

Bit manipulation 2

- single element ✓
- single element 2 ✓
- single element 3
- Max "AND" Pair

7:05AM IST

6:35PM PST

recap :

Given an array of integers, where every element appears twice except for one element which appears once, find that unique element :

ex $a[] = \{ \underline{6}, \underline{9}, \underline{6}, \underline{10}, \underline{9} \}$ $ans = 10$

ex $a[] = \{ \underline{2}, \underline{3}, \underline{5}, \underline{6}, \underline{3}, \underline{6}, \underline{2} \}$ $ans = 5$

```
int findUnique(int a[]) {
    int n = a.Length;
    ans = 0;
    for (i = 0; i < n; i++) {
        ans ^= a[i]
    }
    return ans;
}
```

TC: $O(n)$
SC: $O(1)$

P1 Given an array of integers⁽⁺⁾, where every element appears twice except for two elements which appear once, find the two unique elements: $a[i] \leq 10^9$

ex $a = \{ \underline{2}, \underline{3}, \underline{2}, \underline{5}, \underline{3}, \underline{6}, \underline{7}, \underline{6} \}$ $\{ 5, 7 \}$

ex $a = \{ \underline{1}, \underline{2}, \underline{3}, \underline{1} \}$ $\{ 2, 3 \}$

idea 1

Brute force

for $i = 0 \rightarrow n-1$

for $j = 0 \rightarrow n-1$ $i \neq j$

find dup (i, j)

TC & SC

TC: $O(n^2)$

SC: $O(1)$

2: 0010
3: 0011
5: 0101

6: 0110
7: 0111
1: 0001

0010 8: 2
3: 2 10
b

idea 2

ex a: {2, 3, 2, 5, 3, 6, 7, 6, 1, 1}

a? b? XOR

$$(5 \oplus 7) = 2$$

$$9 \oplus 11 = 2$$

XOR a[i]

$$4 \oplus 6 = \begin{array}{r} 0100 \\ 0110 \\ \hline 0010 = 2 \end{array}$$

① a? ≠ b?

② a? ^ b? ≠ 0

③ a?

groups?
extra mem?

XOR1 ← group A: {a?, 5, 1, 1}

XOR2 ← group B: {b?, 2, 3, 2, 3, 6, 7, 6}

c: 00...1...100
2 10
b

How to find exactly which bit?

Pair<int,int> GetTwoUnique(int a[])

n = a.Len; XOR = 0; → c

for(i=0; i<n; i++) { // a ^ b = XOR

n | XOR = XOR ^ a[i];

XOR all

b = -1

for(i=0; i<32; i++){

if((XOR >> i) & 1 == 1){

b = i;
break;

}

XOR1 = 0 XOR2 = 0

for(i=0; i<n; i++){

if((a[i] >> b) & 1 == 1)

XOR2 = XOR2 ^ a[i]

else

XOR1 = XOR1 ^ a[i]

}

ret pair (XOR1, XOR2)

}

Quiz

Tc: O(n)

Sc: O(1)

P2 Given an array of integers, where every element appears ^{3 times} ~~trice~~ except for one element which appears once, find that unique element:

$$0 \leq a[i] < 10^9$$

ex $A: \{2, 3, 6, 2, 2, 3, 3\}$ ans=6

ex $A: \{5, 7, 5, 9, 7, 11, 11, 7, 5, 11\}$ ans=9

idea 1

for $i = 0 \rightarrow n-1$

for $j = 0 \rightarrow n-1$ $i \neq j$

Count how many times $a[i]$ repeated

$(a[i] == a[j])$

SC: $O(1)$

TC: $O(n^2)$

{5, 7, 5, 9, 7, 11, 11, 7, 5, 11}

	3	2	1	0
5	0	1	0	1
7	0	1	1	1
5	0	1	0	1
9	1	0	0	1
7	0	1	1	1
11	1	0	1	1
11	1	0	1	1
7	0	1	1	1
5	0	1	0	1
11	1	0	1	1

number of repeated
← R

$$\text{GCD}(R, P) = 1$$

add all
bit in Col 2 ← $A_i / R = r$

optional

number of single outlier
reent

~~3x+2~~

4 6 6 10
3x+1 3x 3x 3x+1

7.3

Quiz

Tc: $O(n)$

Sc: $O(1)$

int findUnique3(int a[])

ans = 0

for(i=0; i<32; i++)

Count = 0

for(j=0; j<n; j++)

if(checkBit(a[j], i)) Count++

if(Count % 3 == 1)

ans = ans | (1 << i)

// setBit(ans, i)

ret ans

}

size of var in bit length

P3 Given an array^a of integers, find max value of
 $\max(a[i] \& a[j]), i \neq j, 1 \leq a[i] \leq 10^9$

ex $a = \{16, 9, 11, 10\}$

$16 \& 9 = 0$ $9 \& 11 = 9$ $11 \& 10 = 10$
 $16 \& 11 = 0$ $9 \& 10 = 8$
 $16 \& 10 = 0$

$a \& b = b \& a$
 $16 = 10000$
 $9 = 01001$
 $11 = 01011$
 $10 = 01010$

$\max(a[i], a[j]) = 10 \rightarrow \text{ans}$

idea 1

brute force

$TC: O(n^2)$
 $SC: O(1)$

for $i = 0 \rightarrow n-1$
 for $j = i+1 \rightarrow n-1$
 $\text{ans} = \text{Max}(\text{ans}, a[i] \& a[j])$
 $a_i \quad a_j$

idea 2

① $8 > 7$
 1000
 0111
 MSB
 ② $8 \& 11$
 1000
 0111
 1000
 0111
 ③ $7 \& 15$
 0011
 0011
 0111

ex $a = \{26, 13, 23, 28, 27, 7, 25\}$

	4	3	2	1	0
26	1	1	0	1	0
13	0	1	1	0	1
23	1	0	1	1	1
28	1	1	1	0	0
27	1	1	0	1	1
7	0	0	1	1	1
25	1	1	0	0	1
	1	1	0	1	0

$\max(a_i \& a_j) = \text{ans}$

10110

$\rightarrow \text{ans}$

$O(n)$
TC: $O(30n)$
SC: $O(1)$

```
int maxAnd(int[] a){
```

```
    n = a.length
```

```
    ans = 0
```

```
    for(b = 30; b >= 0; b--){
```

```
        count = 0
```

```
        for(i = 0; i < n; i++){
```

```
            count += (a[i] >> b) & 1
```

```
        }  
        if(count >= 2){
```

```
            ans = ans | (1 << b)
```

```
            for(i = 0; i < n; i++){
```

```
                if((a[i] >> b) & 1 == 0){
```

```
                    a[i] = 0
```

→ makes gray

30

```
        }  
    }  
    return ans
```

check bit?

$N \& (1 \ll i) \neq 0$