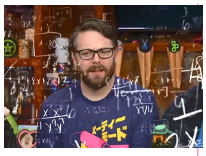
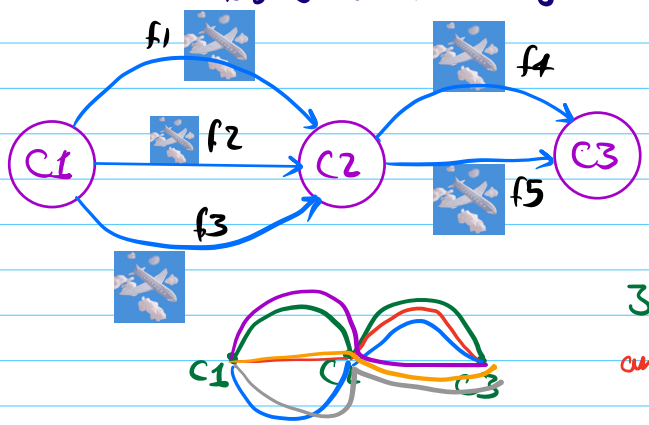


Combinatorics: is about counting!

- 3 introductory quiz
- Permutation
- Combination
- Properties

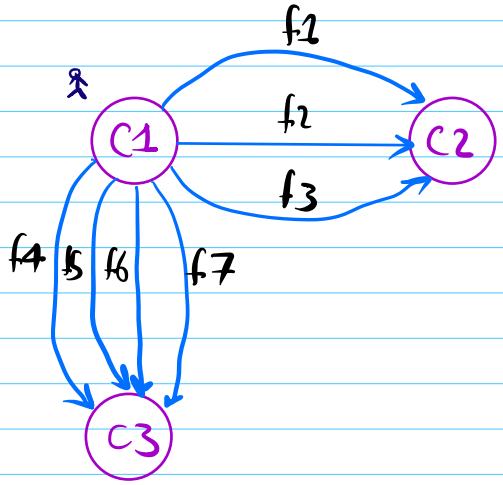
P1- pascal Δ C(n,r) i.M
P2- C(n,r) i.P
P3- Excel Columns

Q1



3x2=6
and

Q2



3+4
or

Q3 In an exam → 3 T/F Questions
→ 2 multiple choice (4 option, only one correct answer)

ex
2x2x2=8

Q										
1	T/F	①	T	T	T	T	F	F	F	F	
2	T/F	②	T	T	F	F	T	T	F	F	#8
3	T/F	③	T	F	T	F	T	F	T	F	

+

Q		
4	a,b,c,d → 4	x → 16#
5	a,b,c,d → 4	

8+16=24

Permutation: ordered arrangement

* what are the number of way (i.e. number of permutations) of arranging n distinct elements?

number of distinct object

ex $n=3$ a, b, c

abc bac cab ans=6

acb bca cba

Q4 $n=4$ | d a t e

$4 \times 3 \times 2 \times 1 = 4! = 24$

$n \longrightarrow n!$

* Find the number of ways to arrange r out of n distinct element.

ex $n=3$ a, b, c

$r=2$

ab ba ca

ac bc cb

$6 = 3 \times 2 \times \cdots$

Q5 $n=4$ a, b, c, d

$r=2$

$4 \times 3 = 12$

$r \leq n$

$n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) \times (n-r) \times \cdots \times 2 \times 1$

n r

0 1 2 $r-1$ 0

* $P(n,r) = \dfrac{n!}{(n-r)!} = {}_nP_r = {}^nP_r = P^n_r$ different notations

choose
Pick ← Combinations: Selection, order doesn't matter

Find the number of ways to select r out of n

ex $n=3$, $r=2$ — a, b, c — ab, cb, ac

$C(n,r) \rightarrow {}_nCr,\ C^n_r,\ {}^nC_r$

① n items → (order) arrange r out of n → $P(n,r)$

=

② n items → select r out of n → $\frac{P(n,r)}{r!}$

ex $n=4$, $r=3$ — a, b, c, d

$\frac{P(4,3)}{3!}$

{a|b|c} {a|c|d} {b|c|d} {a|b|d} — each $3!$

$C(n,r)$ formulas

$P(n,r) = C(n,r) * r!$

$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!} \times \frac{1}{r!} = \frac{n!}{(n-r)!\,r!}$

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

Properties of combinations

(I) # of ways I can select 0 (zero) items out of n

$C(n,0) = 1$

$0! = 1$

(II) # of ways I can select n items out of n

$C(n,n) = 1$

(III) # of ways to select $n-r$ items out of n

$C(n,r) = C(n,n-r)$

$C(10,2) = \frac{10!}{8!\,2!} = \frac{10!}{2!\,8!} = C(10,8)$ (r ↔ $n-r$)

$C(n,r) = \frac{n!}{(n-r)!\,r!}$

Q6 (IV) $\underbrace{C_0^n + C_1^n + C_2^n + \cdots + C_n^n}_{\text{(II)}} = 2^n$ (I)

$\{1,2,3\}$ ✓ ✓ ✓ / x x x

$\{\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$

$C(n',r') = C(n'-1, r'-1) + C(n'-1, r')$

$n' = n+1$, $r' = r+1$

(V) $C(n,r) + C(n,r+1) = C(n+1, r+1)$

$= \frac{n!}{(n-r)!\,r!} + \frac{n!}{(n-r-1)!\,(r+1)!}$ ($(n-r)!$ → $(n-r)(n-r-1)!$; $r!$ → $(r+1)\times r!$)

$5! = 5\times 4!$

@ $n! = n\times(n-1)!$

(b) $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$

verify by yourself

$= \frac{n!}{r!\,(n-r-1)!} \times \left[\frac{1}{(n-r)} + \frac{1}{(r+1)}\right] = \frac{n!}{r!(n-r-1)!}\left[\frac{(r+1)+(n-r)}{(n-r)(r+1)}\right]$ ($n+1$)

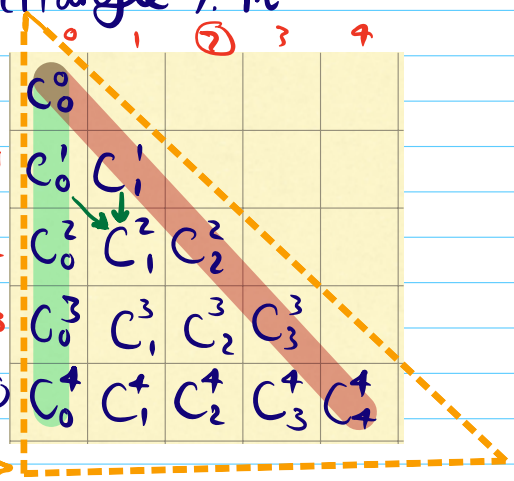
$= \frac{(n+1)!}{(r+1)!\,(n-r)!}$

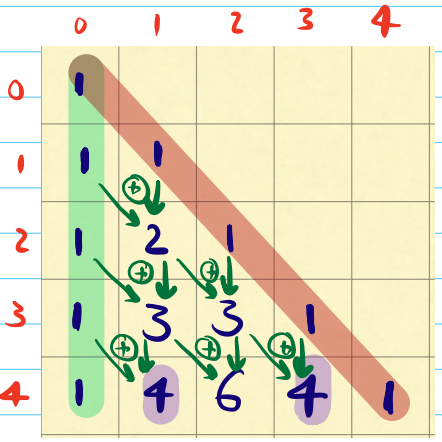
P1 Pascal Triangle: For given n, print n rows of Pascal triangle % m

n = row
r = column

ex 0-4 n=4

n+1





ideal $C_r^n = \frac{n!}{r!(n-r)!}$

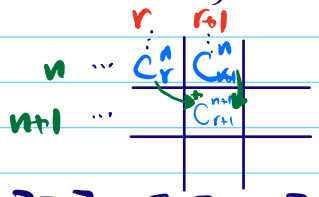
n=30

$5! = 120 \rightarrow$ 3 digits

$10! =$ 7 digits

$20! =$ 19 digits

$C(n,r) + C(n,r+1) = C(n+1, r+1)$



$C[n][r] + C[n][r+1] = C[n+1][r+1]$

$C[n-1][r-1] + C[n-1][r] = C[n][r]$

$n \rightarrow n-1$
$r \rightarrow r-1$

Q7

TC: $O(n^2)$

SC: $O(n^2)$

HW → SC → O(n) reduce ↗

```
void printPascalTriangle(int n){
    long [][] C = new long[n+1][n+1]
    print("1 \n") // C[0][0]  → new line
    for(i=1; i<=n; i++){ //n lines
        C[i][0] = C[i][i] = 1
        print(C[i][0])
        for(int j=1; j<i; j++){
            C[i][j] = (C[i-1][j-1] + C[i-1][j]) % m
            print(C[i][j])
        }
        print(C[i][i] + "\n");
    }
}
```

	j
i=1	1
i=2	2
⋮	⋮
i=n	n

% M

P2 Find C(n,r) % P, P is Prime number

recap from mod

$C(n,r) \% P = \frac{n!}{r!(n-r)!} \% P$

$= \left((n! \% P) \times (r! \% P)^{-1} \times ((n-r)! \% P)^{-1} \right) \% P$

① ② ②' ③

$(a/b) \% P$

$a^{P-1} \% P = 1 \Rightarrow a^{P-2} \% P = (a \% P)^{-1}$

$\frac{1}{a \% P} = ?$

$1/(a \% P)$ $1/(a \% P)$

needs for loop

np ← ① $n! \% P \rightarrow O(n)$

rp ← ② $r! \% P \rightarrow O(r) < O(n) \rightarrow$ fastPower(rp, P-2, P) ②' $O(\log(P))$

nrp ← ③ $(n-r)! \% P \rightarrow O(n-r) < O(n) \rightarrow$ fastPower(nrp, P-2, P) ③' $O(\log(P))$

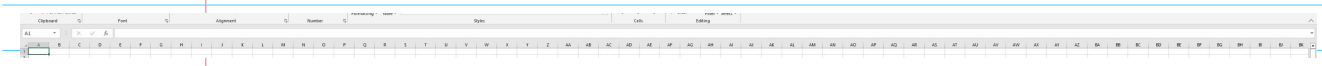
$C(n,r) \% P = [① \times ②' \times ③'] \% P$

vs Pascal Method

TC: $O(3n + 2\log P)$

$O(n + \log P)$

P3 Find Nth column title of a spread sheet:



A, B, C, ..., Z, AA, AB, ..., AZ, BA, BB, ..., ZZ, AAA, AAB
1 2 3 26 27 28 52 53 54 ...

ex N=30 → AD (BD, AC 29, AD 30)

N=50 → AX (AX 50, AY 51)

N=100 → CV

base 26: A ... Z (26); AA ... ZZ (26); AAA ...

base 10: 0 ... 9; 10 11 ...

hint

TC

SC

O(log_26 n)

O(...)

30 | 26 → 1 -1 → A; 4 -1 → D → AD

100 | 26 → 3 -1 → C; 22 -1 → V → CV

0 | 25 → 0 ... ? (26 / 10)

1000 | 26 → 38 -1 | 26 → 1; 12 -1 → L

V	21
W	22
X	23
Y	24
Z	25

base 26 conversion

```
while(d>0){
    q = d/26
    r = d%26
    d = q
    ...
}
```

q-1

r-1

→ append and reverse at end

n → n!

↘ prime factors

20! 21!

A	3	7	8	7	9
B	3!	7!	8!	7!	9!

9! → 4

B2 → # of prime factors of A[i]!

4