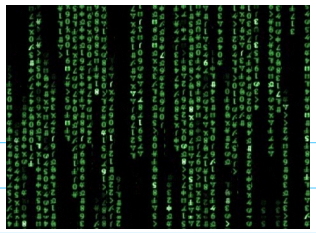
7:05AM IST



2D arrays:

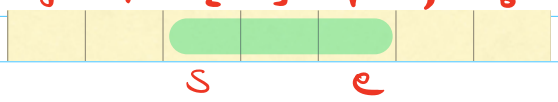
1 - sub matrix sum queries → prefix sum 2D version

2 - sum of all sub matrices

3 - Maximum submatrix sum in Rowwise, Colwise sorted matrix

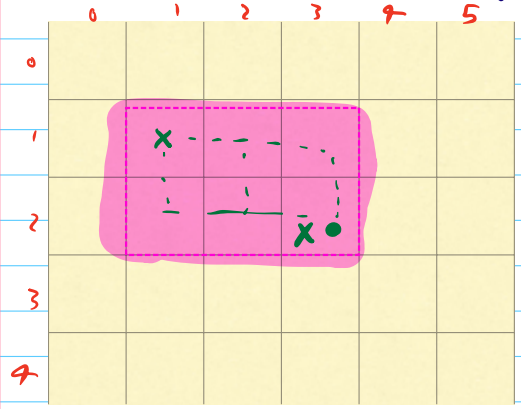
4 - Search in Rowwise, Colwise sorted matrix

Sub array? Continous part of an array



S ← 2,4 → e

Sub matrix? Continous part of a matrix



5x6

{(1,1),(2,3)}
top-left → bottom-right

P1.1 Given a matrix of integers ($M \times N$), find the sum of all the elements in a given sub matrix.

ex

	0	1	2	3	4
0	1	3	5	2	-1
1	4	8	5	0	6
2	10	20	-1	3	5
3	1	5	-5	10	6

{(1,0),(3,2)}

si ei sj ej

{(1,1),(2,4)}

(8+5+0+6) 19

+(20+(-1)+3+5) 27

46 ← ans to Q1

Q1

```
sum=0
for(i=si; i<=sj; i++)
  for(j=ei; j<=ej; j++)
    sum+=a[i][j]
ret sum;
```

TC: O(M x N)

P1.2 Given a matrix of integers (M, N), find the sum of all the elements for multiple submatrix queries (Q).

ex

	0	1	2	3	4
0	1	3	5	2	-1
1	4	8	5	0	6
2	10	20	-1	3	5
3	1	5	-5	10	6

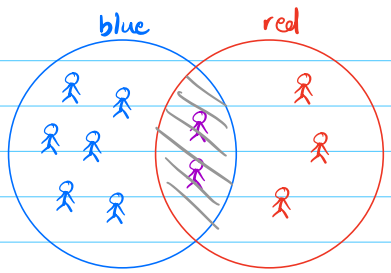
brute force:

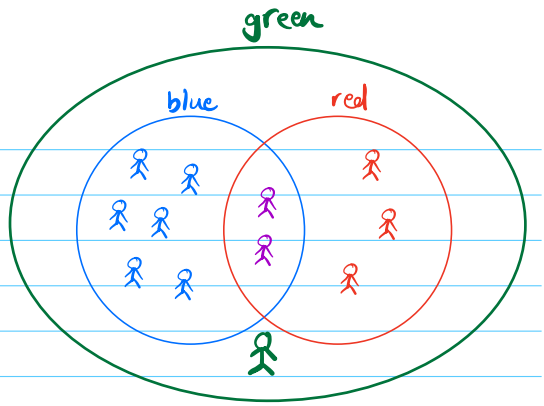
TC: $O(Q \times M \times N)$

similar idea to P.S. in 1D to optimize

inclusion-exclusion principal

detour





👤 = # 🔵 + # 🔴 - # 🔵🔴

8 + 5 - 2 = 11

(13)

🔵 + # 🔴 - # 🔵🔴 + # 🟢

= 12

Prefix sum for 2D matrix

1- how it looks like? visualization, understanding

2- how to calculate? formula

3- how to use? use it to solve problem

1- how it looks like? ✓

a

	0	1	2
0	1	2	3
1	4	5	6

Prefix sum
2D version

PS

	0	1	2
0	ⓐ 1	ⓑ 3	ⓒ 6
1	ⓔ 5	ⓓ 12	ⓕ 21

PS(i,j) → Sum(0,0),(i,j)

ⓐ (0,0),(0,0)

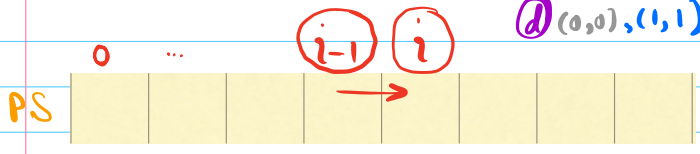
ⓑ (0,0),(0,1)

ⓒ (0,0),(1,0)

ⓓ (0,0),(1,1)

ⓔ (0,0),(0,2)

ⓕ (0,0),(1,2)



PS[i] = PS[i-1] + a[i]

2- how to calculate?

A

	0	1	2	3	4
0	1	3	5	2	-1
1	4	8	5	0	6
2	10	20	1	3	5
3	1	5	-5	10	6

PS →

	0	1	2	3	4
0	1	4	9	11	10
1	5	16	26	28	33
2	15	...	...	...	...
3	16	...	...	...	...

□ = □ + □ − □ + a[3][3]

PS[3][3] = P[2][3] + P[3][2] − P[2][2] + a[3][3]

a[0][0]

PS

linear PS...

TC: O(M×N)

i>=1, <n
j>=1, <m

P(i,j) = P(i-1,j) + P(i,j-1) − P(i-1,j-1) + a[i][j]

{
P(0,0) = a[0][0]
P(0,j) = P(0,j-1) + a[0][j]
P(i,0) = P(i-1,0) + a[i][0]
}

3- how to use?

this is PS not the original array

	0	1	2	3	4
0	1	4	9	11	10
1	5	16	26	28	33
2	15	46	57	62	72
3	16	52	58	73	89

i1 ... i2

P[i2] − P[i1-1]

□ = □ − □ − □ + □

3A

any generic sub matrix sum {(si,sj), (ei,ej)} =

P[ei][ej] − P[si-1][ej] − P[ei][sj-1] + P[si-1][sj-1]

P2 find sum of all submatrices of a given matrix & return its sum?

ex

	0	1
0	9	6
1	5	4

9 + 6 + 5 + 4

14 + 10 + 15 + 9

24

ans

Q2 how many submatrix do we have for a[m][n]?

$\frac{n(n+1)}{2}$

top left (s_i, e_i)

(s_j, e_j) bottom right

$\frac{n(n+1)}{2} \times \frac{m(m+1)}{2}$

↳ total number of sub-mat.s

$m \times n$

3×4

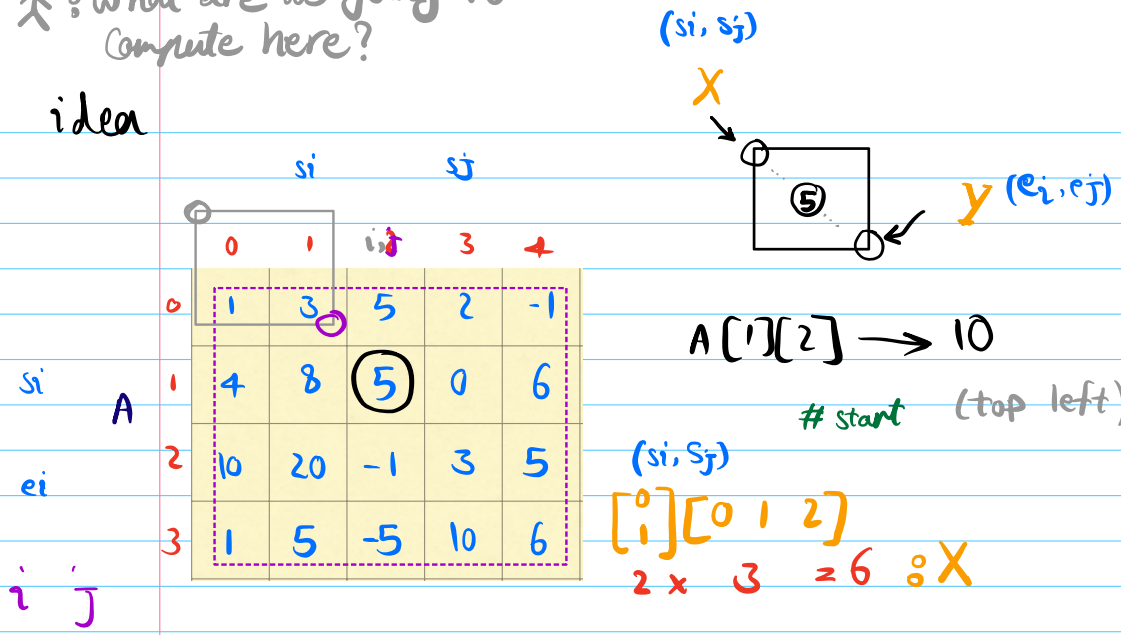
$\frac{m(m+1)}{2}$

60

$O(M \times N + Q) \sim O(M \times N + M^2 \times N^2) \sim O(M^2 N^2)$

: what are we going to compute here?

idea



total : X * Y = 6x9=54

a[i][j] → all submatrix that include a[i][j]

possible ways top left

X → [0 i] [0 j]

(i+1) x (j+1)

possible ways bottom right

[i, n-1] [j, m-1]

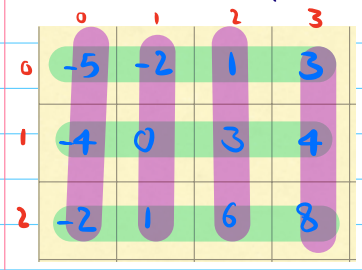
(n-i) x (m-j) : Y

S_{ij} = contribution of a[i][j] = (i+1) x (j+1) x (n-i) x (m-j)

a[i][j] x S_{ij}

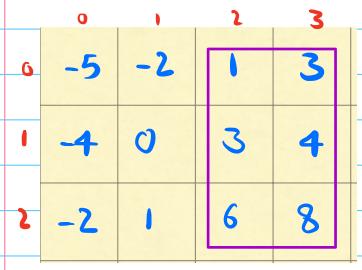
O(MxN)

Rowwise Columnwise sorted matrix:

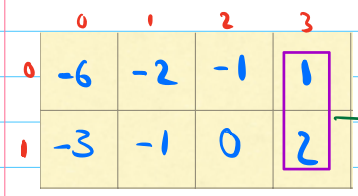


P3 Find maximum non-empty submatrix sum in a row wise column wise sorted matrix.

ex



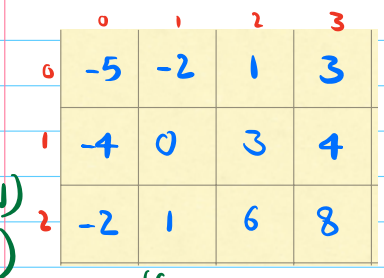
Q3



→ ans

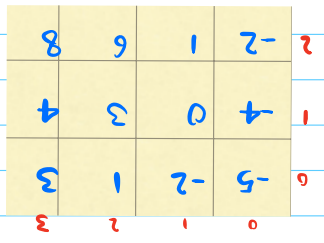
idea1: PS all sub matrix $\frac{n(n+1)}{2} \times \frac{m(m+1)}{2}$ $O(N^2 \times M^2)$

idea2:



suffix sum

rotate



Q4
TC: O(MxN)
SC: O(MxN)

true / false

P4 Check if a given int is present in a row wise column wise sorted matrix.

ex

	0	1	2	3
0	-5	-2	1	3
1	-4	0	3	4
2	-2	1	6	8

ex1: 4 ✓

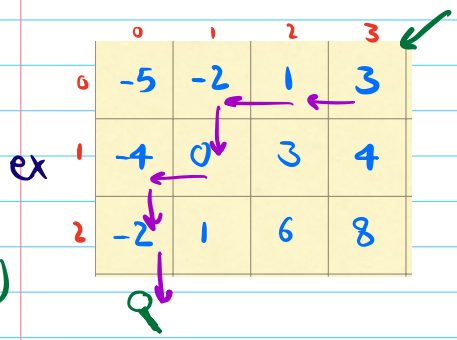
ex2: -4 ✓

ex3: -3 ✗

idea 1 $O(M \times N)$

idea 2

ex



$O(M+N)$

Q5

TC 8