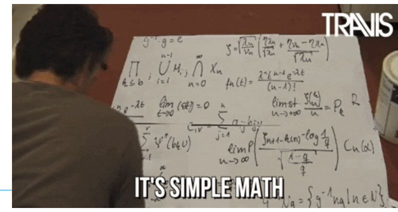
1- GCD intro
2- psudo code for GCD
3- Properties
4- Fast GCD
5- time Complexity (optional)
6- 2problems



GCD Greatest Comman Divisor =(Highest common factor) HCF

ex gcd(15,25) → 5 | 15 = 3 × 5, 25 = 5 × 5

gcd(20,16) → 4 | 20 = 2 × 10 = 2 × 5 × ... 4; 16 = 4 × 2 × 8 ... (20: 2, 10, 5, 4; 16: 4, 2, 8)

gcd(50,115) → 5 | 50: 2, 25, 5, 10; 115: 23, 5

Applications $\frac{15}{25} = \frac{3}{5}$, RSA,

algorithm: int gcd(int a, int b){ a,b>=0

```
int gcd(int a, int b){
    if(a==0 || b==0)
        ret a+b
    ans=1
    for(i=1; i<=min(a,b); i++){
        if(a%i==0 && b%i==0){
            ans=i
        }
    }
    ret ans
}
```

TC: O(min(a,b))
SC: O(1)

0	4
0	1
1	4
2	2
3	
4	
⋮	

0 % 10 = 0
0 = 10 × 0
0 = 12 × 0

$O(\sqrt{n})$ ← first session of intermediate DSA

a&b>=0

Properties

① gcd(a,b) = gcd(b,a)

Q1 ② gcd(1,a) = 1

Q2 ③ gcd(0,a) = a

Euclid theorem   16 vs 6

$\boxed{b >= a}$

④ $gcd(a,b) = gcd(a, b-a)$

$\begin{cases} a = k1 * d \\ b = k2 * d \end{cases}$ d

$b - a = (\cancel{k2 - k1}) d$ $(a-b) \% d = 0$

$k3 >= 0$

Eucleaian theorem or method

$\boxed{b >= a}$

rule → ⑤ $gcd(a,b) = gcd(a, b \% a)$

16 20 $(16,4) \rightarrow (4,16) \rightarrow (4,0)$

$(16,20) = (16,4)$

① → $(4,16) = (4,12)$

$= (4,8)$

$= (4,4)$

$= (4,0)$

$= 4$

$gcd(a, b-a)$ ← Euclid theorem

$= gcd(a, (b-a)-a)$

$= gcd(a, b-a-a-a)$

...

$= gcd(a, b - \underbrace{(a+a+\dots+a)}_{k}) = gcd(a, \underbrace{b-ka}_{>=0})$

$b - ka$

$b \% a$

$= gcd(a, b\%a)$

$g(a,b) = gcd(a, b \% a)$

$gcd(100,72) \overset{①}{=} gcd(72,100)$

$= (72,28)$ ⑤ $= (28,72)$ ① $= (28,16)$ ⑤ $= (16,28)$ ① $= (16,12)$

$= (4,12)$

$= (4,0) = 4$

$b >= a$

main{ gcd(a,b) }

```
              72         100
int gcd(int a, int b){          // recursive
    if(a==0) ret b
    ret gcd(b%a, a)
               [0,a-1]
}
```

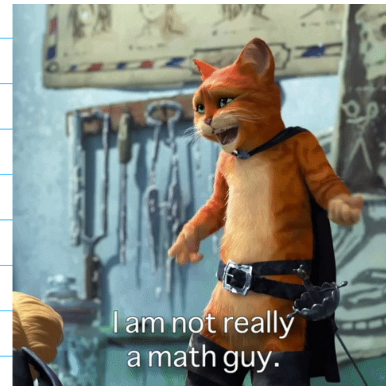
gcd(72,100)

TC:

SC:

Optional 8 → leave for end of session

Time complexity of GCD



$gcd(3,3) \xrightarrow{1\ step} gcd(0,3) \to 3$

$gcd(500,1000) \xrightarrow{1\ step} gcd(0,500) \to 500$

$gcd(\overset{a}{500{,}000}, \overset{b}{1{,}000{,}000{,}000}) \xrightarrow{1\ step}$
$gcd(0, 500{,}000) \to 500{,}000$

$gcd(72,100) \xrightarrow{①} (28,72) \xrightarrow{②} (16,28) \xrightarrow{③} (12,16)$
$\xrightarrow{④} (4,12) \xrightarrow{⑤} (0,4) \to 4$ 5 step

$gcd(a,b) = gcd(b\%a, a) = \ldots$

$T(a,b) = T(b\%a, a) + 1$ looking for upper bound

$\underbrace{b\%a}_{[0,a-1]} < a$

(a,b) $b >= a$

$(b\%a, a)$

Again Optional 8

$a < b/2$; $a = b/2$; $a > b/2$

① $\boxed{b\%a < b/2 \quad [0,a-1]}$

$b\%a = 0$ → done $O(1)$

$b\%a = b - ak$

$b - ak <= b - a$ $\{1, 2, 3, \ldots, k\}$

$b\%a <= \boxed{b - a <= \frac{b}{2}}$

② $\boxed{b\%a <= \frac{b}{2}}$ 100 51 $a > 50\%b$

$b \to \frac{b}{2} \to \frac{b}{4} \to \frac{b}{8} \to \ldots \frac{b}{2^k}$

$TC <= \log_2 b$ larger number

both iterative & recursive → $\boxed{TC <= O(\log_2 b)}$

recursive: $SC: <= O(\log_2 b)$

iterative: $SC: O(1)$

a[]

P1 Given an integer array, find the max gcd of all elements of array after removing exactly one element.

ex a : {9, 18, 98, 12, 30, 60}

3×3   7×7×2

→ 2

→ 1

→ 3 ← ans

→ 1

→ 1

→ 1

a : {20, 64, 24, 100, 50}

brute force?

for i → 0 to n-1

// skip a[i]

for (j → 0 to i-1 and i+1 to n-1) // skip i

ans = gcd(ans, a[j]) → $\log_2 a[i]$

max ans

TC : $O(n^2 \times \log_2 a_j)$

SC : O(1)

ideas 2 :

a : {9, 18, 98, 12, 30, 60}

→ 2

→ 1

→ 3 ← ans

→ 1

→ 1

→ 1

gcd[i-1]

assignment

gcd(9, 18, 98)

= gcd(gcd(9,18), 98)

k = j-1

gcd[k] = gcd(gcd[k-1], a[k])

prefixGcd[k]

suffixGcd[]

n

n TC : $O(n \log x)$

SC : O(n)

for i = 0 → n-1

max ← gcd(prefixGcd(0, i-1), suffixGcd(i+1, n-1))

TC : O(n)

P2 N Playes, playing a game
& each player has a health
of a[i] (for i th player)

if player i attacks player j — 10 → 5 → 0

a) if (a[i] >= a[j]) player j will die!

b) if (a[i] < a[j]) → a[j] = a[j] - a[i] — ~~11~~ 6 ← 5

Find the min health of last surviving player.

ex a : { 10 , 6 }   1 → 2 → 2 game over   ans = 10

2 → 1 : 4, 6

rule #4   10-6   { 4 , 6 }

1 → 2   4, 2

2 → 1   0   6

(10, 6) → (6, 4) → (4, 6-4)
→ (2, 4-2) →

4, 2

2, 2

1 4

2 2, 2

2, 0 → min health

strategy: weak health
stay proactive
and attacks
stronege

GCD(H1, H2)

$\log_2(H_1, H_2)$

ex2,

3Player

$a = \{9, 6, 15\}$ $\quad \{\overset{9-6}{3}, 6, \overset{15-6}{9}\} \longrightarrow$ game == gcd $\{3, \overset{6-3}{3}, \overset{9-3}{6}\} \to \{3, \overset{3-3}{0}, \overset{6-3}{3}\} \to \{3, 0, \overset{3-3}{0}\}$

lie

ans

min health

gcd of whole array → rule#4 & rule#6