A Generalization of the Łoś-Tarski Preservation Theorem

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Joint work with
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Introduction

- Preservation theorems have been one of the earliest areas of study in classical model theory.
- A preservation theorem characterizes (definable) classes of structures closed under a given model theoretic operation.
- Preservation under substructures/extensions (Łoś-Tarski theorem), unions of chains, homomorphisms, etc.
- Most preservation theorems fail in the finite. (The homomorphism preservation theorem is an exception.)
- Recent research (by Atserias, Dawar, Grohe, Kolaitis) has focussed on "recovering" preservation results over special classes of finite structures, like acyclic structures, those with bounded degree, bounded tree-width etc.

Talk Outline

- Preservation under substructures and the Łoś-Tarski theorem
- Preservation under substructures modulo k-cruxes
- Our generalization of the Łoś-Tarski theorem
- Preservation under k-ary covered extensions and a dual form of our generalization
- Our results in the finite model theory setting
- An evaluation of our results

Some assumptions and notation for the talk

Assumptions:

- First Order (FO) logic
- Arbitrary vocabularies (constants, predicates and functions)
- Arbitrary structures

Notations:

- $\forall^* = \forall x_1 \ldots \forall x_n$(quantifier-free formula in $x_1, \ldots x_n$)
- $\exists^k\forall^* = \exists x_1 \ldots \exists x_k \forall y_1 \ldots \forall y_n$(quantifier-free formula in $x_1, \ldots, x_k, y_1, \ldots, y_n$)
- $\mathfrak{A}_1 \subseteq \mathfrak{A}_2$ means $\mathfrak{A}_1$ is a substructure of $\mathfrak{A}_2$. For graphs, $\subseteq$ means *induced subgraph*.
- $\mathrm{U}_{\mathfrak{A}}$ = universe of $\mathfrak{A}$.

Preservation under Substructures

Definition 1 (Pres. under subst.)

A sentence ϕ is said to be preserved under substructures, denoted ϕ is PS, if $\big((M \models \phi) \wedge (N \subseteq M)\big) \rightarrow N \models \phi$.

- E.g.: Consider $\phi = \forall x \forall y E(x, y)$ which describes the class of all cliques.
- Any induced subgraph of a clique is also a clique. Then ϕ is PS.
- In general, every $\forall^*$ sentence is PS.

Theorem 1 (Łoś-Tarski, 1960s)

A FO sentence is PS iff it is equivalent to a $\forall^*$ sentence.

Generalizing preservation under substructures

Preservation under substructures modulo k-cruxes

Definition 2

A sentence ϕ is said to be preserved under substructures modulo k-cruxes, abbreviated ϕ is $PSC(k)$, if for each model M of ϕ, there is a subset C of U_M, of size $\leq k$, s.t. $\big((N \subseteq M) \wedge (C \subseteq \mathsf{U}_N)\big) \rightarrow N \models \phi$.

- The set C is called a k-crux of $\mathfrak{A}$ w.r.t. ϕ. If ϕ is clear from context, we will call C as a k-crux of $\mathfrak{A}$.
- Easy to see that $PS = PSC(0)$.

Example

- Eg. Consider $\phi = \exists x \forall y E(x, y)$.

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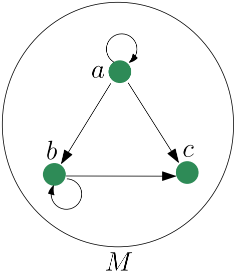

$M \models \phi$

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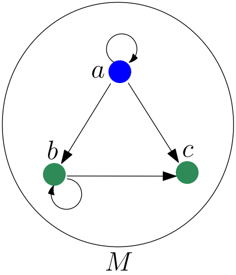

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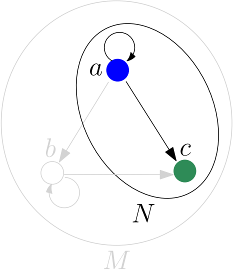

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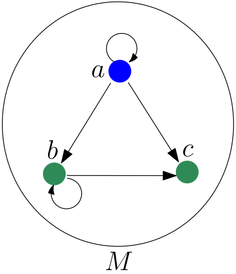

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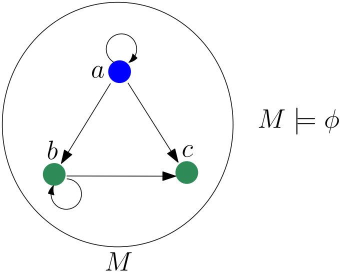

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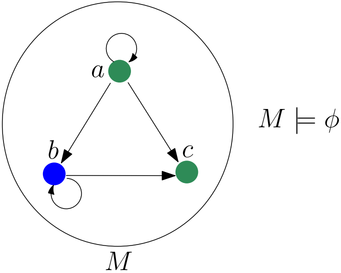

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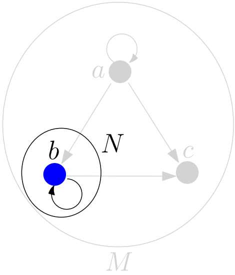

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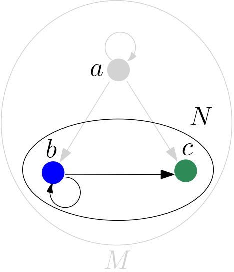

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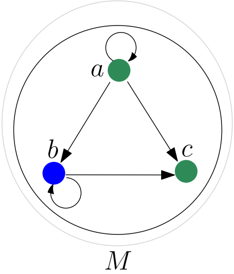

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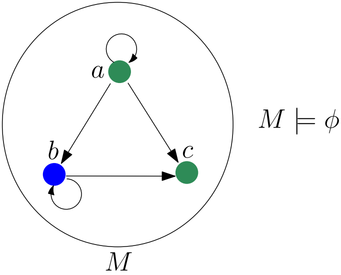

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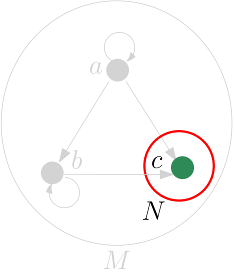

$M \models \phi$
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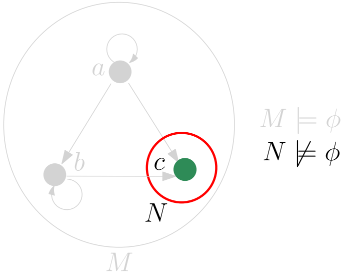

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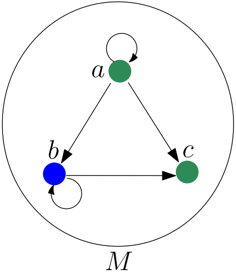

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- There can be 1-cruxes that are not witnesses for x.
- Observe that ϕ is not PS. Then $PS \subsetneq PSC(1)$.

A generalization of the Łoś-Tarski theorem

- Any $\exists^k \forall^*$ sentence ϕ is $PSC(k)$ – the witnesses to the $\exists$ quantifiers of ϕ form a k-crux.

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Theorem 2

An FO sentence is $PSC(k)$ iff it is equivalent to an $\exists^k \forall^*$ sentence.

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- Is the converse true? Yes!

Theorem 2

An FO sentence is $PSC(k)$ iff it is equivalent to an $\exists^k\forall^*$ sentence.

- The case of $k = 0$ is exactly the Łoś-Tarski theorem for sentences.
- The above result is true for arbitrary vocabularies and over any class of structures definable by FO theories.

An Intuitive but Incorrect Attempt at Characterizing $PSC(k)$

- Let $\phi \in PSC(k)$, Vocab(ϕ) $= \tau$ and $\tau_k = \tau \cup \{c_1, \ldots, c_k\}$.
- Let Z be the class of models of ϕ expanded with all k-cruxes. Formally, $Z = \{(M, a_1, \ldots, a_k) \mid M \models \phi$ and $a_1, \ldots, a_k$ is a k-crux of $M\}$.
- Clearly Z is pres. under substr. Then by Łoś-Tarski theorem, Z is captured by a $\forall^*$ sentence. Replace $c_1, \ldots, c_k$ with fresh variables $x_1, \ldots, x_k$ and existentially quantify out the latter.
- Error: Z is assumed FO definable.
- The above proof attempt fails for as simple a sentence as $\phi = \exists x \forall y E(x, y)$. (In fact, Z in this case is not definable by any FO theory too!)

Dualizing $PSC(k)$

Preservation under Extensions

Definition 3

A sentence ϕ is said to be preserved under extensions, denoted ϕ is PE, if $\big((M \models \phi) \wedge (M \subseteq N)\big) \rightarrow N \models \phi$.

- E.g.: Let $\phi = \exists x \exists y E(x, y)$. Easy to see that ϕ is PE.

Following is a duality lemma.

Lemma 3

A sentence ϕ is PS iff $\neg\phi$ is PE.

Theorem 4 (Łoś-Tarski, 1960s)

A FO sentence is PE iff it is equivalent to a $\exists^*$ sentence.

An Alternate Form of Łoś-Tarski Theorem

Definition 4

A structure M is said to be an extension of a collection R of structures, denoted $R \subseteq M$, if for each $N \in R$, we have $N \subseteq M$.

- Easy to check: Preservation under extensions of single structures $\equiv$ Preservation under extensions of collections of structures.
- Then PE can be defined to be preservation under extensions of collections of structures and the Łoś-Tarski theorem statement would still be true.

k-ary Covered Extensions

Definition 5

For $k \in \mathbb{N}$, a structure M is said to be a k-ary covered extension of a non-empty collection R of structures if (i) M is an extension of R, and (ii) for every $A \subseteq U_M$ s.t. $|A| \leq k$, there is a structure in R that contains A. We call R a k-ary cover of M.

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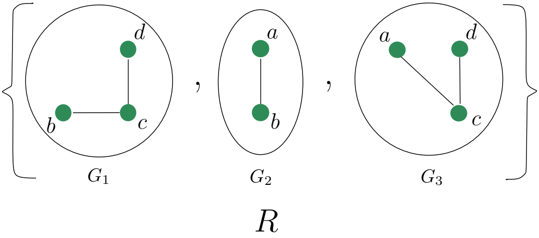

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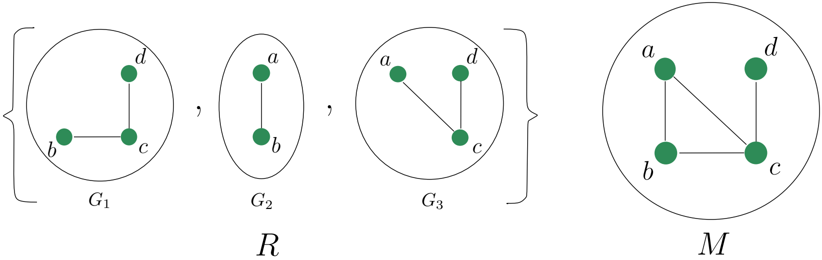

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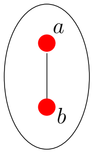
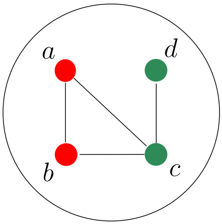


G_2



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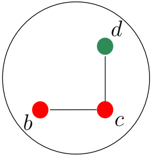
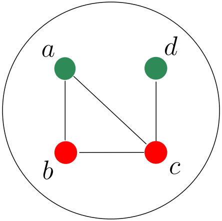


G_1



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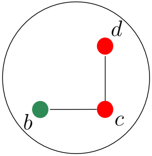

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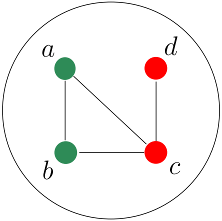

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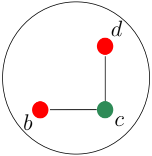

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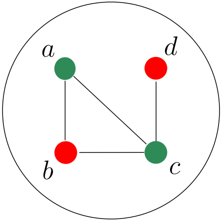

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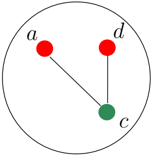

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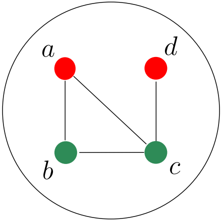

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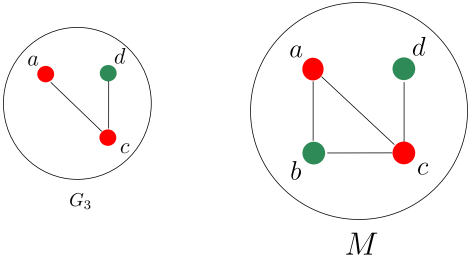

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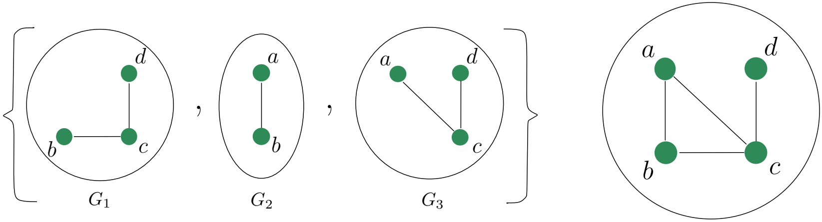

R is a 2-ary cover of M

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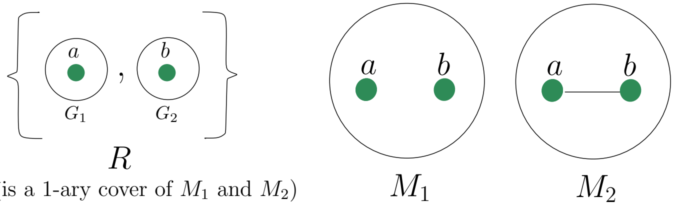

(is a 1-ary cover of M_1 and M_2)

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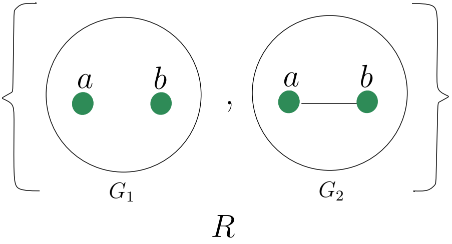

R has no extension!

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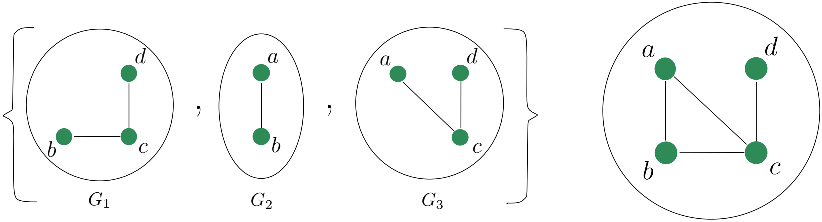

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Preservation under k-ary Covered Extensions ($PCE(k)$)

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Given $k \in \mathbb{N}$, a sentence ϕ is said to be preserved under k-ary covered extensions, denoted $\phi \in PCE(k)$, if for each collection R of models of ϕ, (M is a k-ary covered extension of R) $\rightarrow M \models \phi$.

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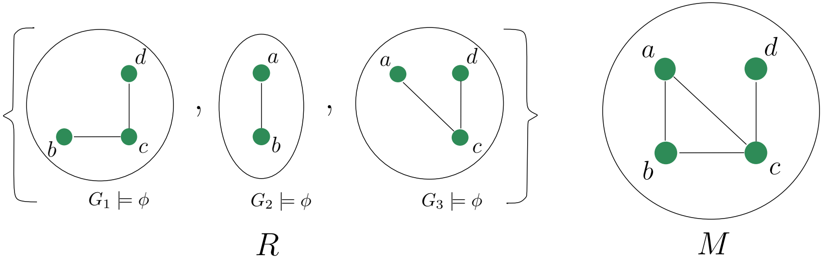

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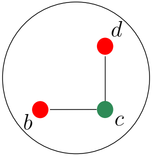
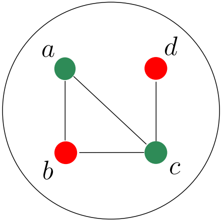


$G_1 \models \phi$



M

Preservation under k-ary Covered Extensions ($PCE(k)$)

Definition 6

Given $k \in \mathbb{N}$, a sentence ϕ is said to be preserved under k-ary covered extensions, denoted $\phi \in PCE(k)$, if for each collection R of models of ϕ, (M is a k-ary covered extension of R) $\rightarrow M \models \phi$.

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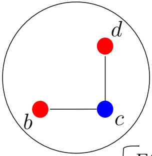
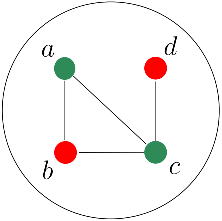


$G_1 \models \begin{cases} E(b, c) \\ E(c, d) \end{cases}$



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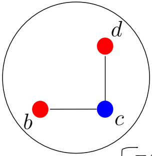

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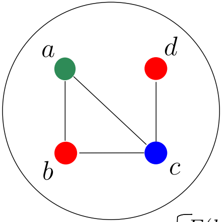

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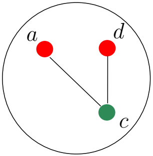
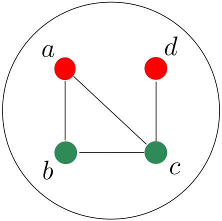


$G_3 \models \phi$



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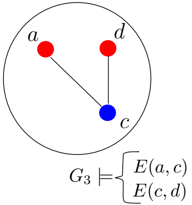

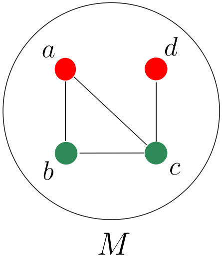

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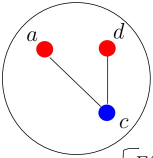

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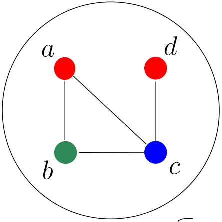

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Preservation under k-ary Covered Extensions ($PCE(k)$)

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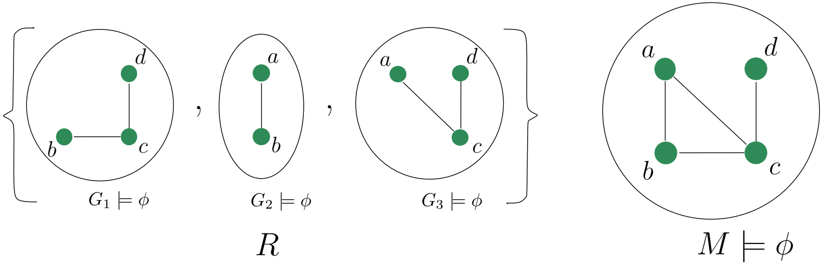
Given $k \in \mathbb{N}$, a sentence ϕ is said to be preserved under k-ary covered extensions, denoted $\phi \in PCE(k)$, if for each collection R of models of ϕ, (M is a k-ary covered extension of R) $\rightarrow M \models \phi$.

- E.g.: $\phi = \forall x \forall y \exists z \big((x = y) \lor E(x, y) \lor (E(x, z) \land E(z, y))\big)$.



The Duality of $PSC(k)$ and $PCE(k)$

Lemma 5

A sentence ϕ is in $PSC(k)$ iff $\neg\phi$ is in $PCE(k)$.

Proof Sketch: Below, $A \subseteq_k B$ means $A \subseteq B$ and $|A| \leq k$.
We show the 'If' direction. The 'Only if' is by a dual argument.

- Suppose $M \models \phi$ and there is no k-core in M.
- Then for each $A \subseteq_k U_M$, there exists $N_A \subseteq M$ containing A s.t. $N_A \models \neg\phi$.
- Then $R = \{N_A \mid A \subseteq_k U_M\}$ forms a k-ary cover of M. Since $\neg\phi \in PCE(k)$, we get $M \models \neg\phi$ – a contradiction.

A Syntactic Characterization of $PCE(k)$

Theorem 6

A sentence ϕ is in $PCE(k)$ iff ϕ is equivalent to a $\forall^k\exists^*$ sentence.

Proof Sketch:

- Let $\Gamma = \{\psi \mid \psi = \forall^k\exists^*(\ldots),\ \phi \rightarrow \psi\}$. Clearly, $\phi \rightarrow \Gamma$.
- Show that $\Gamma \rightarrow \phi$ holds over the class $\mathcal{C}$ of α*-saturated structures*, where $\alpha \geq \omega$.
- Use the fact that every structure has an elementarily equivalent structure in $\mathcal{C}$ to show that $\Gamma \rightarrow \phi$ holds over all structures.
- Finally, by Compactness theorem, the result follows. $\blacksquare$

A Generalization of the Łoś-Tarski Theorem

Theorem 7

1. *A sentence ϕ is $PSC(k)$ iff ϕ is equivalent to a $\exists^k\forall^*$ sentence.*
2. *A sentence ϕ is $PCE(k)$ iff ϕ is equivalent to a $\forall^k\exists^*$ sentence.*

Define $PSC = \bigcup_{k\geq 0} PSC(k)$ and $PCE = \bigcup_{k\geq 0} PCE(k)$. Define $\exists^*\forall^*(\ldots) = \bigcup_{k\geq 0} \exists^k\forall^*(\ldots)$ and $\forall^*\exists^*(\ldots) = \bigcup_{k\geq 0} \forall^k\exists^*(\ldots)$.

Corollary 8

1. *A sentence is PSC iff it is equivalent to a $\exists^*\forall^*$ sentence.*
2. *A sentence is PCE iff it is equivalent to a $\forall^*\exists^*$ sentence.*

- All of the above results hold for arbitrary vocabularies and over any class of structures that is definable by FO theories.

Our results in the finite model theory setting

- Over each of the following classes, Theorem 7 holds (furthermore, in an effective form).
  - Structures over monadic vocabularies
  - Words and trees over any finite alphabet
  - Co-graphs (and various subclasses of it like cliques, complete n-partite graphs, threshold graphs, etc.)
  - Grids of bounded dimension
  - Structures of bounded tree-depth
- These classes are different from those identified by Atserias, Dawar and Grohe [ADG'08], and satisfy the Łoś-Tarski theorem.
- Over the classes identified by [ADG'08], we suspect that Corollary 8 holds (though Theorem 7 does not).

An evaluation of our results

Classical model theory:

- Theorem 7 gives finer characterizations of $\exists^*\forall^*$ and $\forall^*\exists^*$ than those in the literature, which are via notions like unions of ascending chains, intersections of descending chains, Keisler's 1-sandwiches, etc. **None** of the latter notions relate the **count** of quantifiers to any model-theoretic properties.
- Our semantic notions have natural adaptations for higher n, i.e., for each n, we have analogues of our notions that characterize the $\exists^{k_1}\forall^{k_2}\exists^*\forall^{k_3}\exists^* \ldots (n\text{-blocks})$ fragment of FO, for each $k_1, k_2, k_3, \ldots$.
- "[Preservation] theorems contribute more to the inner structure of model theory than they do to applications." – Wilfrid Hodges. Our preservation theorems might contribute to a keener study of the inner structure of model theory.

An evaluation of our results

Finite model theory:

- All aforesaid literature notions become trivial over any class of finite structures, whereas $PSC(k)$ and $PCE(k)$ being finitary and combinatorial, remain non-trivial over finite structures.
- All positive results over finite structures in the context of preservation theorems only characterize prenex FO sentences having one block of quantifiers. We give characterizations for prenex FO sentences having two blocks of quantifiers.
- Our investigations have yielded a connection between well-quasi-orders and logic.
- We further the line of research that investigates the connection between good model-theoretic behaviour of a class of structures and good computational properties of it.

Dhanyavād!

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