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UNIT → III

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Propositional :-

Propositions -

A proposition is a collection of declarative statements that has either a truth value 'true' or a truth value 'False'. A propositional consist of propositional variables and connectives. We denote the propositional variables by capital letters A, B, C ... and so on. The connectives connect the propositional variables. Some examples of propositions are given below:

1 - 'Man is Mortal', it returns truth value 'True'.
2 - '12+9 = 3-2', it returns truth value 'False'.

The following is not a proposition:
'A is less then 2', it is because unless we give a specific value of A, we can not say whether the statement is true or false.

Connectives :-

Generally we used five connectives which are:

1→ OR(V) :-

The OR operation of two proposition A and B (write as A V B) is true if atleast any one of the propositional variable A or B is true.
The truth table is as follows:

A	B	A∨B
T	T	T
T	F	T
F	T	T
F	F	F

2- AND (∧) :-
The AND operation of two propositions A and B (written as A∧B) is true if both the propositional variable A and B is true. The truth table is as follows.

A	B	A∧B
T	T	T
T	F	F
F	T	F
F	F	F

3- Negation (~) :-
The Negation of a proposition A (written as ~A) is false when A is true and is true when A is false. The truth table is as follows:

A	~A
T	F
F	T

4- Implication :- (→)/(⇒)
An implication A→B is the proposition 'If A and B'. It is false if

A is true and B is false. The rest rest cases are true. The truth table is as fallows:

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

I-T, II-F $\Rightarrow$ F

⑤ If and only if ($\Leftrightarrow$):-

$A \Leftrightarrow B$ is bi-conditional logical connective which is true when A and B are same, that is both are false or both are true.

The truth table is as fallows:

A	B	$A \Leftrightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

diff - false
Same - true

Tautology:-

A Tautology is a formula which is always true for every value of its propositional variable.

Q→ Prove that $[(A \to B) \land A] \to$ is a tautology.

Sol :- The truth table is as fallows:

A	B	A → B	(A→B) ∧ A	(A→B) ∧ A] → B
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

as we can see every value of [(A→B) ∧ A] → B is true, it is a tautology.

Q. Construct the truth table of the fallowing functions and check weather it is a tautology and contradiction:
$(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)] \Leftrightarrow [(p \vee q) \wedge (q \vee r) \wedge (r \vee p)]$

Sol: The truth table is as fallows:

p	q	r	p∧q	q∧r	r∧p	(p∧q) ∨ (q∧r) ∨ (r∧p)	p∨q	q∨r	r∨p	(p∨q)∧(q∨r)∧(r∨p)	
T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T	T	T	T	T
T	F	T	F	F	T	T	T	T	T	T	T
T	F	F	F	F	F	F	T	F	T	F	T
F	T	T	F	T	F	T	T	T	T	T	T
F	T	F	F	F	F	F	T	T	F	F	T
F	F	T	F	F	F	F	F	T	T	F	T
F	F	F	F	F	F	F	F	F	F	F	T

as we can see every value of $(p \wedge q) \vee (q \wedge r) \vee (r \wedge p) \Leftrightarrow (p \vee q) \wedge (q \vee r) \wedge (r \vee p)$ are true it is a tautology.

2022
Q.

Contradiction :-

A contradiction is a formula which is always false - for every value of its propositional variable.

Q. Prove that $(A \vee B) \wedge [(\sim A) \wedge (\sim B)$ is a contradiction.

Sol: The truth table is as follows:

A	B	$A \vee B$	$\sim A$	$\sim B$	$(\sim A) \wedge (\sim B)$	$[(A \vee B) \wedge (\sim A) \wedge (\sim B)]$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	F

as we can see every value of $(A \vee B) \wedge (\sim A) \wedge (\sim B)$ is false, it is a contradiction.

Contigency :-

A contigency is a formula which has both some true and some false value for every value of its propositional variable.

Q. Prove that $(A \vee B) \wedge (\sim A)$ is a contigency.

Sol :-

The truth table is as follows:

A	B	$A \vee B$	$\sim A$	$(A \vee B) \wedge (\sim A)$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

as we can see every value of $(A \vee B) \wedge (\sim A)$ has both true and false, it is a contigency.

2023 Q. Show that $(p \leftrightarrow q) \wedge (q \leftrightarrow r) \rightarrow (p \leftrightarrow r)$ is a tautology.

Sol:- The truth table is as follows:

p	q	r	$p \leftrightarrow q$	$q \leftrightarrow r$	$(p \leftrightarrow q) \wedge (q \leftrightarrow r)$	$p \leftrightarrow r$	$(p \leftrightarrow q) \wedge (q \leftrightarrow r) \rightarrow (p \leftrightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	F	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

as we can see every value of $(p \leftrightarrow q) \wedge (q \leftrightarrow r) \rightarrow (p \leftrightarrow r)$ is true, it is a tautology.

Logical equivalence :-

Two statement A and B are logicaly equibalance if any one of the fallowing two conditions hold:

① The truth table of each statement have the same truth values.

② The bi-conditional statement $A \Leftrightarrow B$ is a tautology.

Q3 Prove that $\sim(A \vee B)$ and $[(\sim A) \wedge (\sim B)]$ are equibelent

Sol:- The truth table is as follows:

A	B	$A \vee B$	$\sim(A \vee B)$	$\sim A$	$\sim B$	$(\sim A) \wedge (\sim B)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Here, we can see the truth values of $\sim(A \vee B)$ and $(\sim A) \wedge (\sim B)$ are same, Hence the statements are equibelent.

OR

The truth table is as fallows:

A	B	$\sim(A \vee B)$	$(\sim A)$	$(\sim B)$	$(\sim A) \wedge (\sim B)$	$\sim(A \vee B) \Leftrightarrow (\sim A) \wedge (\sim B)$
T	T	F	F	F	F	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T

Since negation $\sim(A \vee B) \Leftrightarrow (\sim A) \wedge (\sim B)$ is a tautology therefore the Statement are equibalent.

Q.- Prove that equivalence :

$$(p \to q) \to q \equiv p \vee q.$$

Sol The truth table is as fallows:

p	q	$p \to q$	$(p \to q) \to q$	$p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

Here we can see the truth tables of $(p \to q) \to q$ and $(p \vee q)$ are same, Hence the statement are equivalence.

Q. Show that $(p \to q) \wedge (r \to q) \equiv (p \vee r) \to q$.

Sol:- The truth table is as follows:

I T→F F→F

p	q	r	$(p\to q)$	$(r\to q)$	$(p\to q)\wedge(r\to q)$	$p\vee r$	$(p\vee r)\to q$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	F	F	T	F
F	F	F	T	T	T	F	T

Here, we can see the truth values of $(p\to q)\wedge(r\to q)$ and $(p\vee r)\to q$ are same. Hence the statement are equivalence, that is $(p\to q)\wedge(r\to q) \equiv (p\vee r)\to q$

Q. Construct the truth table $(p\Rightarrow q)\vee(q\Rightarrow p)\Leftrightarrow p$ is the proposition: tautology, contradiction or contingency.

Sol:- The truth table is as follows.

p	q	$p\Rightarrow q$	$q\Rightarrow p$	$(p\Rightarrow q)\vee(q\Rightarrow p)$	$(p\Rightarrow q)\vee(q\Rightarrow p)\Leftrightarrow p$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	F	T	T	F	T

Q.→ Define the terms converse, contrapositive and inverse of a proposition. Show that-

$(p \to q) \wedge (r \to q) \equiv (p \vee r) \to q$

Sol.→ The truth table is as follows: I-T II-F

p	q	r	$(p \to q)$	$(r \to q)$	$(p \to q) \wedge (r \to q)$	$(p \vee r)$	$(p \vee r) \to q$	$(p\to q)\wedge(r\to q)\equiv(p\vee r)$
T	T	T	T	T	T	T	T	
T	T	F	T	T	T	T	T	
T	F	T	F	F	F	T	F	
T	F	F	F	T	F	T	F	
F	T	T	T	T	T	T	T	
F	T	F	T	T	T	F	T	
F	F	T	T	F	F	T	F	
F	F	F	T	T	T	F	T	

Sol. Converse, contrapositive and inverse of proposition:-

Implication($\to$) is also called a conditional statement. It has two parts:-

1. Hypothesis, p
2. Conclusion, q

as mentained ideal, it is denoted as $p \to q$

Example of conditional statement:-

"If you do your homework, you will not be punished". Here, "You do your homework" is the hypothesis, p and "you will not be punished" is the conclusion, q.

Inverse- An inverse of the conditional statement is the negation of both the hypotheses and the conclusion. If the statement is "If p, then q", the inverse will be "If not p, then not q". Thus the inverse of $p \to q$ is $\sim p \to \sim q$.

Example - The inverse of "If you do your homework you will not be punished" is "If you do not do your homework, you will be punished".

Converse - The converse of the conditional statement is calculated by interchanging the hypothesis and the conclusion if the statement is "If p, the q", the converse will be "If q, then p". The converse of $p \to q$ is $q \to p$.

Example - The converse of "If you do your homework you will not be punished", is "If you will not be punished, you do not do your homework".

Contrapositive :→

The cotrapositive of the conditional statement is calculated by interchanging the hypothesis and the ~~conclusio~~ conclusion of the inverse statement. If the statement is "If p, then q", the contrapositive will be "If not q, then not p". The contrapositive of $p \to q$ is $\sim q \to \sim p$.

Example -

The contrapositive of "If you do your homework you will not be punish" is "If you are not punished, then you do not do your homework".

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Q→ Prove that conditional proposition and its contrapositive are equebalence, that is $(p \to q) \equiv \sim q \to \sim p$

Sol: The truth table is as fallows:

p	q	$p \to q$	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

as the values in both cases are same, Hence both propositions are equibelent.

Q→ Define the term tautology and contradiction. Show that $(p \to (q \wedge r)) \to (\sim r \to \sim p)$ is a tautology.

Sol: The truth table is as fallows:

p	q	r	$q \wedge r$	$p \to (q \wedge r)$	$\sim r$	$\sim p$	$\sim r \to \sim p$	$(p \to (q \wedge r)) \to (\sim r \to \sim p)$
T	T	T	T	T	F	F	T	T
T	T	F	F	F	T	F	F	T
T	F	T	F	F	F	T	T	T
T	F	F	F	F	T	T	F	T
F	T	T	T	T	F	F	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	F	T	T	T
F	F	F	F	T	T	T	T	T

As we can see every value of $(p \to (q \wedge r)) \to (\sim r \to \sim p)$ is true, it is a tautology.

Q> Prove that the statement $(p \wedge q) \rightarrow (p \vee q)$ is tautology.

Sol> The truth table is as follows:

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

as we can see every value of $(p \wedge q) \rightarrow (p \vee q)$ is true, it is a tautology.

2017
Q> write a following condition statement in symbolic form also give the converse, inverse and contrapositive of the statement "If the flood destroys Mohan's house or the fire destroy Mohan's house, then ~~m~~ Mohan's insurance company will pay him".

Sol> p: The flood destroys Mohan's house or the fire destroy Mohan's house.
q: Mohan's insurance company will pay him.
Therefore, The given conditional statement in symbolic form is $p \rightarrow q$.

Converse> The Converse of "If the flood destroys Mohan's house or the fire destroy Mohan's house, then Mohan's insurence company will pay him" is "If Mohan's insurence Company will pay him, then the flood destroys Mohan's house or the fire destroy Mohan's house".

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Inverse -

The inverse of "If the flood destrayes Mohan's house or the fire destray Mohan's house then Mohan's insurance company will pay him" is "If the flood does not destrayes Mohan's house or the fire do not destory Mohan's house, then Mohan's insurance company will not pay him."

Contrapositive →

The contra-positive of "If the flood destrays Mohan's house or the fire destray Mohan's house then Mohan insurance company will pay him" is "If Mohan's insurance company will not pay him, then the flood does not destrays Mohan's house or the fire do not destray Mohan's house".

Algebra of Propositions :-

① Idempotent laws:

(i) $p \vee p \equiv p$

(ii) $p \wedge p \equiv p$

② Associative laws:

(i) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(ii) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

③ Commutative laws:

(i) $p \vee q \equiv q \vee p$

(ii) $p \wedge q \equiv q \wedge p$

④. Distributive laws:

(i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

⑤. Identity laws:

(i) $p \vee F \equiv p$

(ii) $p \wedge T \equiv p$

(iii) $p \vee T \equiv T$

(iv) $p \wedge F \equiv F$

⑥ Complement laws:

(i) $p \vee \sim p \equiv T$

(ii) $p \wedge \sim p \equiv F$

(iii) $\sim T \equiv F$

(iv) $\sim F \equiv T$

⑦ Involution laws :

$\sim(\sim p) \equiv p$

⑧ DeMorgen's laws :-

(i) $p \vee q \equiv \sim p \wedge \sim q$

(ii) $p \wedge q \equiv \sim p \vee \sim q$

⑨ Absorption laws:

(i) $p \vee (p \wedge q) \equiv p$

(ii) $p \wedge (p \vee q) \equiv p$

Laws of Algebra

Unit 1 → A, B, C
$\downarrow$
$\cup, \cap, ^{-}$

Unit 2 → $a, b, c \in S$
$\downarrow$
$+, *, '$

Unit 3 → p, q, r
$\downarrow$
$\vee, \wedge, \sim$

2019

Q. State Identity law & Demorgen's laws of Algebra of proposition and prove the distributive law of Algebra of proposition.

Sol. Identity laws:

(i) $p \vee F \equiv p$

(ii) $p \wedge T \equiv p$

(iii) $p \vee T \equiv T$

(iv) $p \wedge F \equiv F$

De-Morgon's laws:

(i) $p \vee q \equiv \sim p \wedge \sim q$

(ii) $p \wedge q \equiv \sim p \vee \sim q$

Distributive laws:

(i) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Proof →

The truth table is as follows:

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Here, we can see the truth values of $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are same, Hence the statements are equivalence, that is: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

(ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Proof- Similarly we can prove this result as part(i).

2021
Q: State and proof De-Morgan's laws for proposition using truth table.

De-Morgan's laws :-

(i) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

Proof →

The truth table is as follows:

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Here, we can see the truth values of $\sim(p \vee q)$ and $\sim p \wedge \sim q$ are same, Hence the statement are equivalence, that is $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

(ii) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	F
F	T	F	T	T	F	F
F	F	F	T	T	T	T

Here, we can see the truth values of $\sim(p \wedge q)$ and $\sim p \vee \sim q$ are same, Hence the statement are equivalence, that is $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Q.2 State and prove associative laws for propositions using truth table

Sol:- Associative laws:

(i) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

Proof -

The truth table is as follows:

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$q \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Hence, we can see the truth ~~table~~ values of $(p \vee q) \vee r$ and $p \vee (q \vee r)$ are same. Hence the ~~equivalence~~ statement are equibalence.

(ii) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	F	T	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Q:- State and prove Absorption laws for proposition using truth table.

Sol:- Absorption laws:

(i) $p \vee (p \wedge q) \equiv p$

Proof -

The truth table is as fallows -

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

Hence, we can see the truth values of $p \vee (p \wedge q)$ and p are same, Hence the statement are equibalence.

(ii) $p \wedge (p \vee q) \equiv p$

Proof -

The truth table is as fallows:

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

Hence, we can see the truth values of $p \wedge (p \vee q)$ and p are same,

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Q→ Even the values $p \to q$ is false, determine the value of $(\sim p \vee \sim q) \to q$.

Sol→ The truth table is as follows:

$\leftarrow \vee$ T-T-T-F → F

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \to q$	$(\sim p \vee \sim q) \to q$
T	T	F	F	F	T	T
T	F	F	T	T	$\boxed{F}$	$\boxed{F}$
F	T	T	F	T	T	T
F	F	T	T	T	T	F

From the truth table it follows that $p \to q$ is false, then the value of $(\sim p \vee \sim q) \to q$ is false.

2018

Arguments :-

An argument is a process by which a conclusion is drawn from a set of proposition. The given set of propositions are called premises or hypatheses. The final proposition drived from the given proposition is called conclusion.

Some times an argument is return in the fallowing form. $p_1, p_2, p_3, p_4 - - - p_n$ } Premises

$$\left.\begin{array}{l} p_1 \\ p_2 \\ p_3 \\ p_4 \\ \vdots \\ p_n \end{array}\right\} \text{Premises}$$

$\therefore C \leftarrow$ Conclusion

An argument is said to be logicaly valid argument iff the conjuction of the premises implies the conclusion, that is if the Premises are all true, the conclusion must also ~~~~ be true. The argument which produce a conclusion c from the premises $p_1, p_2, p_3, \ldots p_n$ is valid if and only if the statement $p_1 \wedge p_2 \wedge p_3 \wedge \cdots \wedge p_n \rightarrow c$ is a tautology.

2019 Procedure for testing the validity of an argument using truth table:→

① Idenitify the premises and conclusion of the argument.

② Construct a truth table showing the truth values of all premises and the conclusion.

③ Find the rows (called critical rows) in which all the premises are true.

④ In each critical row, determine wheather the conclusion of the argument is also true.

ⓐ If ineach critical row the conclusion is also true, that the argument form is valid.

ⓑ If there is atleast one critical row inwhich the conclusion is false, the argument form is invalid.

Rules of Inference:- The rules of inference are criteriya for determining the validity of an argument. Any conclusion which is arived by following these rules

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is called a valid conclusion, and the argument is called valid argument. The most familiar type of proof usages to fundamental rules of inference.

2017 Fundamental Rule - 1

If the statement in p assumed as true & also the statement $p \to q$ is accepted as true, then q must be True. Symbalicaly it is written in the following pattern, when we use the familiar symbol.

$$p \to q \Rightarrow \frac{p}{\therefore q}$$

In this presentation of an argument the assertions $p \to q$ and p above the horizontal line are the premises or hypothesic and the assertions q below the line is the conclusion. The Rule stated is known as Modulas ponems or Rule of detachment.

Proof :-

The validity of the argument be seen from the following Truth table:

I - T, II - F ⇒ F

p	q	$p \to q$	p	q
T	T	$\boxed{T}$	T	$\boxed{T}$
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

In this truth table we see that there is only one case in which both premisive are true namely the first case, and that is this case the conclusion is also true, Hence the argument is valid.

2020

Fundamental Rule - 2

when ever the two statement $p \rightarrow q$ and $q \rightarrow r$ are expected as true, then the statement $p \rightarrow r$ is accept as true. Symbolicaly it can be represented as:

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore\ p \rightarrow r \end{array}$$

The argument is known as a hypathetical syllogism.

Broof :-

The truth table of the argument appears in the following table: I-T, II-F ⇒ F

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Both premises are true as seen in the first, fifth, seventh and Eight rows of the truth table, since in each case the conclusion is also true, the argument is valid.

⇒ Additional valid argument:-

There are other valid inferances which state that certain form of argument are valid.

(i) Madul tollen's :-

The argument of the form

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

This argument is valid and is called Madul Tollen's. It can be established by using a truth table.

(ii) The following argument form is valid $\left[\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}\right]$

This argument form is used for making generalizations. If p is true, then more generally p or q is true for any other statement q.

p	q	p∨q
T	T	T
T	F	T

(iii) Disjunctive Syllogism:-

The following statement

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form is valid:

$$\left[\begin{array}{c} p \vee q \\ \sim q \\ \hline \therefore\ p \end{array}\right] \text{ or } \left[\begin{array}{c} p \vee q \\ \sim p \\ \hline \therefore\ q \end{array}\right]$$

This argument states that when there are two posibility and one can rule one out, the other must be the case.

(iv). Simplification :-

$$\begin{array}{c} p \wedge q \\ \hline \therefore\ p \end{array} \quad \text{or} \quad \begin{array}{c} p \wedge q \\ \hline \therefore\ q \end{array}$$

(v). Conjuction :-

$$\begin{array}{c} p \\ q \\ \hline \therefore\ p \wedge q \end{array}$$

(vi) Constructive dilemma :-

$$\begin{array}{c} (p \rightarrow q) \wedge (r \rightarrow s) \\ p \vee r \\ \hline \therefore\ q \vee s \end{array}$$

(vii) Destructive dilemma :-

$$\begin{array}{c} (p \rightarrow q) \wedge (r \rightarrow s) \\ \sim q \vee \sim s \\ \hline \sim p \vee \sim r \end{array}$$

Q.3 Show that $\sim r$ is a valid conclusion from the premises:

$p \rightarrow \sim q,\ r \rightarrow p,\ q$

(i) With truth table

(ii) Without truth table

Sol: (i) With truth table –

The truth table is as follows:

← Premises → ← Conclusion →

p	q	r	$\sim q$	$p \to \sim q$	$r \to p$	q	$\sim r$
T	T	T	F	F	T	T	F
T	T	F	F	F	T	T	T
T	F	T	T	T	T	F	F
T	F	F	T	T	T	F	T
F	T	T	F	T	F	T	F
F	T	F	F	T	T	T	T
F	F	T	T	T	F	F	F
F	F	F	T	T	T	F	T

Here, premises $p \to \sim q$, $r \to p$, q are true simatanusly in row 6, where $\sim r$ is also true. This proves the ~~following~~ validity of the conclusion.

(ii) Without using truth table:-

The valid argument for deducting $\sim r$ from the given premises is given as a sequence.

1. $r \to p$ — premise (Given)
2. $p \to \sim q$ — Premise (Given)
3. $r \to \sim q$ — Hypothetical syllogism using 1 and 2
4. q — Premise (Given)
5. $\sim r$ — Modus tollens using 3 and 4

Thus we can conclude $\sim r$ from the given premises.

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Q. Show that s is a valid conclusion from the premises:

$p \to q$, $p \to r$, $\sim(q \wedge r)$ and $s \vee p$.

using truth table -

Sol:- The truth table is as fallows:

I-T, II-F→F ← Premises → * (conclusion)

p	q	r	s	$q\wedge r$	$p\to q$	$p\to r$	$\sim(q\wedge r)$	$s\vee p$	s
T	T	T	T	T	T	T	F	T	T
T	T	T	F	T	T	T	F	T	F
T	T	F	T	F	T	F	T	T	T
T	T	F	F	F	T	F	T	T	F
T	F	T	T	F	F	T	T	T	T
T	F	T	F	F	F	T	T	T	F
T	F	F	T	F	F	F	T	T	T
T	F	F	F	F	F	F	T	T	F
F	T	T	T	T	T	T	F	T	T
F	T	T	F	T	T	T	F	F	F
F	T	F	T	F	T	T	T	T	T
F	T	F	F	F	T	T	T	F	F
F	F	T	T	F	T	T	T	T	T
F	F	T	F	F	T	T	T	F	F
F	F	F	T	F	T	T	T	T	T
F	F	F	F	F	T	T	T	F	F

Hence, premises $p\to q$, $p\to r$, $\sim(q\wedge r)$ and $s\vee p$ are true simutanusly in row 11, 13 and 15, where s is also true.

(ii) Without truth table:

The valid argument for deducting s from the given premises is given as a sequence:

1. $p\to q$ Premise (Given)
2. $p\to r$ Premise (Given)
3. $(p\to q)\wedge(p\to r)$ Conjunction using 1 and 2
4. $\sim(q\wedge r)$ Premise (Given)
5. $\sim q\vee\sim r$ Demorgan's law using 4

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⑥	$\sim p \vee \sim p$	Destructive dilemma using 3 and 5

$$(p \to q) \wedge (r \to s)$$
$$\underline{\sim q \vee \sim s}$$
$$\therefore \sim p \vee \sim r$$

⑦	$\sim p$	Idempotent law using 6
⑧	$s \vee p$	Premise (Given)
⑨	s	Dijunctive syllogism using 7 and 8

Thus, we can conclude s from the given primise.

Qs. Define the term argument. Prove the validity of the fallowing argument "If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. ~~the~~ Therefore, either I will not get the job or I will not work hard".

Sol:- **Argument :**

An argument is a process by which a conclusion is drawn from a set of proposition. The given set of proposition are called premises or hypotheses. The final proposition is called conclusion. Some times an argument is return in the fallowing form.

$$\left.\begin{array}{l} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{array}\right\} \text{Premises}$$
$$\overline{\therefore c \leftarrow \text{conclusion}}$$

Let p: I get the job
q: I work hard
r: I get promoted
s: I will be happy

Then the given argument can be return in symbolic form as:

$$\begin{array}{c} (p \wedge q) \rightarrow r \\ r \rightarrow s \\ \hline \therefore \sim p \vee \sim q \end{array}$$

Proof:-
1- $(p \wedge q) \rightarrow r$ Premise (Given)
2- $r \rightarrow s$ Premise (Given)
3- $(p \wedge q) \rightarrow s$ Hypothetical syllogism using 1 & 2
4- $\sim s$ Premise (Given)
5- $\sim (p \wedge q)$ Modus tollens using 3 and 4
6. $\sim p \vee \sim q$ Demorgan's law using 5

Thus we can conclude $\sim p$ or $\sim q$ from the given premises, Hence the argument is valid

2019 Q3 → Consider p: He is intelligent, q: He is tall, be two proposition. Write each of the following statement in symbolic form using p and q:
(i) He is tall but not intelligent.
(ii) He is neither tall nor intelligent.
(iii) He is intelligent or He is tall.

Sol → Here, p: He is intelligent
q: He is tall

(i) $q \wedge \sim p$
(ii) $\sim q \wedge \sim p$
(iii) $p \vee q$

Universal quantification :→

Many mathematical assert that a property is true for all values of a variable in a particular domain, called the universe of discourse. Such a statement is expressed using a universal quantification. The universal quantification of a proposition function is the proposition that asserts $P(x)$ is true for all posible value of x.

It is denoted in many ways:

Let ($p(x)$ be the statement defined on a set A.

$p(x)$ is true for all $x \in A$.

or

$p(x), \forall x \in A$

or $\forall x, p(x)$

or $\forall x \in A : p(x)$

symbollicaly for all is denoted by $\forall$ and is called universal quantifiers.

It is also equibalence to the statement $[x : x \in A, p(x)] = A$.

Example:-

① The proposition for all natural numbers $(n+5<4)$ is true.

since $[n : n+5 \cancel{>} 4] = \{1, 2, 3 ----\} = N$

② The proposition for all $(\forall n \in N) . (n+3>7)$ is false since $[n : n \in N, n+3>7] = \{5, 6, 7, ----\} \neq N$

Existential quantifier:-

Let $P(x)$ be the statement function on a set A. The existential quantification of $P(x)$ is the statement that there exist atleast one value $x \in A$ for which $p(x)$ is true. It is denoted in the following ways:

$p(x)$ is true for some $x \in A$

or $\exists x \in A,\ p(x)$

or $\exists x,\ p(x)$

or $\exists x \in A : p(x)$

The symbol $\exists$ is called the existential quantifier. It is also equebelent to the statement : $[x : x \in A : p(x)] \neq \phi$.

Example:- ① The proposition $(\exists n \in N) \cdot (n+2 < 6)$ is true.

since $[n : n \in N : n+2 < 6] = \{1, 2, 3\} \neq \phi$

② The proposition $(\exists n \in N) \cdot (n+7 < 5)$ is false.
since $[n : n \in N : n+7 < 5] = \phi$

Q. If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ determine the truth value of each of the following statement:

(i) $(\forall x \in A)\ x+4 < 15$

(ii) $[\exists x \in A)\ x+4 = 10$

(iii) $(\forall x \in A)\ x+4 \leq 10$

Sol:- (i) Since $[x : x \in A,\ x+4 < 15] = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$= A,$

Therefore the truth value of this statement is true.

(ii) $(\exists x \in A)\ x+4 = 10$

Since $[x : x \in A : x+4 = 10] = \{6\} \neq \phi$,

Therefore the truth value of this statement is true

(iii) $(\forall x \in A)\ x+4 \leq 10$

Since $[x : x \in A, x+4 \leq 10] = \{1, 2, 3, 4, 5, 6\} \neq A$,

$(1+4 \leq 10)$

The truth value of this statement is false.

2022

Q3 If $K(x)$: x is student, $M(x)$: x is clever, $N(x)$: x is succesful.

Express the following using quantifiers:

(i) There exist a student

(ii) Some students are clever

(iii) Some students are not succesful.

Sol:- (i) There exist a student -

$\exists x (K(x)\ \cancel{\wedge M(x)})$

(ii) There exists an x such that x is student and x is clever.

$\exists x (K(x) \wedge M(x))$

(iii) There exists an x such that x is student and x is not succesful.

$\exists x (K(x) \wedge \sim N(x))$.

Q? What do you mean by quantifiers with suitable example?

Sol:- Quantifier :-

Most of the statement in mathematic use terms such as "for all" (for every), and for sum and number. for example - consider the following statements:

(i) For every triangle the sum of angle is 180°

(ii) No natural number is negative.

(iii) Some students are poor.

when all the variables in a propositional function are asigned values, the resulting statement has a truth value. How ever there is another important way called quantification to creat a proposition from a propositional function. There are two types of quantification namely universal quantification and existential quantification.

2020

Q.) What do you mean by Bound and free variable with example?

Sol:- If a quantifier is applied on the variable x, then this occurence of the variable is called bounded and the variable is called bound variable. The quantified variables, either by $\forall$ or $\exists$ there exist are bound by values of universe of discourse on which the function is defined. An occurance of a variable that is not bound by a quantifier is said to be free.

Example:-

Consider $\forall x P(x, y)$. x is a bound variable, where as y is a free variable since quantifier is applied only on x not on y.

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Negation of compound statement:-

① Negation of conjunction:-

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

Q> Negate the statement "Paris is in france and London is in England".

Sol:- The negation of the given statement is " Paris is not france or London is not in England

2022

Q> Negate the statement " He is poor and ~~[illegible]~~ laborious".

Sol:- The negation of the given statement is " He is not poor or he is not labourious".

2→ Negation of disjunction :-

$$\sim(P \vee q) \equiv \sim P \wedge \sim q$$

Q> Negate the statement " Ram is in class XI or Arun is class XII ".

Sol:- The negation of given statement is " Ram is not in class XI and Arun is not in class XII ".

3→ Negation of Conditional :-

$$\sim(p \rightarrow q) \equiv P \wedge \sim q$$

Q> Negate the statement " If it is raining then the game is cancelled ".

Sol:- The negation of given statement is " It is raining and the game is not cancelled ".

4→ Negation of Biconditional:-

$$\sim(p \Leftrightarrow q) \equiv p \Leftrightarrow \sim q$$

Q→ Negate the statement " He swims iff the water is warm".

Sol:- The Negation of given statement is " He swims iff the water is not warm".

Q→ Show that t is a valid conclusion from the premises.

$p \to q$, $q \to r$, $r \to s$, $\sim s$ and $p \vee t$.

Sol:- The valid argument for deducting t from the given premises is given as a sequence:

①	$p \to q$	Premises (Given)
②	$q \to r$	Premises (Given)
③	$p \to r$	hypothetical syllogism using 1 and 2
④	$r \to s$	Premises (Given)
⑤	$p \to s$	Hypothetical syllogism using 3 & 4
⑥	$\sim s$	Premises (Given)
⑦	$\sim p$	Modus tollens using 5 and 6
⑧	$p \vee t$	Premises (Given)
⑨	t	Disjunctive syllogism using 7 and 8

Thus we can conclude t from the given premises. Hence t is a valid conclusion.

Q→ Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premise

$P \vee Q$, $Q \to R$, $P \to M$ and $\sim M$.

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Sol: (1) $P \to M$ Premise (Given)

(2) $\sim M$ Premise (Given)

(3) $\sim P$ Modus tollens using 1 and 2

(4) $P \vee Q$ Premise (Given)

(5) Q Disjunctive syllogism using 3 and 4

(6) $Q \to R$ Premise (Given)

(7) R Modus Pones using 5 and 6

(8) $R \wedge (P \vee Q)$ Conjunction 4 and 7.

Thus we can conclude $R \wedge (P \vee Q)$ from the given premises. Hence $R \wedge (P \vee Q)$ is a valid conclusion.