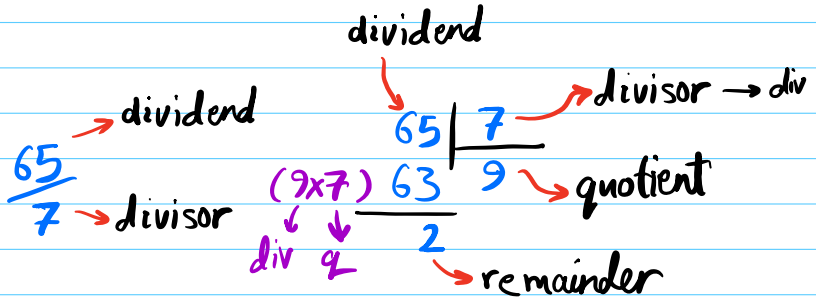
Topics: - %. operator
- python vs c/c++/java...
- mod arithmatics
- 2 problems

Modular Arithmatics



modulo operator ← %. basics

n %. a = Remainder whe dividing n by a ✓

dividend = quotient x divisor + remainder
9 x 7 + 2 = 65

r = dividend − greatest mult. of div <= dividend
65 63

10 %. 4 = 2 = 10 − (greatest mult. of 4 <= 10) = 2
8

13 %. 5 = 3 = 13 − (greatest mult. of 5 <= 13) = 3
10

Quiz $150 \,\%\, 11 = 150 - (\text{greatest mult. of } 11 \le 150) = 7$ (143)

Quiz $100 \,\%\, 7 = 100 - (\text{greatest mult. of } 7 \le 100) = 2$ (98)

Quiz $-40 \,\%\, 7 = -40 - (\text{greatest mult. of } 7 \le -40) = +2$ (−42)

Quiz

$-42 < -40 < -35$

$7 \times -6 = -42$

$-7 \times 5 = -35$

-5 ✗

Quiz $-60 \,\%\, 9 = -60 - (\text{greatest mult. of } 9 \le -60) = +3$ (−63)

$-9 \times 6 = -54$

$-9 \times 7 = -63$

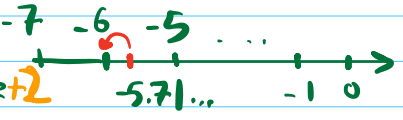
% in Python VS C/C++/Java/C#/JS

	Python	Java
$-40 \,\%\, 7 = +2$	+2	$-5 \to +7 = 2$
$-60 \,\%\, 9 = +3$	+3	$-6 \to +9 = 3$
$100 \,\%\, 7 = +2$	+2	$+2 \to +0 = +2$

```
r = n % a
if(r<0)
  r = r + a
:
```

Python $-40 - \left\lfloor \frac{-40}{7} \right\rfloor \times 7$ → floor

$-40 - (-5.71\ldots) \times 7 = -40 - (-42) = +2$ ($-5.71\ldots \to -6$)



Java $-40 - \left\lfloor \frac{-40}{7} \right\rfloor \times 7 = -40 - (-5 \times 7) = -40 - (-35) = -5$ ($-5.71\ldots$)

Why using %? Applications?

limit input/output to a required range

hashing ADS

LLD/HLD
Consistent hashing

MD5
SHA 256
HTTPS
SSH

10^9+7
largest prime num. fits in int

998789... $\begin{matrix}-\infty \\ +\infty\end{matrix}$ | %10 → [0-9]

$\begin{matrix}-\infty \\ +\infty\end{matrix}$ | %P → [0 to P-1]

$(2+3)\%7 = [(2\%7)+(3\%7)]$ ✓

Modular Arithmetics (%)

Combine with +, −, *, /

① $(a+b)\%P = [(a\%P)+(b\%P)]\%P$

a	b	P
8	6	10
5	4	6

$14\%10 \neq 14[8\%10 + 6\%10]$

$9\%6 \neq [5\%6 + 4\%6] \rightarrow 9$

② $(a*b)\%P = [(a\%P)*(b\%P)]\%P$

$(6\times8)\%10$ → 8

$[(6\%10)*(8\%10)]\%10$ → 8

Advance DSA class

combine with another %

III $(a \,\%\, p) \,\%\, p = a \,\%\, p$

$[0, p-1]$

IV $(\overbrace{a \,\%\, p}^{X} * \overbrace{b}^{Y}) \,\%\, p \overset{\checkmark}{=} (a * b) \,\%\, p$ — why?

let $X = a \,\%\, p$, $Y = b$

$(X * Y) \,\%\, p \overset{II}{=} [(X \,\%\, p) * (Y \,\%\, p)] \,\%\, p$

$= ((a \,\%\, p) \,\%\, p * b \,\%\, p) \,\%\, p$ (III)

$= (a \,\%\, p * b \,\%\, p) \,\%\, p$ (II)

$= (a * b) \,\%\, p$

Quiz

Divisibility Rules

2, 3, 4, 5, 6, ~~7~~ → optional HW
8, 9

✓3 ⁞3 : sum of digits should be divisable by 3

✓9 ⁞9 : sum of digits should be divisable by 9

✓4 ⁞4 : Last 2 digits should be divisable by 4

✓8 ⁞8 : Last 3 digits should be divisable by 8

---

intuiton for ⁞3

$(2475)_{10} \,⁞3 = (2\times10^3+4\times10^2+7\times10^1+5\times10^0)\,⁞3$

$=((2\times10^3)⁞3+(4\times10^2)⁞3+(7\times10^1)⁞3+(5\times10^0)⁞3)\,⁞3$

$[2⁞3\times10^3⁞3]\,⁞3$

$=(2⁞3\times1+4⁞3\times1+7⁞3\times1+5⁞3\times1)\,⁞3$

$=[(2+4+7+5)⁞3]⁞3=(2+4+7+5)⁞3$

obs1	obs.2
$10⁞3=1$	$10⁞9=1$
$100⁞3=1$	$100⁞9=1$
$10000⁞3=1$	$10000⁞9=1$
$10^n⁞3=1$	$10^n⁞9=1$

intuiton for ⁞4

$(2475)_{10} \,⁞4 = (2\times10^3+4\times10^2+7\times10^1+5\times10^0)\,⁞4$

$=((2\times10^3)⁞4+(4\times10^2)⁞4+(7\times10^1)⁞4+(5\times10^0)⁞4)\,⁞4$

$=75⁞4$

obs3.

$10⁞4=2\neq0$

$100⁞4=0$

$1000⁞4=(100\times10)⁞4=0$

obs 4

$10 \,\%\, 8 = 2 \neq 0$

$100 \,\%\, 8 = 4 \neq 0$

$1000 \,\%\, 8 = 0$ all multiples of 1000 are divisible by 8

estimates

reminder * int (signed 32 bit range) $\sim +2 \times 10^9$ to -2×10^9

from last session * long (signed 64 bit range) $\simeq +9 \times 10^{18}$ to -9×10^{18}

P1 Given int a, n, p, calculate $a^n \,\%\, p$ without built-in functions (inbuilt)

Constraints: $1 <= a <= 10^9$, $2 <= p <= 10^9$, $1 <= n <= 10^5$

$\left[\underbrace{(10^9)^{100000}}_{x} \right] \,\%\, 10^9 \rightarrow$ int

5 min ideas?

$x = (10^9 \times 10^9 \times \cdots \times 10^9)$ (100000 terms)

$a^n \% p = (a \times a \times a \times \cdots \times a) \% p$ (n times)

$n \leftarrow 4$, $a \leftarrow 2 \% 3 \rightarrow p = 1$

```
int powmod1(int a, int n, int p){
    for(i=1;i<=n;i++){
        a = a*a   → ✗
    }
    ret a%p   → overflow before hand
}
```

wrong

i	a → 2
1	4
2	16
3	256
4	2^{16}

$2^{16} \% 3$

```
int powmod2(int a, int n, int p){
    long ans = 1
    for(i=1;i<=n;i++){
        ans = (ans*a)%p
    }
    ret ans%p
}
```

→ extra

Correct

O(n) : TC

$1 <= a <= 10^9$
$2 <= p <= 10^9$
$1 <= n <= 10^5$

$[(ans \% p) * (a \% p)] \% p$

$10^9 \quad 10^9 \quad 10^{18}$

P2 Given a number in array format a[], calculate a[] %. P. Each a[i] represent a single digit

Constraints:

$1 <= N <= 10^5$

$0 <= a[i] <= 9$

$2 <= P <= 10^9$

a[5]:

0	1	2	3	4
6	2	3	4	3

ex , P=49

(62343) %.49 =15 ← ans

int

a[4]:

0	1	2	3
2	4	3	7

ex , P=16

(2437) %.16 →

19 ··· 209453

10000 digits

idea?

hint? digit

100000

6 x 10

$(62343)\%P = (6\times10^4 + 2\times10^3 + 3\times10^2 + 4\times10^1 + 3\times10^0)\%P$

ex

0	1	2	3	4
6	2	3	4	3

a[] →

$a[i]\times10^{n-i-1}$

3×10^0

$+ 4\times10^1$

$+ 3\times10^2$

$+ 2\times10^3$

6×10^4

$(6\times10^4)\%P$

$\%P$

$\%P$

9×10^{10000}

$(6\%P \times 10^{10000}\%P)\%P$

0	1	2	3	4
6	2	3	4	3

$\frac{n(n+1)}{2}$

$O(n) + O(n-1) + O(n-2) + \cdots$

$TC \Rightarrow O(n^2)$

$[6\times10^4 + 2\times10^3 + 3\times10^2 + 4\times10^1 + 3\times10^0]\%P$

$[6\times10^4\%P + 2\times10^3\%P + 3\times10^2\%P + 4\times10^1\%P + 3\times10^0\%P]\%P$

$10^0\%P = t$

$10^1\%P = t$

$(10^1\%P \times 1\%P)\%P$

$10^2\%P = t$

$(10\%P \times 10\%P)\%P$

$10^3\%P = t$

$(10\times10^2)\%P$

$(10\%P)\times(10^2\%P)\%P$

$10^4\%P$

$(10\times10^3)\%P$

$[(10\%P)\times(10^3\%P)]\%P$?

...

```
int arrmod(int a[], int P){
    int n=a.Len
    long t=1
    long sum=0                    // reverse
    for(i=n-1;i>=0;i--)
        sum=sum+(a[i]*t)%P
        t=(t*10)%P = (t*(10%P)%P)
    }
    ret sum%P
}
```

TC: O(n)

SC: O(1)

Len

$1 <= N <= 10^5$

$0 <= a[i] <= 9$

$2 <= P <= 10^9$