Questions For Practice

Calculus of Complex functions

Analyticity Testing

Are the following functions analytic? Check using C-R eq^ns.

1. $f(z) = e^{-2x}(\cos 2y - i \sin 2y)$
2. $f(z) = Re(z^2) - i\, Im(z^2)$
3. $f(z) = \frac{i}{z^8}$
4. $f(z) = \frac{8\bar{z}^2}{z^3 + 4\bar{z}^2 z}$
5. $f(z) = \cos x \cosh y - i \sin x \sinh y$.

Are the following functions ~~analytic~~ harmonic? If yes, find a corresponding analytic function $f(z) = u + iv$

1. $u = xy$
2. $u = \frac{x}{x^2+y^2}$
3. $v = (2x+1)y$
4. $v = e^x \sin y$
5. $v = -e^{-2x} \sin 2y$

Determine a and b so that the given function is harmonic and find a conjugate harmonic

1. $u = e^{\pi x} \cos ay$
2. $u = ax^3 + bxy$
3. $u = \cos ax \cosh 2y$
4. $u = \cos hx \cos y$

~~5. $v = e^{2x} \sin 2y$~~

Find e^z in the form of $u+iv$ and $|e^z|$ if z equals

1. $2\pi i(1+i)$
2. $2 + 3\pi i$
3. $\sqrt{2} + \frac{1}{2}\pi i$

Write in exponential form

1. $4+3i$ 2. -6.3 3. $1+i$ 4. e^{z^2} 5. e^{z^3}

Find all solutions of the followings

1. $e^z = 1$ 2. $e^z = 0$ 3. $e^z = -2$

Find in the form $u+iv$

1. $\cos i$ 2. $\sin i$ 3. $\cosh(-1+2i)$ 4. $\cos(-2-i)$

5. $\sin 5i$ 6. $\cos\left(\frac{1}{2}\pi - \pi i\right)$

Find all solutions of the following eqns

1. $\cosh z = 0$ 2. $\sinh z = 0$

Find $\operatorname{Ln} z$ when z equals

1. -11 2. $4-4i$ 3. $0.6+0.8i$

4. ei

Find $\ln z$ when z equals

1. $\ln|1|$ 2. $\ln(e^i)$

Solve for z

1. $\ln z = 4-3i$ 2. $\ln z = 0.6+0.4i$

Find the principal value of the following

1. $(1+i)^{1-i}$ 2. $(-3)^{3-i}$ 3. $(-1)^{2-i}$

Complex Integration

1. $\int_C \operatorname{Re} z \, dz$, C: the shortest path from $1+i$ to $3+3i$
2. $\int_C e^z dz$: C the shortest path from πi to $2\pi i$
3. $\int_C z e^{z^2} dz$: C from 1 along the axes to i
4. $\int_C \sec^2 z \, dz$, any path from $\frac{\pi}{4}$ to $\frac{\pi i}{4}$

Question

Integrate $f(z)$ counterclockwise around the unit circle ($|z|=1$). Indicate whether Cauchy thm applies.

1. $f(z) = e^{-z^2}$
2. $f(z) = \frac{1}{2z-1}$
3. $f(z) = \frac{1}{z^4 - 1.1}$
4. $f(z) = \operatorname{Im} z$
5. $f(z) = \frac{1}{|z|^2}$
6. $f(z) = z^3 + z$

2.

Evaluate the integrals checking whether Cauchy thm applies or not

1. $\oint_C \frac{dz}{z-3i}$, C: $|z| = \pi$ Counterclockwise
2. $\oint_C \frac{e^z}{z} dz$, C consists of $|z|=2$ counterclockwise and $|z|=1$ clockwise
3. $\oint_C \frac{\cos z}{z} dz$, C consists of $|z|=1$ counterclockwise & $|z|=3$ clockwise.

4. $\oint_C \frac{\sin z}{z+2iz}\,dz$, $C: |z-4-2i| = 5.5$ clockwise.

Integration using Cauchy's Integral formula

1. $\oint_C \frac{z^2}{z^2-1}\,dz$, $C: |z+1| = 1$

2. $\oint_C \frac{z^2}{z^2-1}\,dz$, $C: |z+i| = 1.4$

3. $\oint_C \frac{\cos 3z}{6z}\,dz$, $C: |z| = 1$

4. $\oint_C \frac{z^3}{2z-i}\,dz$, $C: |z| = 1$

5. $\oint_C \frac{dz}{z^2+4}$ $C: 4x^2 + (y-2)^2 = 4$

6. $\oint_C \frac{z+2}{z-2}\,dz$, $C: |z-1| = 2$

7. $\oint_C \frac{\cosh(z^2-\pi i)}{z-\pi i}\,dz$, C: the boundary of the square with vertices $\pm 2, \pm 4i$

8. $\oint_C \frac{\ln(z+1)}{z^2+1}\,dz$, $C: |z-i| = 1.4$

9. $\oint_C \frac{\exp z^2}{z^2(z-1-i)}\,dz$, C consists of $|z| = 2$ Counterclockwise and $|z| = 1$ clockwise