Basic Maths, Kinematics, Units & Dimensions

(Solution)

14.04.2020

Total Ques : 30 **MM:120**

Origin

A JEE Prep Hub

1. A

Given in the question that x=0. Thus
$t^3 - 4t^2 + 3t = 0$
$t(t-1)(t-3) = 0$
so, t=0,1,3
Now acceleration
$x = t^3 - 4t^2 + 3t$
$\frac{dx}{dt} = 3t^2 - 8t + 3$
$\frac{d^x}{dt^2} = 6t - 8$
So,
At t=0, $a_0 = -8$
At t=1, $a_1 = -2$
At t=3, $a_3 = 10$

2. B

Area under the graph represents the change in velocity so
$v_{max} - v_0 = at_0 + at_0 + \frac{1}{2}(at_0) + \frac{1}{2}(2a \times 2t_0)$
$v_{max} - v_0 = 2at_0 + \frac{1}{2}(at_0) + 2at_0 = \frac{9}{2}(at_0)$
$v_{max} = v_0 + \frac{9}{2}(at_0)$

3. A

$a = mx - \frac{v_0^2}{x_0}$

Now $a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$

$v\frac{dv}{dx} = mx - \frac{v_0^2}{x_0}$

$vdv = (mx - \frac{v_0^2}{x_0})dx$

$\int_{v_0}^{0} vdv = \int_{0}^{x_0}(mx - \frac{v_0^2}{x_0})dx$

$-\frac{v_0^2}{2} = \frac{mx_0^2}{2} - v_0^2$

or

$m = \frac{v_0^2}{x_0^2}$

4. A

When the boat moves from A to B

Speed of the boat= $\eta + 1$

Let t_1 be the time taken

Then $\eta + 1 = \frac{AB}{t_1}$

Similarly when the boat moves from point B to A

Speed of the boat= $\eta - 1$

Let t_2 be the time taken

Then $\eta - 1 = \frac{AB}{t_2}$

Average Velocity

$= \frac{2AB}{t_1+t_2}$

Substituting the values from above

Average velocity= $\eta^2 - 1$

5. B

For X, position of stone is given by
$a = g = 9.8m/sec^2$
$x = v_0 t + \frac{1}{2}at^2$
or
$x = 0 + \frac{1}{2} \times 9.8 \times 4$
=19.6 m
Velocity
$v = v_0 + at$
or v=19.6 m/sec

6. C

Distance moved by the stone in 3 sec
$x = v_0 t + \frac{1}{2}at^2$
or,
x=44 m (downward)
Distance moved by the elevator in 3 sec
=10*3=30 m (upward)
So position w.r.t to Man Y=44+30=70 m/sec

Now Velocity of stone at t=3
=9.8 *3=29m/sec(downward)
Velocity of elevator=10 m/sec(upward)

So velocity of stone w.r.t to Man Y
=29 + 10(since they are opposite direction) =39m/sec

7. A

Man X is at rest. So acceleration will be 9.8 m/s^2
Man Y is moving with constant velocity. Again since it is inertial frame, acceleration will be 9.8 m/s^2

8. A

Answer (a)
Displacement- As the lift is coming in downward direction, displacement will be negative. i.e. x<0
Velocity- As displacement is negative, velocity must be negative. i.e. v<0
Acceleration- The lift is about to stop at 4th floor, hence the motion is regarding. i.e. a > 0

9. B

Answer (b)
Here we are given the instantaneous speed v which satisfies $0 \leq v < v_0$. Instananeous speed just tell the magnitude of the instananeous velocity.So instananeous velocity can be negative
Also It means instantaneous speed is increasing or decreasing. So the velocity will be also increasing or decreasing. But here it could happen in any direction.So option (a) ,(b) and (d) may not be correct.
Now maximum displacement can happen with maximum velocity v_0 but again it can happen in positive or negative direction.
So maximum displacement in + direction v_0T and maximum displacement in negative direction as $-v_0T$
Hence The displacement x in time T satisfies �$v_0T < x < v_0T$.

10. B

Since the graph is a straight line
So for A
$a = -k_1 x$
Where k_1 is the constant of the straight line
Now $a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$
So
$v\frac{dv}{dx} = -k_1 x$
$vdv = (-k_1 x)dx$
Integrating both sides with in the limit $x = 0$ and $x = x_0$
$v^2 = v_0^2 - k_1 x_0^2$
Similarly for the particle B
$v^2 = v_0^2 - k_2 x_0^2$
Now from the graph it is clear that
$k_2 > k_1$
So Particle B has the high Magnitude of the velocity

11. b

Answer is (b)

12. C

$y = 2t + t^2 - 2t^3$
Velocity is given
$v = \frac{dy}{dt} = 2 + 2t - 6t^2$
$a = \frac{dv}{dt} = 2 - 12t$
Now acceleration is zero
$2 - 12t = 0$
$t = \frac{1}{6}$
Putting this value velocity equation
$v = 2 + \frac{2}{6} - 6(\frac{1}{6})^2 = 2 + \frac{1}{3} - \frac{1}{6} = \frac{13}{6}$

13. B

If a body is projected with a given velocity u at angle θ and $(90 - \theta)$ to the horizontal, it will have same range R given by
$R = \frac{u^2 sin 2\theta}{g}$
The corresponding times if flight are
$t_1 = \frac{2u sin\theta}{g}$
$t_2 = \frac{2u sin(90-\theta)}{g} = \frac{2u cos\theta}{g}$
$t_1 t_2 = \frac{2u^2(2 sin\theta cos\theta)}{g^2} = \frac{2u^2 sin 2\theta}{g^2} = \frac{2R}{g}$

14. B

Given

u=4**i**+3**j** m/s

a=.4**i**+.3**j** m/s^2

So velocity vector at any time t

v=u+at = 4**i**+3**j**+(.4**i**+.3**j**)t =(4+.4t)**i**+(3+.3t)**j**

So velocity at 10 sec

v=8**i**+6**j**

|v|=10

displacement vector at any time t

s=ut+(1/2)at^2 =(4**i**+3**j**)t+(1/2)(.4**i**+.3j)t^2 =**i**(4t+.2t^2)+**j**(3t+.15t^2)

Hence Answer is (a) and (b)

15. A

For projectile A

$y = a_1 x - b_1 x^2$

for y=0

$x = 0$ and $x = \frac{a_1}{b_1}$

Similarly for Projectile B

$y = a_2 x - b_2 x^2$

for y=0

$x = 0$ and $x = \frac{a_2}{b_2}$

For the range to be same

$\frac{a_1}{b_1} = \frac{a_2}{b_2}$

16. d

This question can be solved in two ways
Method 1:
Given
$x = 36t$

$y = 48t - 4.9t^2$

For a body projected with velocity u at an angle θ with the horizontal ,the x and y displacement is given by
$x = (ucos\theta)t$
$y = (usin\theta)t - \frac{gt^2}{2}$
Comparing this the given equation we have
$ucos\theta = 36$
$usin\theta = 48$
Squaring and adding we get
$u^2(cos^\theta + sin^2\theta) = 3600$
so u=60 m/s

Method 2:
Given
$x = 36t$

$y = 48t - 4.9t^2$

$v_x = \frac{dx}{dt} = 36$
$v_y = \frac{dy}{dt} = 48 - 9.8t$
So intial velocity can be found substituting t=0 in both the equation
$v_x = 36$
$v_y = 48$
So net velocity= $\sqrt{362 + 482)} = 60$ m/s

17. A

Range is given by

$R = \frac{u^2 sin2\theta}{g}$ ----1

and time taken by

$T = \frac{2usin\theta}{g}$ ----2

Now $3R = \frac{u_c^2 sin2\theta}{g}$ ---3

$T_c = \frac{2u_c sin\theta}{g}$ ---4

Dividing 1 by 3

$\frac{1}{3} = (\frac{u}{u_c})^2$ ----5

Dividing 2 by 4

$\frac{T}{T_c} = \frac{u}{u_c}$ ---6

From 5 and 6

$\frac{T}{T_c} = \frac{1}{\sqrt{3}}$

$T_c = \frac{T}{\sqrt{3}}$

18. A

19. D

20. A

21. B

22. B

Let h be the total distance travelled in n second then,

$h = \frac{1}{2}gn^2$

In the last second i.e., (n-1)th second distance covered by it is

h^' = \frac {h}{2} = \frac {1}{4} gn^2

hence

$\frac{1}{2}g(n-1)^2 = \frac{1}{4}gn^2$

Solving for n we now get a Quadratic equation in n

$n^2 - 4n + 2 = 0$

Positive root of this equation is

$n = 2 + \sqrt{2}$

23.

$x = \frac{a}{b}(1 - e^{-bt})$

Velocity

$v = \frac{dx}{dt} = ae^{-bt}$

Acceleration

$w = \frac{dv}{dt} = -abe^{-bt}$

So, at t=0

$v = a$ and $w = -ab$

Also

$v = \frac{a}{e^{bt}}$

So with time ,velocity decreases

$v = -\frac{ab}{e^{bt}}$

Similarly acceleration decreases with time and maximum acceleration is -ab

Now at t=0

$x = 0$

when $t- > \infty$

$x = \frac{a}{b}$

So,

$0 \leq x \leq \frac{a}{b}$

All the four options are correct

24. B

$$\left[\frac{ax\,c}{bt^2}\right] = \frac{MLT^{-2} \times MLT^{-2}}{MLT^{-2}} = MLT^{-2}$$

25. D

26. D

27. D

28. B

For equilibrium, net resultant force must be zero. These forces form a closed traingle such that

$$F_1 \tilde{\ } F_2 \leq F_3 \leq F_1 + F_2 \Rightarrow 2N \leq F_3 \leq 8N$$

29. B

30. B

$$12y = x^3 \Rightarrow 12dy = 3x^2dx \Rightarrow \frac{dy}{dt} = \left(\frac{x}{2}\right)^2 \left(\frac{dx}{dt}\right)$$

Therefore for $\left(\frac{x}{2}\right)^2 > 1$ or $x > 2$, y– coordinate changes at faster rate.