Topics: -Bin encoding of nums.
- B2D & D2B
- Bit ops
- ops properties
- Left & right shift

Bit Manipulation 1

Decimal Number system {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} Base 10

$342 \rightarrow 300 + 40 + 2 = 3\times10^{2} + 4\times10^{1} + 2\times10^{0}$

$(2563)_{10} \rightarrow 2000 + 500 + 60 + 3 = 2\times10^{3} + 5\times10^{2} + 6\times10^{1} + 3\times10^{0}$

10 → base

Binary Number System {0, 1} Base 2

$110_{2} \rightarrow 1\times2^{2} + 1\times2^{1} + 0\times2^{0} = 6$

$1011_{2} \rightarrow 1\times2^{3} + 0\times2^{2} + 1\times2^{1} + 1\times2^{0} = 11$

decimal numbers vs binary Numbers counting:

Decimal						
0	10	20	90	100	110	
1	11	21	91	101	111	
2	12	22	92	102	112	...
⋮	⋮	⋮	⋮	⋮	⋮	
9	19	29	99	109	119	

10 base

Binary					
0	10	100	110	1000 → 8	
1	11	101	111	1001 → 9	

$1000_{2} = 1\times2^{3} + 0\times2^{2} + 0\times2^{1} + 0\times2^{0}$

Binary to Decimal 8

ex $10\,11\,0_2 = \underbrace{0\times2^0}_{0} + \underbrace{1\times2^1}_{2} + \underbrace{1\times2^2}_{4} + \underbrace{0\times2^3}_{0} + \underbrace{1\times2^4}_{16} = 22$

(4 3 2 1 0 ←)

2^n	
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
⋮	256
⋮	⋮

Quiz

6 5 4 3 2 1 0
1 0 1 1 0 1 0

0×2^0
$1\times2^1 = 2$
0×2^2
$1\times2^3 = 8$
$1\times2^4 = 16$
0×2^5
$1\times2^6 = 64$

ans = 90

ex 6,5,4,3,2,1,0
1 0 1 1 0 1 0
64 + 16 + 8 + 2 = 90

ex $(1\,0\,2\,0\,1\,0)_2$ ✗ wrong

Decimal to Binary

ex 20

2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

10100
4 3 2 1 0
16 + 4 = 20

2	n	r → remainder
2	q	→ quotient
2		

decimal #

Quiz feedback

more time for Quiz 60+sec

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

101101
5 4 3 2 1 0
32 + 8 + 4 + 1 = 45

Additions

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ } \\ 3\ 6\ 8 \\ +\ 4\ 5\ 3 \\ \hline 8\ 2\ 1 \end{array}$$

$6+3=9$ (I)

$8+3=11$ (II)

(I) $\begin{cases} 0+0=0 \\ 1+0=1 \\ 0+1=1 \end{cases}$

(II) $\{ 1+1=10$

ex:

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ \ } \\ 1\ 0\ 1 \\ +\ 0\ 1\ 1 \\ \hline 1\ 0\ 0\ 0 \\ {\scriptstyle 3\ 2\ 1\ 0} \end{array}$$

$101 \rightarrow 5$

$011 \rightarrow 3$ +

$1000 \rightarrow 8$ ✓

index

	4	3	2	1	0	
		1	1			
	1	0	1	1	0	$\rightarrow 22$
(+)	0	0	1	1	1	$\rightarrow 7$
	1	1	1	0	1	$\rightarrow 16+8+4+1=29$
	4	3	2	1	0	

Quiz

↓

Bug report

Quiz sometimes doesn't show for some users

↓

2 users

base 2 & binary one sigle digit in base 2 & bit

Bitwise operations

→ {AND, OR, NOT, XOR, Left shift, right shift}

& | ! or ~ ^ « »

single bits

addition (+) ignore the carry

A	B	A&B	A|B	!A	A^B
0	0	0	0	1	0
0	1	0	1	1	1
1	0	0	1	0	1
1	1	1	1	0	0

0 ← unset bit

1 ← set bit

0 : false

1 : true

Bitwise operation on decimal numbers:

5 → 101

6 → 110

100 → 4

5 & 6 = 4

A : 20 → 0 1 0 1 0 0 (5 4 3 2 1 0)

B : 45 → 1 0 1 1 0 1

111101 → 61

A|B

20 | 45 = 61

A : 92 → 0 1 0 1 1 1 0 0 (7 6 5 4 3 2 1 0)

B : 154 → 1 0 0 1 1 0 1 0

00011000

A&B = 24

!92 → 0 1 0 1 1 1 0 0

10100011 → 128+32+3 = 163

A : 92 → 0 1 0 1 1 1 0 0

B : 154 → 1 0 0 1 1 0 1 0

178

A^B → 1 1 0 0 0 1 1 0 → 128+64+6

Quiz

60 sec is not enough

A ← 20 → 0 1 0 1 0 0 (5 4 3 2 1 0)

B ← 45 → 1 0 1 1 0 1

111001 (5 4 3 2 1 0)

$0\times2^5+1\times2^4+0\times2^3+1\times2^2+0\times2^1+0$

$1\times2^5+0\times2^4+1\times2^3+1\times2^2+0\times2^1+1$

= 32+16+8+1

= 57 ✓

break

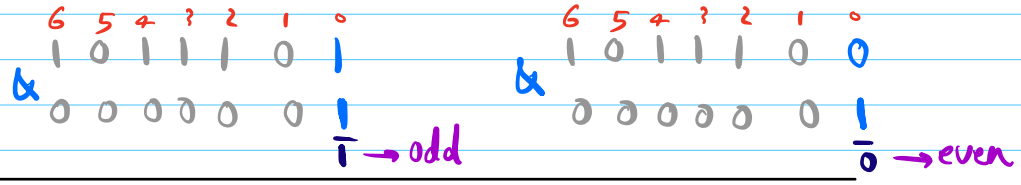
Properties

1) A & 1 = ?

A=10
1 & 1010
0000 → 0

9 1001
& 1 0001
0001 → 1

A & 1 { 0 even, 1 odd ✓



2) A & 0 = 0

A 101
& 000
000

3) A & A = A

101
& 101
101

4) A | 0 = A

101
OR 000
101

5) A | A = A

101
OR 101
101

6) A ^ 0 = A ✓

101
^ 000
101

7) A ^ A = 0

101
101
000

$a+b=b+a$

8) Commutative property

$a \& b = b \& a$

$a \mid b = b \mid a$

$a \wedge b = b \wedge a$

9) Associative Property $(a+b)+c = a+(b+c)$

$(a \& b) \& c = a \& (b \& c) = a \& b \& c$

$(a \mid b) \mid c = a \mid (b \mid c) = a \mid b \mid c$

$(a \wedge b) \wedge c = a \wedge (b \wedge c) = a \wedge b \wedge c$

8* and 9*

ex $a \wedge b \wedge c \wedge a \wedge b = (a \wedge a) \wedge (b \wedge b) \wedge c = 0 \wedge 0 \wedge c$

$a \wedge a = 0$ (7*) $\quad b \wedge b = 0$ (7*) $\quad 0 \wedge 0$ (7* or 6*) $= 0$ (6*) $= c$

idea

$1 \wedge 3 \wedge 5 \wedge 6 \wedge 1 \wedge 5 \wedge 3 = 6$

P1 Given an array of integers, where every element appears twise except for one element which appears onces, find that unique element: SC: O(1)

ex a[]: {6, 9, 6, 10, 9} → ans = 10

ex a[]: {2, 3, 5, 6, 3, 6, 2} → ans = 5

```
int findUnique(int a[]){
    int n = a.Len
    ans = 0
    for(i=0; i<n; i++){
n |     ans ^= a[i]   // ans = ans ^ a[i]
    }
    ret ans
}
```

Quiz*
TC & O(n)
SC: O(1)

Left shift : <<

a << 3 ← shift three times left

int | 4 Bytes | 8 Bytes | 1 Bytes

a = 45

	7	6	5	4	3	2	1	0	
	0	0	1	0	1	1	0	1	= 45
a << 1	~~0~~	0	1	0	1	1	0	1	= 90
a << 2	~~0~~	1	0	1	1	0	1	0	= 180
a << 3	~~0~~	0	1	1	0	1	0	0	= 104

(x2, x2, x2)

3 2 1 0
0000 0010 → 2
0000 0100 → 4

left (<<) is "x 2" shift

180 x 2 = 360 360 ≠ 104

360 = 104 + 256

256 = 2^8

Right shift >>

a = 20	0 0 0 1 0 1 0 0	= 20
a >> 1	0 0 0 0 1 0 1 0 ~~0~~	= 10
a >> 2	0 0 0 0 0 1 0 1 ~~0~~	= 5
a >> 3	0 0 0 0 0 0 1 0 ~~1~~	= 2
a >> 4	0 0 0 0 0 0 0 1 ~~0~~	= 1
a >> 5	0 0 0 0 0 0 0 0 ~~1~~	= 0

(/2, /2, /2, /2, /2)

$\log_2 n$ ← right shift − 1