1. **(d)**

Let points $P(x,y)$, $A(a+b, b-a)$, $B(a-b, a+b)$.

According to Question, $PA = PB$, *i.e.*, $PA^2 = PB^2$

$\Rightarrow (a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$

$\Rightarrow (a+b)^2 + x^2 - 2x(a+b) + (b-a)^2 + y^2 - 2y(b-a)$

$= (a-b)^2 + x^2 - 2x(a-b) + (a+b)^2 + y^2 - 2y(a+b)$

$\Rightarrow 2x(a-b-a-b) = 2y(b-a-a-b)$

$\Rightarrow -4bx = -4ay \Rightarrow bx - ay = 0$

2. **(a)**

Let A $(4,0); B(-1,-1); C(3,5)$ then

$AB = \sqrt{26},\ AC = \sqrt{26},\ BC = \sqrt{52}$; *i.e.* $AB = AC$

So triangle is isosceles and also $(BC)^2 = (AB)^2 + (AC)^2$.

Hence ΔABC is right angled isosceles triangle

3. **(b)**

Let the vertices of the triangle be (0, 0), A (a, 0) and B (0, b)

Let O′, A′ and B′ be the reflection of O, A and B in the opposite sides of the triangle

$\Rightarrow$ A′ ≡ (-a, 0) and B′ (0, -b)

$\Rightarrow$ area of Δ O′AB′ = $\frac{1}{2} . 3k\sqrt{a^2+b^2}$

$\Rightarrow$ area of ΔOAB = $\frac{1}{2}\ k\sqrt{a^2+b^2}$ $\Rightarrow$ ratio = 3: 1

4. **(a)**

$\frac{\text{Area of }\Delta DBC}{\text{Area of }\Delta ABC} = \frac{1}{2}$

$= \frac{\frac{1}{2}[x(5+2)-3(-2-3x)+4(3x-5)]}{\frac{1}{2}[6(5+2)-3(-2+3)+4(3-5)]} = \frac{1}{2}$

$\Rightarrow \frac{7x+6+9x+12x-20}{7x-3-8} = \frac{1}{2}$

$\Rightarrow x = \frac{11}{8}$

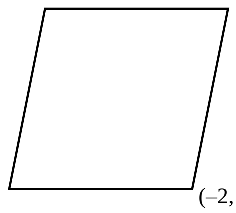
5. **(c)**

D = A + C - B

D = (2 + 5 - 8, - 2 + 7 - 4)

D = (-1, 1)

6. **(b)**



$\alpha - 2 = 3 - 6 \Rightarrow \alpha = -1$

$\beta + 1 = -4 + 5 \Rightarrow \beta = 0$

7. **(a)**

Let vertices are

A (2, 1),B(3, -2), C(α, α + 3)

Area = 5

$\Rightarrow \frac{1}{2}\begin{vmatrix} \alpha & \alpha+3 & 1 \\ 2 & 1 & 1 \\ 3 & -2 & 1 \end{vmatrix} = \pm 5$

$\Rightarrow 3\alpha + \alpha + 3 - 7 = \pm 10$

$\Rightarrow 4\alpha - 4 = \pm 10$

$\Rightarrow \alpha = \frac{7}{2}$, $\alpha = \frac{-3}{2}$

$\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

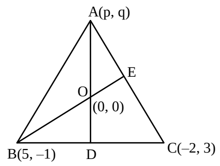
8. **(d)**

AB = BC = CA = $\sqrt{3}$

triangle is equilateral triangle. $\therefore$ Distance = 0

9. **(a)**

Slope of BC = $-\frac{4}{7}$;Slope of AD = $\frac{q}{p}$; $m_1m_2 = -1$



$\left(\frac{-4}{7}\right)\frac{q}{p} = -1 \Rightarrow 7p = 4q$ … (1)

slope of CA = $\frac{3}{p+2}$

slope of BE

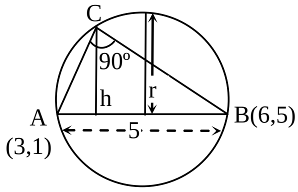
= slope of BO = $-\frac{1}{5}$

But $m_1m_2 = -1$; $\left(\frac{q-3}{p+2}\right)\left(-\frac{1}{5}\right) = -1$

$\Rightarrow 5p - q + 13 = 0$ …(2)

solving (1) and (2), we get p = -4, q = -7

10. **(a)**



AB = $\sqrt{(3-6)^2+(1-5)^2}$ = $\sqrt{9+16}$ = 5

2r = 5 ⇒ r = 5/2 = 2.5 ⇒ ΔABC = 7

$\frac{1}{2}\times AB\times h = 7 \Rightarrow h = \frac{2\times 7}{5} \Rightarrow h = 2.8$

h > r ; which is not possible hence no point

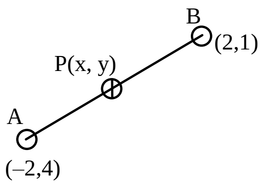
11. (a)

$\Delta = \frac{1}{2}\begin{Bmatrix} 4 & 3 & -5 & -3 & -3 & 4 \\ 1 & 6 & 1 & -3 & 0 & 1 \end{Bmatrix}$

$\Delta = \frac{1}{2}$ [24 - 3 + 3 + 30 + 15 + 3 - 9 - 3]

$\Delta = \frac{1}{2}$ | 60 | = 30 unit^2

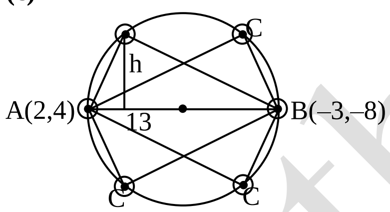
12. (c)



(PA + PB) = AB

(AB) = $\sqrt{16+9}=5$ hence locus is the line segment AB

13. (c)

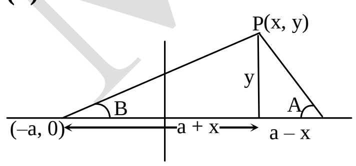


AB = $\sqrt{(5)^2+(4+8)^2}$ = 13

2r = 13 ⇒ r = 13/2

ΔABC = $\frac{1}{2}\times 13\times h = \frac{41}{2} \Rightarrow h = \frac{41}{13} = 3\frac{2}{13}$

14. (b)



$\tan A = \frac{y}{a-x}$; $\tan B = \frac{y}{a+x}$

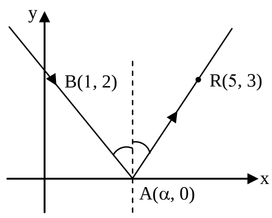
tan A + tan B = λ

$\frac{y}{a-x}+\frac{y}{a+x} = \lambda \Rightarrow \frac{y.2a}{a^2-x^2} = \lambda$

$a^2 - x^2 = \frac{2a}{\lambda}y \Rightarrow x^2 = a^2 - \frac{2a}{\lambda}y$ Parabola.

15. (a)

Let, the coordinates of point A is (α, 0)



Now $-m_{AB} = m_{AR}$

If AR makes an angle θ with +ve x-axis, then AB makes (π - θ), therefore

$-m_{AB} = m_{AR}$

$-\left(\frac{0-2}{\alpha-1}\right) = \left(\frac{0-3}{\alpha-5}\right)$

⇒ 2(α - 5) = -3(α -1)

$\Rightarrow \alpha = \frac{13}{5}$ ∴ A is $\left(\frac{13}{5},0\right)$

16. (c)

If (x_1, y_1) and (x_2,y_2) are two vertices of an equilateral triangle have integral co-ordinate then third vertex will be

$$\left(\frac{(x_1+x_2)\pm(y_1-y_2)\sqrt{3}}{2},\frac{(y_1+y_2)\mp\sqrt{3}(x_1-x_2)}{2}\right)$$

Irrational co-ordinates.

17. (a)

Area of Δ = $\frac{1}{2}\left|a(c-b)+b(a-c)+c(b-a)\right|$

= $\frac{1}{2}\left|ac-ab+ba-bc+cb-ca\right|$ = 0

18. (d)

A $\left(1,\sqrt{3}\right)$;B (0, 0) ; C(2, 0)

AB = 2 ; BC = 2; CA = 2

∴ABC is equilateral triangle

∴ incentre and centroid coincide

∴ incentre ≡ $\left(\frac{1+0+2}{3},\frac{\sqrt{3}+0+0}{3}\right) \equiv \left(1,\frac{1}{\sqrt{3}}\right)$

19. (d)

$\left(-1,-\frac{\pi}{3}\right)$

x = r cos θ and y = r sin θ

x = (-1) cos $\left(-\frac{\pi}{3}\right)$

$\Rightarrow y = (-1)\sin\left(-\frac{\pi}{3}\right)$

$x = (-1)\left(\frac{1}{2}\right)$ $\qquad \Rightarrow y = (-1)\left(-\frac{\sqrt{3}}{2}\right)$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

20. (b)

Ratio = $-\left(\frac{-1+1-4}{5+7-4}\right) = \frac{4}{8} = \frac{1}{2}$

1. (a)

Equation of a line, perpendicular to $\sqrt{3}\sin\theta + 2\cos\theta = \frac{4}{r}$ is $\sqrt{3}\sin\left(\frac{\pi}{2}+\theta\right) + 2\cos\left(\frac{\pi}{2}+\theta\right) = \frac{k}{r}$

It is passing through $\left(-1, \frac{\pi}{2}\right)$. Hence,

$\sqrt{3}\sin\pi + 2\cos\pi = k/-1 \Rightarrow k = 2$

$\therefore \sqrt{3}\cos\theta - 2\sin\theta = \frac{2}{r} \Rightarrow 2 = \sqrt{3}\,r\cos\theta - 2r\sin\theta$.

2. (c)

Since the lines are concurrent -2 = m(a + m)

$\Rightarrow m^2 + am + 2 = 0$.

Since m is real, $a^2 \geq 8$, $|a| \geq 2\sqrt{2}$.

Hence the least value of $|a|$ is $2\sqrt{2}$

3. (d)

Slope of line passes through (4, 3) and (2, k)

$m_1 = \frac{k-3}{2-4} = \frac{k-3}{-2}$

slope of line y = 2x + 3 is $m_2 = 2$

lines are perpendicular

$\therefore m_1 m_2 = -1$

$\Rightarrow \left(\frac{k-3}{-2}\right)(2) = -1 \Rightarrow k = 4$

4. (b)

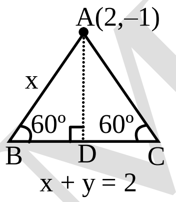
Let side AB is x

$\therefore$ length AD = $\frac{|2-1-2|}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

In ΔABD, $\sin 60^0 = \frac{1}{\sqrt{2}x}$

$\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}x} \Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}}$

area of equilateral $\Delta = \frac{\sqrt{3}}{4}\,\frac{2}{6} = \frac{\sqrt{3}}{6}$



5. (a)

The point $(2\sqrt{3}, -1)$ lie on line $y = \sqrt{3}x - 7$

$\therefore$ locus of point is straight line perpendicular to given line passing through given point

i.e. $x + \frac{y}{\sqrt{3}} = 1$

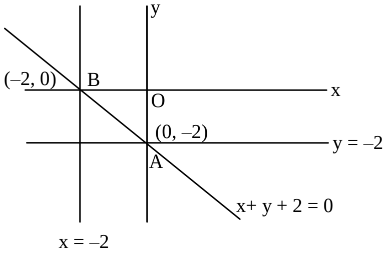
$\therefore$ Area enclosed = $\frac{\sqrt{3}\times 1}{2} = \frac{\sqrt{3}}{2}$

6. (d)

$x(y + 2) + 2(y + 2) = 0 \Rightarrow x = -2, y = -2$

Circumcentre of ΔOAB is mid point of AB

$\equiv (-1, -1)$



7. (a)

2 3

(3, –1) P(h, k) (8, 9)

$h = 5, k = 3, m = 1$

8. (c)

$y - mx = \pm a\sqrt{1+m^2}$; $y - nx = \pm a\sqrt{1+n^2}$

These are set of parallel line and distance betweenparallel lines are equal. So figure is rhombus.

9. (d)

lines given first equation are $2x - 3y = 0$, $3x + 4y = 0$ lines given second equation are $5x - 2y = 0$, $3x + 4y = 0$ hence required line is parallel to $3x + 4y = 0$

10. (a)

Since origin and the point $(a^2, a + 1)$ lie on the same side of both the lines, therefore we have

$3a^2 - (a + 1) + 1 > 0$

i.e. $a(3a - 1) > 0$

gives $a \in (-\infty , 0) \cup \left(\frac{1}{3}, \infty\right)$

and $a^2 + 2(a + 1) - 5 < 0$

i.e. $a^2 + 2a - 3 < 0$

i.e. $(a - 1)(a + 3) < 0$

gives $a \in (-3, 1)$

Intersection of the above inequalities gives

$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$.

11. (c)

We have $3x + 4y = 9$

i.e $x = \frac{9-4y}{3} = 3 - \frac{4y}{3}$

Thus, points lying on the above line and having integral coordinates are given by

$P \equiv (3 - 4k, 3k)$ $k \in I$

If P also lies on $y + mx - 1 = 0$, then we have

$3k + m(3 - 4k) - 1 = 0$

gives $m = \frac{3k-1}{4k-3}$, $k \in I$

For m to be an integer, we have

$|4k - 3| \le |3k - 1|$

i.e. $-(3k - 1) \le (4k - 3) \le (3k - 1)$

i.e. $\frac{4}{7} \le k \le 2$

There are only two integral values of k lying in the above integral, viz. k = 1, 2, Hence, there are only two integral values of m.

12. (a)

We $x^2 \le r^2 [\because r^2 = x^2 + y^2]$

i.e. $2x^2 \le 2r^2$

i.e. $r^2 + r^4 \le 2r^2 [\because 2x^2 = r^2 + r^4 \text{ given}]$

i.e. $r^4 - r^2 \le 0$

i.e. $r^2 (r^2 - 1) \le 0$

i.e. $0 \le r^2 \le 1$

i.e. $0 \le r \le 1$ [$\because$ r is a +ve quantity]

Also, we can see that the given curve is symmetrical about the X-axis as well as the Y-axis (replacing x by -x or y by -y does not change the equation).

Thus, if (h, k) is a point on the curve then (-h, k), (h, - k) and (-h, -k) are also points on the curve, all of which have the same distance from the origin.

However, there is only one point (0, 0) whose r = 0 and two points (1, 0) and (-1, 0) whose r = 1.

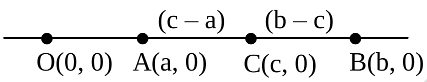
Hence, there are exactly four points on the given curve for every 0 < r < 1.0

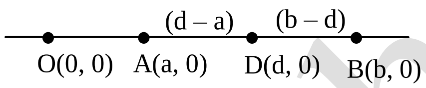
13. (b)

We have

$\frac{CA}{CB} + \frac{DA}{DB} = 0$ i.e. $\left|\frac{c-a}{b-c}\right| + \left|\frac{d-a}{b-d}\right| = 0$

i.e. $\frac{c-a}{b-c} \pm \frac{d-a}{b-d} = 0$





Taking +ve sign, we have

(a + b) (c + d) = 2(ab + cd) Taking -ve sign, we have (a - b) (c - d) = 0

Which is not possible if the four points are distinct.

14. (d)

If possible equation of line is y + 5 = m(x - 4)

$\Rightarrow$ y - mx + 4m + 5 = 0

then $\left|\frac{3+2m+4m+5}{\sqrt{1+m^2}}\right| = 12$

$\Rightarrow (8 + 6m)^2 = 144 (1 + m^2)$

$\Rightarrow 27 m^2 - 24 m + 20 = 0$

Here discriminate < 0

there for no such line possible for real value of m.

15. (a)

The equation of any straight line passing through (3, - 2) is y + 2 = m (x - 3)
...(i)

The slope of the given line is $-\sqrt{3}$

So, $\tan 60^0 = \left|\dfrac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}\right| \Rightarrow m = 0$ or $\sqrt{3}$

Put in equation (i) we get,

$y+2=0,\ \ \sqrt{3}x-y=2+3\sqrt{3}$

16. (c)

$\because\ a_1a_2 + b_1b_2 = \dfrac{1}{ab'} + \dfrac{1}{a'b} = 0$

Therefore, the lines are perpendicular.

17. (c)

$4x+ 3y = 11$ …(i)

and $8x + 6y = 15$

$\Rightarrow 4x + 3y = 15/2$ …(ii)

$d = \left|\dfrac{11-\frac{15}{2}}{\sqrt{16+9}}\right| = \dfrac{7}{10}$

18. (c)

$y = m_1x + c_1$ & $y = m_2 x + c_2$

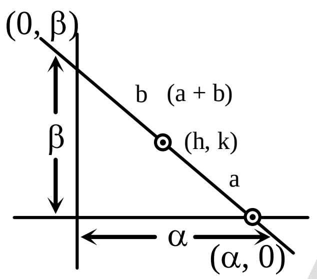
Equation of conic section

$(m_1 x - y + c_1)(m_2 x - y + c_2) + \lambda xy = 0$

Represents circle coffe. of x^2 = coffe. of y^2

$m_1m_2 = 1$

19. (c)



$\alpha^2 + \beta^2 = (a + b)^2$

$h = \dfrac{b\alpha}{a+b} \Rightarrow \alpha = \dfrac{h(a+b)}{b}$

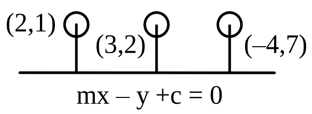
$k = \dfrac{a\beta}{a+b} \Rightarrow \beta = \dfrac{k(a+b)}{a}$

$\dfrac{h^2(a+b)^2}{b^2} + \dfrac{k^2(a+b)^2}{a^2} = (a + b)^2$

locus of (h, k) is

$\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$, ellipse

20. (d)



$P_1 + P_2 + P_3 = 0$

$$\frac{2m-1+c}{\sqrt{1+m^2}}+\frac{3m-2+c}{\sqrt{1+m^2}}+\frac{(-4m-7+c)}{\sqrt{1+m^2}}=0$$

2m + 3m - 4m - 1 - 2 - 7 + 3c = 0

m - 10 + 3c = 0   mx - y + c = 0

$\frac{1}{x}=\frac{-10}{-y}=\frac{3}{1}$ x = 1/3, y = 10/3

1. **(b)**

$\because\ f(x,y) \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ Now,

$f(0,\lambda) \equiv \lambda^2 + 2f\lambda + c = 0$ and its roots are 2, 2.

$\therefore\ 2+2 = -2f,\ 2\times 2 = c,$ *i.e.* $f = -2,\ c = 4$

$f(\lambda,0) \equiv \lambda^2 + 2g\lambda + c = 0,$ and its roots are $\frac{4}{5}, 5.$

$\therefore\ \frac{4}{5} + 5 = -2g,\ \frac{4}{5}\times 5 = c,$ *i.e.,* $g = \frac{-29}{10},\ c = 4$.

Hence, centre of the circle $= (-g, -f) = \left(\frac{29}{10}, 2\right)$

2. **(b)**

The polar of the point $\left(5, -\frac{1}{2}\right)$ is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

$\Rightarrow\ 5x - \frac{1}{2}y - 2(x+5) + 0 + 0 = 0$

$\Rightarrow\ 3x - \frac{y}{2} - 10 = 0\ \Rightarrow\ 6x - y - 20 = 0$

3. **(d)**

$C_1 = (-a, 0),\ \ r_1 = \sqrt{a^2 - c}\,;\ C_2 = (0, -b),\ \ r_2 = \sqrt{b^2 - c}\,;$

$C_1C_2 = \sqrt{a^2 + b^2}$

$\because$ Circles touch each other, therefore $r_1 + r_2 = C_1C_2$

$\Rightarrow\ \sqrt{a^2 - c} + \sqrt{b^2 - c} = \sqrt{a^2 + b^2}$

$\Rightarrow\ a^2b^2 - b^2c - a^2c = 0$

Multiplying by $\frac{1}{a^2b^2c^2}$, we get $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$.

4. **(d)**

The equation of the common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 = 16$ is $x + y = 4$ which meets the circle $x^2 + y^2 = 16$ at points A(4, 0) and B(0, 4). Obviously OA⊥OB, where O is the origin.

Hence the common chord AB makes a right angle at the centre of the circle $x^2 + y^2 = 16$

5. **(c)**

The line $5x - 2y + 6 = 0$ meets the y-axis at (0, 3). We have now to find the length of the tangent from Q (0, 3) to the given circle. Hence

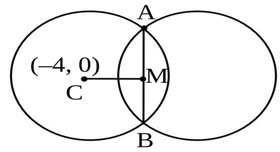
$PQ = \sqrt{0 + 3^2 + 0 + 6\times 3 - 2} = 5$

6. **(b)**

Common chord is $S_1 - S_2 = 0$

$4x - \mu y + 1 = 0$

$\therefore AC = \sqrt{15}$



$AM = \frac{AB}{2} = \sqrt{6}$

$\therefore CM = 3$

$\therefore\ \frac{|-16 + 1|}{\sqrt{16 + \mu^2}} = 3$

$\Rightarrow \mu = \pm 3$

7. **(a)**

The equations of the given circles are

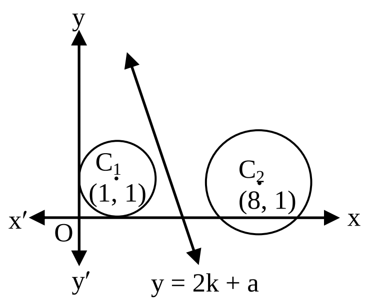
$x^2 + y^2 - 2x - 2y + 1 = 0$ … (1)

and, $x^2 + y^2 - 16x - 2y + 61 = 0$ … (2)

The coordinates of the centres and radii of these two circle are $C_1(1, 1)$, $r_1 = 1$ and $C_2(8, 1)$, $r_2 = 2$ respectively.

For the line $y = 2x + a$ not to touch or intersect circle (1), we must have

$\left|\frac{1 + a}{\sqrt{5}}\right| > 1$ [Length of perpendicular from centre C_1 > radius r_1]



$\Rightarrow |a + 1| > \sqrt{5}$

$\Rightarrow a \in (-\infty, -\sqrt{5} - 1) \cup (\sqrt{5} - 1, \infty)$ …(3)

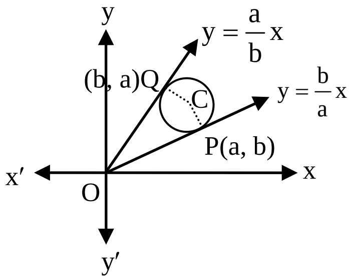
Similarly, for the line $y = 2x + a$ not to touch or intersect circle (2), we must have

$\left|\frac{15 + a}{\sqrt{5}}\right| > 2$

$\Rightarrow |15 + a| > 2\sqrt{5}$

$\Rightarrow a \in (-\infty, -15 - 2\sqrt{5}, -15 + 2\sqrt{5})$ …(4)

The line $y = 2x + a$ will be between the circles, if their centres C_1 and C_2 are on the opposite sides of it.



$\therefore (2 - 1 + a)\,(16 - 1 + a) < 0$

$\Rightarrow (a + 1)\,(a + 15) < 0$

$\Rightarrow a \in (-15, -1)$ … (5)

From equations (3), (4) and (5), we get

$a \in (-15 + 2\sqrt{5}\,,\ -\sqrt{5} - 1)$.

8. (a)

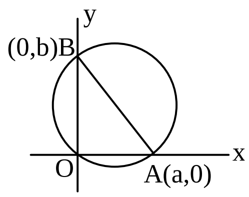
$a + 1 = b - 1$ & $a = 2b \Rightarrow a = -4,\ b = -2$

$\Rightarrow$ Circle $-3x^2 - 3y^2 - 6x + 9y - 3 = 0$

or

$x^2 + y^2 + 2x - 3y + 1 = 0$

9. (d)



Let centroid is (α, β)

$\alpha = a/3 \Rightarrow a = 3\alpha$, $\beta = b/3 \Rightarrow b = 3\beta$

radius $= \frac{AB}{2}$

$3k = \frac{\sqrt{a^2 + b^2}}{2}$

$\Rightarrow 6k = \sqrt{9\alpha^2 + 9\beta^2}$

$\alpha^2 + \beta^2 = 4k^2$

10. (b)

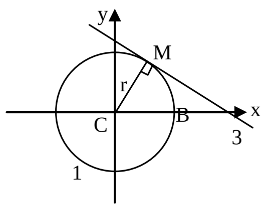
Put y = 0; $x^2 - 4x + \lambda = 0$ give equal roots so D = 0

$\Rightarrow 16 - 4\lambda = 0 \Rightarrow \lambda = 4$

11. (c)

$x^2 + y^2 = 9$ ...(1)

$3x + 4y = 0$ ...(2)



Equation of the line parallel to the line (2) is

$\Rightarrow 3x + 4y + \lambda = 0$ ...(3)

$\because$ line (3) touches the circle (1)

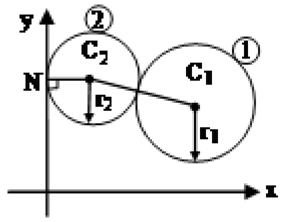
$\Rightarrow CM = 8 \Rightarrow CM = \left|\frac{0 + 0 + \lambda}{5}\right| = 3 \Rightarrow \lambda = \pm 15$

By (3) $\Rightarrow$ equation of tangent is $\Rightarrow 3x + 4y = 15$

12. (d)

$x^2 + y^2 - 6x - 6y + 14 = 0$ ...(1)

$\Rightarrow C_1(3, 3)$, $r_1 = 2$



Locus of centre $C_2(h, k)$ = ?

$\because C_2N = r_2 = |h| \Rightarrow C_1C_2 = (r_1 + r_2)$

$\because$ Both the circles touches each other externally.

13. (b)

$S = x^2 + y^2 - 6x - 10y + p = 0$

P(1, 4)

$g^2 - c < 0 \Rightarrow 9 - p < 0 \Rightarrow p - 9 > 0$ ...(i)

$f^2 - c < 0 \Rightarrow 25 - p < 0 \Rightarrow p - 25 > 0$ ...(ii)

$S_1 < 0$

$1 + 16 - 6 - 40 + p < 0 \Rightarrow$ $p - 29 < 0$

$p < 29$ ...(iii)

comparing (i), (ii) & (iii)

$25 < p < 29$

14. (a)

$x^2 + y^2 - 6x + 2y - 8 = 0$ ...... (i)

centre (3, - 1)

Then, coordinates of the centre will satisfy the equation of the diameter.

15. (c)

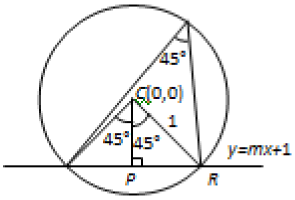
Given circle is $x^2 + y^2 = 1,$ C (0,0) and radius = 1 and chord is $y = mx + 1$

$\cos 45^o = \frac{CP}{CR}$; CP = Perpendicular distance from (0, 0) to chord $y = mx + 1$

$CP = \frac{1}{\sqrt{m^2 + 1}}$ (CR = radius =1)

$\cos 45^o = \frac{\frac{1}{\sqrt{m^2+1}}}{1} \Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{m^2 + 1}}$

$\Rightarrow m^2 + 1 = 2 \Rightarrow m = \pm 1.$



16. (b)

The centres of the given circles are $(-\lambda_i, 0)\ (i = 1, 2, 3)$

The distances from the origin to the centres are $\lambda_i\ (i = 1, 2, 3).$

It is given that $\lambda_2^2 = \lambda_1\lambda_3$.

Let $P(h, k)$ be any point on the circle $x^2 + y^2 = c^2$, then, $h^2 + k^2 = c^2$

Now, L_i = length of the tangent from (h, k) to

$x^2 + y^2 + 2\lambda_i x - c^2 = 0 = \sqrt{h^2 + k^2 + 2\lambda_i h - c^2} =$

$\sqrt{c^2 + 2\lambda_i h - c^2} = \sqrt{2\lambda_i h}$

$[\because\ h^2 + k^2 = c^2$ and $i = 1, 2, 3]$

Therefore, $L_2^2 = 2\lambda_2 h = 2h(\sqrt{\lambda_1\lambda_3})$

$[\because\ \lambda_2^2 = \lambda_1\lambda_3]$

$= \sqrt{2h\lambda_1}\sqrt{2h\lambda_3} = L_1L_3$. Hence, L_1, L_2, L_3 are in *G.P*

17. (b)

The line joining (4, 3) and (2, 1) is also along a diameter. So, the centre is the intersection of the diameters $2x - y = 2$ and $y - 3 = (x - 4)$. Solving these, the centre = (1, 0)

$\therefore$ Radius = Distance between (1, 0) and (2, 1) = $\sqrt{2}.$

$\therefore$ Equation of circle $(x-1)^2 + y^2 = (\sqrt{2})^2$

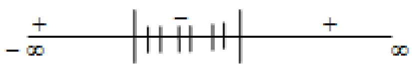
$\Rightarrow x^2 + y^2 - 2x - 1 = 0$

18. (b)

$\sqrt{\lambda^2 + (1-\lambda)^2 + 1} < 3$

$\lambda^2 + \lambda^2 - 2\lambda + 1 + 1 < 9$

$2\lambda^2 - 2\lambda - 7 < 0$



$\dfrac{1-\sqrt{15}}{2}$ $\quad$ $\dfrac{1+\sqrt{15}}{2}$

$-1.45 < \lambda < 2.45$

-1, 0, 1, 2

19. (d)

Here, $r^2 + r_1^2 + r_2^2 + a^2 + b^2 + c^2$

$= r^2 + (r_1 + r_2 + r_3)^2 - 2(r_1r_2 + r_2r_3 + r_1r_3)$

$+ (a + b + c)^2 - 2(ab + bc + ca)$

$= r^2 + (4R + r)^2 - 2s^2 + 4s^2 - 2(ab + bc + ca)$

$= 2r^2 + 16R^2 + 8rR + 2s^2 - 2(ab + bc + ca)$

$= 16R^2 + \dfrac{2\Delta^2}{s^2} + \dfrac{8\Delta R}{s} + 2s^2 - 2(ab + bc + ca)$

$= 16R^2 + \dfrac{2(s-a)(s-b)(s-c)}{s} + 2abc + 2s^2 - 2(ab + bc + ca)$

$= 16R^2 + 2s^2 - s.\ 4s + 2(ab + bc + ca) + 2s^2 - 2(ab + bc + ca)$

$= 16R^2$.

20. (a)

The perimeter of a regular polygon of n sides = $2nr\sin\dfrac{\pi}{n}$

If the radius of the circle is a then $2\pi a = 2nr\sin\dfrac{\pi}{n}$ ...(i)

Now area of polygon = $n\left(\dfrac{1}{2} \times 2r\sin\dfrac{\pi}{n} r\cos\dfrac{\pi}{n}\right)$

$\Rightarrow$ ratio of areas of circle and polygon

$= \dfrac{\pi a^2}{nr\sin\dfrac{\pi}{n}\cos\dfrac{\pi}{n}} = \left(\tan\dfrac{\pi}{n}\right) : \dfrac{\pi}{n}$ (from (i))