



deeplearning.ai

# Regularizing your neural network

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## Regularization

# Logistic regression

$$\min_{w,b} J(w,b) \quad \rightarrow w \in \mathbb{R}^{n_x}, b \in \mathbb{R}$$

$$J(w,b) = \frac{1}{M} \sum_{i=1}^M L(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2m} \|w\|_2^2$$

L2 Regularization

$$\|w\|_2^2 = \sum_{j=1}^{n_x} w_j^2 = w^T w$$

Q Why not regularize "b" as well?  $\rightarrow \frac{\lambda}{2m} b^2$

well you're doing this to ↓  
overfitting of your params  
params are w, b

$$\dim(w) = (n_x \times 1)$$

$$\dim(b) = (1, \times 1)$$

$\Rightarrow$  the overfitting would likely  
be happening due to w & not  
b (more params to tweak in w)

Why Regularization?

$\hookrightarrow$  to ↓ overfitting

- Getting more data can also ↓ overfitting

L1 Regularization

This is when we add  $\frac{\lambda}{2m} \|w\|_1$

ie,  $\frac{\lambda}{m} \sum_{j=1}^{n_x} |w_j|$  to the eqn

If L1 is used, w ends up being  
sparse  $\Rightarrow$  many 0s in w  
 $\Rightarrow$  helps in storage of params  
as most values are 0s, so you  
need to store only non-zeros  
& their positions

L2 is used more often than L1

$\lambda$  = Regularization param  
 $\hookrightarrow$  find value via dev set

# Neural network

$$J(w^{[1]}, b^{[1]}, \dots, \underbrace{w^{[L]}, b^{[L]}}_{\substack{\text{\# layers in} \\ \text{NN}}} = \frac{1}{M} \sum_{i=1}^M L(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2M} \sum_{j=1}^L \underline{\|w^{[j]}\|_2^2}$$

for each  $j$ th layer  $\rightarrow \underline{\|w^{[j]}\|_2^2} = \sum_{i=1}^{n^{[L-1]}} \sum_{k=1}^{n^{[L]}} (w_{ik}^{[j]})^2$  } For each  $w$  at layer  $j$ , find  $\sum$  of  $Sq$ , then do for all layers

aka "Frobenius Norm"

why are the limits  $n^{[L]}$  &  $n^{[L-1]}$   
Remember  $\dim(w)$  from before:  $(n^{[L]}, n^{[L-1]})$   
 $\downarrow$  # hidden units in current layer  
 $\downarrow$  # hidden units in prev layer

So now how does Back prop change w/ the Addition of Regularization?

$$\frac{\partial J}{\partial w^{[L]}} = ?$$

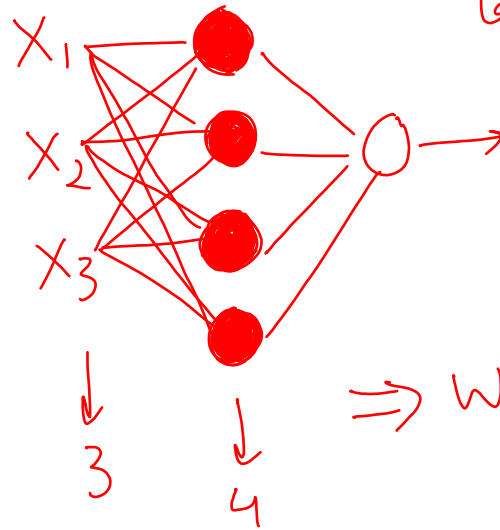
now,  $\underline{\underline{dw^{[L]}}} = \text{old } dw + \frac{\lambda}{M} w^{[L]}$

update

$$w^{[L]} = w^{[L]} - \alpha \cdot \underline{\underline{dw^{[L]}}}$$

The new  $dw^{[L]}$  makes the update further  $\downarrow$  by  $\frac{\alpha \lambda}{M} w^{[L]}$

$\therefore$  we call gradient descent w/ Regularization "WEIGHT DECAY"

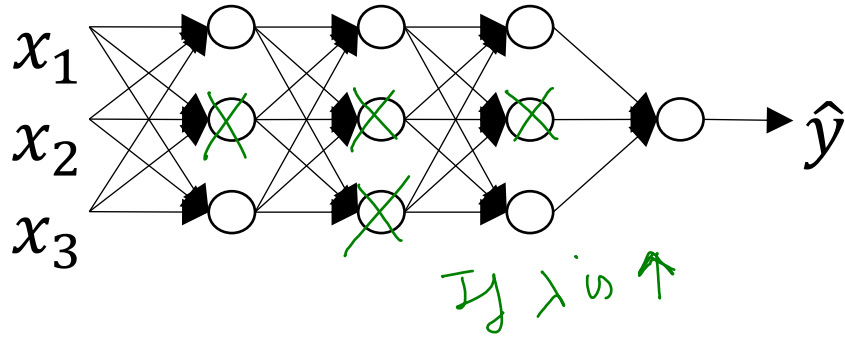


$$\Rightarrow w^{[1]} = (4 \times 3)$$

# How does regularization prevent overfitting?

$$J(w^{[L]}, b^{[L]}) = \frac{1}{M} \sum L(y^{(i)}, \hat{y}^{(i)}) + \underbrace{\frac{\lambda}{2M} \sum_{i=1}^L \|w^{[i]}\|_F^2}_{\text{why does this } \downarrow \text{ overfitting?}}$$

All layers

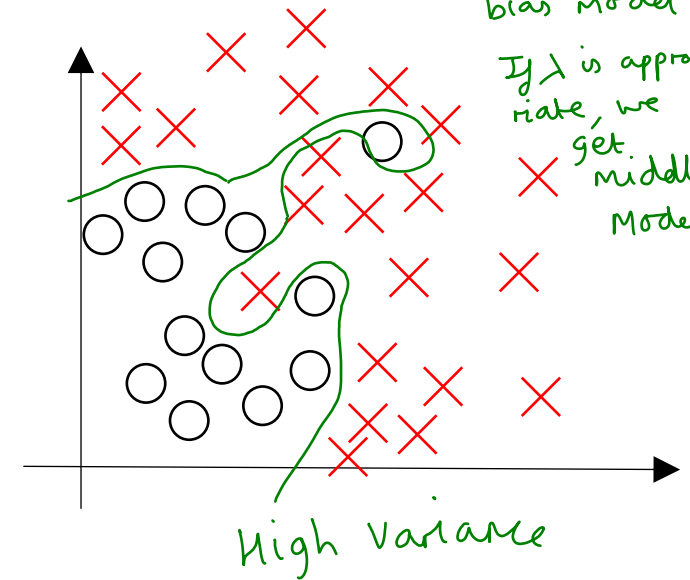
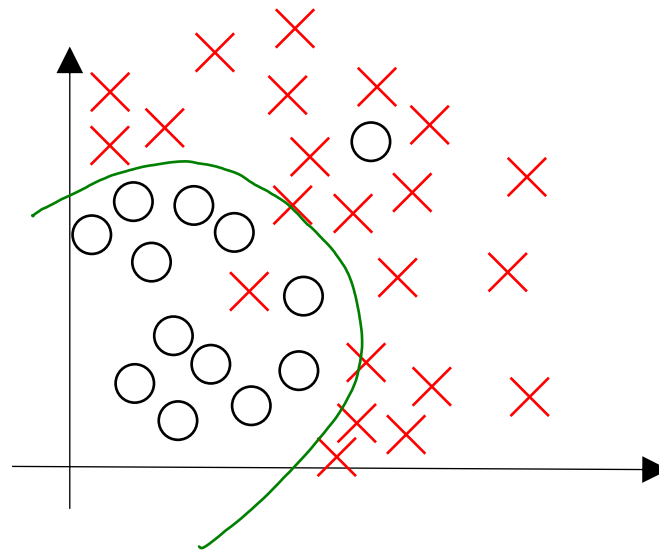
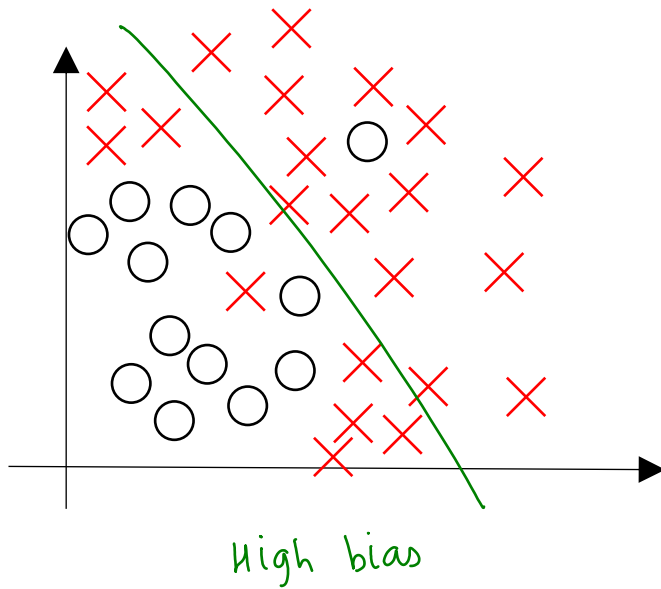


If  $\lambda$  is high,  
then many weights  
w in diff layers  
become close to 0  
to  $\downarrow$  cost func

$\Rightarrow$  Simpler  
model

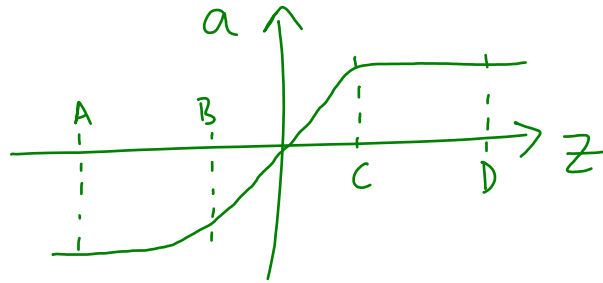
If  $\lambda$  too high,  
we get high  
bias model

If  $\lambda$  is approp-  
riate, we  
get middle  
model



# How does regularization prevent overfitting?

Assume, we use tanh Activation func



$$g(z) = a$$
$$g(z) = \tanh(z)$$

Now If we  $\uparrow \lambda$ ,  $w^{[L]} \downarrow$

If  $w^{[L]} \downarrow$   $z^{[L]} \downarrow$ , why?

$$z^{[L]} = w^{[L]} \cdot a^{[L-1]} + b^{[L]}$$
$$\Rightarrow z \propto w$$

If  $z \downarrow$  and let's assume  $z$  goes from being  
in Range  $C \rightarrow D$  or Range  $A \rightarrow B$   
to Range  $B \rightarrow C$

then we can see derivative of  $z$  " $dz$ "  
is linear,  $\Rightarrow$  every layer is computing a linear  
eqn  $\Rightarrow$  model has become simpler

$\Rightarrow$  If  $\lambda \uparrow$ , we get simpler  
model

$g'(z)$