will start 7:05AM IST

Time Complexity 1

Topics:
1. $\log_2 a$ ✓
2. intervals ✓
3. progressions ✓
4. iterations Quizzes ✓
5. intro to big O ✓

Corrected: whats app group . https://chat.whatsapp.com/IBt9FwPqNsk6Ne7GrlYBut

Celebrate errors: → check this pdf after the session

https://drive.google.com/file/d/1Y8rU1vjwTYZz2FMBBhZKDyHSbXTeP4s5/view?usp=sharing

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Quiz(HW) how many times can we divide n ($n>0$, int) by 2 till reaching 1

↳ $\log_2 n$

ex 128

×2 64 /2
×2 32 /2
×2 16 /2
×2 8 /2
×2 4 /2
×2 2 /2
1

$2\times2\times2\times2\times2\times2\times2 = 128$

$2^7 = 128$ (b ← 7, a ← 2, c → 128)

$b = \log_a c$

$c = 128$

$a = 2$ ← divide by 2

ans $= 7 = \log_2 128$ $\log_2 128$

$64 < 100 < 128$

$\log_2 64$ $\log_2 128$

6 7

$6 < \log_2 100 < 7$

$\log_2 199103$

100 /2
50 /2
25 /2
12 /2
6 /2
3 /2
1

Intervals

$[a,b]$ → inclusive

$10 - 3 + 1 = 7 + 1$

$b \quad a$

$[3,10] \rightarrow 3, 4, 5, 6, 7, 8, 9, 10$

(a,b) → exclusive $(3,10) \Rightarrow 4, 5, 6, 7, 8, 9$

$[0,b]$ $b+1$

$[a,b]$ $b-a+1$

Quiz

Quiz

arithmatic progression (series)

$a = 4$, $d = 3$

$4 \xrightarrow{+3} 7 \xrightarrow{+3} 10 \xrightarrow{+3} 13 \xrightarrow{+3} 16 \xrightarrow{+3} 19 \quad 22 \quad 25$

$a + a+d + a+2d + a+3d + a+4d + \cdots + a+(n-1)d$

How to derive formula?

$$\text{Sum} = \frac{n}{2}\left[2a+(n-1)d\right]$$ → n terms not n+1

$$S = \frac{nx(n+1)}{2}$$

$a=1$, $d=1$

$S = 1 + 2 + 3 + 4 + 5 \cdots + n \rightarrow \frac{n}{2}\left[2x1+(n-1)1\right]$

$= \frac{n}{2}\left[2+(n-1)\right]$

$= \frac{n^2+n}{2} = \frac{n(n+1)}{2}$

$1+2+3+\cdots+10$

$\frac{10 \times 11}{2} = 55$

geometric progression (series)

r

$\times 2 \quad \times 2 \quad \times 2$

$S = 5 + 10 + 20 + 40 + 80 + ? + ? + ?$

$160 \quad 320 \quad 640$

$a \quad a \quad ar \quad ar^2 \quad ar^3 \quad \cdots \quad ar^{n-1}$

$$S = \frac{a(r^n-1)}{r-1} \qquad r \neq 1$$

intro end of this session

O notation, time complexity, space complexity → next session

in this sisson we focus on the number of iterations.

ex

```
int f(int N){
    s = 0;
    for(int i = 1; i <= N; i++){
        s += i;
    }
    return s;
}
```

#iter. = [1, N]

O(n)

[a, b] b-a+1

1 N

N-1+1 = N ✓ → ans

around 10 quiz!

Please do not answer in chat, wait for the quiz and answer from the quiz choices 🙏

Quiz4

```
for(i=1; i<=N; i++)
{
    S=S+i;
}
```

T

i	S
1	...
2	...
3	...
4	...
5	...
6	

5

N=5

[1, n]

ans = n

O(n)

Please do not answer in chat, wait for the quiz and answer from the quiz choices 🙏

Quiz5

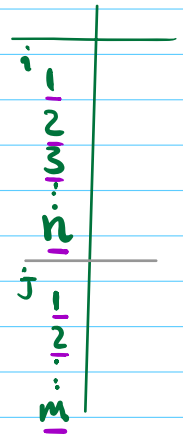
↓

remove 0 from answers

```
func(){
    for(int i = 1; i <= N; i++){
        if(i % 2 == 0){
            print(i);
        }
    }
    for(int j = 1; j <= M; j++){
        if(j % 2 == 0){
            print(j);
        }
    }
}
```

n iteration

m iteration



ans = n+m

O(n+m)

iterations

0 based with #

```
for(int i = 0; i <= 100; i++){
    s = s + i + i^2;
}
```

[a, b] b-a+1

[0 100]

101

O(1)

Constant Complexity

101 × n^0

Quiz6

```
for(i=1; i*i <= N; i++)
{
    X++
    ...
}
```

√16 = 4

i	i x i	N → 16
1	1	✓
2	4	✓
3	9	✓
4	16	✓
5	25	x

i = 1, 2, ..., î → î x î <= n ⇒ î^2 <= n ⇒ i <= √n = ans

O(√n)

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Quiz 7

```
N>0
i=N;
while(i>1)
{
    i=i/2;
}
```

i_a → n, i_a ; i_b → i_b

i_a	itr#	i_b
$n/2^0$	1	$n/2$
$n/2^1$	2	$n/4$
$n/2^2$	3	$n/8$
$n/2^3$	4	$n/16$
⋮	⋮	⋮
	k	1
		0

$\frac{n}{2^k} = 1$; $n = 2^k$; $k = \log_2 n$; int(1/2) = 0

$O(\log_2 n)$; $\log_2 n$

Quiz 8

```
for(i=0; i<=N; i = i*2)
{
    ...
}
```

i_a ; i_b → 0×2

O notation not defined

i_a	itr	i_b	n = 10
0	1	0	
0	2	0	
0	3	0	
0	4	0	
	⋮		
	$+\infty$ → ans		

Quiz 9

```
for(i=1; i<N; i=i*2)
{
    X++
    ...
}
```

i_a ; i_b

i_a	itr	i_b	
1	1	2	2^1
2	2	4	2^2
4	3	8	2^3
8	4	16	2^4
⋮	⋮	⋮	⋮
	k → ans	N	2^k
		N < i_b	

$N = 2^k$ (k → b, 2 → a) ; $k = ?$; $k = \log_N$ (N → c) ; $b = \log_a c$

$k = \log_2 N$ ans

$O(\log_2 N)$

break

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Quiz 10

```
for(i=1; i<=10; i++){
    for(j=1; j<=N; j++){
    / X++ ........./
    }
}
```

[1,N] = N-1+1 = N

10 × N = ans

O(N)

i	j
1	[1,N] +
2	[1,N] +
3	[1,N] +
⋮	
10	[1,N] +
11	x

Quiz 11

```
for(i=1; i<=N; i++){
    for(j=1; j<=N; j++){
        ...
    }
}
```

ans = N × N = N^2

$O(N^2)$

i	j
1	[1,N] → N itr
2	[1,N]
3	[1,N]
⋮	⋮
N	[1,N]

N

Quiz 12

```
for(i=1; i <= N; i++){
    for(j=1; j <= N; j = j*2){
        ...
    }
}
```

Q9: inner loop → $\log_2 N$

ans = $N \times \log_2 N$

$O(N \log_2 N)$

i	j
1	$\log_2 N$ +
2	$\log_2 N$ +
⋮	
N	$\log_2 N$ +

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Quiz 13

```
for(i=1; i<=N; i++){
    for(j=1; j<=(2^i); j++)
    {
        X++
        ...
    }
}
```

i	j	itr
1	[1, 2]	2
2	[1, 2²]	4
3	[1, 2³]	8
⋮	⋮	⋮
N	[1, 2^N]	2^N

$S = 2 + 4 + 8 + \cdots + 2^N$

$= 2^1 + 2^2 + 2^3 + \cdots + 2^N$

$a = 2$, $r = 2$

$S = \frac{a(r^n - 1)}{r - 1}$

$\frac{2(2^n - 1)}{2 - 1} = 2^{n+1} - 2$ ← and

$O(2^n)$

$2^1 \times 2^n - 2$

Quiz 14

45 sec is not enough

```
for(i=N ; i > 0 ; i = i/2 ) {
    for(j = 1; j <= i ; j++) {
        // print(i+j) //
    }
}
```

i	j	itr
N	[1, N]	N
N/2	[1, N/2]	N/2
N/4	[1, N/4]	N/4
⋮		⋮
1	[1, 1]	1

x x

$S = N + \frac{N}{2} + \frac{N}{4} + \cdots + \frac{N}{N}$

$= \frac{N}{2^0} + \frac{N}{2^1} + \frac{N}{2^2} + \cdots + \frac{N}{2^k}$ } 1 | $N =$

$= N \times 2^{-0} + N \times 2^{-1} + N \times 2^{-2} + N \times 2^{-3} + \cdots + N \times 2^{-k}$

$= N\left(\frac{1}{2^0} + \frac{1}{2^1} + \cdots + \frac{1}{2^k}\right) = N\left[\frac{1\left(1 - \left(\frac{1}{2}\right)^{k+1}\right)}{1 - \frac{1}{2}}\right] = 2N\left(1 - \frac{1}{2^{k+1}}\right)$

$S = \frac{a(r^n - 1)}{r - 1}$

$a = 1$, $r = \frac{1}{2}$

$\frac{\frac{1}{2}}{\frac{1}{2}} = 2$ ✓

$= 2N\left(1 - \frac{1}{2^k \times 2}\right) = 2N\left(1 - \frac{1}{N \times 2}\right)$

($2^k = N$)

$= 2N\left(\frac{2 \times N - 1}{2 \times N}\right) = 2N - 1$

$O(N)$

$\lim_{n \to \infty}$

Big O notations

why? math proof?

1) # iterations based on input size
2) omit all lower terms
3) omit all constants coeff.

ex $2n^3 + 3n^2 + 5n^0$ iterations $O(n^3)$

ex $4n^2 + 3n + 6\sqrt{n} + 8\log n + 90n^0$ ($\sqrt{n} = n^{1/2}$) $O(n^2)$

Common expression orders:

$$\log n < \sqrt{N} < N < N\log n < N\sqrt{N} < N^2 < N^3 < \cdots < 2^n < N! < N^N$$

2^{10} 1024

$S = a + a+d + a+2d + a+3d + a+4d + \cdots + a+(n-1)d$

$S = a+(n-1)d + \cdots + a+2d + a+d + a$

$a+(n-1)d$

$(2a+(n-1)d) + (2a+(n-1)d) + \cdots + \cdots + (2a+(n-1)d)$

$(n) \times (2a+(n-1)d) = 2S$

$S = \dfrac{n[2a+(n-1)d]}{2}$

$\text{Sum} = \frac{n}{2}[2a+(n-1)d]$

$S = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$

both sides $\times r$ $\times r$

$rS = ar + ar^2 + ar^3 + \cdots + ar^n$

$rS - S = -a + ar^n = ar^n - a$

$S(r-1) = a(r^n-1) \Rightarrow S = \dfrac{a(r^n-1)}{r-1}$ ✓

```
for( i=1; i<=N; i+=2){
   for(j=1; j<=i ;j++)
```

1 3 5 7 ... N