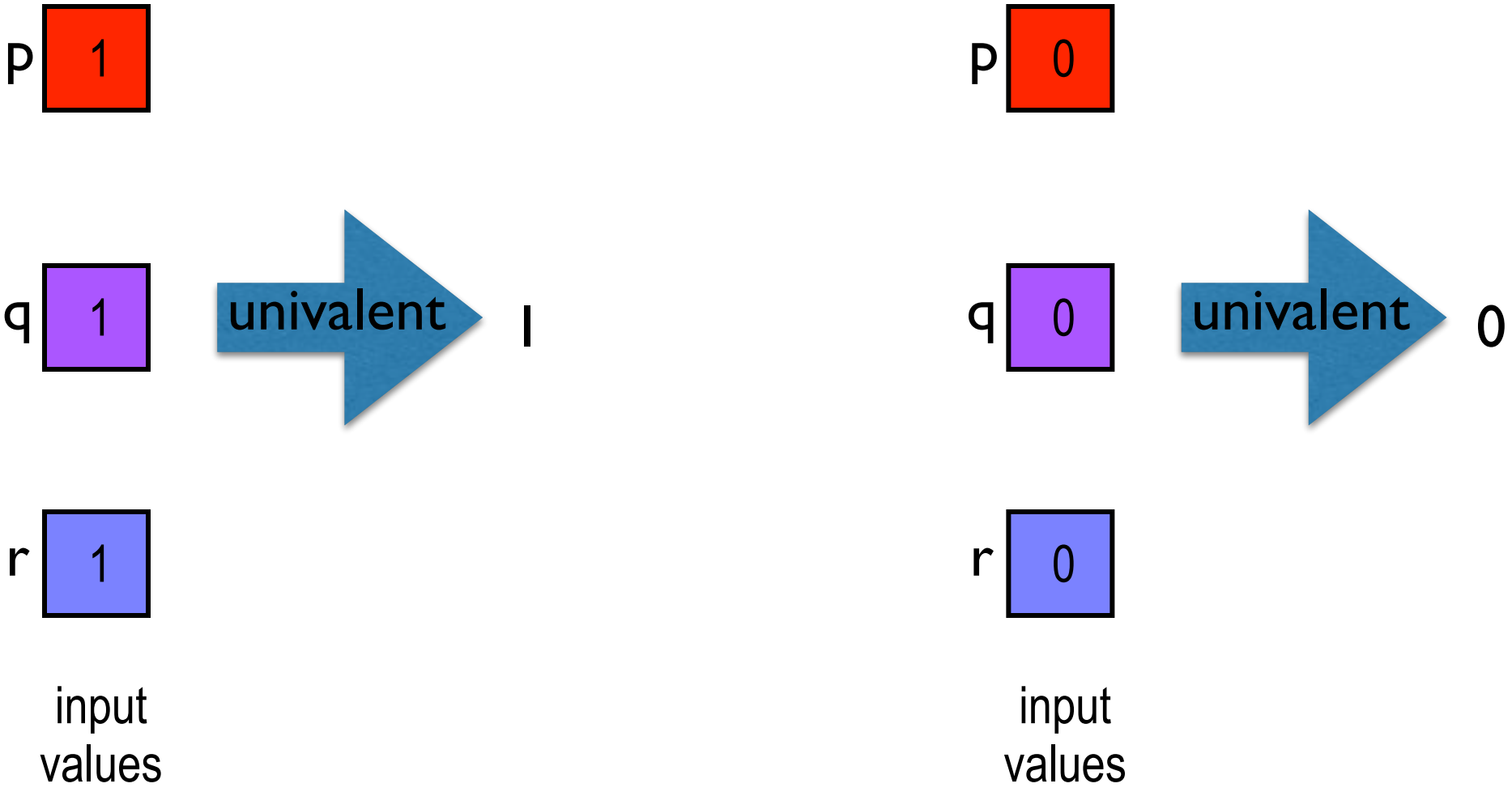
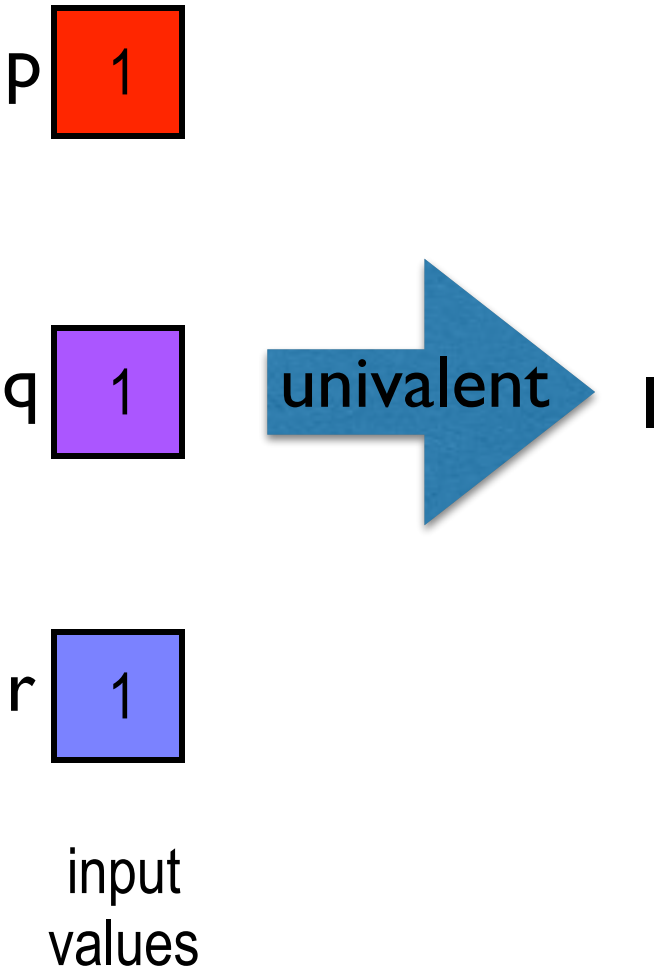


Assume only univalent configurations exist..



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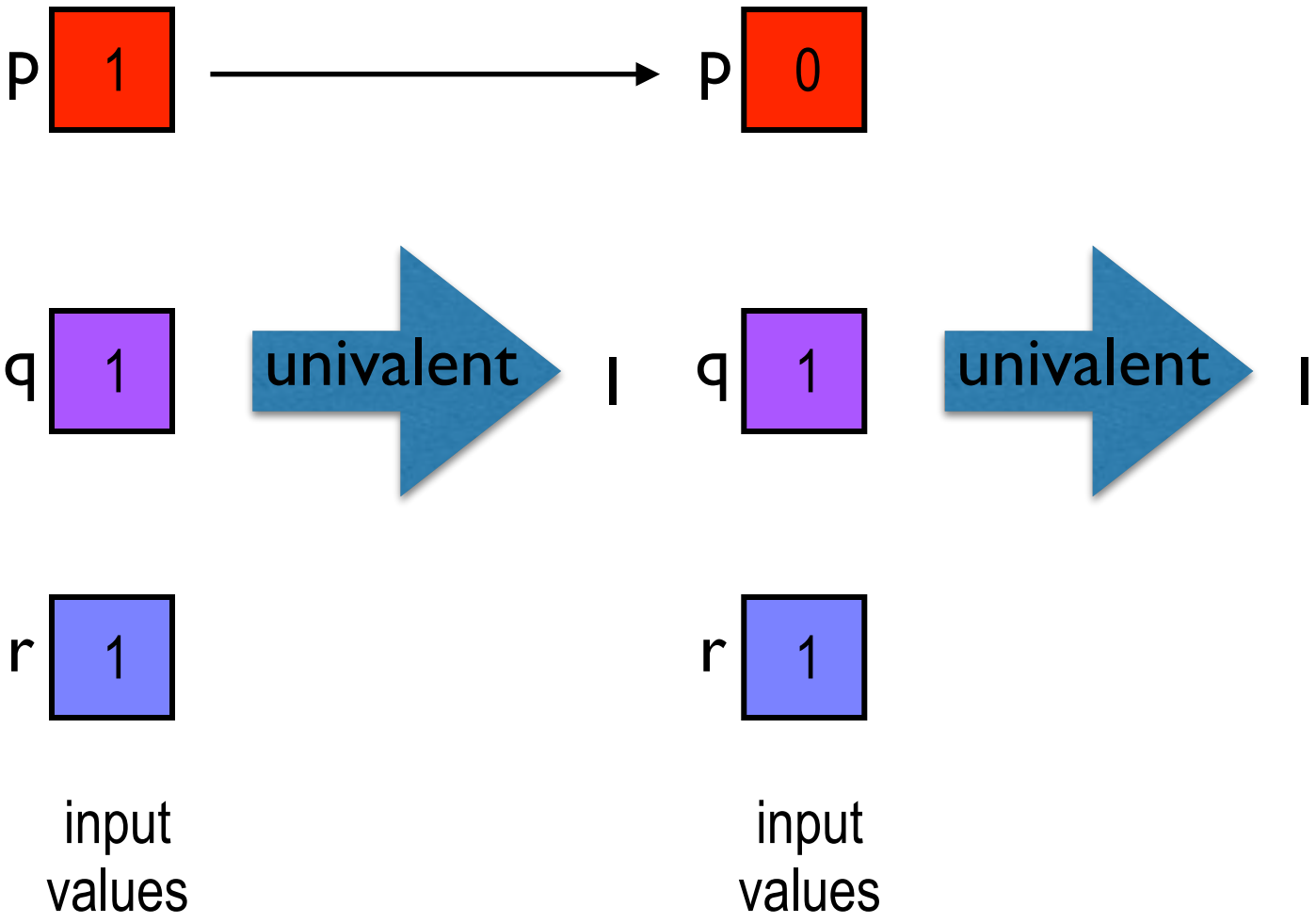

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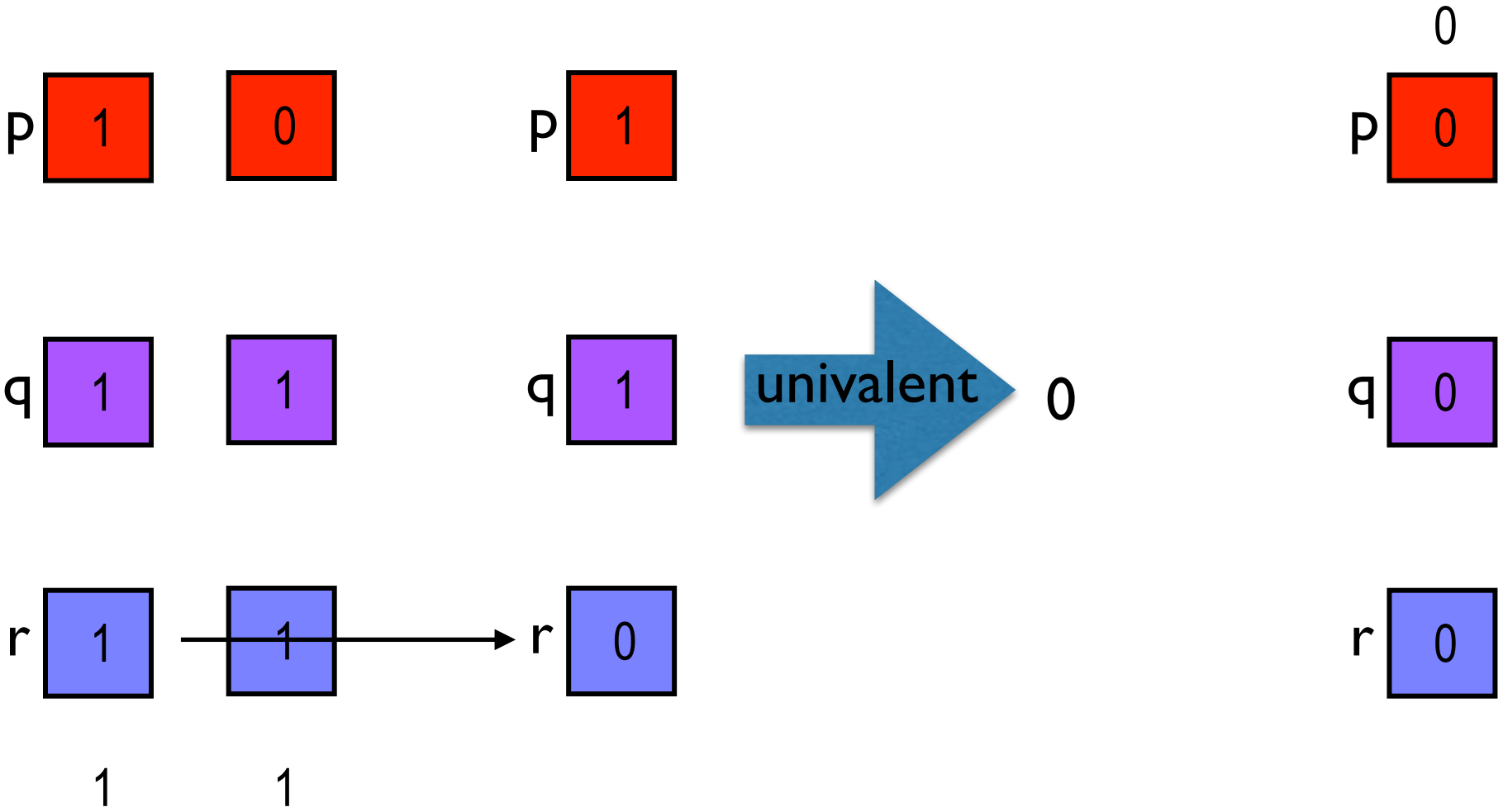

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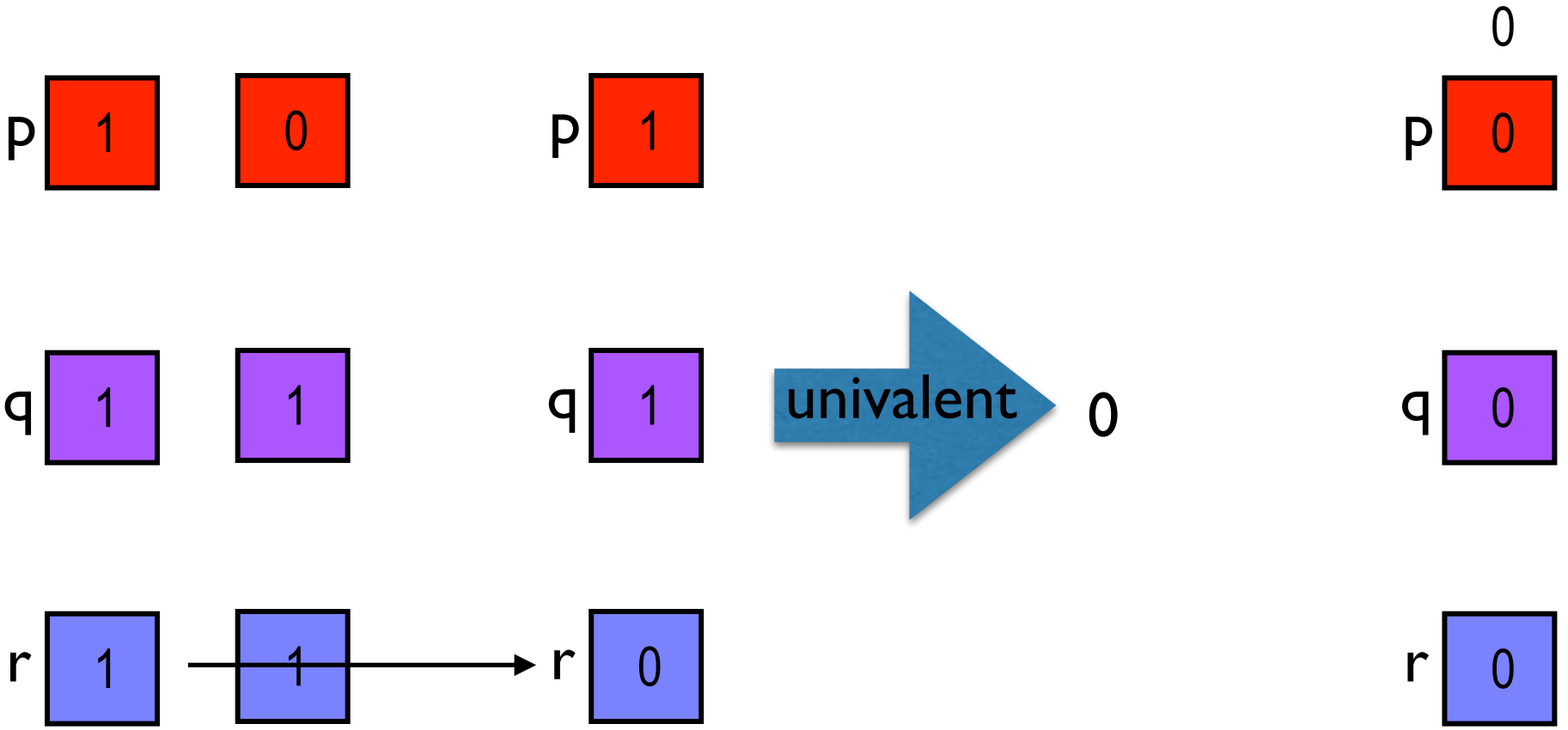
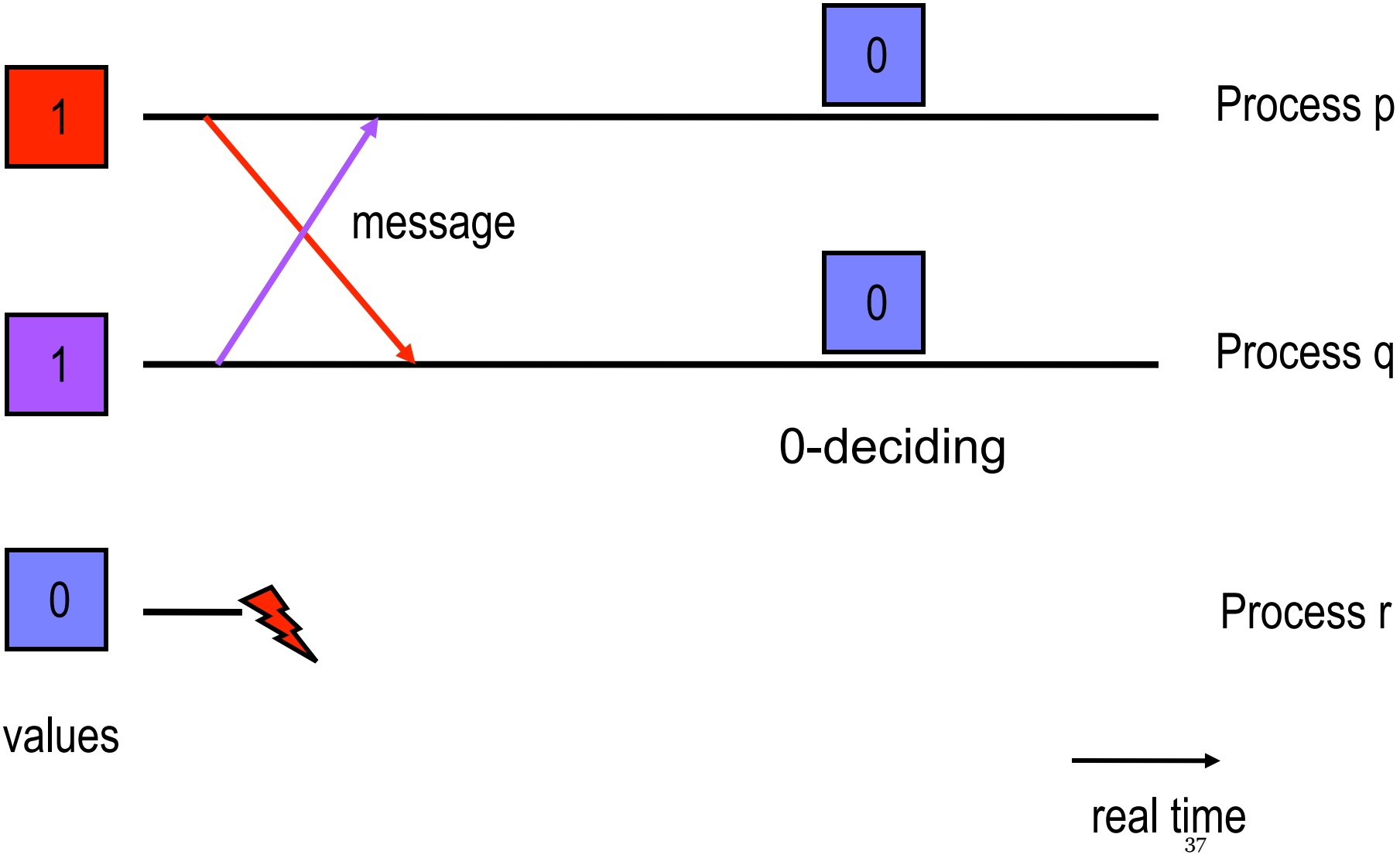
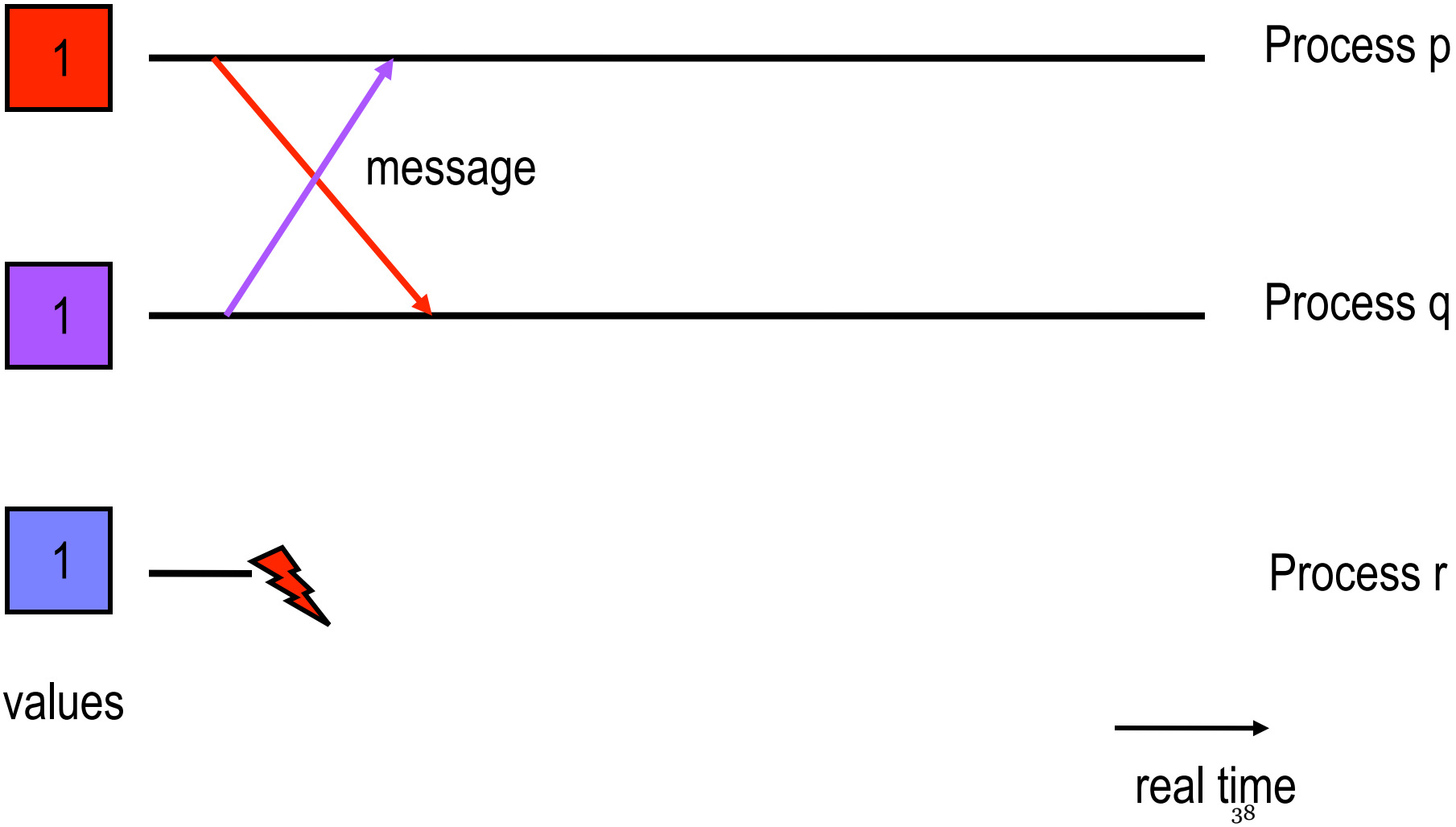


We will find a single input change for which the decision switches from 1 to 0

Run 2: Permitted by specification



Run 3: Must be 1-deciding



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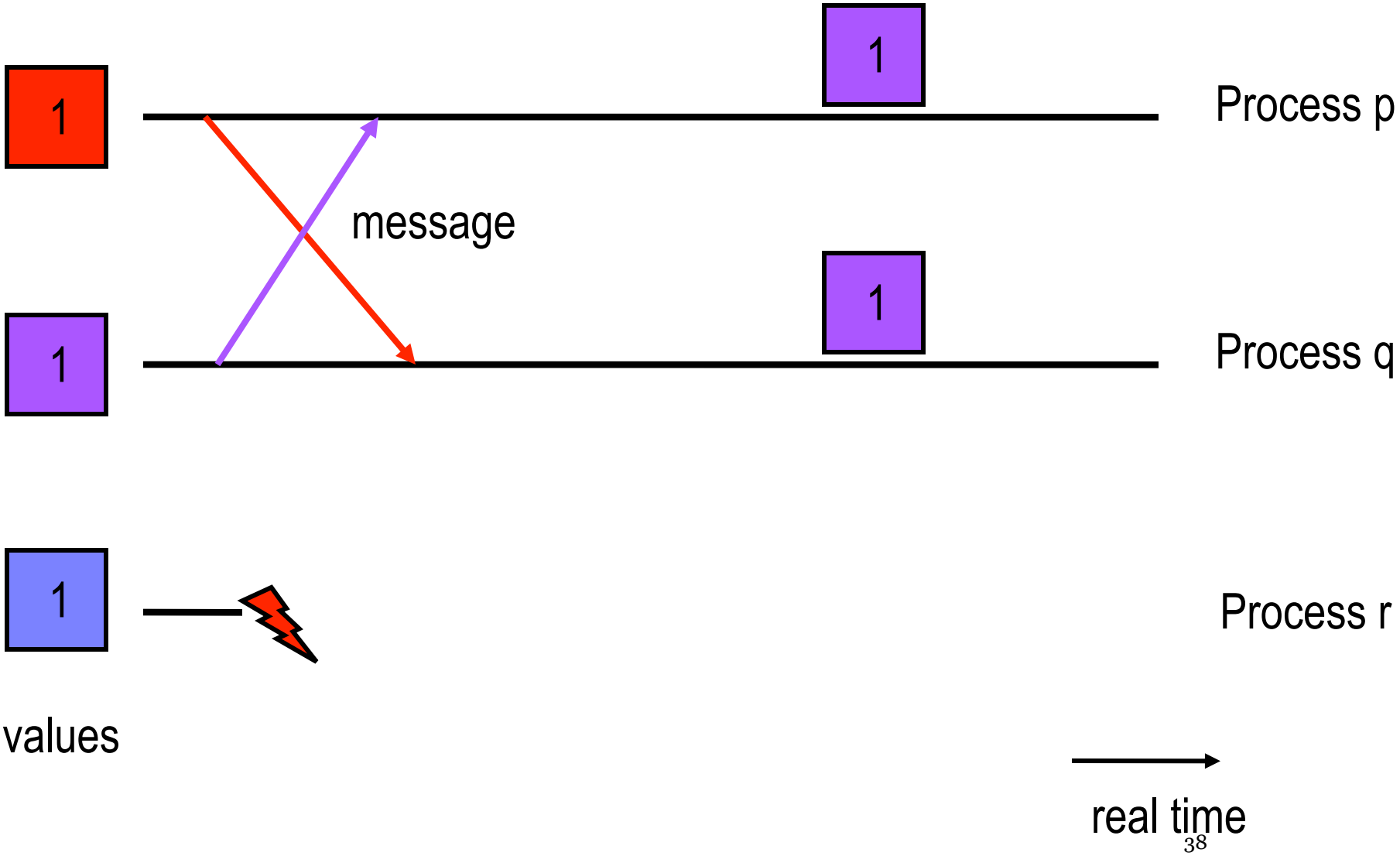
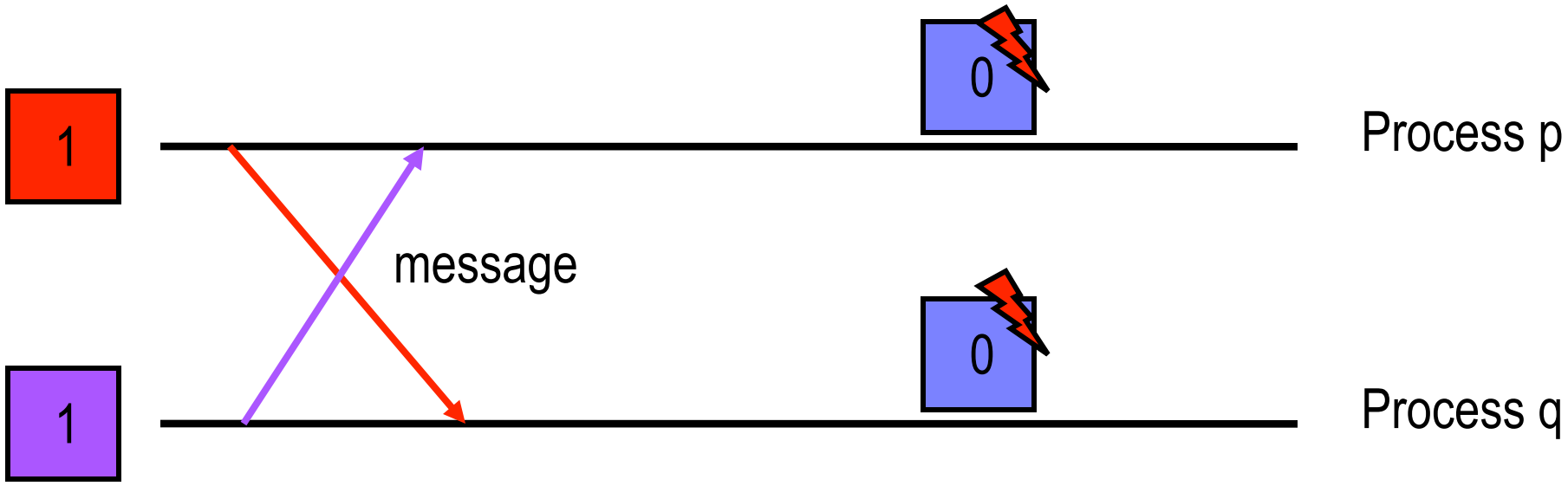
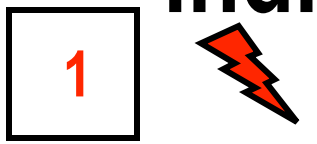


Run 2 is incorrect!



indistinguishable by p and q from run 3



Process r

$\Rightarrow$ 1,1,0 is bivalent

values

real time

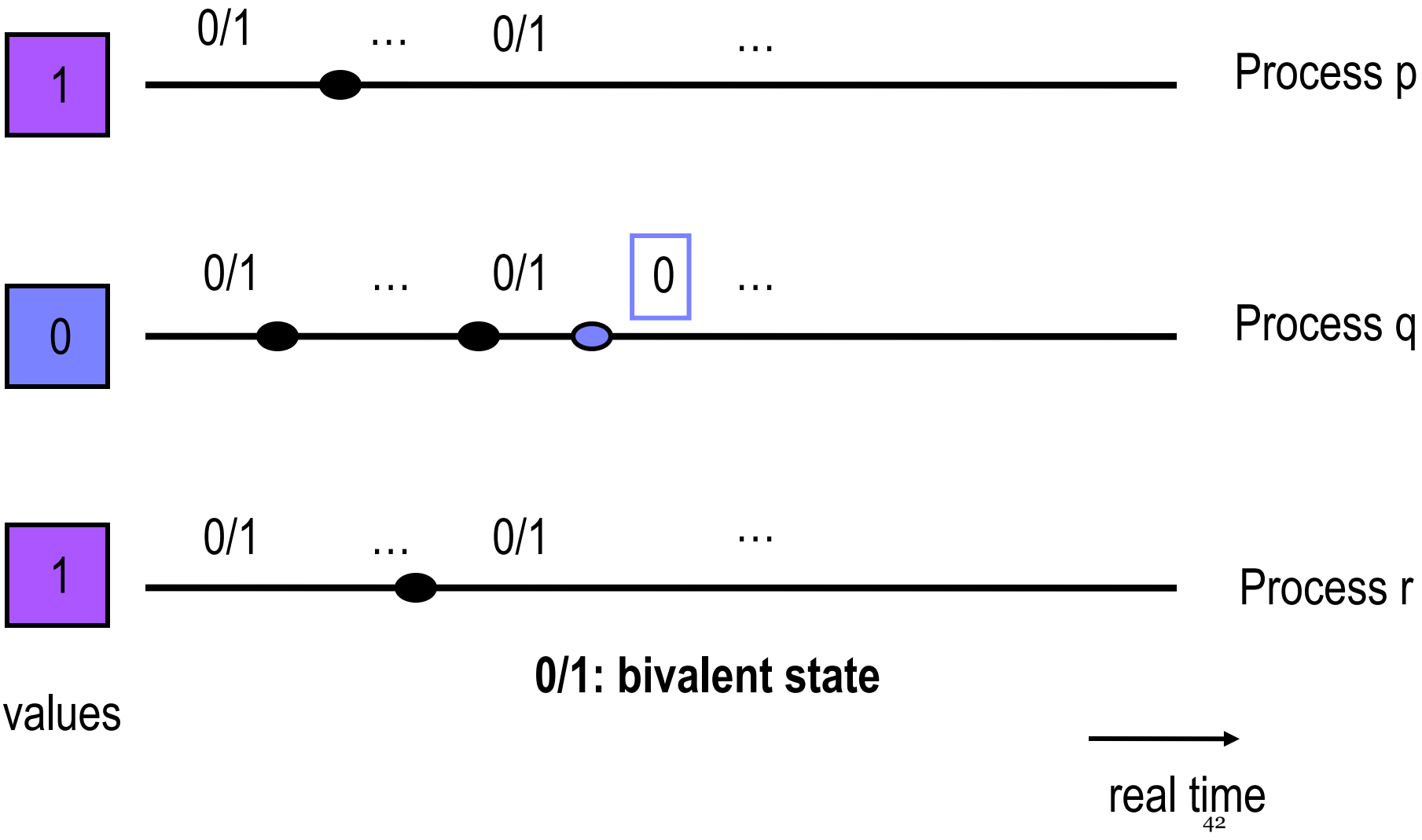
Many seemingly easy problems are impossible to solve!

Impossibility Result (FLP1985)

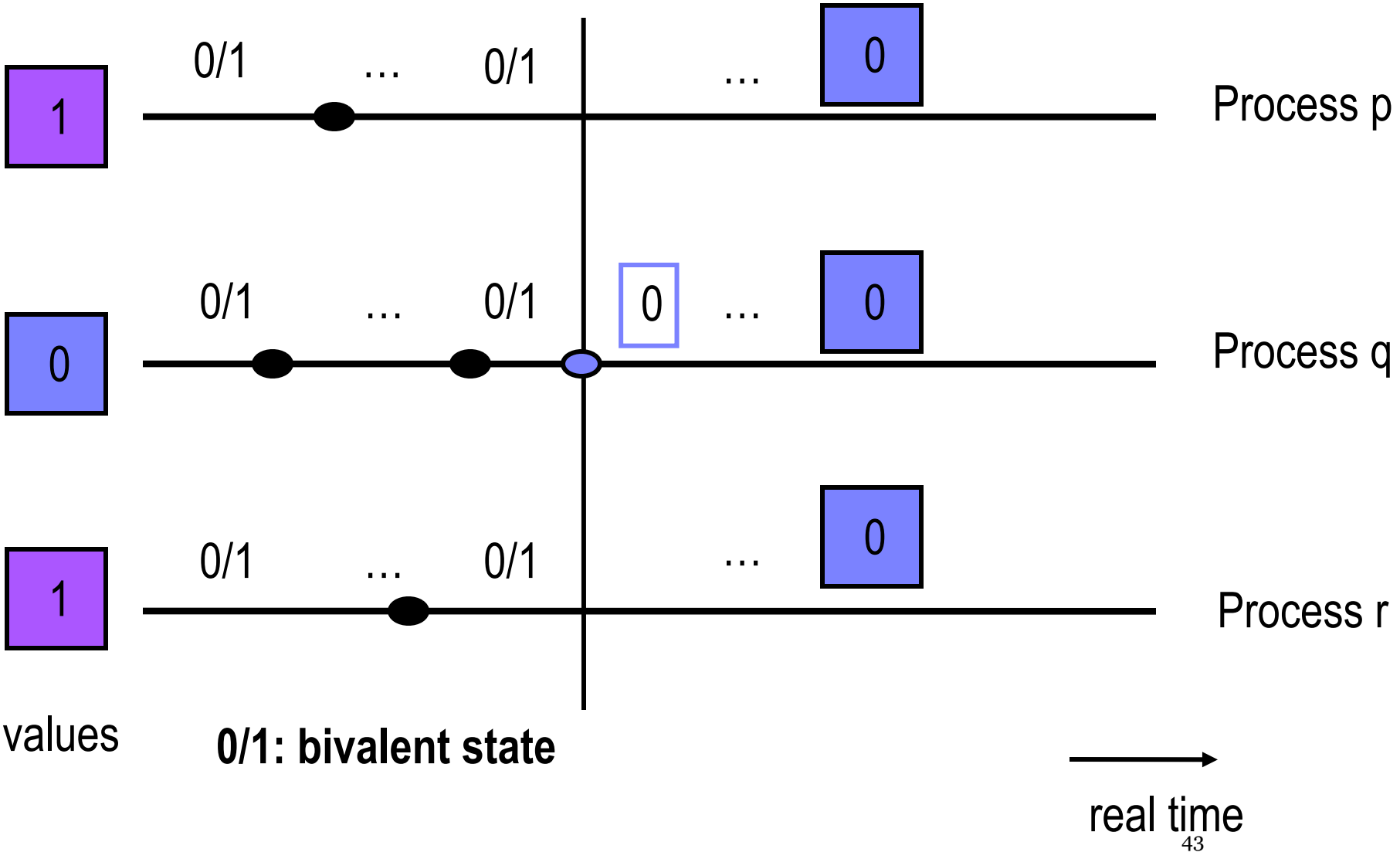
Consensus is impossible to solve even in systems with

- ***Reliable channels***
  - Messages are delivered unless receiver crashes
- ***At most one process crashes***
- ***BUT no bounds on relative speed of processes, no clocks, no failure detectors***

Intuition


1
0/1
…
0/1
…
Process p
0
0/1
…
0/1
0
…
Process q
1
0/1
…
0/1
…
Process r
0/1: bivalent state
values
real time

Why Impossible?



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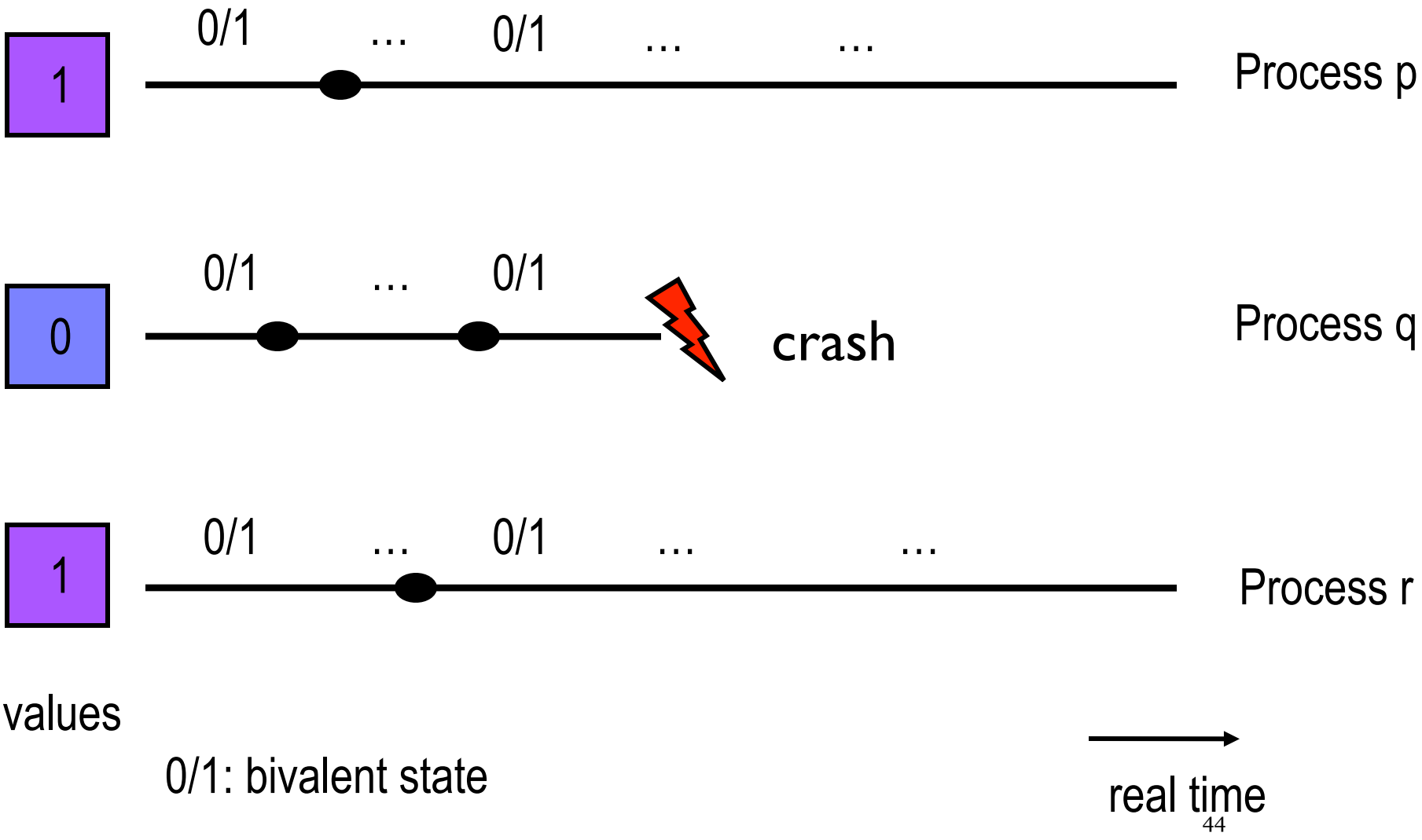

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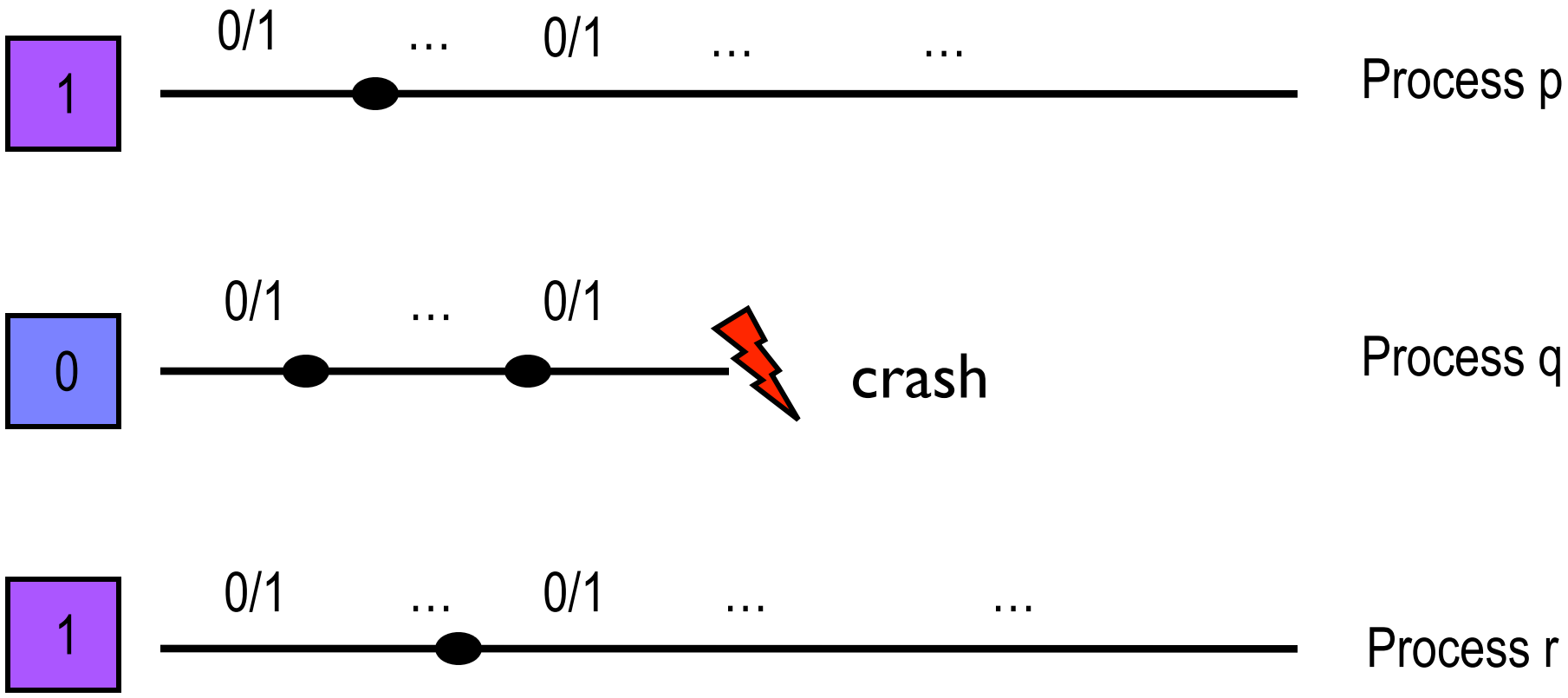
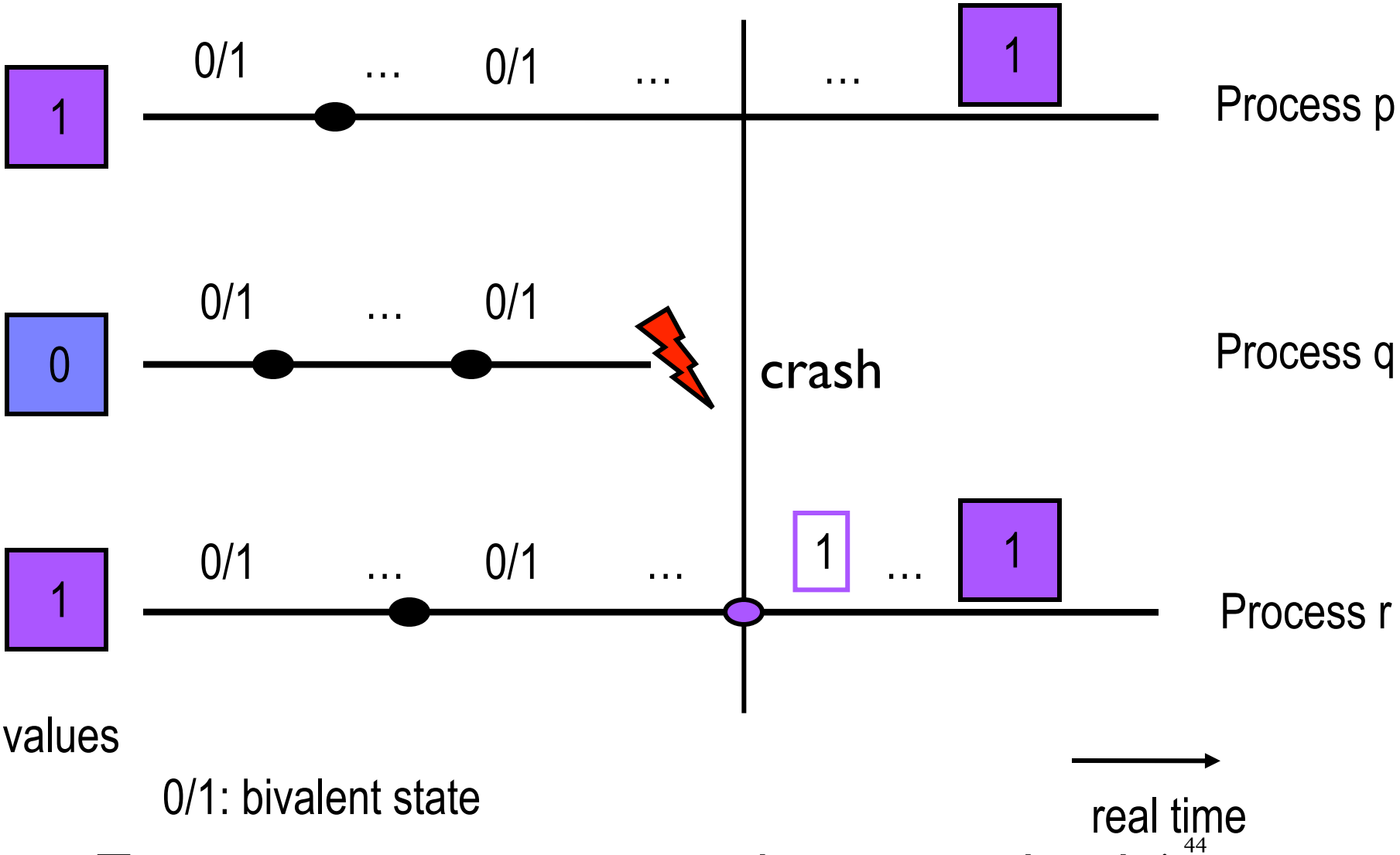


Termination requirement: others must decide!

Why Impossible?



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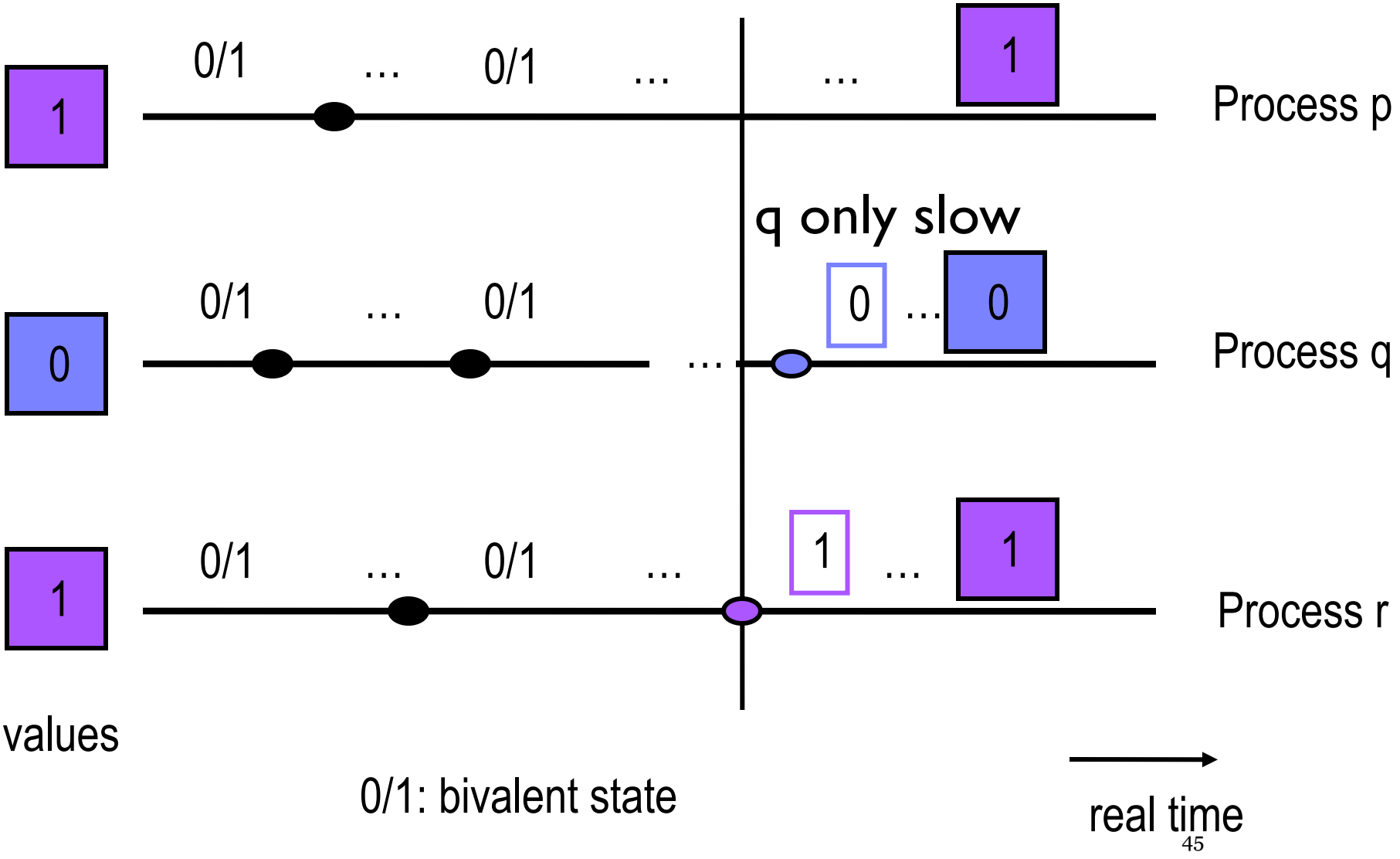


Note

Note:

- This is only the intuition

Question:

- how can we circumvent this impossibility result?

Potential approaches:

- use a **failure detector**, or
- do not always guarantee termination, or …

Failure Detector

- Minimum Consensus Protocol:
  - Wait to get message from all processes that have not crashed!
- Problems:
  - some processes might have received messages from crashed processes! Processes might decide on different values!
- Possible approach:
  - process p needs to ensure that all non-crashed processes learned the same values before p decides !

Paxos

Consensus Protocol

Paxos

Implementation of consensus

- used in several commercial apps (e.g., Google Chubby)

Very efficient algorithm for achieving

- consensus in a message-passing system

First described by

- Lesli Lamport in 1990 in a tech report

Guarantees safety, i.e.,

- agreement and validity

Termination

- if there is a long enough interval in which the system behaves "well".

System Model

Processes communicate

- through messages

Messages are asynchronous

- no bounds on the transmission delay, but
- eventually delivered between **correct** processes

Processes can

- **restart and remember**

Roles

There are three basic roles of a process:

- **Proposers** that propose a value for consensus;
- **Acceptors** that choose the consensus value;
- **Learners** that learn the consensus value.

A single process

- may take on multiple roles.

Paxos - Idea

A proposer attempts to gain acceptance

- by the majority of acceptors

Agreement is enforced

- by allowing only one value to be accepted by majority

Validity is enforced

- by allowing only input values to be proposed

Non-Termination

- possible with more than two proposals, or if processes fail.
- **Try to minimize by**
  - by proposers can restart the protocol (i.e., propose again)
  - acceptors can be released from old acceptance
  - unique proposal numbers and a primary proposer

Paxos - Idea

Proposal number

- is attached to each proposal

Safety properties are dependent

- only on distinct, **increasing proposal numbers**
- Can be done with timestamp or polling

Messages are

- **Prepare(n)** – n is the proposal number
- **Accept(n,v)** – v is the value

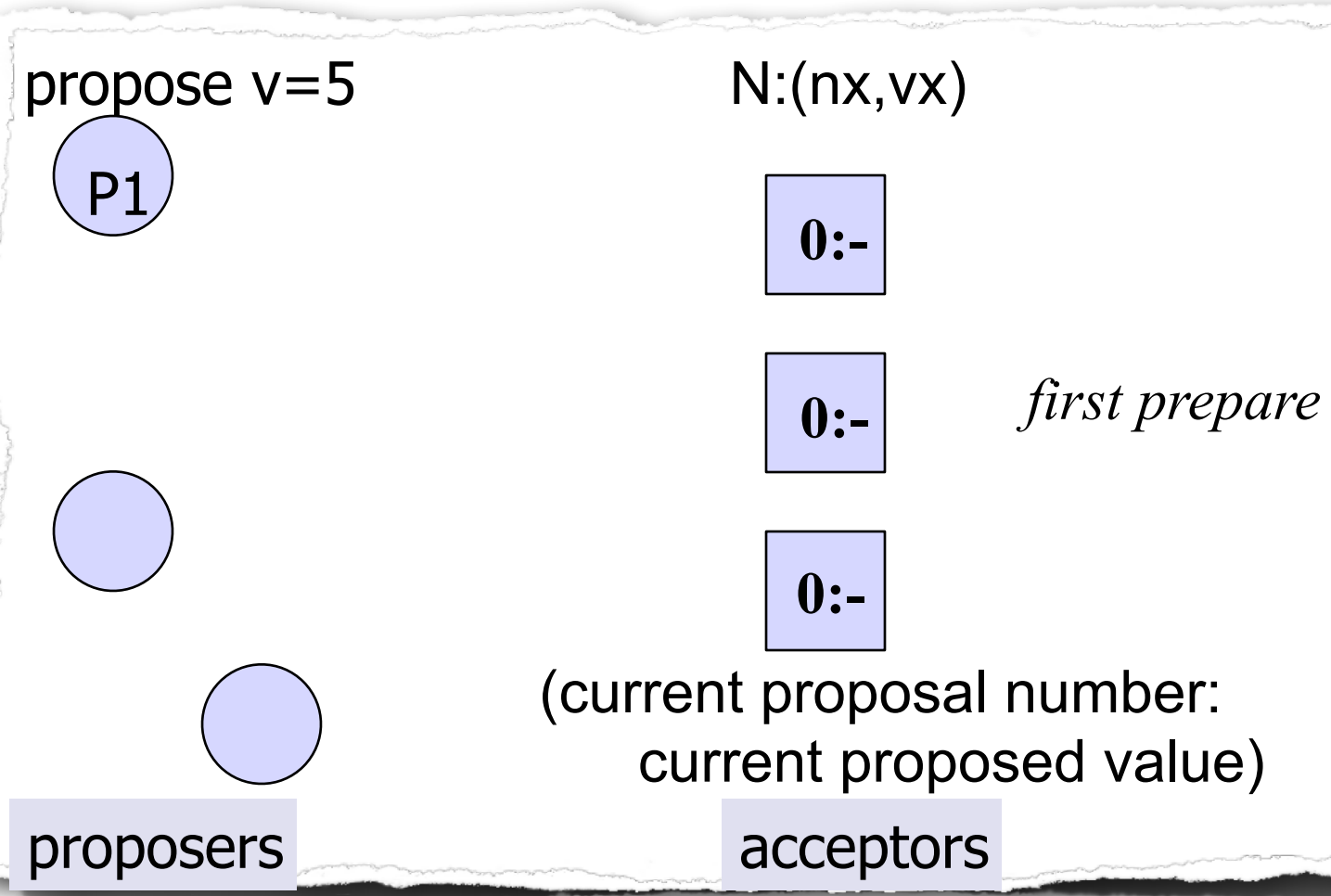
Paxos - Algorithm

- **Phase 1: Preparation**
  - ensure that we do not revise any previous decision

- **Proposer sends**
    - **prepare(n)** to all or a majority of acceptors ;
    - e.g., n = (time(), unique process id);

- **Each Acceptor compares n to the**
  - highest numbered proposal N which it has responded to.
  - If n > N then
    - responds with **ack(n,(nx,vx))**
    - N := n
  - **vx** is the value with the highest proposal number **nx** accepted so far (might be empty), and
  - **ack** is a promise to never **accept** proposal number less than N

Preparation



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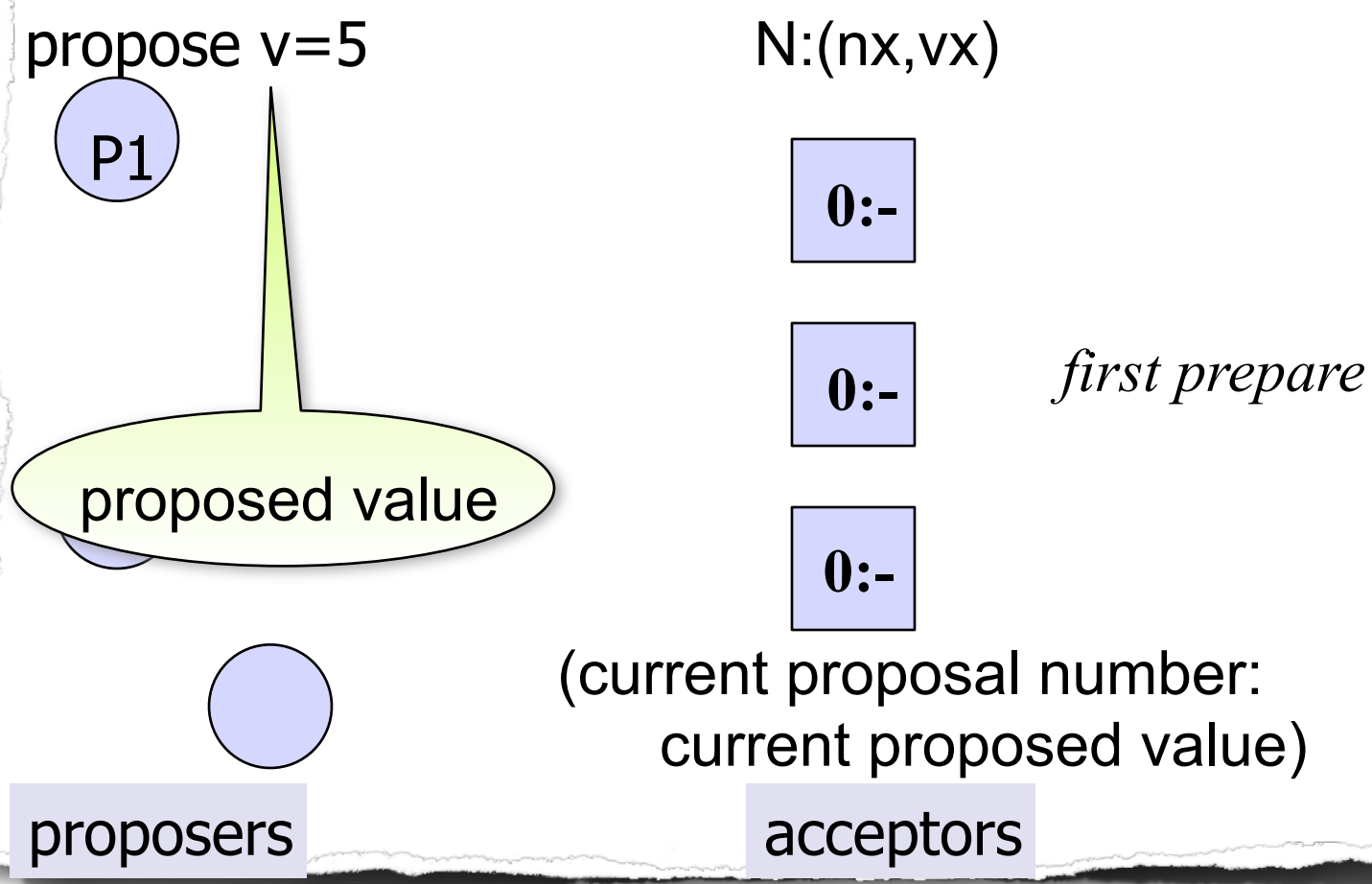

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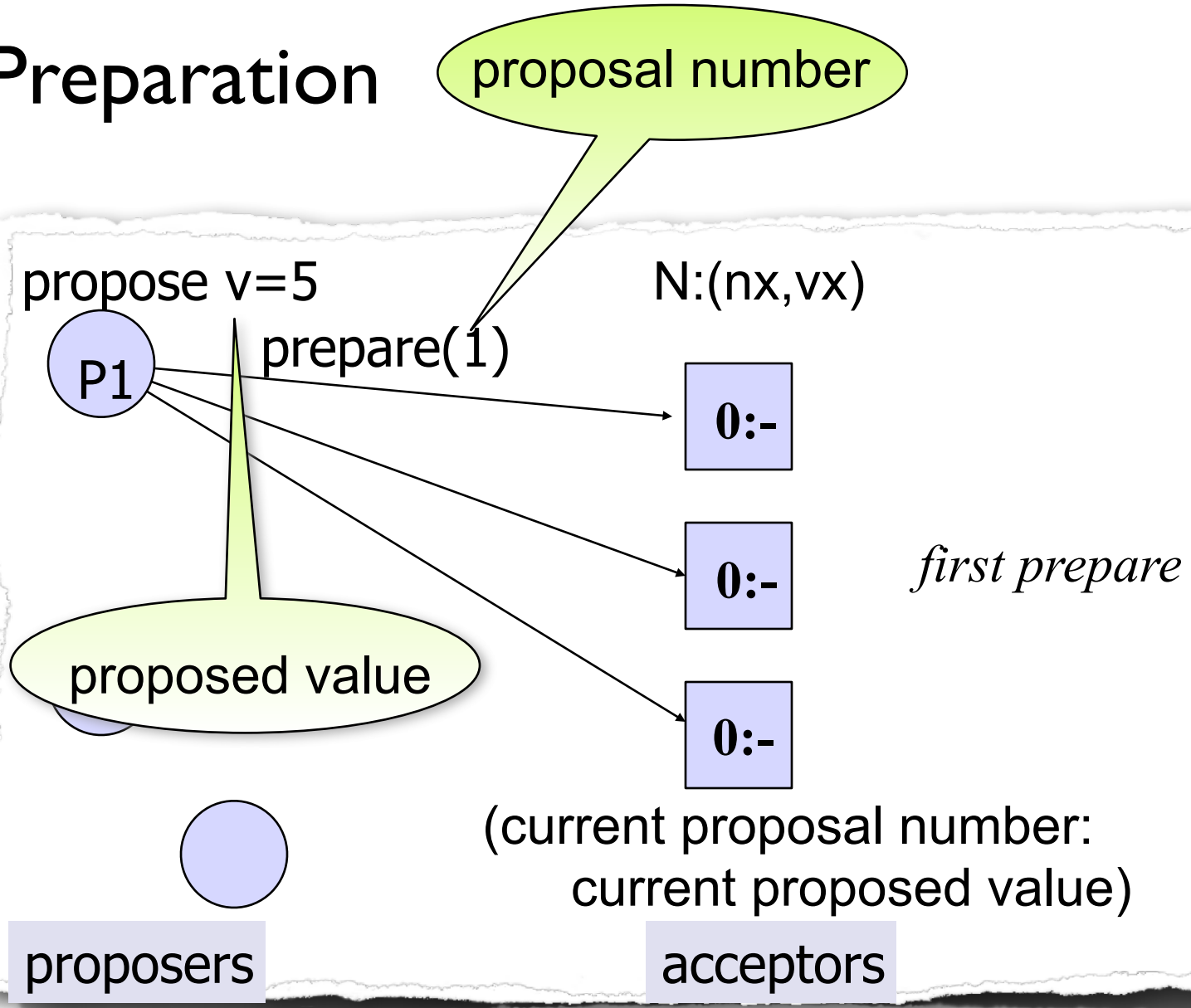

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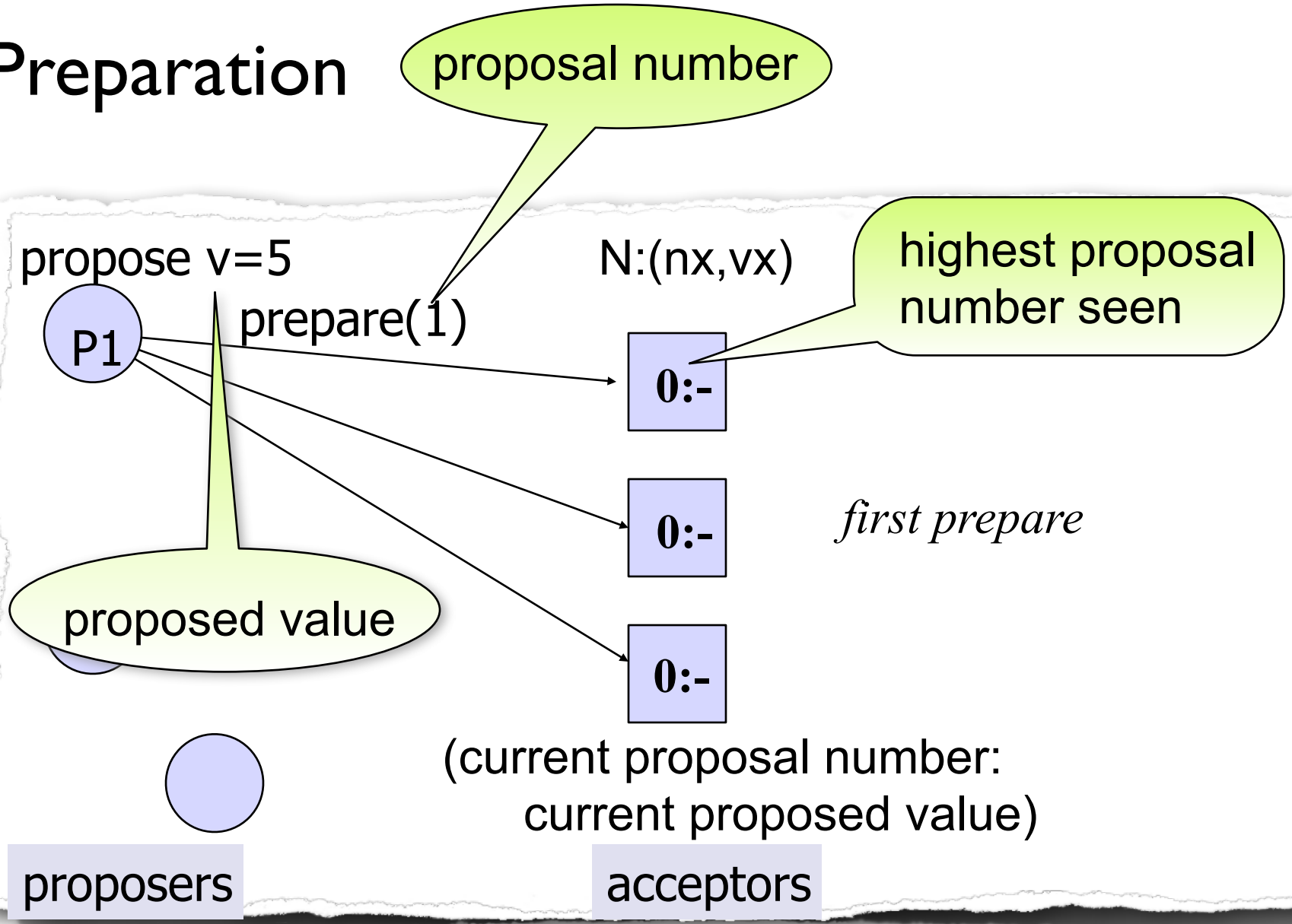

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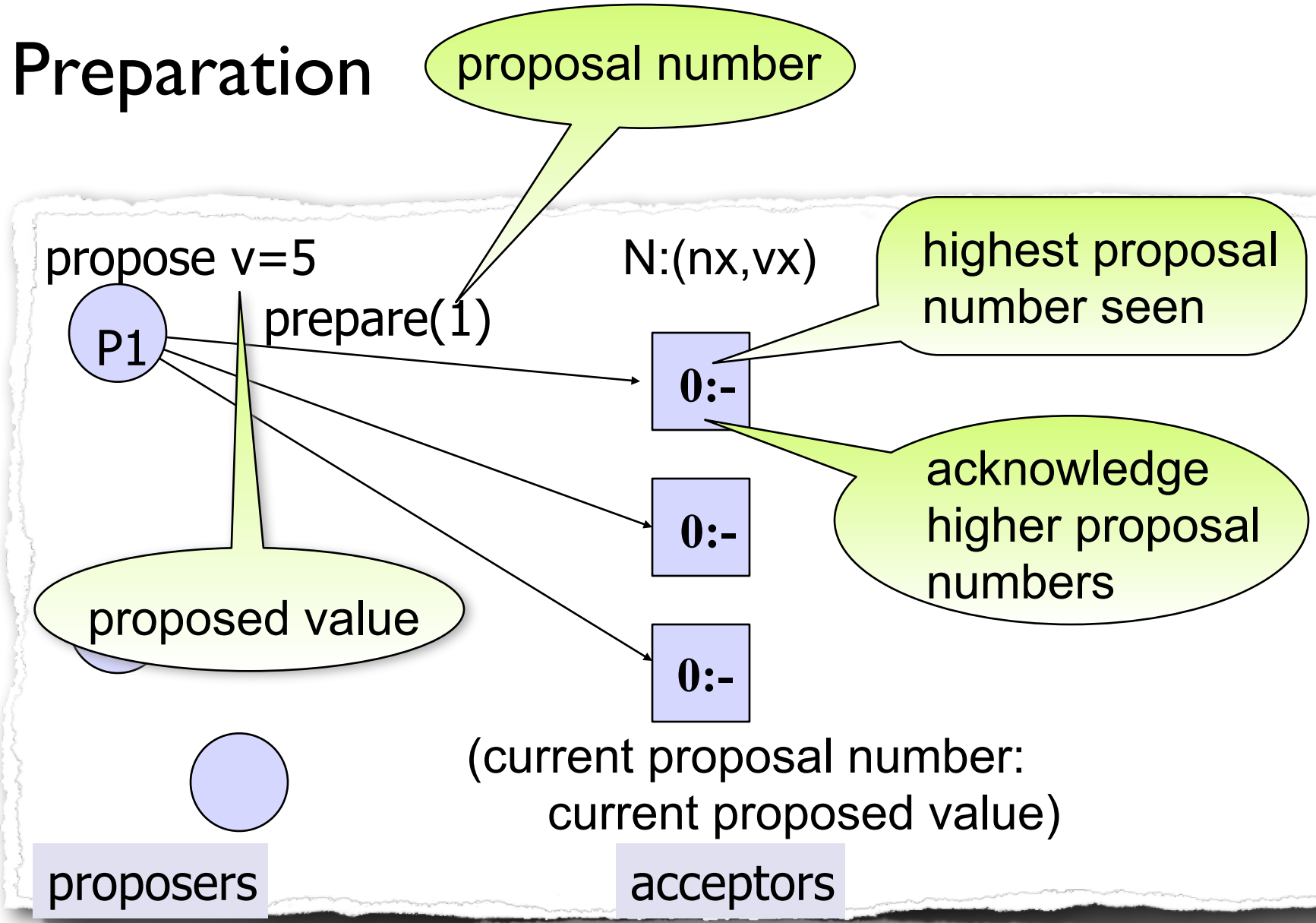

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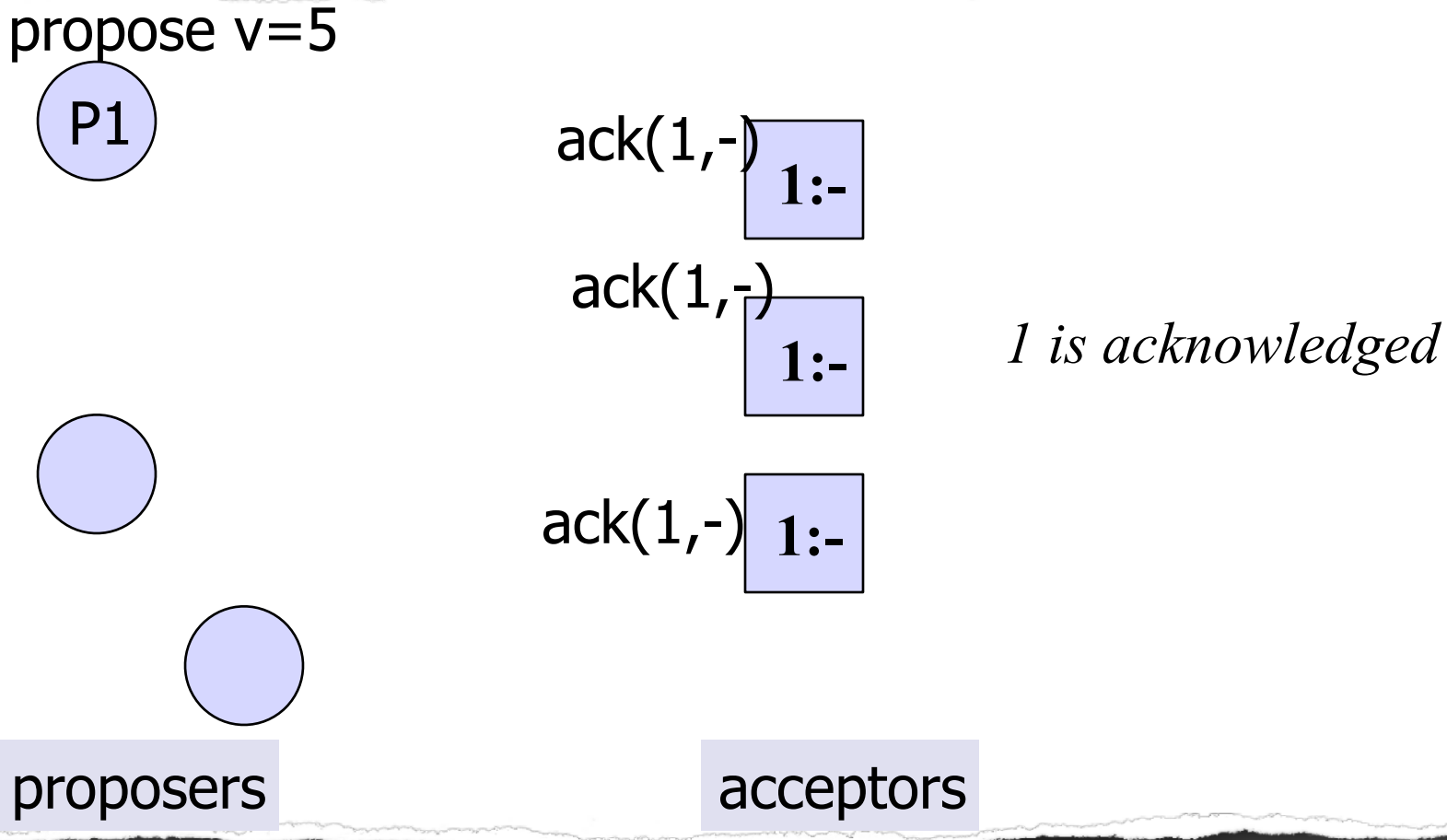

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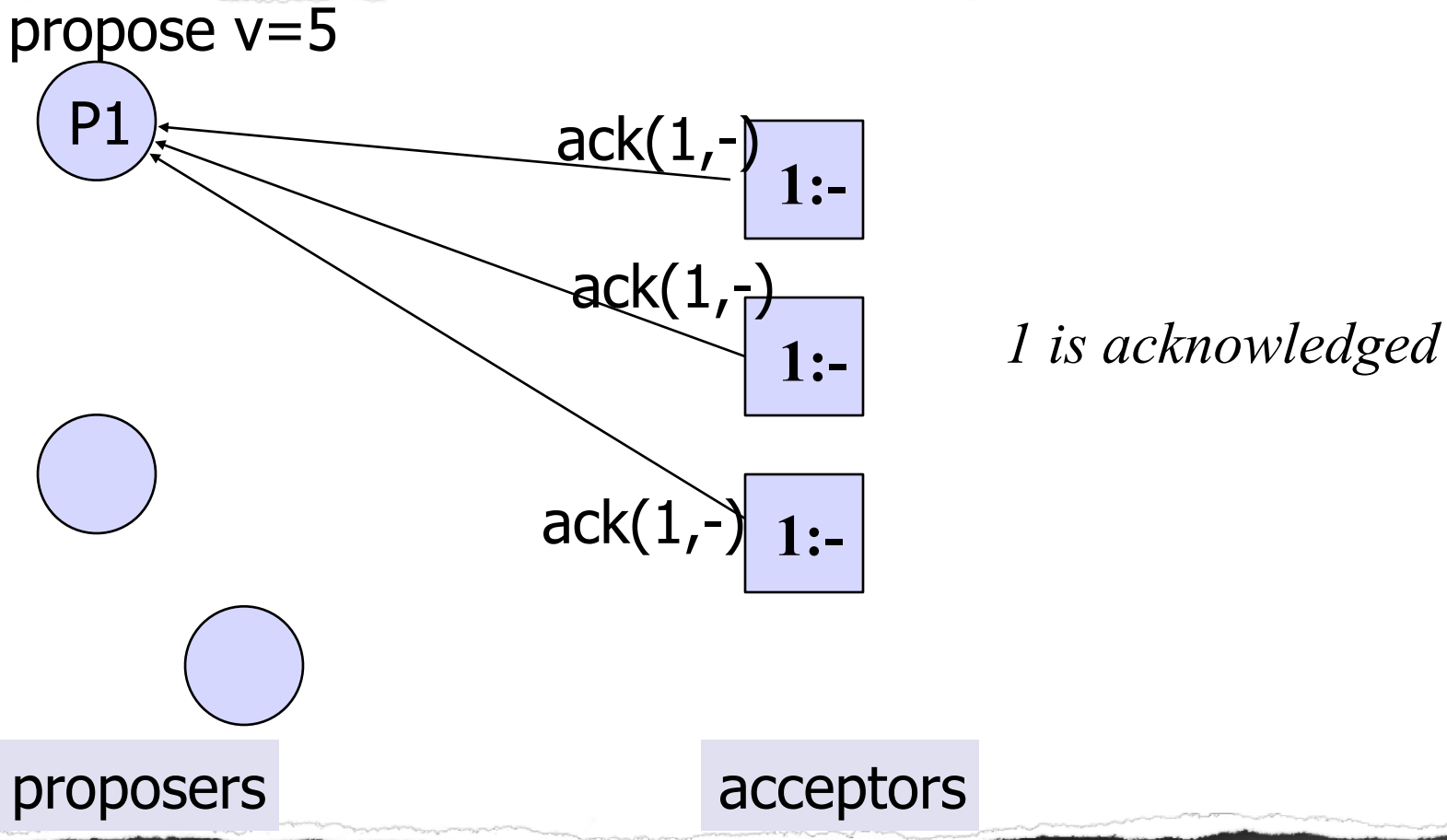
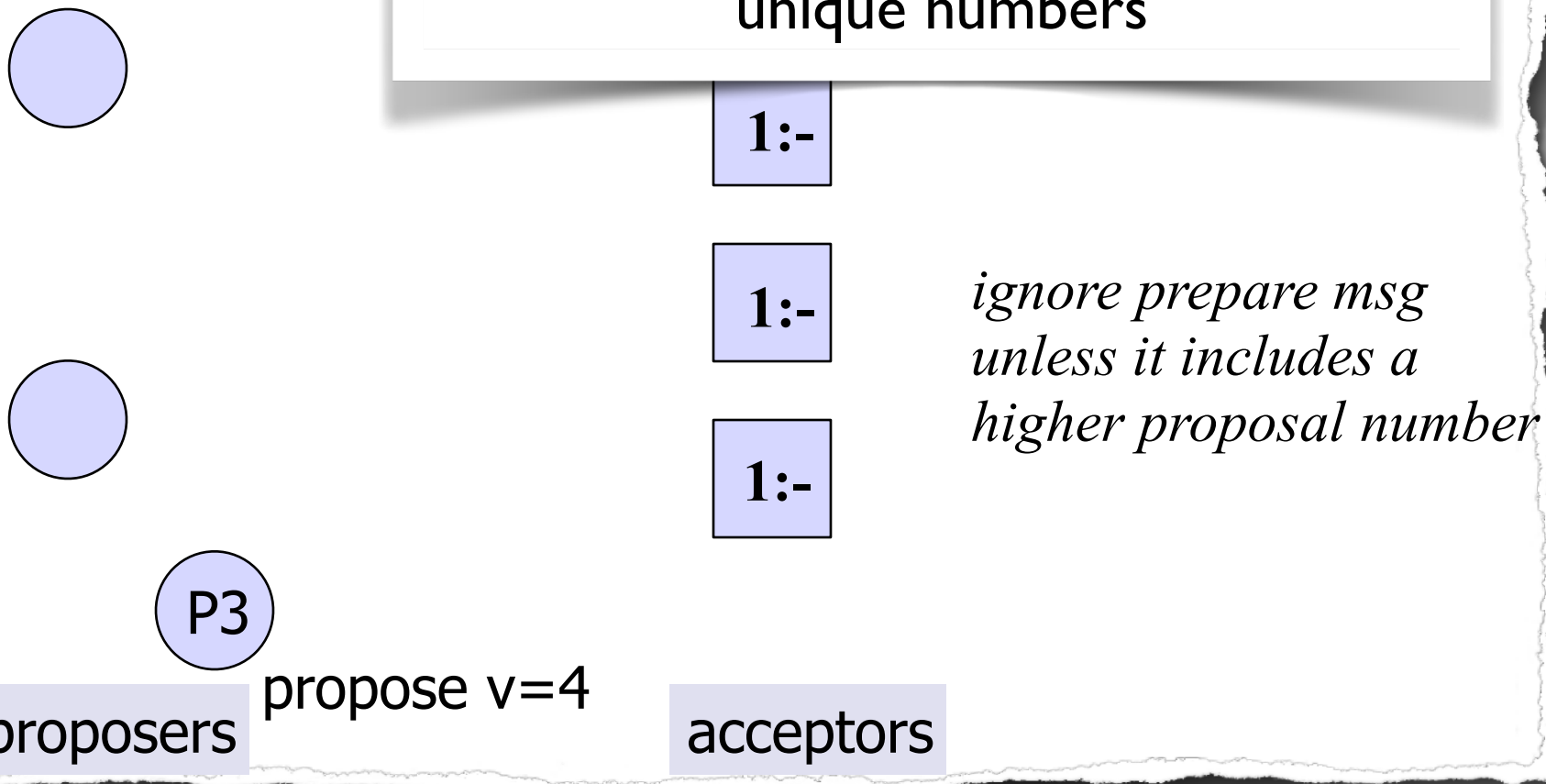


1 is acknowledged

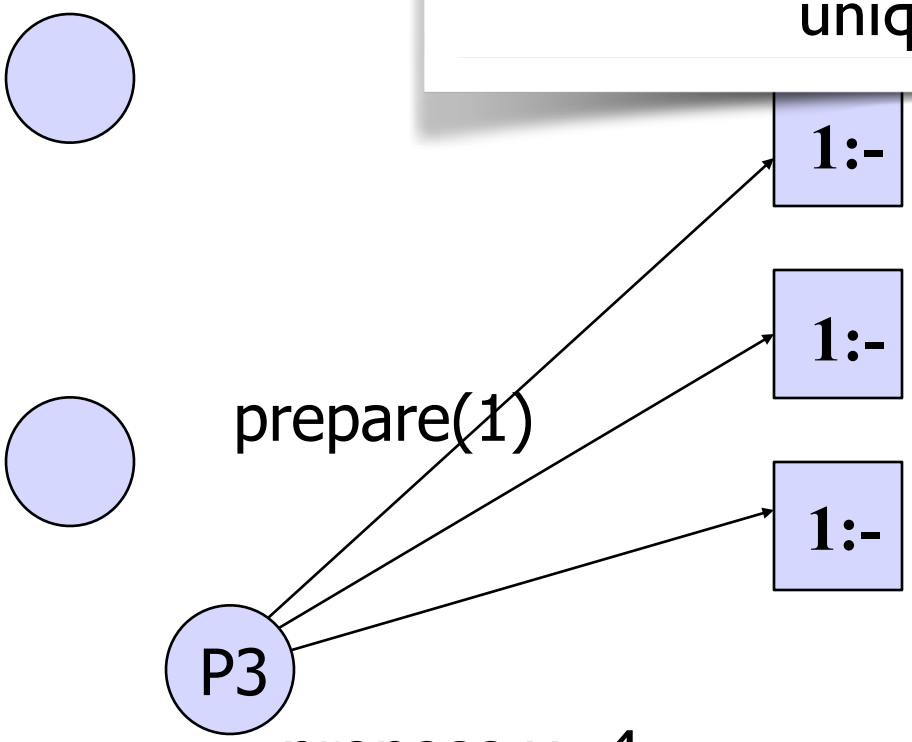
Preparation

proposal numbers may not be unique - but implementations should aim for unique numbers



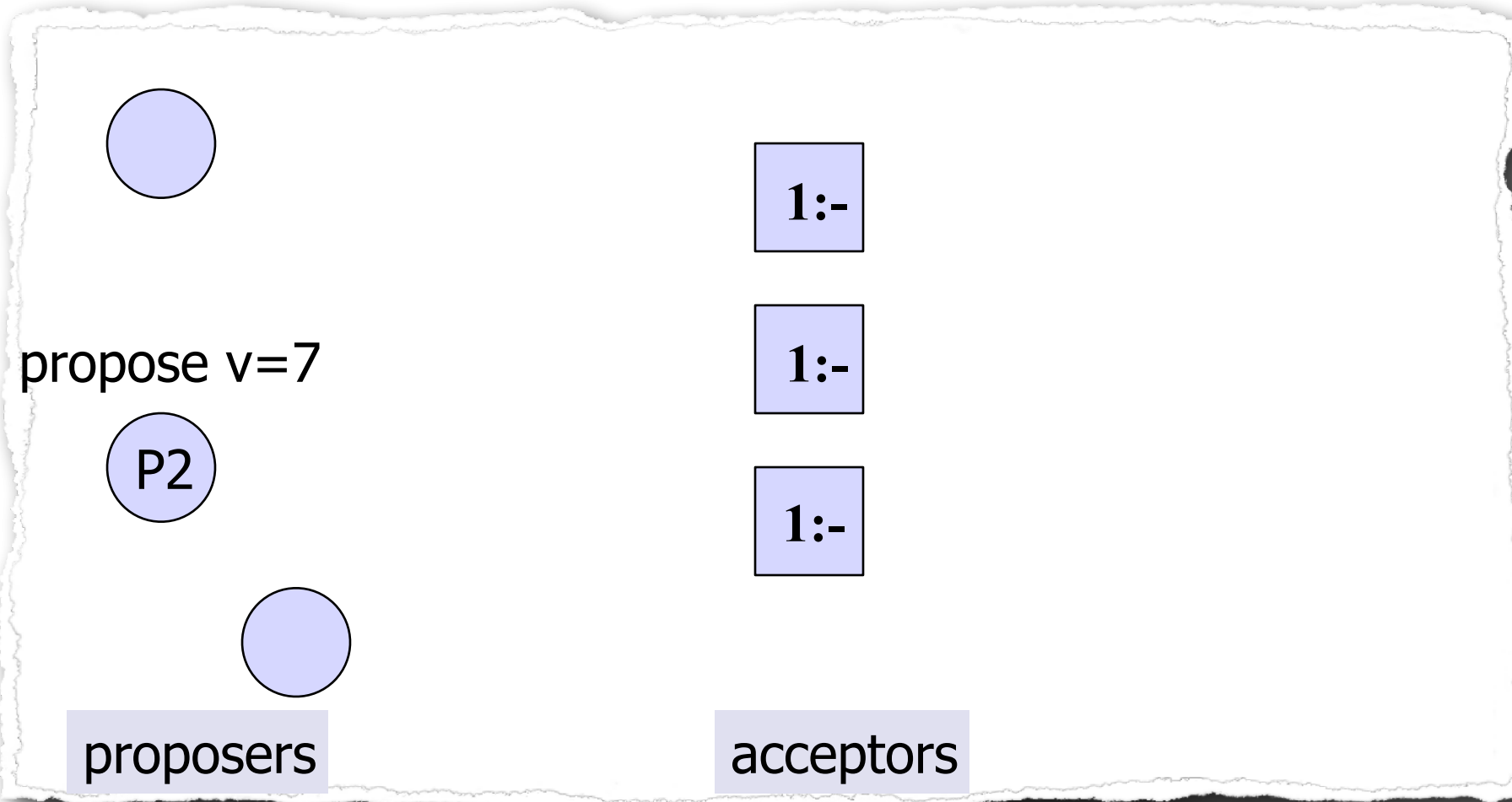
Preparation

proposal numbers may not be unique - but implementations should aim for unique numbers

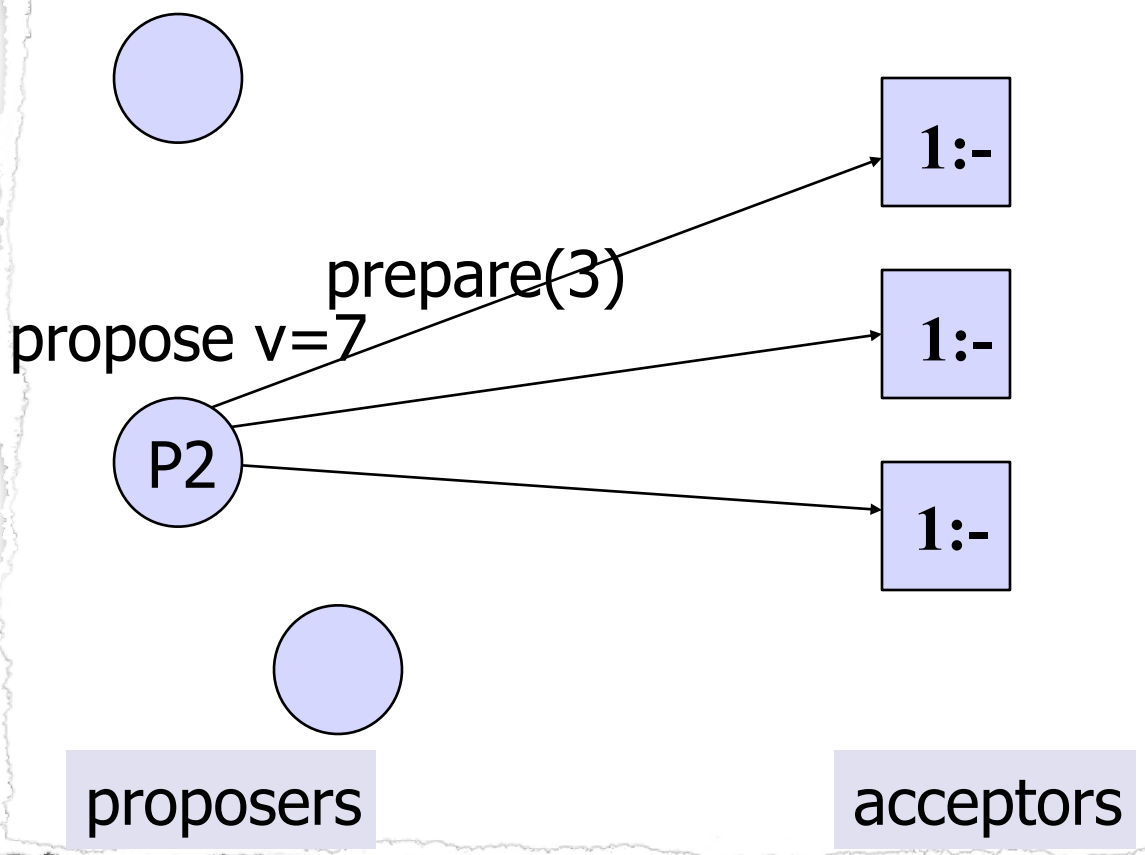


ignore prepare msg unless it includes a higher proposal number

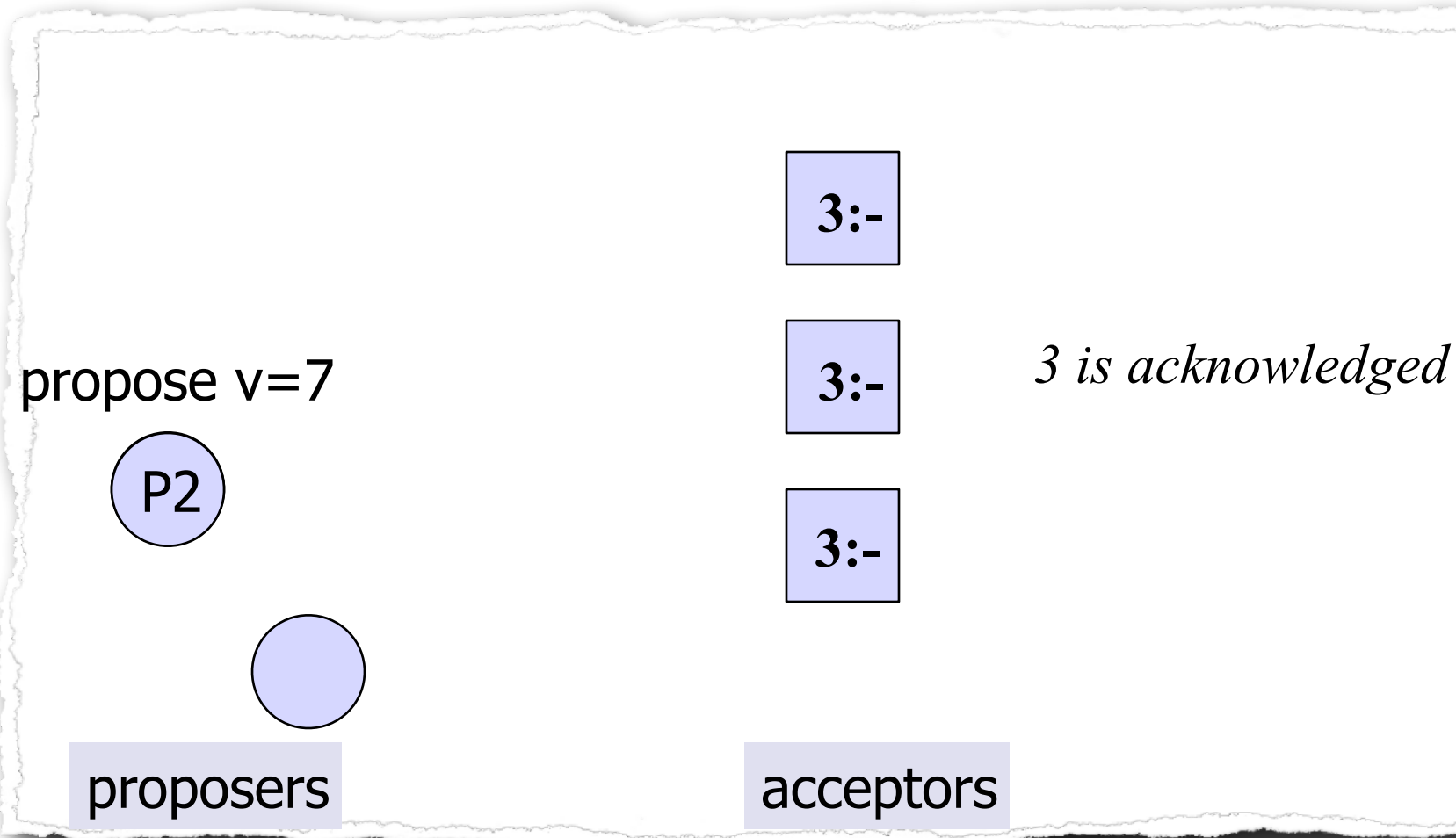
Preparation



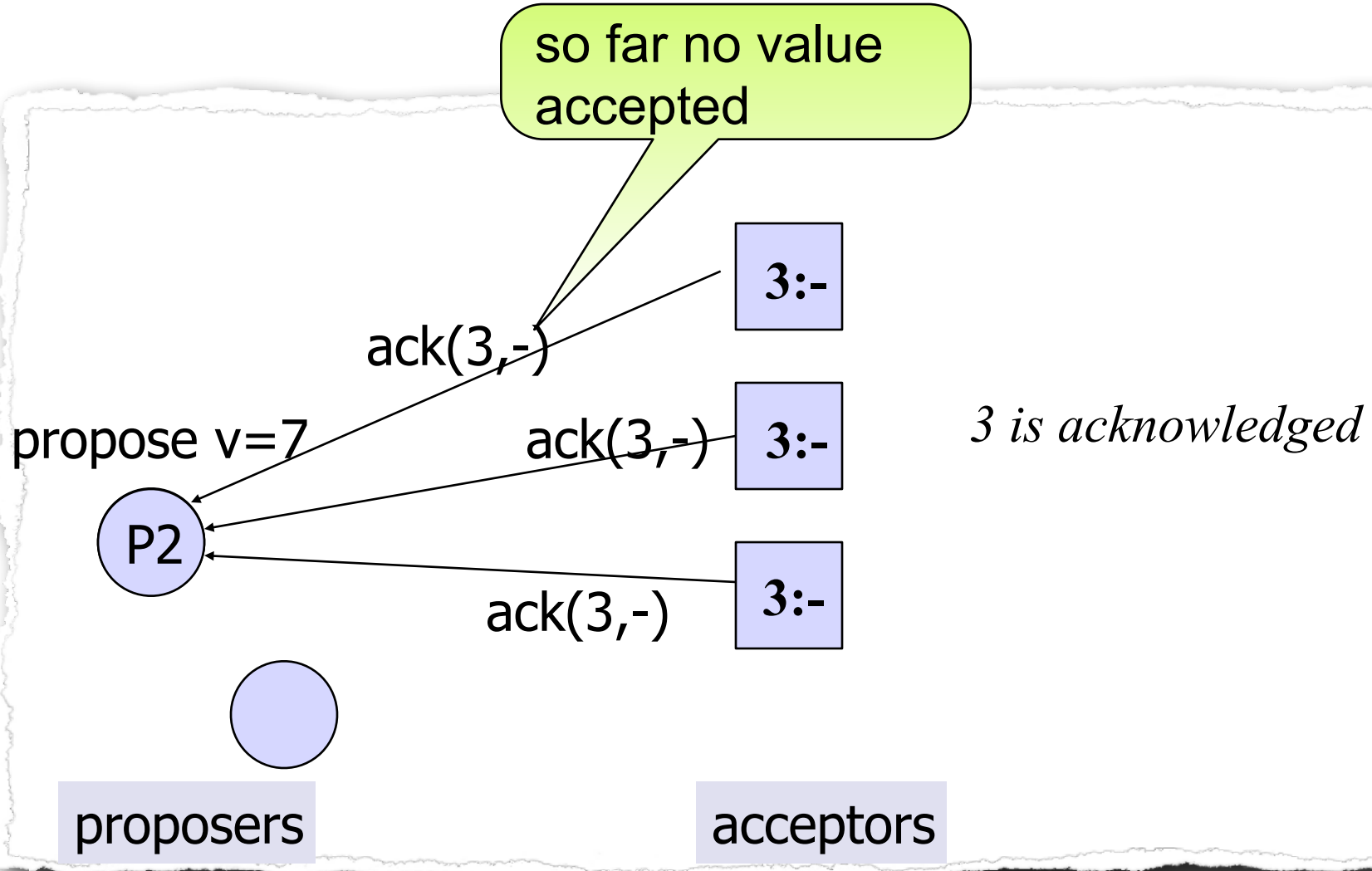
Preparation



Preparation



Preparation



3 is acknowledged

Phase 2:

- Proposer waits for acks from majority of acceptors.
  - If exactly one **ack(n,(nx, vx))** contained a value,
    - sends **accept(n,vx)** to all acceptors
  - **If multiple acks contained value,**
    - send value with highest prev. proposal number nx
  - **Otherwise**,
    - sends **accept(n,own-value)** to all acceptors

Phase 2:

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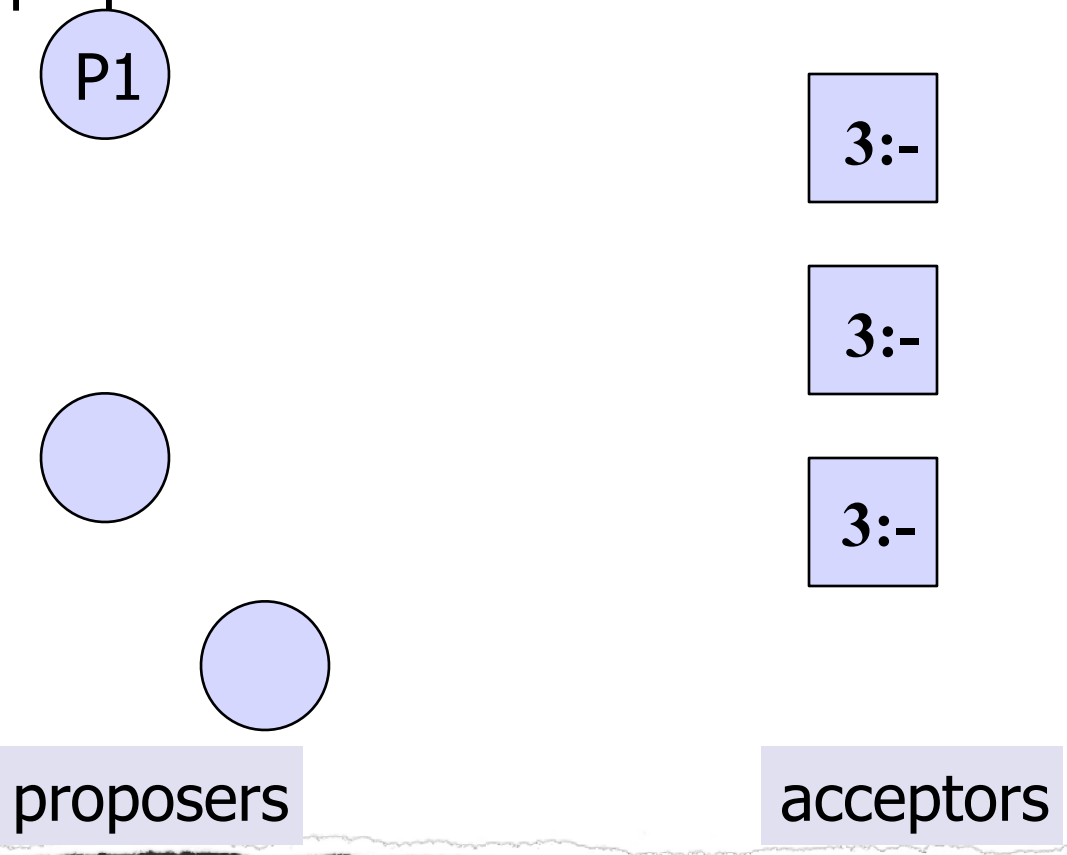
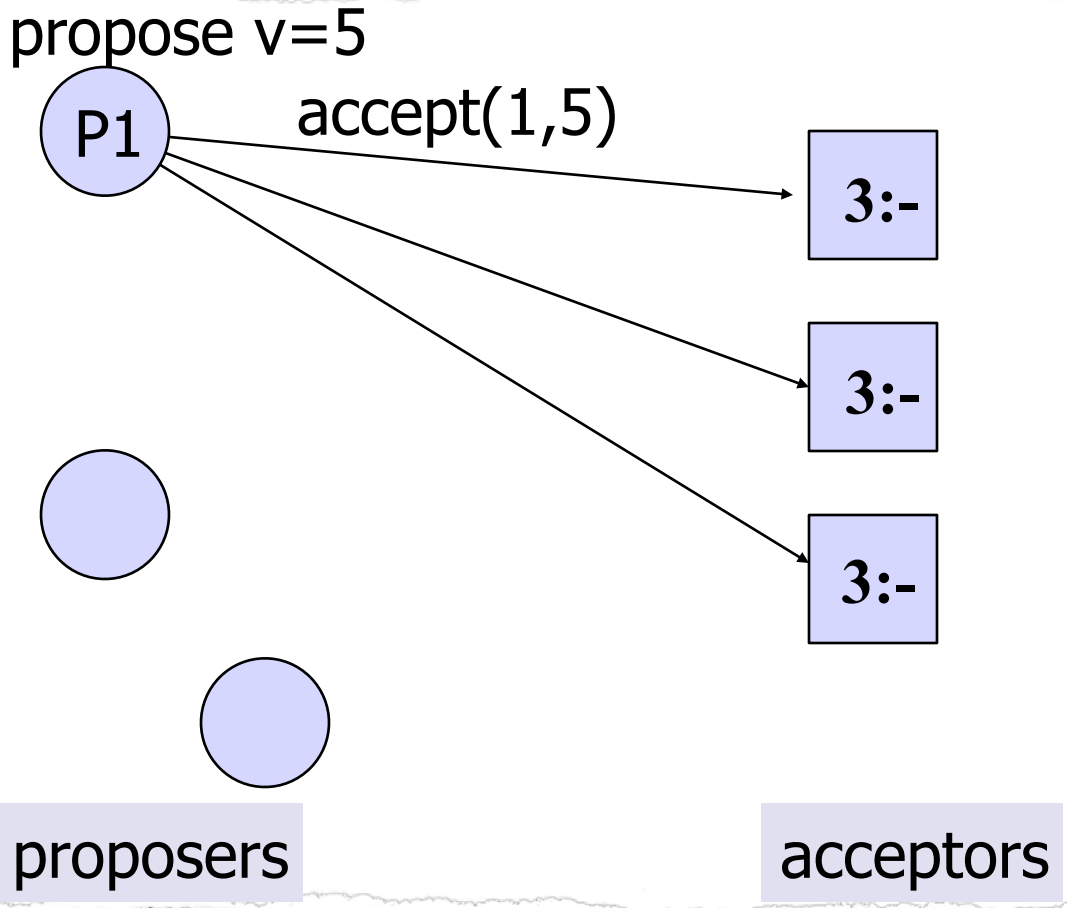
  - **Otherwise**,
    - sends **accept(n,own-value)** to all acceptors

- Upon receiving **accept(n,v)**
  - an acceptor accepts v **unless**
    - it has already received prepare(n') for some n' > n (i.e., N > n) or an accept(n',v') for some n' >= n (i.e., N >= n)
  - on acceptance, it sets:
    - N := n
    - nx := n, vx := v

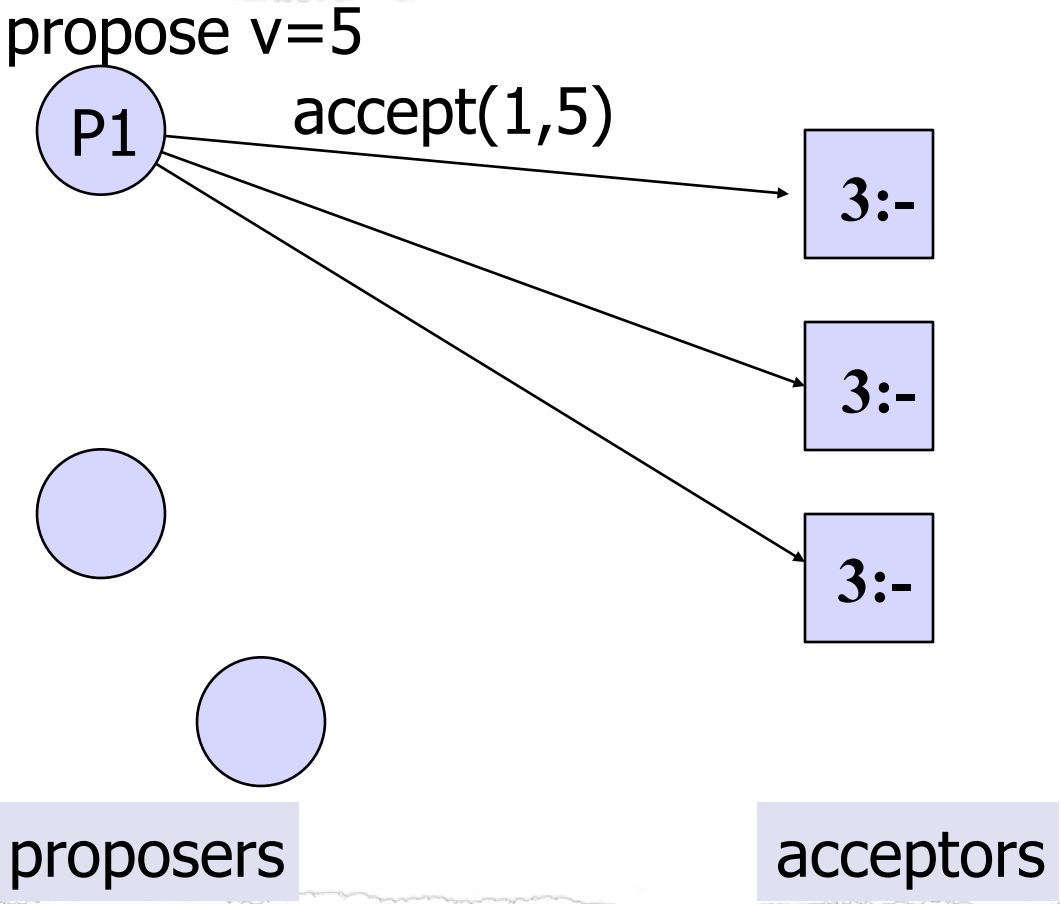
Phase 2:



Phase 2:

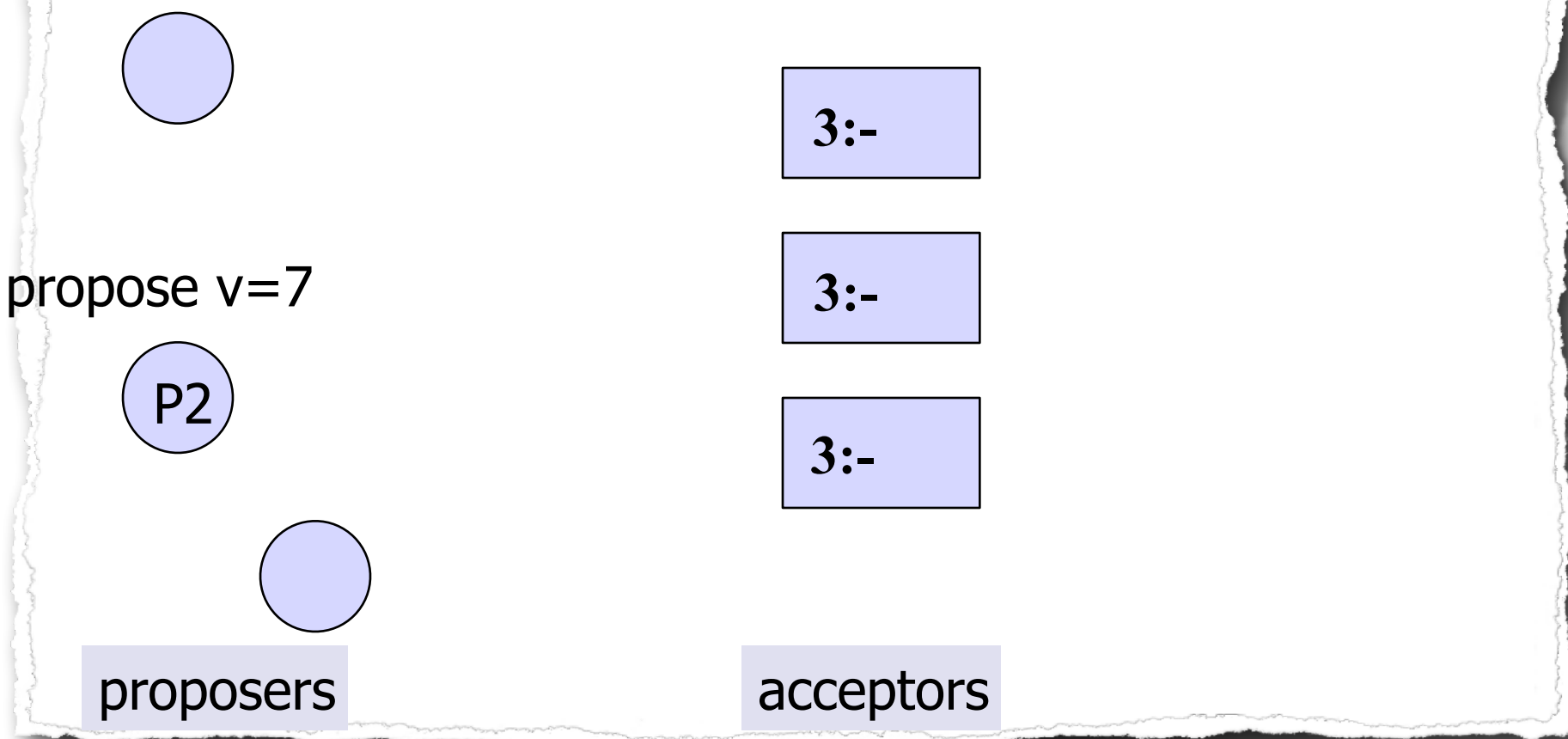


Phase 2:

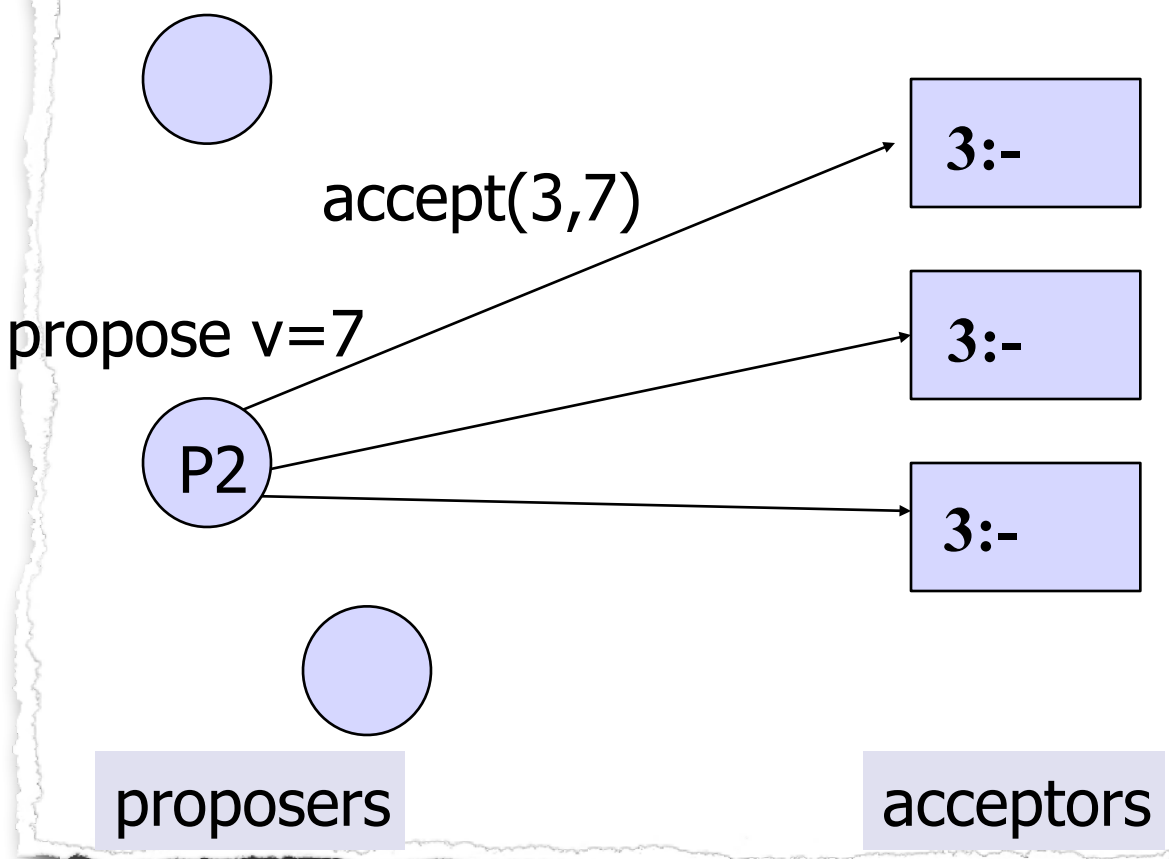


accept is ignored since proposal number is too small!

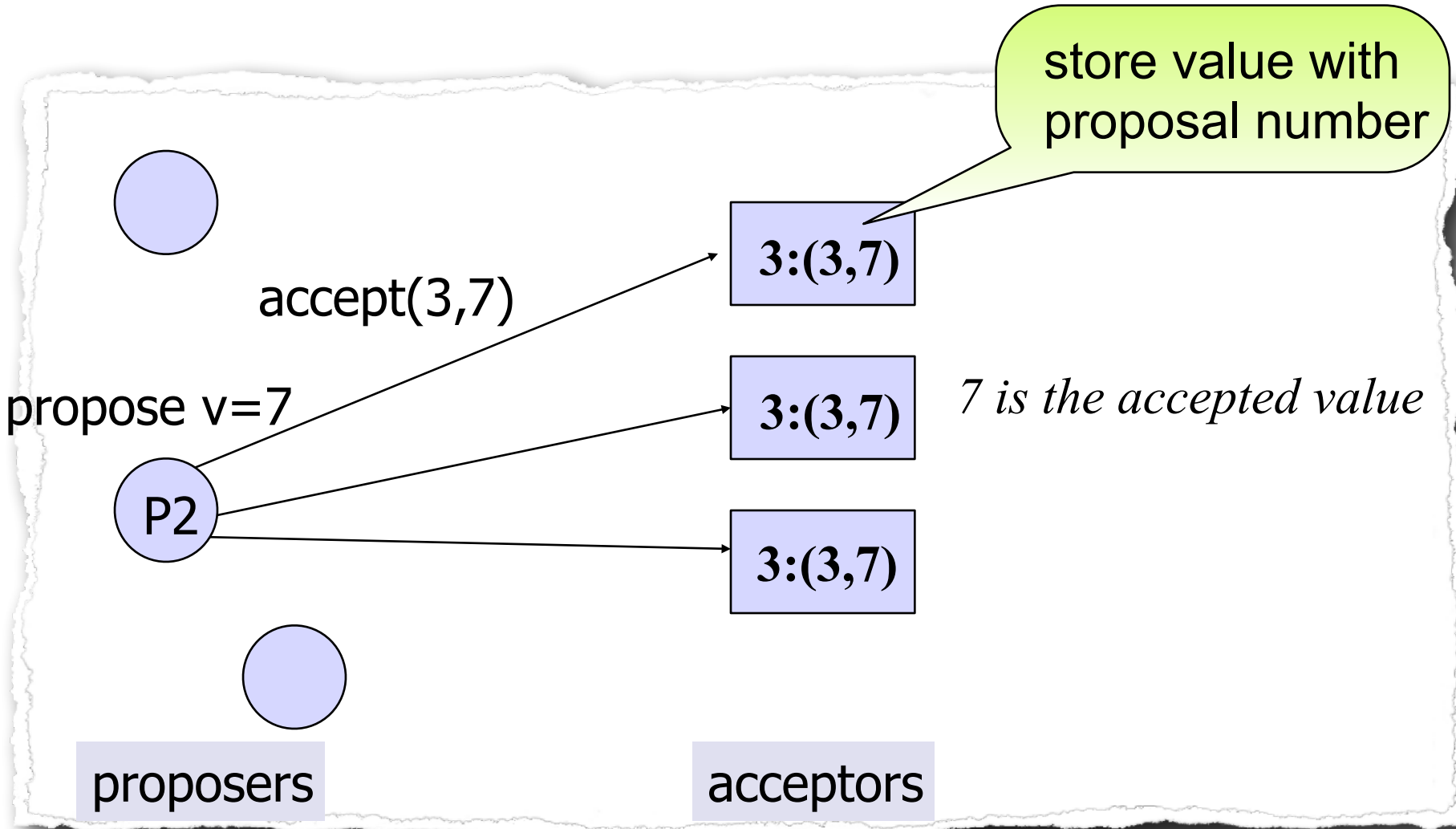
Phase 2:



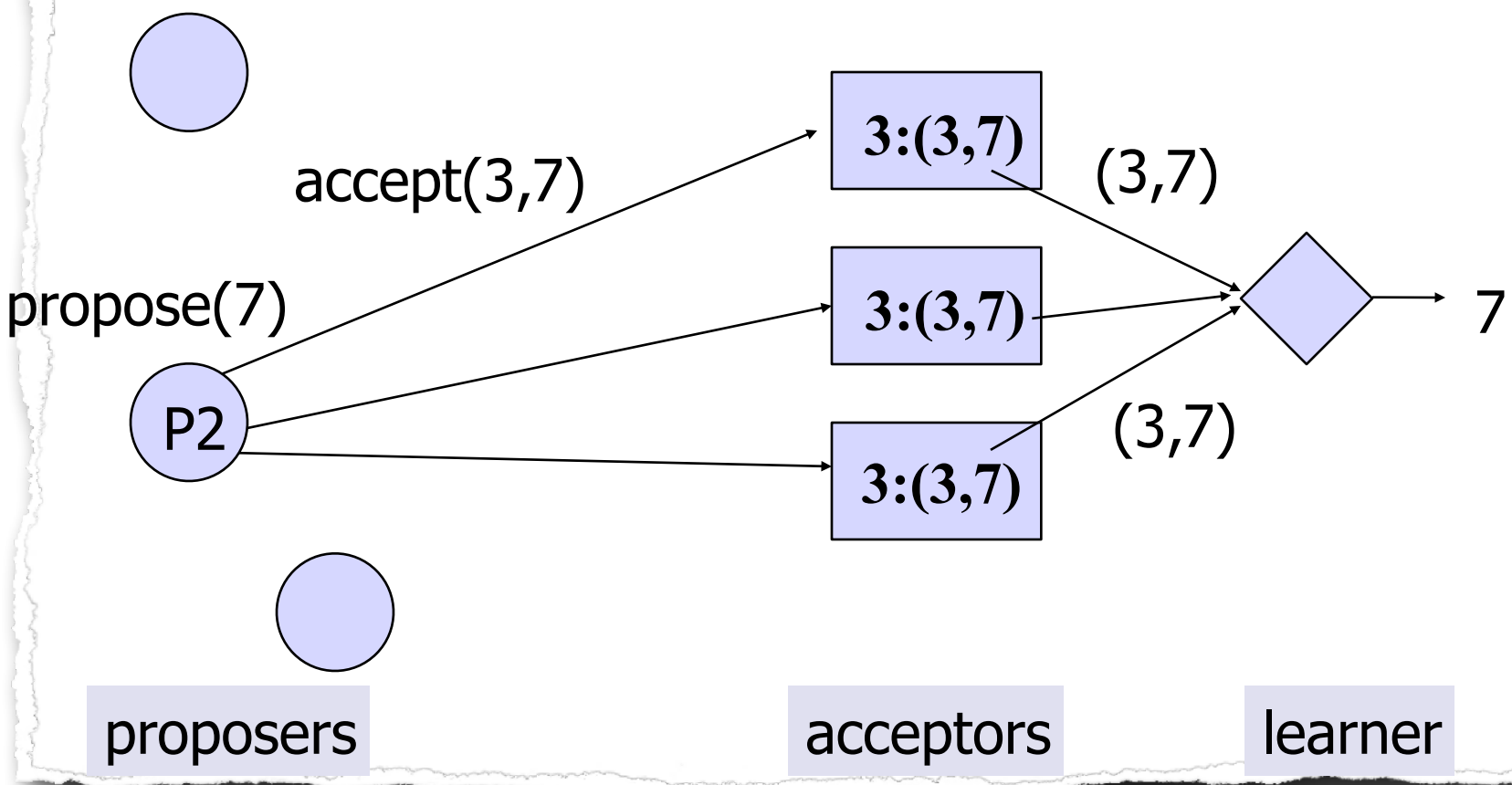
Phase 2:



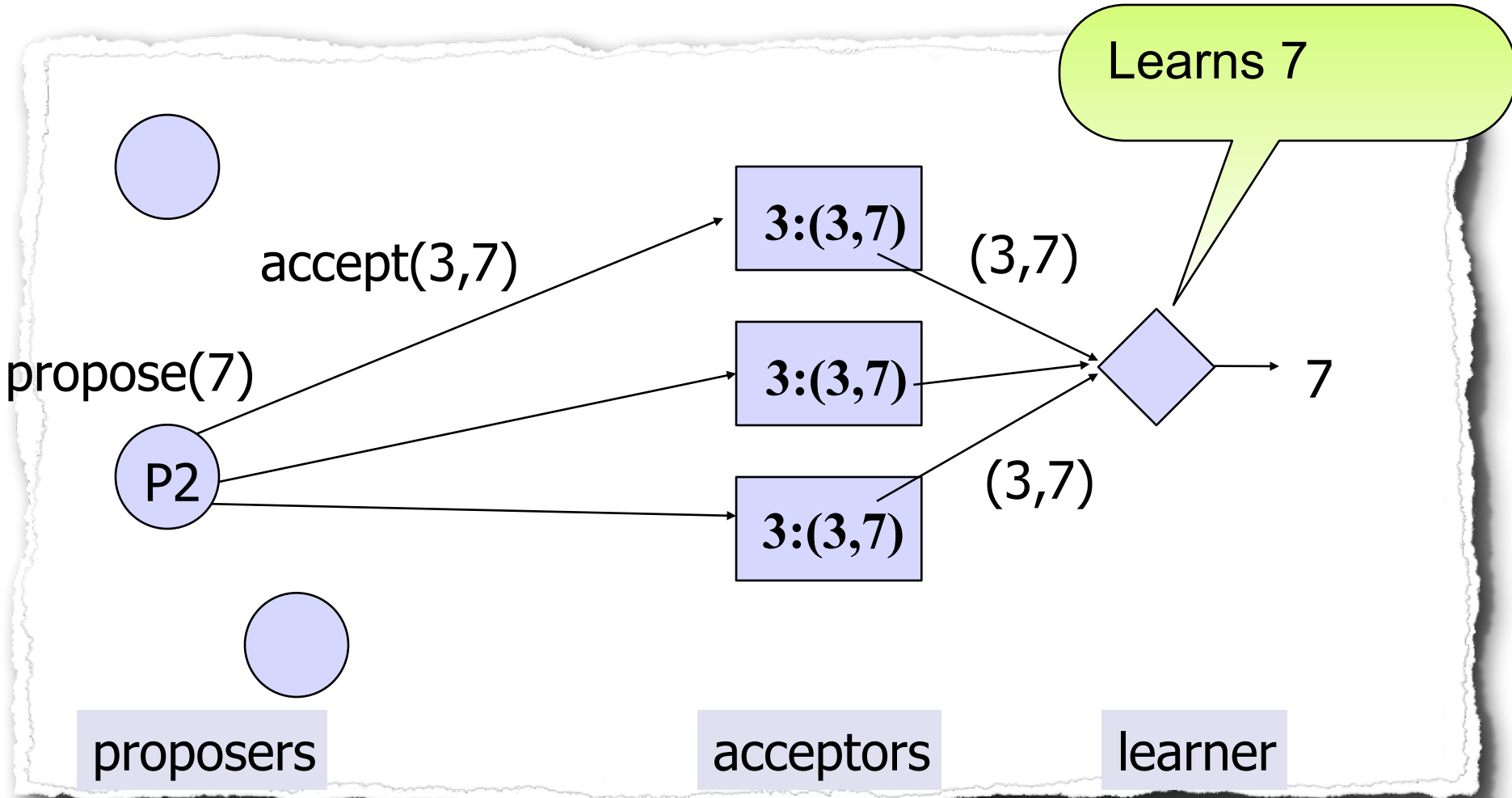
Phase 2:


store value with proposal number
accept(3,7)
propose v=7
P2
3:(3,7)
3:(3,7)
3:(3,7)
7 is the accepted value
proposers
acceptors

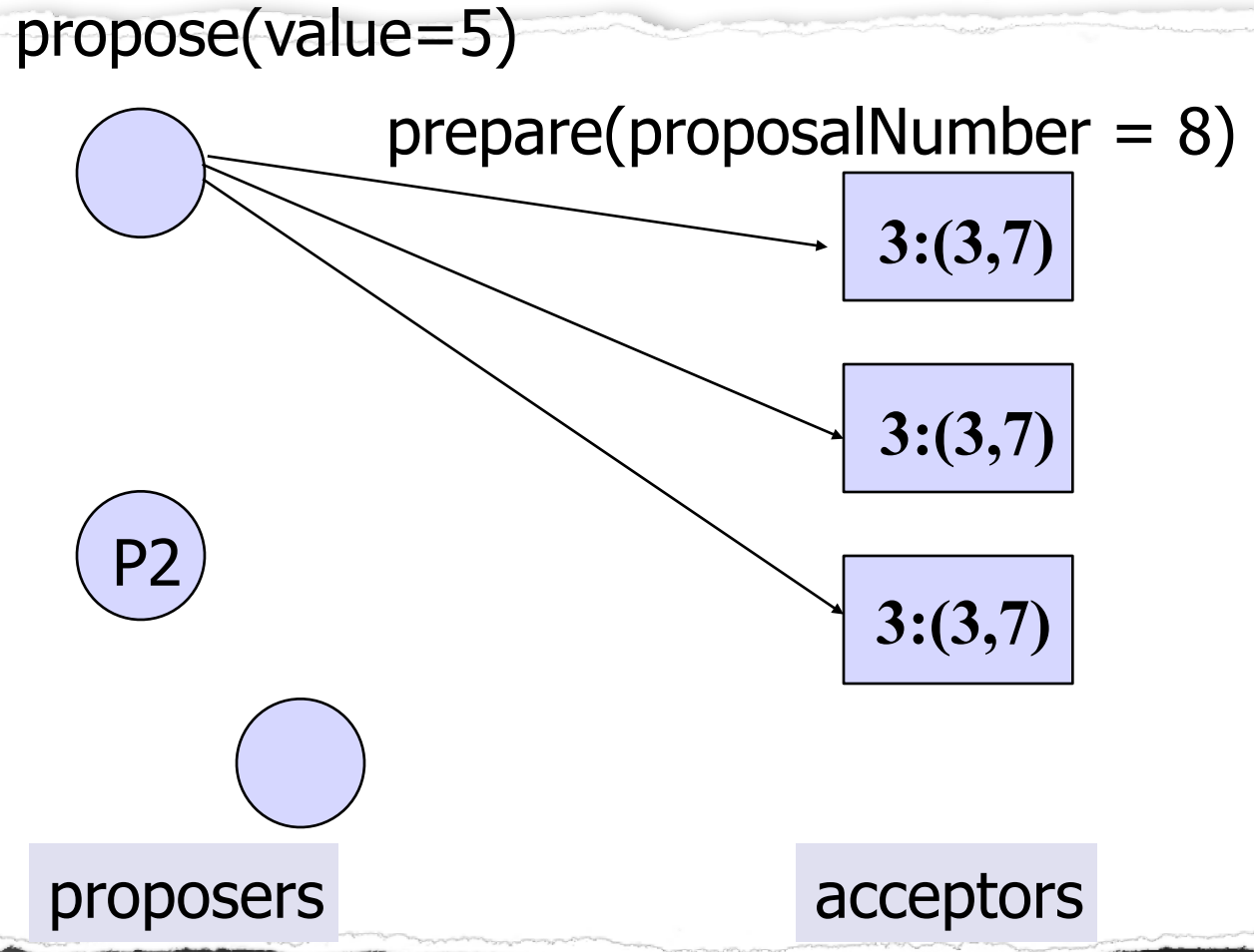
Phase 2:


accept(3,7)
propose(7)
P2
3:(3,7)
3:(3,7)
3:(3,7)
(3,7)
(3,7)
7
proposers
acceptors
learner

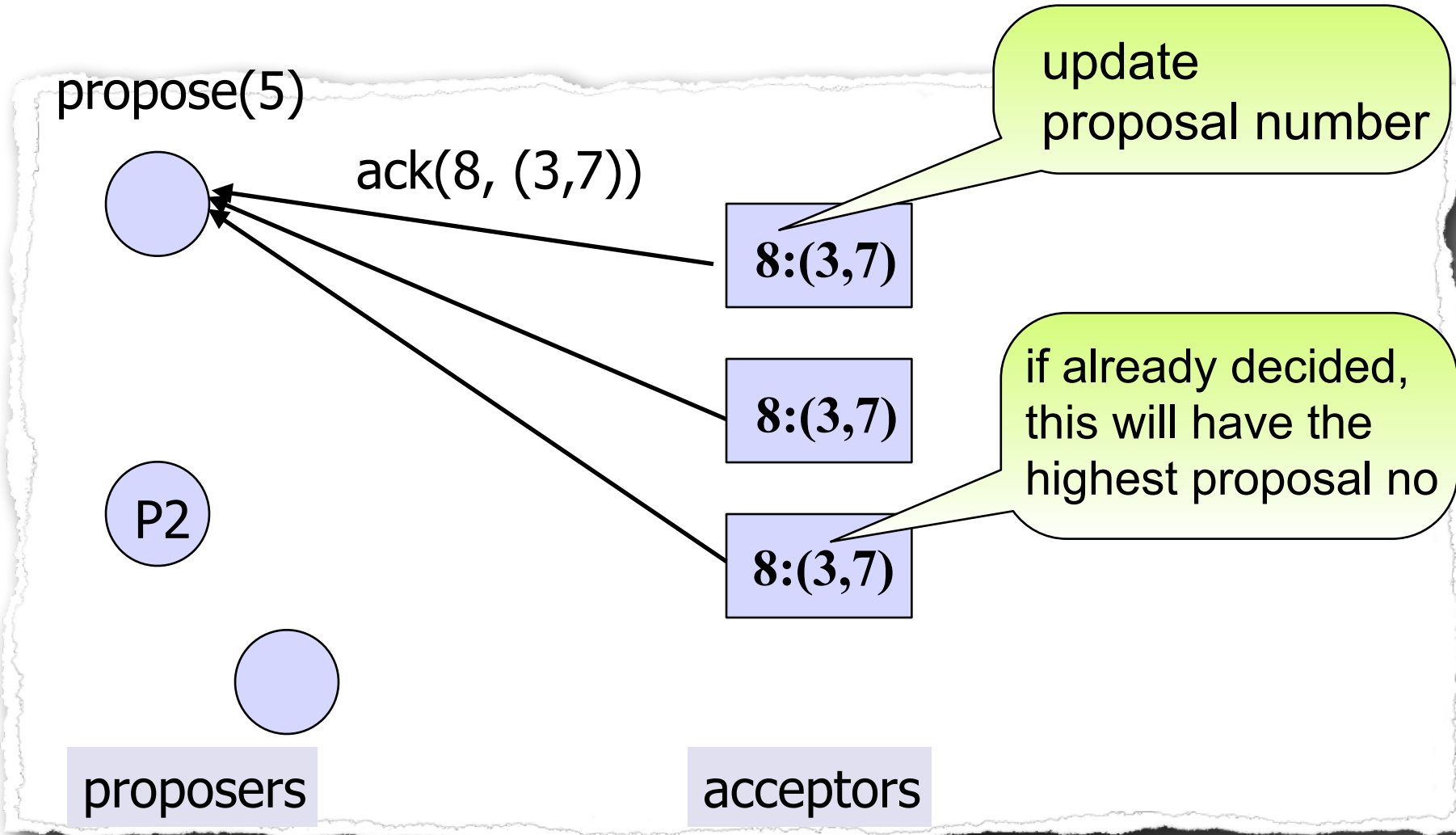
Phase 2:


Learns 7
accept(3,7)
propose(7)
P2
3:(3,7)
3:(3,7)
3:(3,7)
(3,7)
(3,7)
7
proposers
acceptors
learner

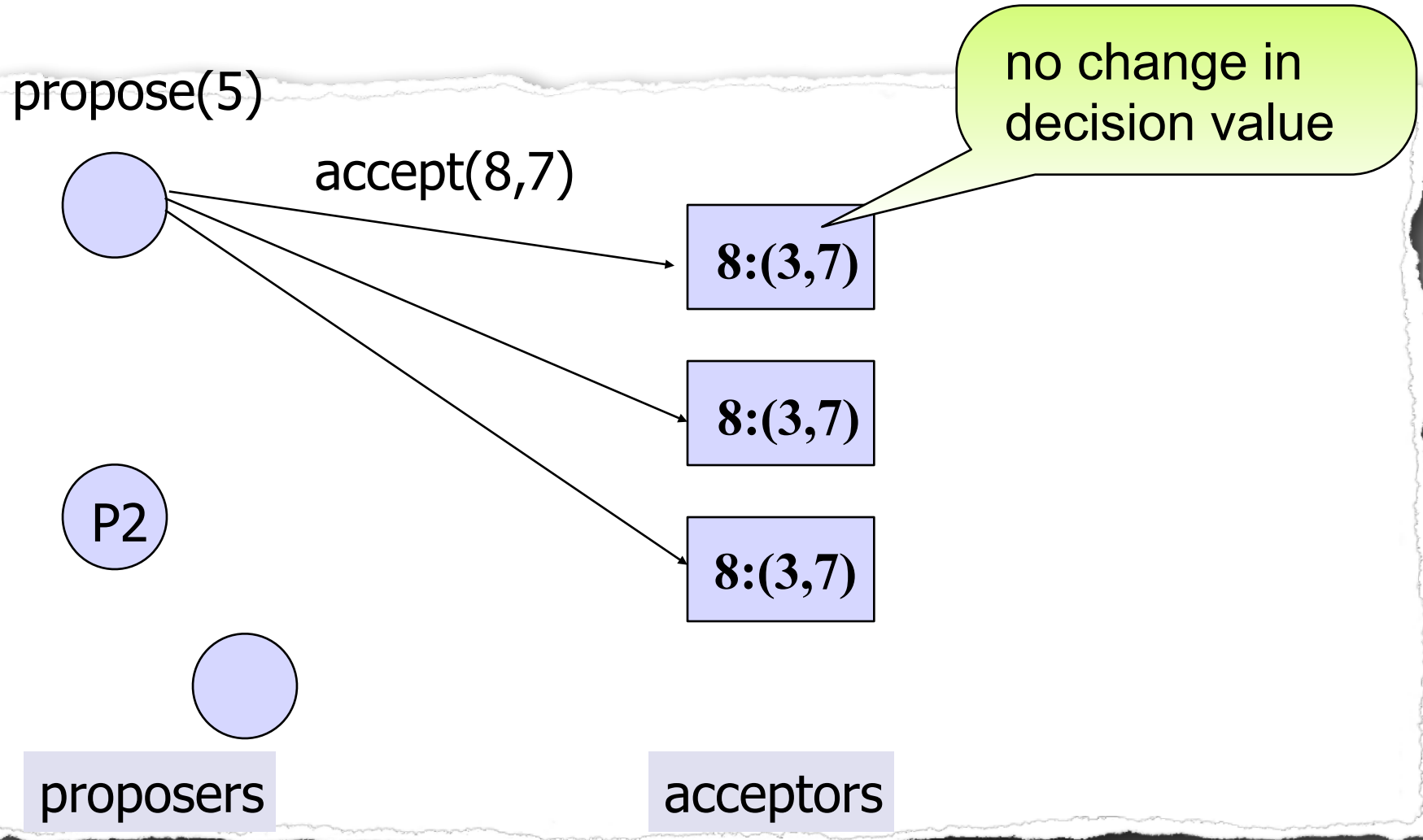
Phase 1:


propose(value=5)
prepare(proposalNumber = 8)
3:(3,7)
3:(3,7)
3:(3,7)
P2
proposers
acceptors

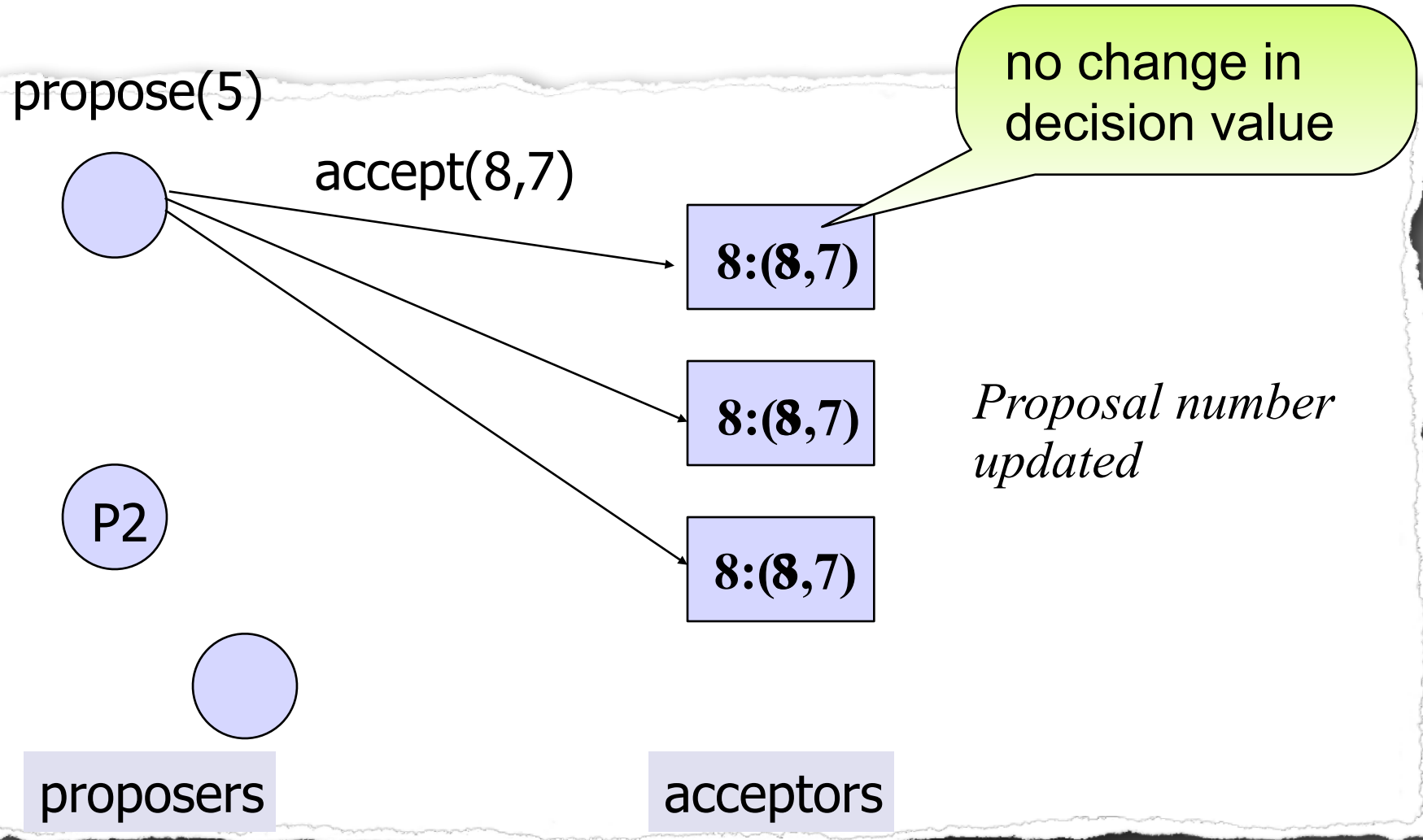
Phase 1:


propose(5)
ack(8, (3,7))
update
proposal number
8:(3,7)
8:(3,7)
8:(3,7)
if already decided,
this will have the
highest proposal no
P2
proposers
acceptors

Phase 1:


propose(5)
accept(8,7)
no change in decision value
8:(3,7)
8:(3,7)
8:(3,7)
P2
proposers
acceptors

Phase 1:

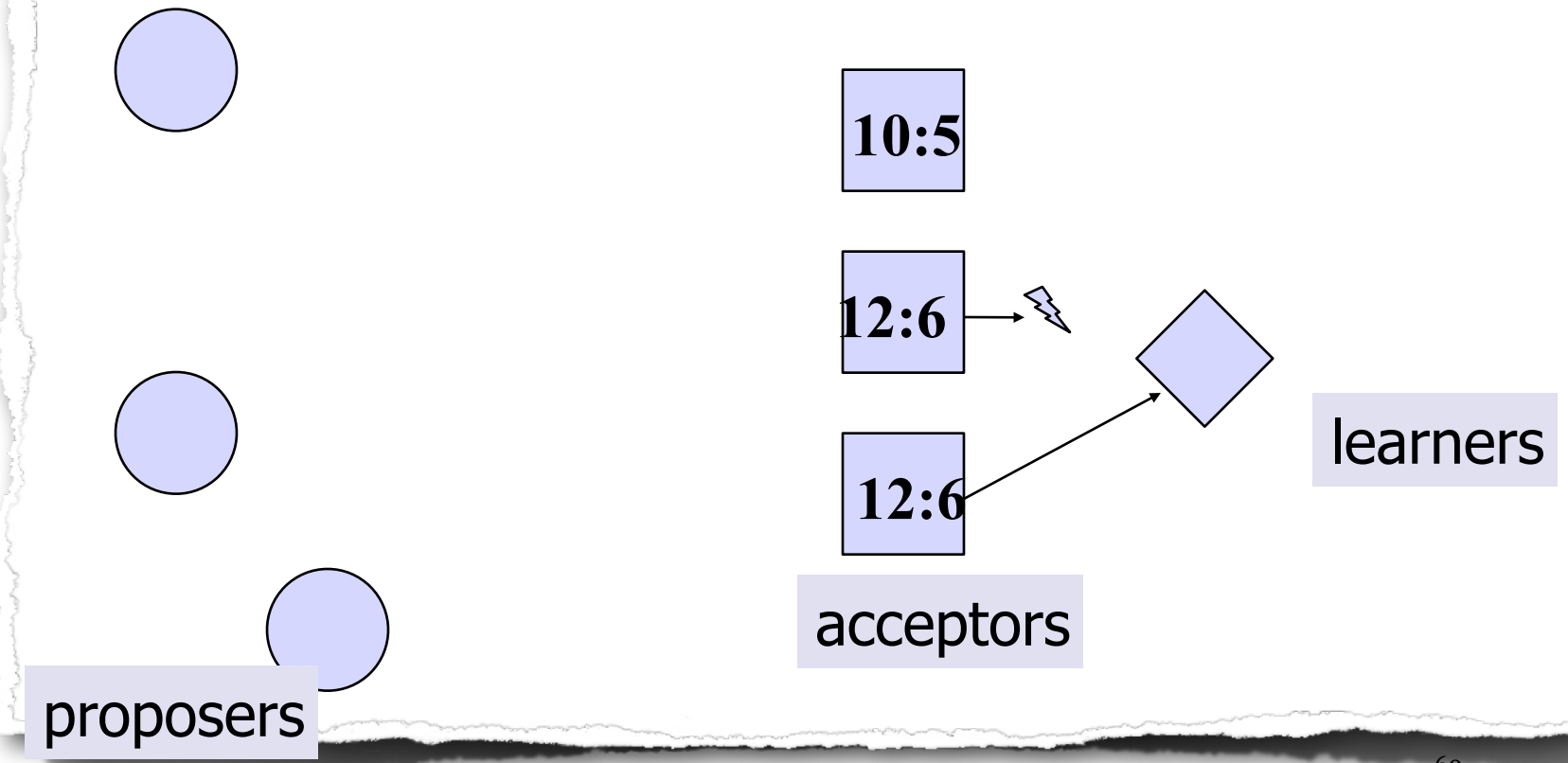

propose(5)
accept(8,7)
no change in decision value
8:(8,7)
8:(8,7)
8:(8,7)
P2
Proposal number updated
proposers
acceptors

Learning a Chosen Value (I)

- A learner must find out that a proposal has been accepted by a majority of acceptors.
  - Each acceptor sends a message to each learner whenever it accepts a proposal.
  - When a learner receives the same message from a majority of acceptors, then it knows that the value in these messages was chosen.
  - Can have a *distinguished learner* (or set of such learners) that take on this role, and can inform other learners when a value has been chosen.

Learning a Chosen Value

- Due to message loss, a learner may not know that a value has been chosen.



Learning a Chosen Value

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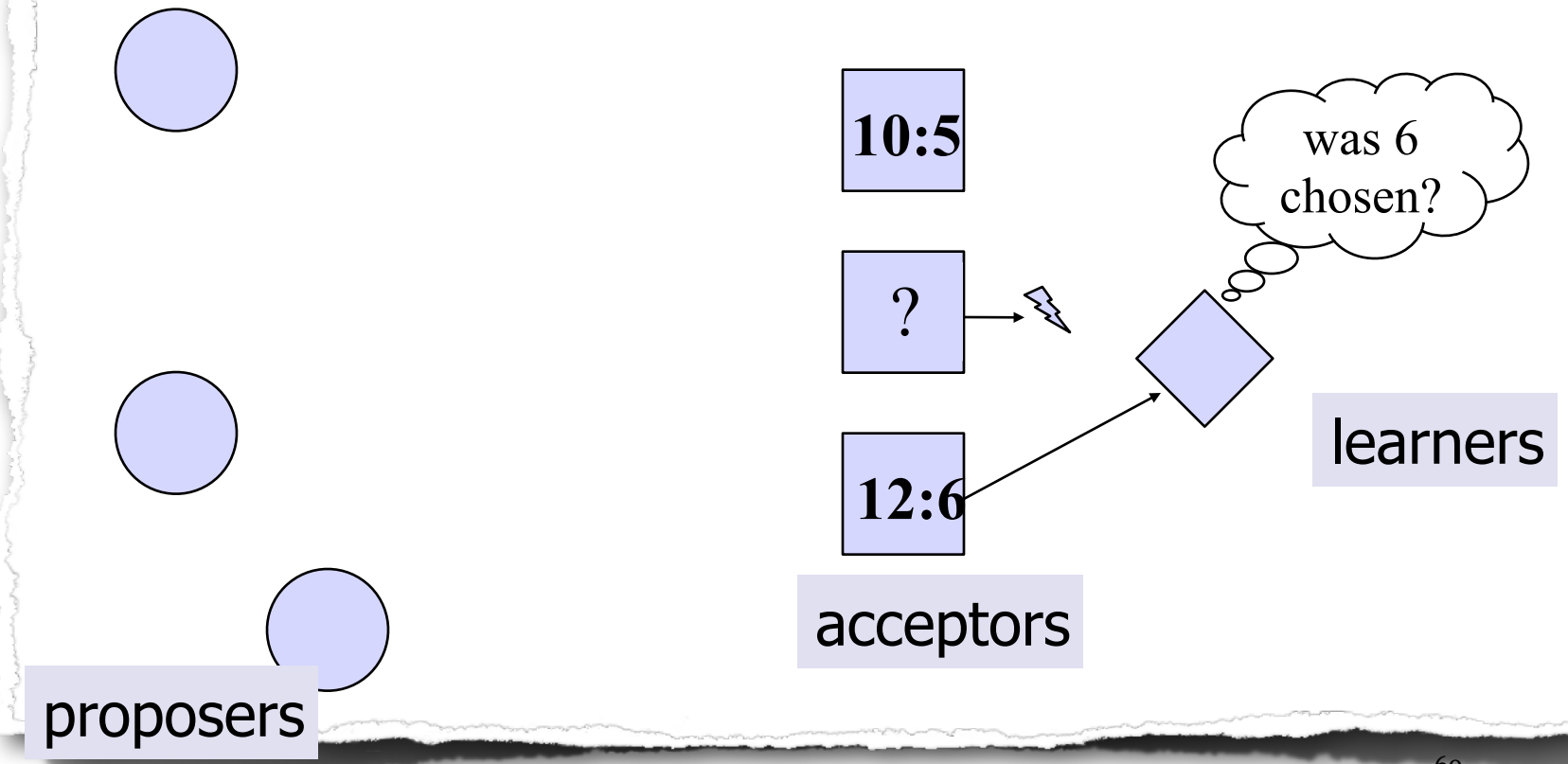

Learning a Chosen Value

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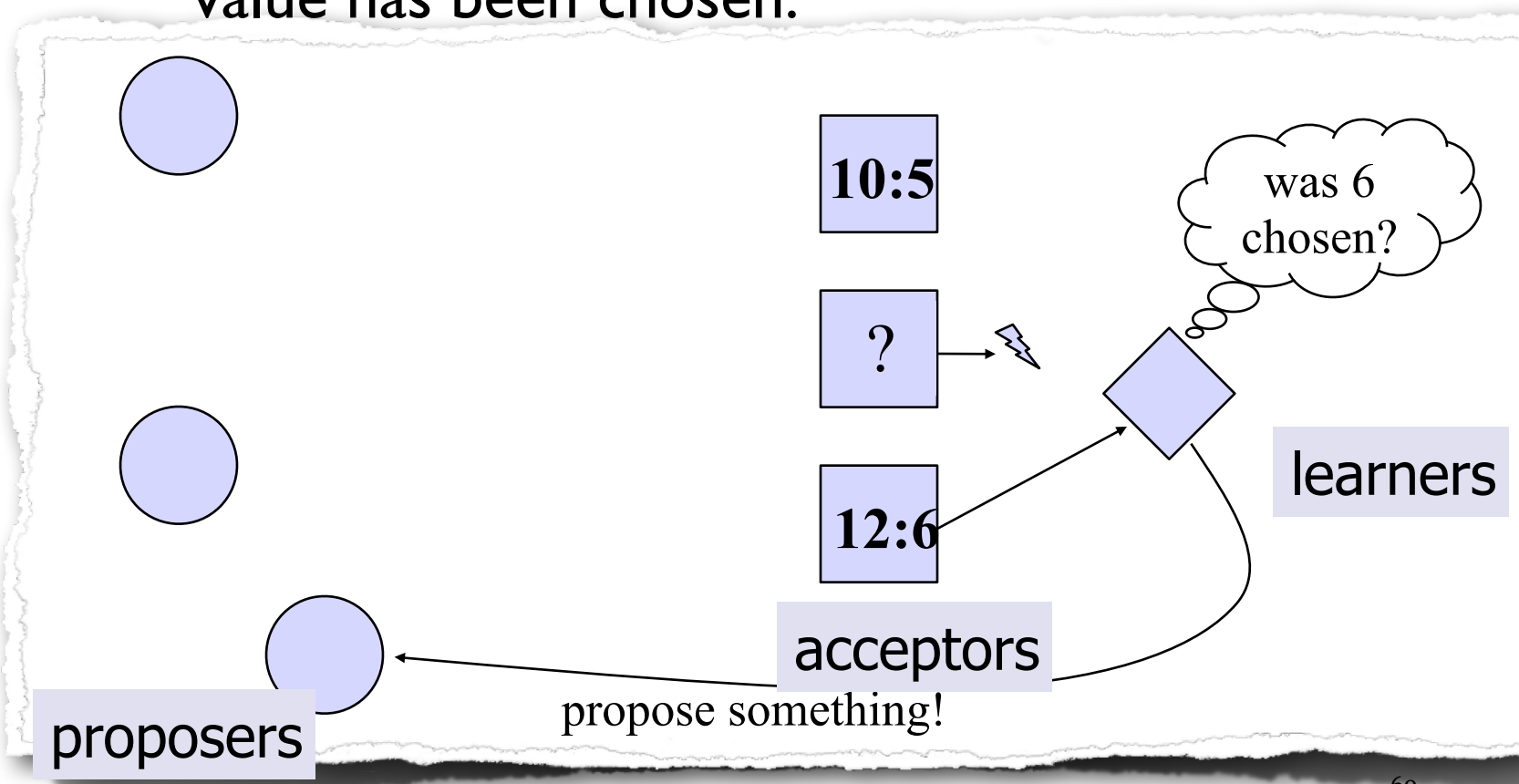


Why does it work?

- safety -

Validity

- *accept* message either
  - contains local value of proposer, or
  - value returned by acceptor (via ack)
- Value returned by acceptor via ack
  - is a value received via accept
- Hence,
  - chosen value was proposed by a proposer

Agreement

- To show:
  - all learners learn the same value!
- Intuition:
  - say, learners L1, L2 learn values v1, v2
  - this means, that a majority of acceptors
    - accepted messages *accept*(n1, v1) & *accept*(n2,v2)
  - we know
    - some proposer P1, sent *accept*(n1,v1)
    - some proposer P2, sent *accept*(n2,v2)
  - if (n1 == n2) then
    - majority of acceptors acked n1(=n2) to P1 **and** P2
    - this means that P1 = P2 because each proposal number is acked at most once and majorities overlap!

- An acceptor accepts an accept message with proposal numbered n
  if and only if it has not responded to a prepare message with a number n' > n
  and has not accepted an accept message with a number n‘ >= n.

Agreement…

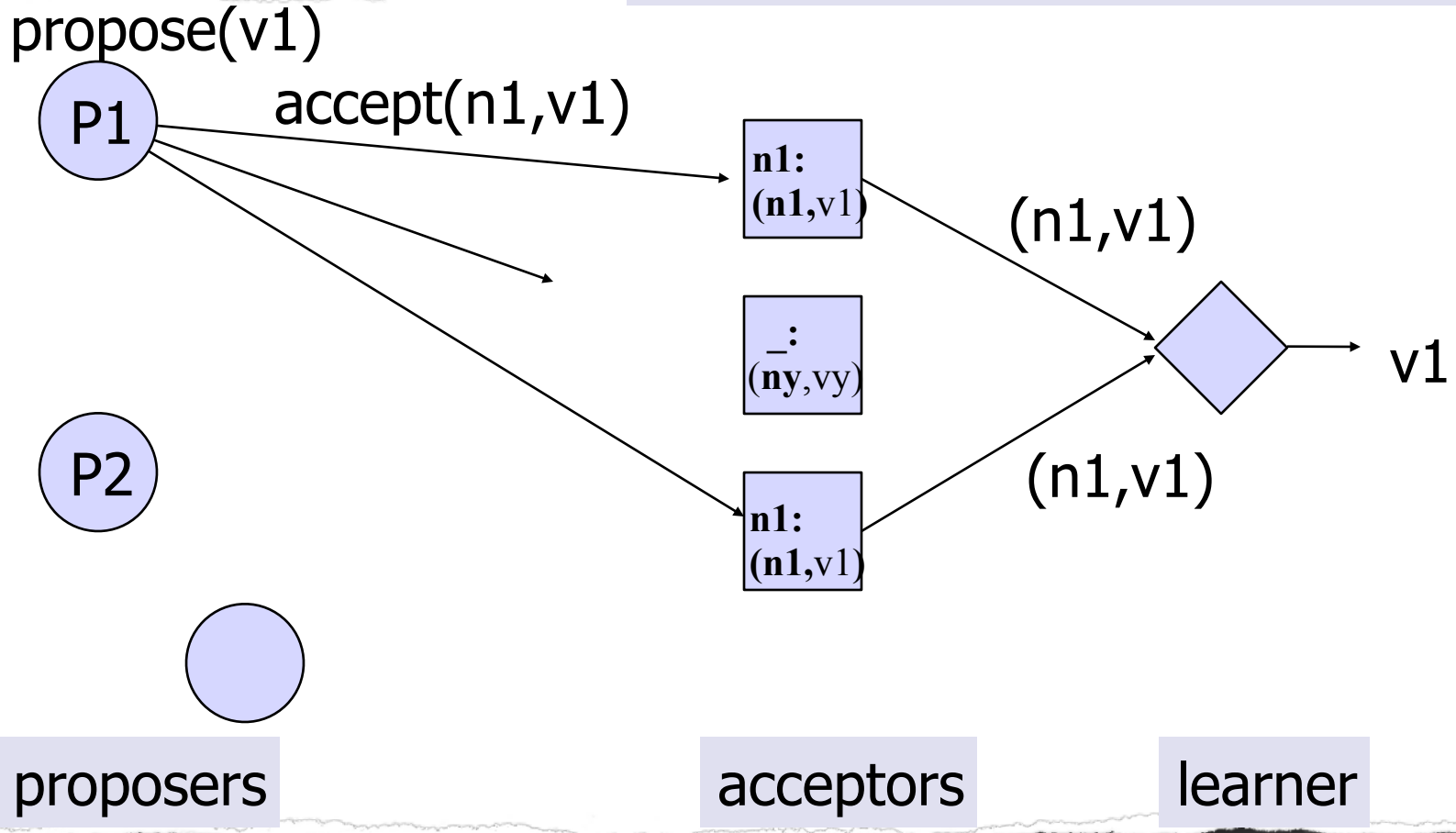
- Ok, assume n1 < n2:
  - proposer P1 got ack(n1, v1') from majority
  - proposer P2 got ack(n2, v2') from majority
- And
  - proposer P1 succeeded to send accept to at least one acceptor A1 such that P2 got
    - ack from A1 after A1 received accept(n1,v1),
    - A1 will reply with ack(n2,(nx,v1)), and
    - nx is highest proposal number in acks
  - Why??

Why…

- **P1's prepare(n1) was ack'ed by majority**
  - this means that no other proposer was able to get an ack(n,_) with n > n1 from maj beforehand
- Hence,
  - the next proposer Px that gets an ack
    - from a majority, will get (n1, v1)
  - will hence send an accept with
    - value v1
  - and all accepts will contain v1 from now on

In pictures

accept(n1,v1) accepted by majority
=>
ny *< n1 because otherwise for majority of acceptors would N > n1*



In pictures

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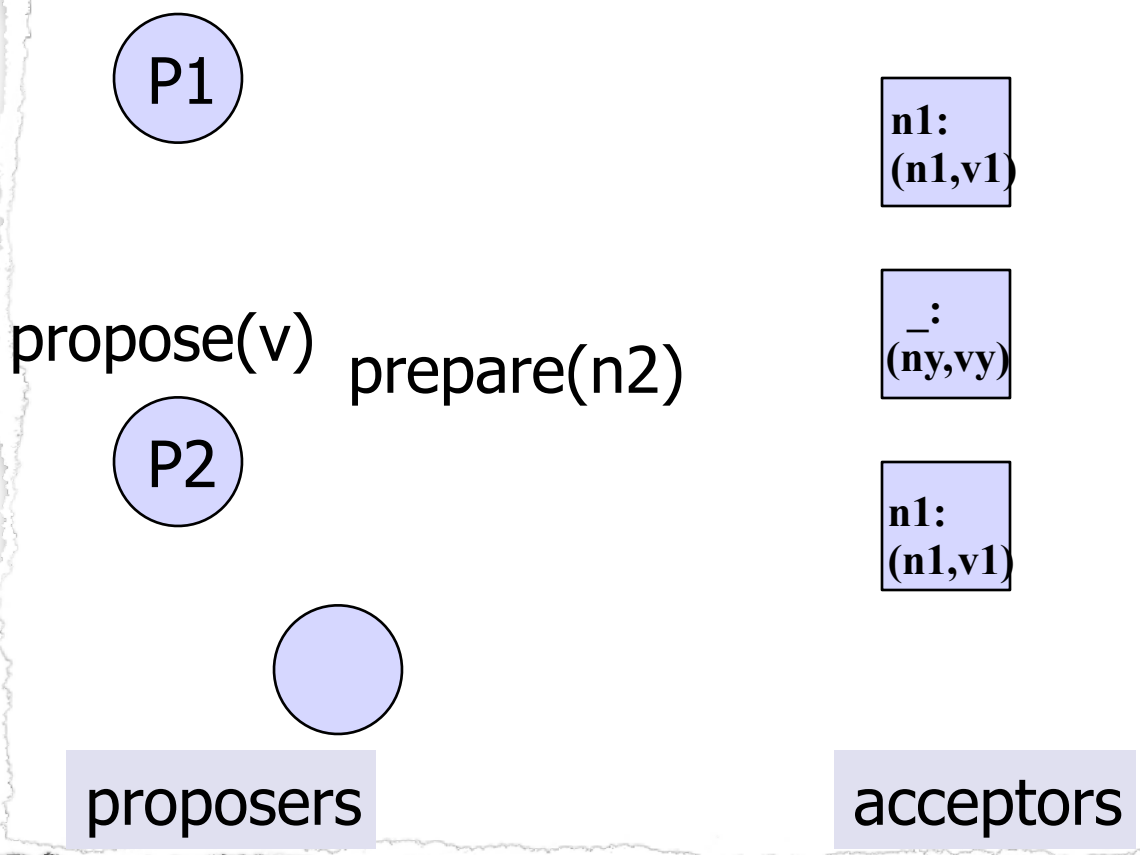

In pictures

accept(n1,v1) accepted by majority
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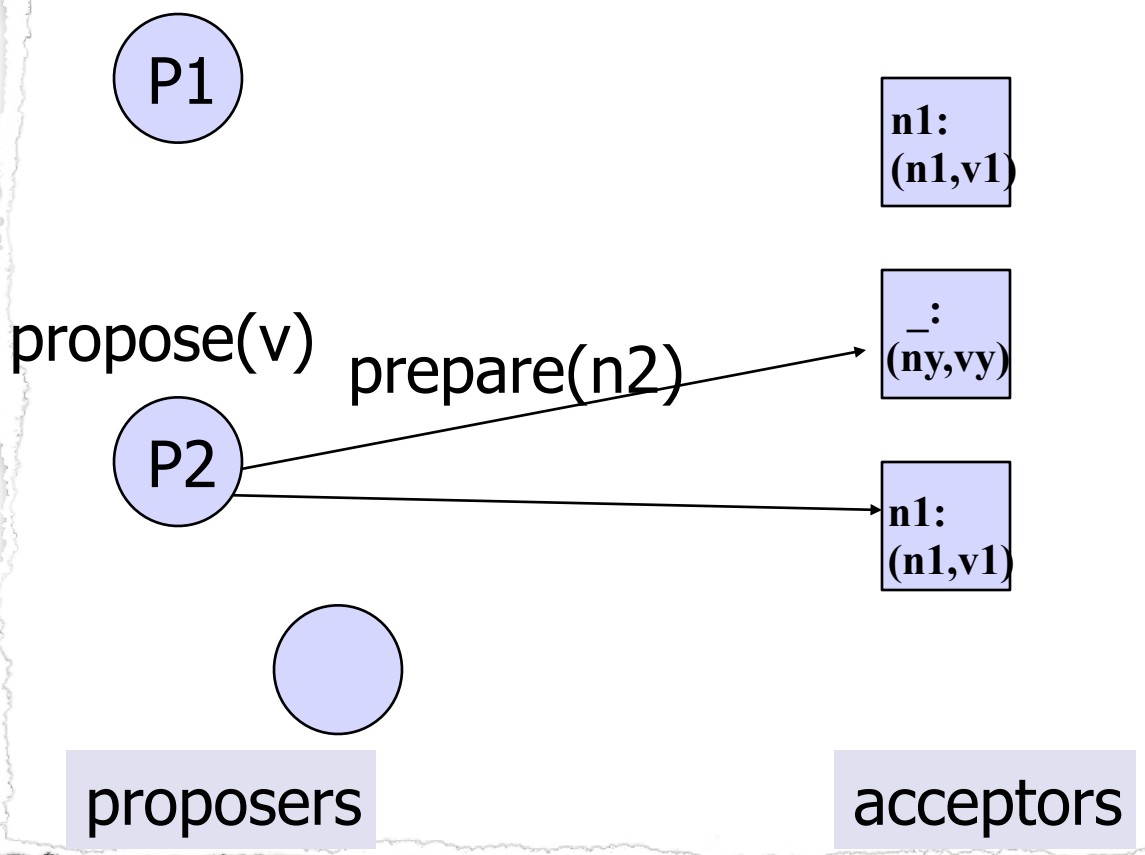
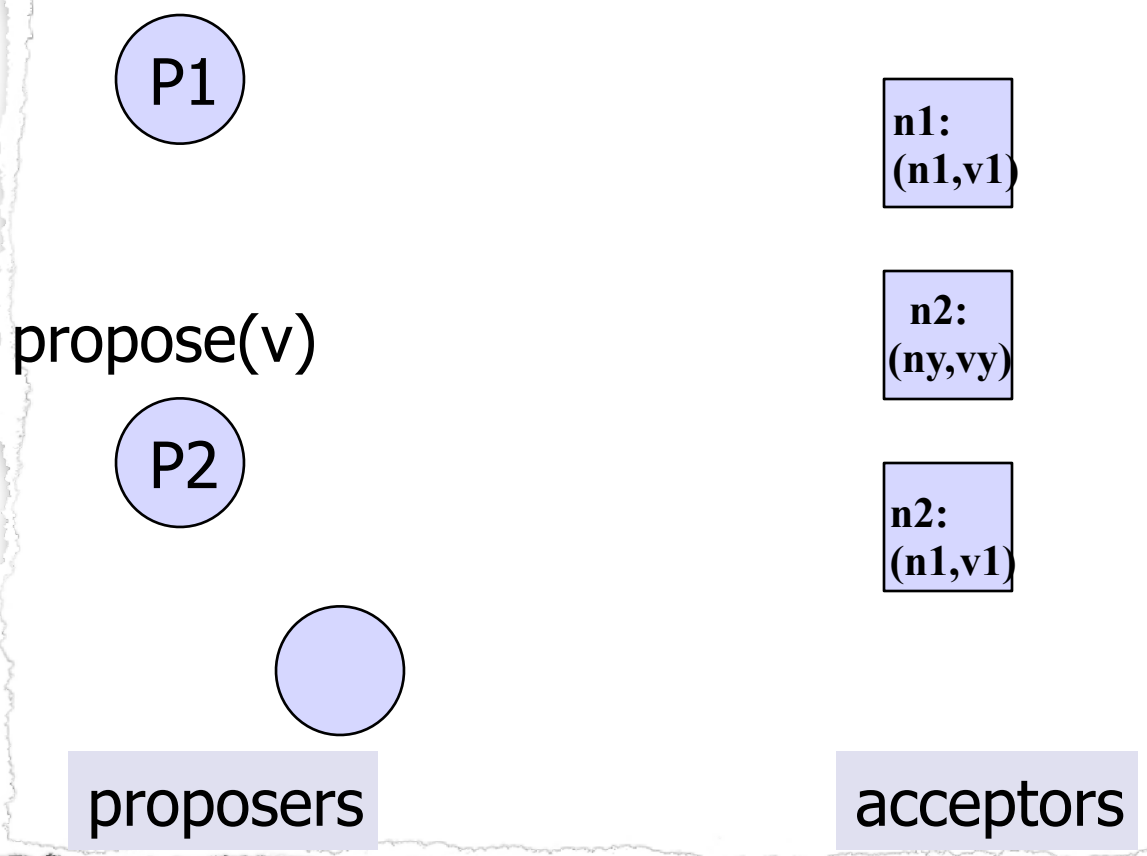


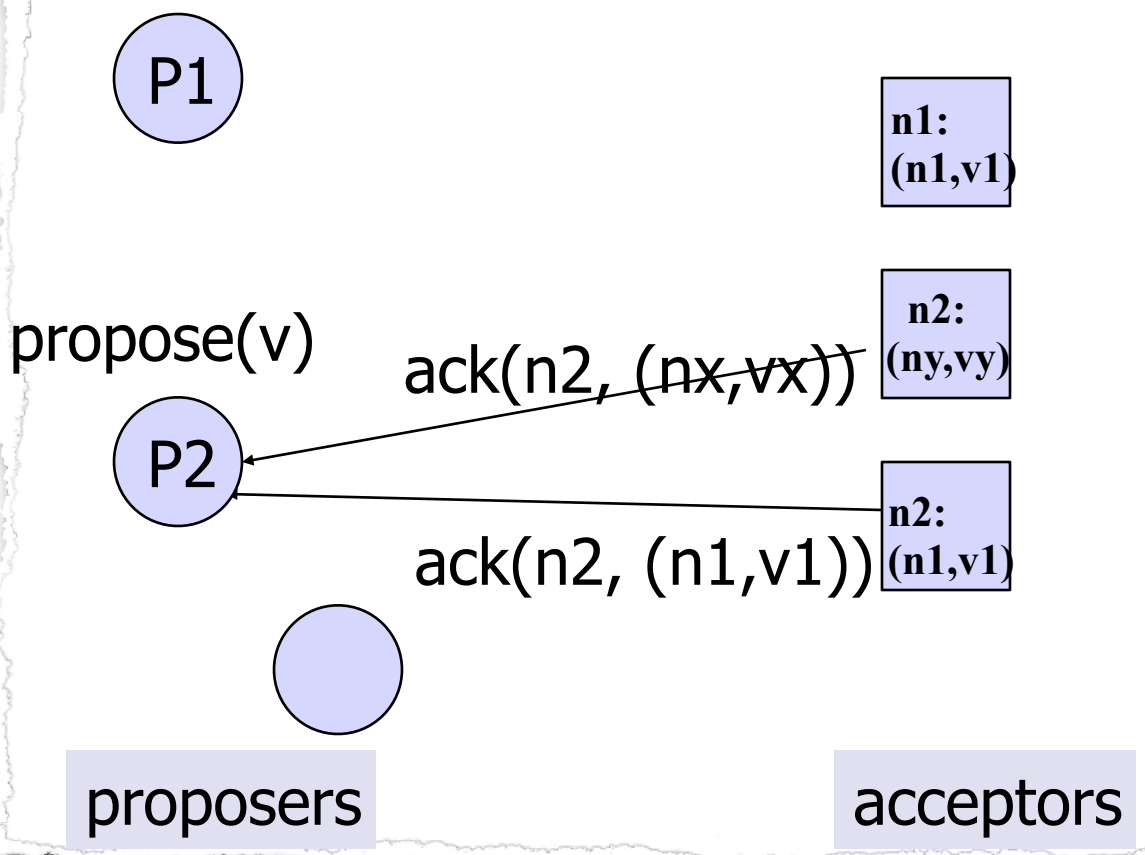
In pictures

accept(n1,v1) accepted by majority
*=> **ny < n1***



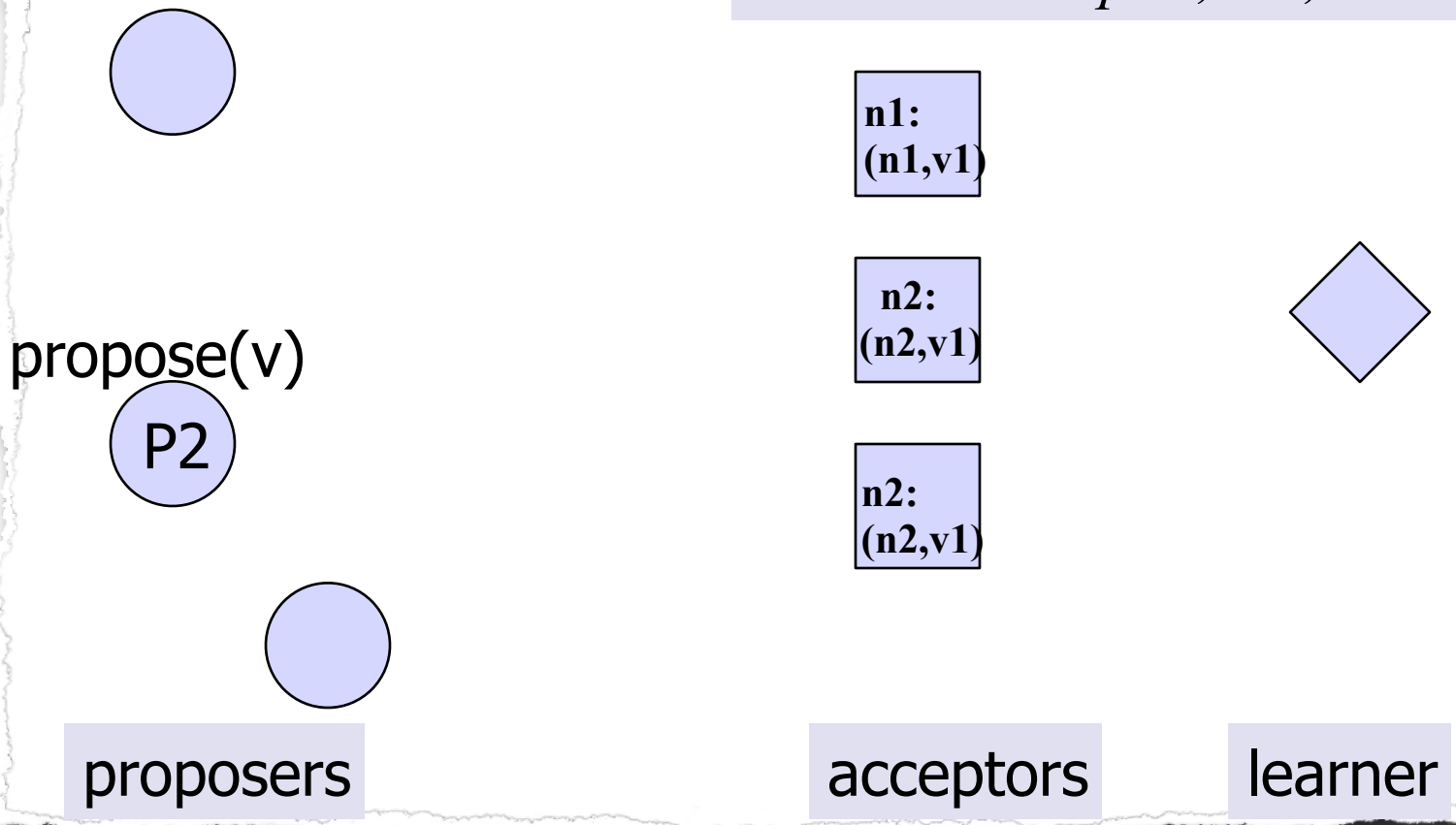
In pictures

accept(n1,v1) accepted by majority
=> ***ny < n1***



In pictures

accept(n2,v2) accepted by majority && n2 > n1
=>
P2 received ack(n2, (n1,v1)) from at least one acceptor, i.e., v2 = v1.



In pictures

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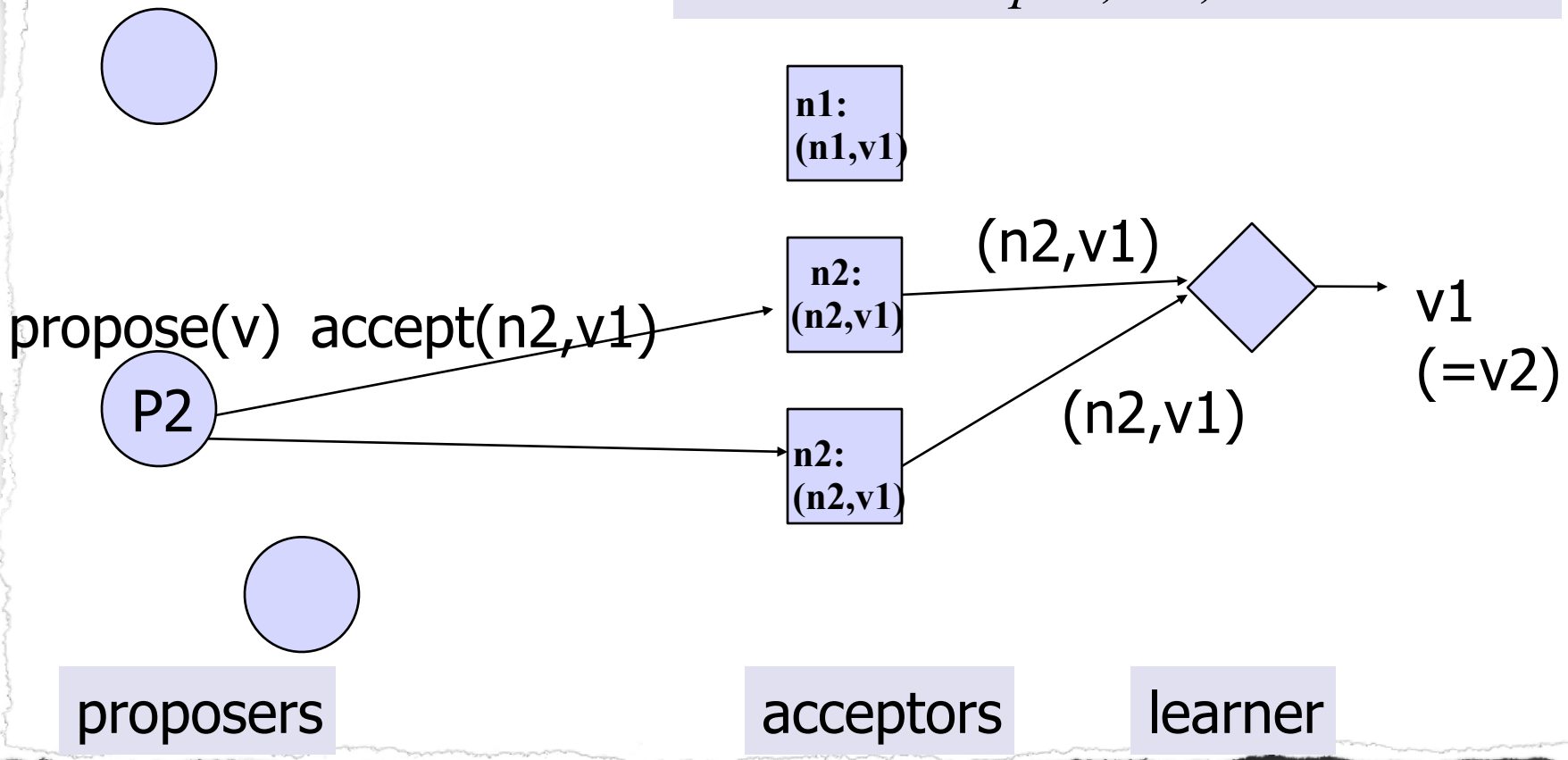
P2 received ack(n2, (n1,v1)) from at least one acceptor, i.e., v2 = v1.



Termination?

Races

Races between proposers:

- processes race each other by increasing proposal numbers

Approach:

- assign a primary proposer
- elect a new one if old primary is slow / crashed

Conclusion

Consensus is a fundamental problem

- in distributed systems

Impossible to solve in asynchronous distributed systems

BUT

- practical solutions like PAXOS exist