UNIT → I

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Set Theory :→

Sets ↦ A set is a well define collection of object. Each ~~etom~~ object belonging to a set is called an element or a number of this set.

We genraly use capital letters A, B, C, X, Y, Z etc to donate sets and lower case letters a, b, c, x, y, z, etc to donates eliments of a set.

If an element x belong to a set A, then we write $x \in A$.

If an object y is known as element of a set B, Then we write $y \notin B$.

Equality of Sets :→

Two sets A and B are said to be equal if the containt some element. we write

$$A = B$$

If the set A and B are equal and A is not equal to B if the sets $A \neq B$ are not equal.

for example →

$$\{a, b, c\} = \{b, a, c\}$$

$$\{2, 3, 5\} \neq \{2, 3, 7\}$$

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Subsets:

Let A and B be to sets. If every element of A is also an element of 'B', then 'A' is called a subsets of 'B'.
We also say that 'A' is contained in 'B' or that B contains A. In symbals, we write

$A \subseteq B$ or $B \supseteq A$

We say 'A' is not a subset of 'B' if atlest one element of 'A' does not belong to 'B' and we write it as-

$A \not\subseteq B$

For example →

Consider the sets

$A = \{1, 3, 4, 5\}$

$B = \{1, 2, 3, 5\}$

and $C = \{2, 3\}$

Then $C \subseteq B$ but $C \not\subseteq A$

Proper Subset:

Any Subsets 'A' is said to be proper Subset of another Set 'B' if A is a subset of 'B', but there is atlest one element of B which ~~that~~ does not belong 'A'. that is:-

If $A \subseteq B$ but $A \neq B$. It is written as $A \subset B$.

for example,

If $A = \{1, 5\}$

$B = \{1, 5, 6\}$

$C = \{1, 6, 5\}$

Then,
'A' and 'B' are both Subset of 'C'. but 'A' is a proper Subsets of C, where as B is not a proper subsets of C.
Since,

$B = C$

Empty Set :-
These set which contains no element is called the empty set (or null set or void set) and is denoted by { } or ϕ.

Since ϕ has no element, Therefore empty set is a subset of every set.

Singleton Set :→
A set which contains atzeley one element is called singleton set.
for example,
{3} is a singleton set.

Universal Set :→
In any mathematical discusson, we usualy consider all this set to be Subset of a fixed Set called the universal set, denoted by U.
For example,
In studying human population, the universal set consist of all human in the world.

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Power Set :-

If 'X' is any Set then the Set of all subsets of 'X' is called the power set of 'X', denoted by $P(X)$.

Thus,

$$P(X) = \{ A : A \subseteq X \}$$

If X contains n element then $P(X)$ contains 2^n elements.

For example,

Let $X = \{1, 2, 3\}$

Since X has 3 element, therefor $P(X)$ has $2^3 = 8$ elements.

Then,

~~$P(X) = \phi, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\},$~~

$P(X) = \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

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Q→ Define the power set. If $A = \{1, 2, 3\}$, find $P(A)$ and $n\{P(A)\}$.

Sol→ Power Set ↦ If 'X' is any sets then the Sets of all subsets of X is called the power Set of 'X', denoted by $P(X)$.

Thus,

$$P(X) = \{ A : A \subseteq X \}$$

If n contain n element then $P(X)$ Contains 2^n element.

$n\{P(A)\} = \{\{n\}, \{n\}, \{2n\}, \{3n\}, \{n, 2n\}, \{n, 3n\},$
$\{2n, 3n\}, \{n, 2n\}, 3n\}.$

Finite and Infinite Sets :- A set is said to be finite if it contains a finite No. of distinct elements. is said to be infinite if it is not finite.

Example ①

Let $A = \{1, 3, 5, 7, 9\}$. then A is finite because it contains 5 distinte element.

Ex-2 → Let $B = \{1, 2, 3, 4, ---\}$, then B is an infinite set.

2019 Q → Write down all possible subsets of $A = \{2, 3\}$ and $B = \{a, b, c\}$

Subsets of 'A' are $\phi, \{2\}, \{3\}, \{2,3\}$

Subsets of 'B' are $\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,c\}$
$\{b,c\}$

Basic operations on Sets-

- Union of Sets.

symbolicaly

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

For eg. let $A = \{2, 3, 5\}$
$B = \{3, 7, 9\}$

$A \cup B = \{2, 3, 5, 7, 9\}$

- Intersection of Sets.

Let 'A' & 'B' two sets The intersection of 'A' & 'B' is the set of all elements which are both in 'A' & 'B'. We denote the intersection of 'A' & 'B' by $A \cap B$. Symbolicaly,

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

For examample,

$$\text{let } A = \{6, 8, 10\}$$
$$B = \{10, 12, 14\}$$
$$\text{then } A \cap B = \{10\}$$

- Complement of a Set.

Let 'A' be a subset of a universal subset U. The set of all those elements of U which are not in 'A' is called the complement of 'A' and is denoted by U−A or simply 'A''.
Symbolically,

$$A' = U - A = \{x : x \in U \text{ and } x \notin A\}$$

For example,

$$\text{Let } U = \{a, b, c, d, e, f\}$$
$$A = \{a, b, c, d\} \text{ then}$$
$$A' = U - A = \{e, f\}$$

Difference of Sets :-

The difference A − B of two sets 'A' & 'B' is the set of element which belong to 'A'. But which do not belong to 'B'.
Thus,

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

For example,

Let $A = \{w, x, y, z\}$

and $B = \{x, z, v\}$

then

~~A~~ $A - B = \{w, y\}$

$B - A = \{v\}$

- **Symmetric Difference :->**

The symmetric difference of two sets 'A' & 'B', denoted by $A \Delta B$ or $A \oplus B$ is the set of element that belong to 'A' or 'B', but not to both 'A' & 'B'.
It is also called the Boolean sum of 2 sets.

It is easy to say that

$A \oplus B = (A - B) \cup (B - A)$ or $(A \cup B) - (A \cap B)$

$= \{x : x \in$ adzedly 1 of 'A' and 'B'$\}$

Q-> Define Symmetric difference of two sets. If $A = \{2, 3, 4\}$, and $B = \{3, 4, 5, 6\}$, Find $A \oplus B$.

Sol-> we are given that

$A = \{2, 3, 4\}$

and $B = \{3, 4, 5, 6\}$

Then $A - B = \{2\}$

$B - A = \{5, 6\}$

$A \oplus B = (A \cdot B) \cup (B - A)$

$= \{2\} \cup \{5, 6\}$

$\boxed{A \oplus B = \{2, 5, 6\}}$

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Laws of Algebra Sets :-

Let 'A' & 'B' and 'C' be a Subsets of an universal Set 'U'. Then the operation defined on sets satisfy the following property :

① Idempotent Laws -

(a) $A \cup A = A$

(b) $A \cap A = A$

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2 - Associative laws -

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

Proof -

Let x be an element of $(A \cup B) \cup C$ then

$x \in (A \cup B) \cup C \Rightarrow (x \in A \text{ or } x \in B) \text{ or } (x \in C)$

$\Rightarrow x \in A \text{ or } (x \in B \text{ or } x \in C)$

$\Rightarrow x \in A \text{ or } x \in B \cup C$

$\Rightarrow x \in A \cup (B \cup C)$

$\therefore x \in (A \cup B) \cup C \Rightarrow x \in A \cup (B \cup C)$

Thus,

$(A \cup B) \cup C \subseteq A \cup (B \cup C)$ —— ①

conversaly,

Let x be any element of $(A \cup B) \cup C$. Then

$x \in A \cup (B \cup C) \Rightarrow x \in A \text{ or } (x \in B \text{ or } x \in C)$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ or } x \in C$

$\Rightarrow x \in A \cup x \in B \text{ or } x \in C$

$\Rightarrow x \in (A \cup B) \cup C$

$\therefore x \in A \cup C (B \cup C) \Rightarrow x \in (A \cup B) \cup C$

Thus, $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ —— ②

from ① & ②, we get

$$\boxed{(A \cup B) \cup C = A \cup (B \cup C)}$$

(b)- $(A \cap B) \cap C = A \cap (B \cap C)$

Proof -

Similarly, we can prove that this result with the help of part 'a'.

(3)- <u>Commutative Laws</u> →

(a) $A \cup B = B \cup A$

<u>Proof</u>:-

Let x be an element of $A \cup B$. Then

$$x \in A \cup B \Rightarrow x \in A \text{ or } x \in B$$
$$\Rightarrow x \in B \text{ or } x \in A$$
$$\Rightarrow x \in B \cup A$$

Therefore,

$$x \in A \cup B \Rightarrow x \in B \cup A$$

Thus,

$$A \cup B \subseteq B \cup A \quad \text{—— (1)}$$

Conversely, let x be any element of $B \cup A$. Then

$$x \in B \cup A \Rightarrow x \in B \text{ or } x \in A$$
$$\Rightarrow x \in A \text{ or } x \in B$$

$\therefore x \in B \cup A \Rightarrow x \in A \cup B$

Thus,

$$B \cup A \subseteq A \cup B \quad \text{—— (2)}$$

from (1) & (2), we get

$$\boxed{A \cup B = B \cup A}$$

(b) $A \cap B = B \cap A$

<u>Proof</u>:-

Similarly, we can prove that this result with the help of part 'a'.

(4). <u>Distributive laws</u> →

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(5). <u>Identity laws</u> -

(a). $A \cup \phi = A$

(b). $A \cap U = A$

(c). $A \cup U = U$

(d). $A \cap \phi = \phi$

(6). <u>Involution Laws</u> →

(a) $(A')' = A$

(7). <u>Complement Laws</u> -

(a). $A \cup A' = U$

(b). $A \cap A' = \phi$

(c). $U' = \phi$

(d). $\phi' = U$

(8). <u>Demorgan's Laws</u> →

(a) $(A \cup B)' = A' \cap B$

Proof -

Let x be any arbitory element of the Set $(A \cup B)'$. Then

$\therefore x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\therefore \Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$

$\therefore x \in (A \cup B)' \Rightarrow x \in A' \cap B'$

Thus, $(A \cup B)' \subseteq A' \cap B'$ —— ①

Conversly,

x be any arbitary element of the set $A' \cap B'$. Thus

$x \in A' \cap B' \Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in (A \cup B)'$

$\therefore x \in A' \cap B' \Rightarrow x \in (A \cup B)'$

Thus $A' \cap B' \subseteq (A \cup B)'$ —— ②

from '1' & '2', we get

$\boxed{(A \cup B)' = A' \cap B'}$

(b) $(A \cap B)' = A' \cup B'$

Proof -

Let x be any arbitary element of the set $(A \cap B)'$. Then

$x \in (A \cap B)' \Rightarrow x \notin (A \cap B)$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A' \cup B'$

$\therefore x (A \cap B)' \Rightarrow x \in A' \cup B'$ —— ①

Conversaly, let x be any arbitary element of the set $A' \cup B'$. Them, $x \in A' \cup B' \Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \in (A \cap B)'$

$\therefore x \in A' \cup B' \Rightarrow x \in A(A \cap B)'$

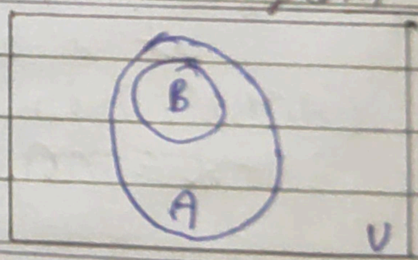
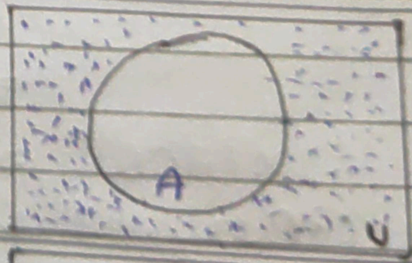
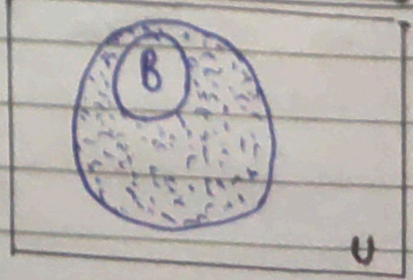
Thus

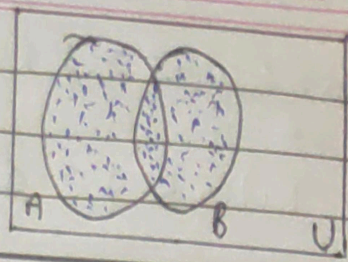
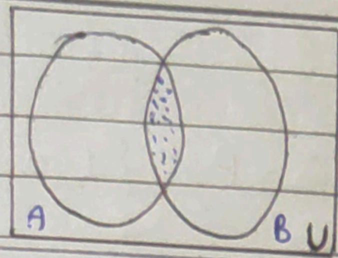
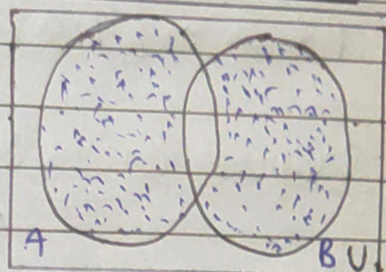
$A' \cup B' \subseteq (A \cap B)'$ —— ②

from ① & ②, we get

$$\boxed{(A \cap B)' = A' \cup B'}.$$

<u>Venn Diagram</u> → A ven diagram is a pictorial representation of sets which are used to show relationship btween Sets and also the operation on them. The universal Sets is represented by the interior of a ractangle and its subsets are represented by circular area drawn with in the rectangle. The venn diagram of Set operation are sone in the following table-

Set operation	Symbol	Venn Diagram
Set B is a proper Subset of 'A'	$B \subset A$	
The complement of Set 'A'	A'	
The diffrence of Set 'A' & 'B'	$A - B$	

The union of sets 'A' & 'B'	$A \cup B$	
The intersection of sets 'A' & 'B'	$A \cap B$	
The symetric difference of sets 'A' & 'B'	$A \oplus B$	

Multi Set →

Multi set are sets where an element can ocur as a member more then once.

For example,

$A = \{a, a, a, b, b, b, c\}$

and $B = \{a, a, a, a, b, b, b, d, d\}$ are multisets

The Multisets 'A' & 'B' can also be written as–

$A = \{3 \cdot a,\ 2 \cdot b,\ 1 \cdot c\}$

and $B = \{4 \cdot a,\ 3 \cdot b,\ 2 \cdot d\}$

Q→ Let P & Q be two multisets $\{3 \cdot a, 2b, 1 \cdot c\}$ & $\{4 \cdot a, 3 \cdot b, 2 \cdot d\}$ respectively. Find

(a) $P \cup Q$
(b) $P \cap Q$
(c) $P - Q$
(d) $Q - P$
(e) $P + Q$

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Sol→(a). $P \cup Q = \{4 \cdot a, 3 \cdot b, 1 \cdot c, 2 \cdot d\}$

(b). $P \cap Q = \{3 \cdot a, 2 \cdot b\}$

(c). $P - Q = \{1 \cdot c\}$

(d). $Q - P = \{1 \cdot a, 1 \cdot b, 2 \cdot d\}$

(e) $P + Q = \{7 \cdot a, 5 \cdot b, 1 \cdot c, 2 \cdot d\}$

Ordered pairs :- An ordered pair is a pair of object whose components occur in a special order. It is return by listing the two component in the specifyed order, shapretting by a, And inclusing the pair in parenthesis. In the orderd pair (a, b), 'a' is called the first component and 'b', the seconds component.

Cartesian product :-
Let 'A' & 'B' be Sets. Cartesion product of 'A' & 'B', denoted by $A \times B$ is defined as.

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

that the $A \times B$ is the set of all possible ordered pairs whose first component comes from 'A' and whose second component come 'B'.

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Q→ Define the cartesion product of sets. If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{2, 4, 6, 8\}$ and $B = \{3, 5, 6, 7\}$ then find $A \times B$, $A - B$?

Sol:-

$A \times B = \{(2,3), (2,5), (2,6), (2,7), (4,3), (4,5), (4,6)$
$(4,7), (6,3), (6,5), (6,6), (6,7), (8,3), (8,5) (8,6)$
$(8,7)\}$

$A - B = \{2, 4, 8\}$

$B - A = \{3, 5, 7\}$

2021

Q→ What is the cordinelty of the set? Find the cardinelty of the set $\{1, \{2, \phi, \{\phi\}\}, \{\phi\}\}$

Sol:- Cardinality of the Set :-

The cardinality of a set 'A' is the number of element in the set 'A'. It is denoted by $|A|$ or $n(A)$.

Let

$A = \{1, \{2, \phi, \{\phi\}\}, \{\phi\}\}$,

Then,

Cardinality of the Set $A = n(A)$
$= 3$

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Q→ For any set A & B, prove that
$P(A \cap B) = P(A) \cap P(B)$.

Sol:- Let $A = \{1, 2\}$
$B = \{1, 2, 3\}$

Then $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

and $A \cap B = \{1, 2\}$

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Now L.H.S = $P(A \cap B)$
$= \{\phi, \{1\}, \{2\}, \{1,2\}\}$

and R.H.S = $P(A) \cap P(B)$
$= \{\phi, \{1\}, \{2\}, \{1,2\}\} \cap \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$
$= \{\phi, \{1\}, \{2\}, \{1,2\}\}$
L.H.S = R.HS

$\boxed{P(A \cap B) = P(A) \cap P(B)}$

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Q→ Show that the for any two sets 'A' & 'B' in the set theory:
$A - (A \cap B) = A - B$

Sol ⇒ Let $A = \{a, b, c, d, e\}$
and $B = \{a, c, e, g\}$
Then L.H.S = $A - (A \cap B)$
$= \{a, b, c, d, e\} - \{a, c, e\}$
$= \{b, d\}$
and R.H.S = $A - B$
$= \{a, b, c, d, e\} - \{a, c, e, g\}$
$= (b, d)\}$
L.H.S = R.H.S
$\boxed{A - (A \cap B) = A - B}$

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Relation :-

Let 'A' & 'B' be two Sets. A binary operation from 'A' to 'B' is a subset of $A \times B$. symbolicaly 'R' is a Relation from 'A' to 'B' if $R \subseteq A \times B$.

If R is a Relation from A to B and if the ordered pair $(a, b) \in R$ then, we say that the element 'a' is Related to the element 'b' by 'R' and we also write aRb. If 'x' is non related to 'y', we write $(x, y) \notin R$.

Relation on a Set :-

Let 'R' be a Relation from $A \times B$. If $A = B$ then we say that 'R' is a relation on the Set 'A' insted of saying that 'R' is a Relation from 'A' to 'A'. Thus a relation on a Set 'A' is a Subset of $A \times A$

Example-1 → Let $A = \{1, 2, 3\}$ and $B = \{p, q\}$, then $R = \{(1, p), (2, q), (3, q)\}$ is a relation from A to B.

Example-2 → Let 'I' with the Set of intizers. Define the following relation < on 'I'.
xRy if and only if $x < y$
Then $R = \{\{x, y\} : (x, y) \in I$ and $x < y\}$

Example-3 → Let $A = \{1, 2, \cdots 12\}$ define the following relation 'R' on 'A':
aRb if and only if a devides b.
Then $R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (1,10), (1,11), (1,12), (2,2), (2,4), (2,6), (2,8), (2,10)$

(2,12), (3,3), (3,6), (3,9), (3,12), (4,4), (4,8), (4,12), (5,5), (5,10), (6,6), (6,12), (7,7), (8,8), (9,9), (10,10), (11,11), (12,12) }

<u>Operations on Relations</u> →

Since binary Relation are sets of ordered pairs all set operation can be done on Relations; The Resulting sets contain ordered pairs and are, therefore Relations. If 'R' and 'S' denote two relation, then.

① Union of 'R' and 'S':

$$x(R \cup S)y = xRy \vee xSy$$

② Intersection of 'R' and 'S':

$$x(R \cap S)y = xRy \wedge xSy$$

③ Difference of 'R' and 'S':

$$x(R-S)y = xRy \wedge x \not S y$$

④ Complement of R:

$xR'y = x \not R y$ where R' is the complement of 'R'.

<u>Q</u> ⇒ If $A = \{x, y, z\}$, $B = \{X, Y, Z\}$, $C = \{x, y\}$ and $D = \{Y, Z\}$ ~~are~~ is a Relation from 'A' to 'B' defined by $R = \{(x, X), (x, Y), (y, Z)\}$ and 'S' is a Relation from 'C' to 'D' defined by $S = \{(x, Y), (y, Z)\}$ find R', $R \cup S$, $R \cap S$ and $R - S$.

Sol → $R' = \{(x,Z),(y,X),(y,Y),(z,X),(z,Y),(z,Z)\}$

$R \cup S = \{(x,X),(x,Y),(y,Z)\}$

$R \cap S = \{(x,Y),(y,Z)\}$

and $R - S = \{(x,X)\}$

Properties of Relations :-

① Reflexive relations →

A Relation 'R' on a set 'A' is said to be reflexive relation if $(a,a) \in R, \forall a \in A$
Thus Relation 'R' is not reflexive if there exist an $a \in A$ such that $(a,a) \notin R$.

Example 1 →

Let A with the set of positive intizer, define a relation 'R' on 'A' as followed. aRb if and only if a devides 'b'.

Since every intizer always devides itself, 'R' is a Reflexive Relation.

Example-2 → Let $A = \{1,2,3,4\}$; Then $R = \{(1,1)(2,3),(2,4),(3,3),(4,1),(4,4)\}$ on A is not reflexive Since $2 \in A$ but $(2,2) \notin R$.

② Symmetric Relations :→ A Relation 'R' on a Set 'A' is said to be symetric Relation. If $(a,b) \in R \Rightarrow (b,a) \in R$.

Thus 'R' is symetric if whenever aRb then bRa.
A Relation 'R' is not symetric if there exist $a, b \in A$ such that $(a,b) \in R$ but $(b,a) \notin R$.

EX-1→ Let $A = \{a, b, c\}$ and let $R_1 = \{\}$, $R_2 = \{(a,a),(b,b)\}$, $R_3 = \{(a,b), (b,a)\}$, $R_4 = A \times A$

Then all four R_1, R_2, R_3 and R_4 are symetric relation. The relation 'R_1' is called empty relation on 'A'. while the relation 'R_4' which is equal to $A \times A$ is called universal relation on 'A'.

Example-2→

Let A with the set of positive intizer. Define a relation 'R' on 'A' as follows:

$(a,b) \in R$ if and only if $a \geq b$.

Then the relation R is not symetric because $(10, 9) \in R$ but $(9, 10) \notin R$.

③ Antisymmetric relation :-

A Relation 'R' on a set 'A' is said to be antisymmetric relation if $(a,b) \in R$ and $(b,a) \in R$ then $a = b$.

A Relation 'R' on a set 'A' is not antisymetric if there exist element $a, b \in A$ such that $(a, b) \in R$ and $(b,a) \in R$ but $a \neq b$.

Example 1 →
Let $A = \{1, 2, 3\}$ and let Relation 'R' on 'A' is given by
$$R = \{(1,2), (2,2), (2,1)\}$$
Then 'R' is an antisymmetric Relation.

(5) ~~Transetive Relation~~ :→
Example 2 → Let $A = \{a, b, c\}$ and let 'R' on 'A' is given by $R = \{(a,b), (a,c), (c,a)\}$
Then 'R' is not antisymetric relation.

(4) Transetive Relation :→
A Relation 'R' on a set 'A' is said to be transitive relation if.
$(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$.
A Relation 'R' on a Set 'A' is not transetive if there exist element $a, b, c \in A$, Not necesurly disting, such that
$(a,b) \in R$ and $(b,c) \in R$ but $(a,c) \notin R$

Example 1 →
Let 'A' with the Set of pasetive intizers. Define a Relation 'R' on 'A' as fallows aRb if and only if a divides 'b'.

If 'R' is a transetive realation because if 'a' devides 'b' and 'b' devides 'c' then 'a' devides 'c'.

Example 2 →

Let $A = \{a, b, B\}$, where $b = B$ and a relation 'R' on 'A' is given by

$R = \{(a, b), (a, B), (b, B), (B, a)\}$

Then 'R' is not transitive because $(a, b) \in R$ and $(b, B) \in R$ but $(a, B) \notin R$.

Equivalence Relation :-

Let A be a non empty set and let 'R' be a Relation on 'A'. Then 'R' is said to be equivalent Relation if it is-

① Reflexive, i.e. for every $a \in A$, aRa.
② Symetric, i.e aRb then bRa.
③ Transitive, i.e. aRb and bRc then aRc.

The equivalence Relation is usualy denoted by this symbol $\sim$

Example →

The following are some example of equivalence relation.

(i) Equality of number on the set of real number.
(ii) Equality of subsets of a universal set.

2020
Q→ Show that the relation "xRy iff $(x-y)$ is divisible by 3" is an equivalence Relation on the set of intizer.

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Sol→ ① R is reflexive
since for any $a \in I$, $a-a=0$ which is divisible by 3, therefore $(a,a) \in R$. Thus R is reflexive.

② R is symmetric:
for any $a, b \in I$, If $a-b$ is divisible by 3.
then $b-a$ is also divisible by 3.
Thus, $(a,b) \in R$ then $(b,a) \in R$.
Hence R is symmetric.

③ R is transitive:
For any $a, b, c \in I$, Let $(a,b) \in R$ and $(b,c) \in R$
⇒ Both $(a-b)$ and $(b-c)$ are divisible by 3.
⇒ $(a-b)+(b-c)$ is divisible by 3.
⇒ $(a-c)$ is divisible by 3.
⇒ aRc
⇒ $(a,c) \in R$
Thus 'R' is transitive.
Since 'R' is reflexive, symmetric and transitive.
'R' is an equivalence Relation.

Q→ If 'R' and 'S' are two equivalence Relation on a Set 'A', Prove that $R \cap S$ is an equivalence relation.

Sol→ It is given that 'R' and 'S' are equivalence Relation on 'A'. We have to prove that $R \cap S$ is an equivalence Relation.

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2018
Q.→

Sol →

① $R \cap S$ is reflexive :
Let $a \in A$ then
$(a,a) \in R$ and $(a,a) \in S$
{∵ 'R' and 'S' are reflexive}

$\Rightarrow (a,a) \in R \cap S$
Thus $(a,a) \in R \cap S, \forall a \in A$
So, $R \cap S$ is Reflexive Relation.

② $R \cap S$ is symetric :
Let $a, b \in A$ such that $(a,b) \in R \cap S$
Since $(a,b) \in R \cap S$
$\Rightarrow (a,b) \in R$ and $(a,b) \in S$.
$\Rightarrow (b,a) \in R$ and $(b,a) \in S$
{∵ 'R' and 'S' are symetric}
$\Rightarrow (b,a) \in R \cap S$
Thus $(a,b) \in R \cap S \Rightarrow (b,a) \in R \cap S$
∴ $R \cap S$ is Symetric Relation.

③ $R \cap S$ is transitive –
Let $a, b, c \in A$ such that $(a,b) \in R \cap S$ and $(b,c) \in R \cap S$
Since, $(a,b) \in R \cap S$ and $(b,c) \in R \cap S$
$\Rightarrow (a,b) \in R$ and $(a,b) \in S$, $(b,c) \in R$ and $(b,c) \in S$
$\Rightarrow (a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ {∵ R is transitive}
and $(a,b) \in S$ and $(b,c) \in S \Rightarrow (a,c) \in S$ {∵ S is transitive}
Now, $(a,c) \in R$ and $(a,c) \in S \Rightarrow (a,c) \in R \cap S$
Thus $(a,b) \in R \cap S$ and $(b,c) \in R \cap S \Rightarrow (a,c) \in R \cap S$
∴ $R \cap S$ is transitive relation.
Since $R \cap S$ is reflexive, symetric and transitive relation $R \cap S$ is an equabelence Relation on 'A'.

2018

Q.3 Define the composite Relation. And let set $A = \{1, 2, 3\}$, $B = \{p, q, r\}$, $C = \{x, y, z\}$ and the relation are $R = \{(1,p), (1,r), (2,q), (3,q)\}$ and $S = \{(p,y), (q,x), (r,z)\}$, then compute $R \circ S$.

Sol → <u>Composit Relation</u> :-

Let 'A', 'B' and 'C' be sets and let 'R' be a relation 'A' to 'B' and let 'S' be a Relation from 'B' to 'C', i.e.

$R \subseteq A \times B$ and $S \subseteq B \times C$. Then the composition of 'R' and 'S' denoted by $R \circ S$, is the Relation from 'A' to 'C' define by setting $(a,c) \in R \circ S$ if and only if there exist an element $b \in B$ such that $(a,b) \in R$ and $(b,c) \in S$.

Supose that 'R' is a relation on a set 'A', that is '$R \subseteq A \times A$'. Then $R \circ R$', The composition of 'R' with itself is always defined and '$R \circ R$' is sometimes denoted by 'R^2'. similarly,

$$R^3 = R^2 \circ R = R \circ R \circ R$$ and show on.

we are given that $A = \{1, 2, 3\}$

$B = \{p, q, r\}$

and $C = \{x, y, z\}$

also relation 'R' on $A \times B$ is

$R = \{(1,p), (1,r), (2,q), (3,q)\}$

and relation S on $B \times C$ is

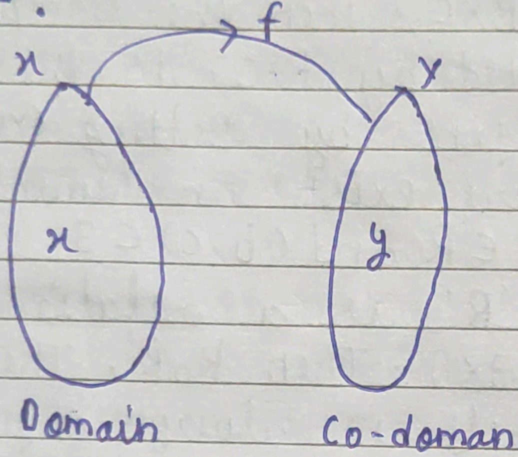
$S = \{(p,y), (q,x), (r,z)\}$

then $R \circ S = \{(1,y), (1,r), (1,z), (2,x), (3,x)\}$

Functions →

Let 'X' and 'Y' be two Sets. A function 'X' to 'Y' is a rule whic Associates to each element 'x' in 'X' a unique element 'y' of 'Y'. The function 'f' from 'X' to 'Y' is denated by $f: X \longrightarrow Y$.

If 'f' is a function from 'X' to 'Y', that is $f: X \to Y$ then this set 'X' is called domain of the function 'f' and 'Y' is called co-domain of f.



The element $x \in X$ is called an argument of the function and the element '$y \in Y$' which the function 'f' associate '$x \in X$' is denated by $f(x)$ and is called the image of 'x' under 'f'. This set $f(x): x \in X$ is called the range of 'f'.

Function as sets of ordered pairs →

If 'X' and 'Y' be any two Sets then a function 'f' from 'X' to 'Y' is a subset 'f' of '$X \times X$' Satisfying the fallowing two conditions:

(i) For each $x \in X$, $(x, y) \in f$ for some $y \in Y$.
(ii) If $(x, y) \in f$ and $(x, z) \in f$ then $y = z$

Classification of functions :-

Function can be classifyed can be into two group.

(1) Algebric function :→ A function which consist of a finite number of terms involving power and roots of the independent veriable 'x' and the four fundamental operation of addition, subtraction multiplication and divison is called Algebric function. Three particular cases of Algebric function are:

$$y = f(x) = x^2 + 5x + 3$$

y ↓ depended Variable

$x^2 + 5x + 3$ ↘ Independed variable.

(i) Polynomial function →

A function of the form $a_0 x^n + a_1 x^n + ---- + a_n$, where 'n' is a positive intizer and $a_0, a_1 ---a_n$ are real constant and $a_0 \neq 0$ is called a polynomial of 'x' in degree 'n'.

For example →, $f(x) = 2x^3 + 5x^2 + 7x - 3$ is a polynomial of degree 3.

(ii) Rational function :-

A function of the form $\frac{f(x)}{g(x)}$, where $f(x)$ are polynomial in x $g(x) \neq 0$ is called a rational function.

For ex, $F(x) = \frac{x^2 + 2x + 1}{x+1}$ is a Rational function.

(iii) Irrational function →

The function involving radicals are called irrational functions.
For ex,
$f(x) = \sqrt[3]{x} + 5$ is an irrational function.

(2) → Transcondental ~~fun~~ function →

A function which is known Algebric is called Transcendental function.

(i) Trigonometric functions :-

The six function $\sin x$, $\operatorname{cosec} x$, $\cos x$, $\sec x$, $\tan x$, $\cot x$, where the angle x is mesaured in radian are called Trigonometric function.

(ii) Inverse Trigonometric functions :- The six function $\sin^{-1}x$, $\operatorname{cosec}^{-1}x$, $\cos^{-1}x$, $\sec^{-1}x$, $\tan^{-1}x$, $\cot^{-1}x$ are called inverse Trigonometric functions.

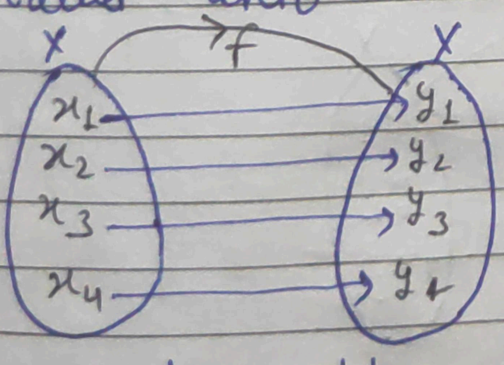
(iii), **Exponential functions :-**
A function $f(x) = a^x (a>0)$ satisfing the law $a^1 = a$ and $a^x \cdot a^y = a^{x+y}$ is called the exponential functions.

(iv) **Logarithm Functions :-** The Inverse of the exponential function is called the Logarithem functions. $\because (\log_e = 1)$

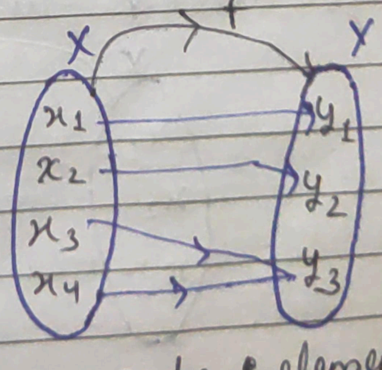
So,
if $y = a^x (a>0, a=1, x \in R, y>0)$ then
$\log_a y = \log_a a^x \Rightarrow x \log_a y$ is called functions.

Type of functions :-

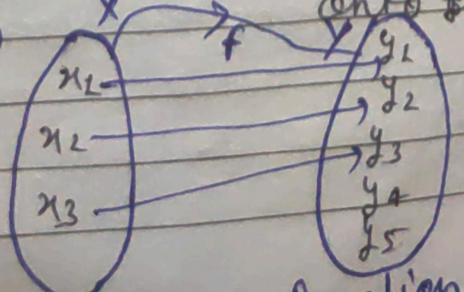
Onto and into functions :-
A function $f: X \to Y$ is called onto (or surjective) if range of '$f = Y$', i.e- Each element of 'Y' is the image of some element of 'X'.
If $f: X \to \cancel{X} Y$ is known onto then it is called into.



onto function



onto & element



into function.

One - to - one function :→

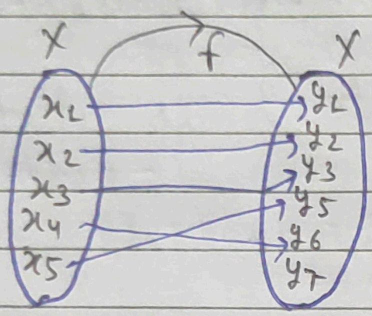
A function $f: X \to Y$ is called one to-one (or injective) if distint element of 'X' have distint images under 'f'. In other words 'f' one-to-one.

If

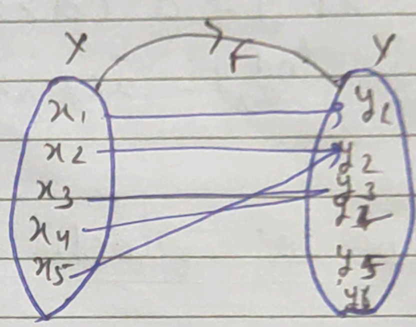
$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

or equivalently,

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$



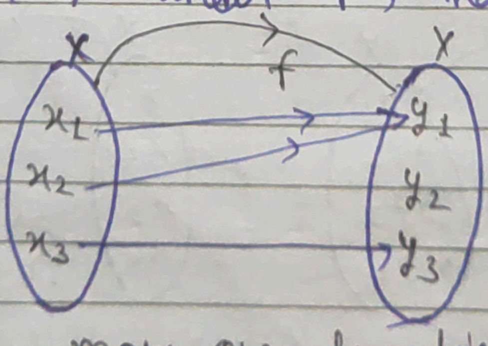
one to one function



Not one to one functions.

Many - one - Function →

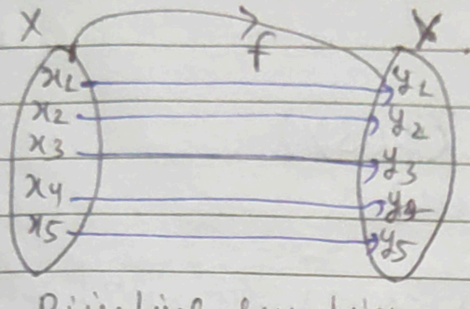
A function $f: X \to Y$ is said to be many-one if atlest two distint element in 'X' have the same image in Y under f, that is -



many one functions

Bijective function →

A function $f: X \to Y$ is said to be bijective if 'f' is one-to-one and onto. A Bijective function is also called one-to-one correspondance.



Bijective function

2020 Q. Define the function. AND also explain the varrious types of function.

Sol→

Composite function –

Let f be a function from 'X' to 'Y' and let g be a function from 'Y' to 'Z'. Then we composite of the function f and g denoted by gof is a maping from 'X' to Z define by

$$(g \circ f)(x) = g(f(x)), \ \forall \ x \in X.$$

Q. Let $X = \{1, 2, 3\}$ and 'f' and 'g' be functions from 'X' to 'X' given by: $f = \{(1,2), (2,3), (3,1)\}$ $g = \{(1,1), (2,2), (3,1)\}$. Find fog and gof and also show that $fog \neq gof$

Sol: Given that, $f: X \to X$

and $g: X \to X$

$\therefore (fog)(x) = f(g(x)), \ \forall\, x \in X$

Now, $(fog)(1) = f(g(1))$

$= f(1)$

$\Rightarrow (fog)(1) = 2$

$(fog)(2) = f(g(2))$

$= f(2)$

$(fog)(2) = 3$

and $(fog)(3) = f(g(3))$

$= f(1)$

$\Rightarrow (fog)(3) = 2$

$\therefore \boxed{fog = \{(1,2),(2,3),(3,2)\}}$ — ①

now,

$(gof)(1) = g(f(1)$

$= g(2)$

$\Rightarrow (gof)(1) = 2$

$(gof)(2) = g(f(2))$

$= g(3)$

$\Rightarrow \boxed{(gof)(2) = 1}$

and $(gof)(3) = g(f(3))$

$= g(1)$

$\Rightarrow (gof)(3) = 1$

$\boxed{gof = \{(1,2),(2,1),(3,1)\}}$ — ②

from eqn ① & ②, we ~~get~~ see that

$\boxed{fog \neq gof}$.

2017 Q.→ Let if two fallowing function be define on set of real no. we as:
$f(x) = 2x+3$ and $g(x) = x^2+1$ find the $(f\circ g)(x)$.

Sol→ we have,

$$(f\circ g)(x) = f(g(x))$$
$$= f(x^2+1)$$
$$= 2(x^2+1)+3$$
$$\boxed{(f\circ g)(x) = 2x^2+5}$$

2017 Q.→ Let 'f' and 'g' : $R \to R$, we defined as fallowed:

$$f(x) = x+2,\; g(x) = \frac{1}{x^2+1}$$ comput $f\circ g(x)$.

Sol→ we have

$$f\circ g(x) = f(g(x))$$
$$= f\left(\frac{1}{x^2+1}\right)$$
$$= \frac{1}{x^2+1} + 2$$
$$\Rightarrow \boxed{f\circ g(x) = \frac{2x^2+3}{x^2+1}}$$

Identity - function -

A Function $I : X \to X$ is called an identity function if.

$$I(x) = x,\ \forall\, x \in X$$

under identity function, each element is mapped on itself.

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2019 Q→ Define the inverse function with example.

Sol→

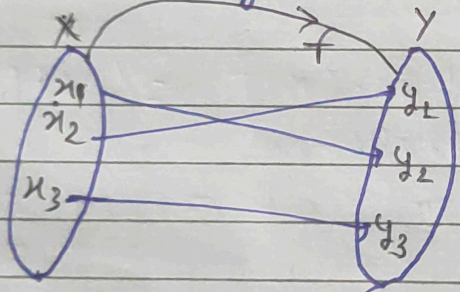
Inverse function :→

Let $f: X \to Y$ be a bijective function. The function $f^{-1}: Y \to X$ defined by $f^{-1}(y) = x$, where $f(x) = y$, is called the inverse function of 'X'. (1)

It can be sum easly that if f is bijective then f^{-1} is also bijective. More over $f^{-1} o f$ & $f o f^{-1}$ both are identity function of 'X' and 'Y' respectivly. (2)

Example-

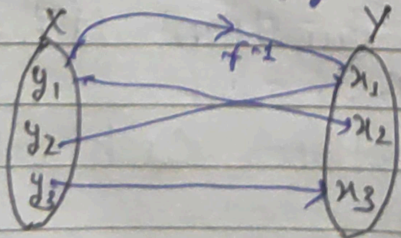
Let the function $f: X \to Y$ be define by the digram as soon in the figer. (3)



then f is bijective.

Therefore,

'f^{-1}', The inverse function, exist. One can describe 'f^{-1}': $Y \to X$ by the diagram as soon in the figer.



Notice that if we sent the arrows in the opposite direction in the digram of 'f', we essentialy have the diagram of 'f^{-1}'.

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Operations of function :→

① **Addition of two functions :-**

Let $f(x)$ and $g(x)$ be two functions then there addition will be a function $(f+g)(x)$ is defined as:

$$(f+g)(x) = f(x) + g(x).$$

② **Subtraction of two functions :-**

Let $f(x)$ and $g(x)$ be two functions then there Subtraction is defined as:

$$(f-g)(x) = f(x) - g(x).$$

③ **Multiplication of two functions :-**

If $f(x)$ and $g(x)$ are two functions then there multiplication denots by:

$(f \cdot g)(x)$ = is defined as:

$$(f \cdot g)(x) = [f(x)] \cdot [g(x)]$$

④ **Divison of two functions :-**

If $f(x)$ and $g(x)$ are two functions then there divison denoted as $(f/g)(x)$ is defined as:

$$(f/g)(x) = f(x) / g(x).$$

Q→ Given $f(x) = 3x+2$ and $g(x) = 4-5x$ find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, $(f/g)(x)$.

Sol→

Given that:

$$f(x) = 3x+2$$

and $g(x) = 4-5x$

we have

$$(f+g)(x) = f(x) + g(x)$$
$$= 3x+2+4-5x$$
$$\boxed{(f+g)(x) = -2x+6}$$

$$(f-g)(x) = f(x) - g(x)$$
$$= (3x+2) - (4+5x)$$
$$= 3x+2-4+5x$$
$$\boxed{(f-g)(x) = 8x-2}$$

$$(f\cdot g)(x) = f(x)\cdot \cancel{f}\, g(x)$$
$$= (3x+2)\cdot(4-5x)$$
$$\boxed{(f\cdot g)(x) = 15x^2+2x+8}$$

and

$$(f/g)(x) = \frac{f(x)}{g(x)}$$
$$= \frac{(3x+2)}{(4-5x)} \times \frac{(4+5x)}{(4+5x)}$$
$$\boxed{(f/g)(x) = \frac{15x^2+22x+8}{16-25x^2}}$$

find

Recursively defined functions :-

Sometime it is dificult to define an object explicitly. How ever, it may be easy to define this object in terms of itsef. This process is called Recursion. Recursion refers to several related consept is computer science and mathematics. One can use recursion to define sequences, functions, and sets. The sequences 1, 3, 9, 27, ... for example, can be defined explicitly by the formula $S(n) = 3^n$ for all integers $n \geq 0$, but the sequence can also be defined recursively as fallowes.

(i) $S(0) = 1$

(ii) $S(n+1) = 3S(n)$ for all integers $n \geq 0$

(iii) Here (ii) is the salient feature of recursion namly the feature of self- refrence.

2022
Q.→ Define the function and explain the diffrence between function and relation with example.

Sol. Function :-

Let 'X' and 'Y' be two sets. A function 'X' to 'Y' is a rule whic Associates to each element 'x' in 'X' a unique element 'y' of 'Y'. The function 'f' from 'X' to 'Y' is denated by $f: X \rightarrow Y$.

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Date 26/10/23
2022
Q→

Difference between function & relation.

Let 'A' and 'B' be two sets. Let 'f' be a function from 'A' to 'B'. Then 'f' ~~be a function~~ is a subsets of $A \times B$ satisfing the fallowing to condition:

① For each $a \in A$, the ordered pair $(a,b) \in f$ for some $b \in B$.

② If $(a,b) \in f$ and $(a,c) \in f$, then $b = c$

On the other hand, Every subset of $A \times B$ is a relation from 'A' to 'B'. Thus every function is a relation but every relation is not necessarly a function. In a relation relation from 'A' to 'B' an element of 'A' may be related to more then one element in 'B'. Also there may be some element of 'A' which may not be related to any element of 'B'. But in a function from 'A' to 'B' each element of 'A' must be associated to one and only one element of 'B'.

Example→

Let $'A' = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Then $R = \{(1,a), (2,a), (3,a), (4,a)\}$ is both relation & function from A to B. But $S = \{(1,a), (2,b), (1,c), (3,a)\}$ is a relation from A to B but not a function from A to B because the element 1 of A is associated to different elements a and c of B.

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2022 Q→

If $X = \{1, 2, 3\}$, $Y = \{p, q\}$ and $Z = \{a, b\}$ and the function f and g are defined as:
$f: X \rightarrow Y$ be $f = \{(1, p), (2, p), (3, q)\}$, $g: Y \rightarrow Z$ be $g = \{(p, a), (q, b)\}$ then find fog and gof.

Sol:-

It is given that

$f: X \rightarrow Y$

and $f: Y \rightarrow Z$

$\therefore (gof)(x) = g(f(x)), \ \forall x \in X$

Now,

$(gof)(1) = g(f(1))$

$= g(p)$

$\Rightarrow (gof)(1) = a$

$(gof)(2) = g(f(2))$

$= \cancel{g(p)}\ g(p)$

$\Rightarrow (gof)(2) = a$

and $(gof)(3) = g(f(3))$

$= \cancel{g(}\ g(q)$

$\Rightarrow (gof)(3) = b$

$$\boxed{gof = \{(1, a), (2, a), (3, b)\}}$$

Similarly $(fog)(x) = f(g(x)), \ \forall x \in Z$

Now,

$(fog)(a) = f(g(a))$

$= f(p)$

$= 1$ or 2

$\Rightarrow (fog)(a) = 1$

or $(fog)(a) = 2$

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and $(f\circ g)(b) = f(g(b))$
$= f(q)$
$\Rightarrow (f\circ g)(b) = 3$

$\therefore$ $\boxed{\begin{array}{l} f\circ g = \{(a,1), (b,3)\} \\ \text{or } f\circ g = \{(a,2), (b,3)\} \end{array}}$

Partial order Relation:-

A binary relation 'R' defined on a set 'A' is called Partial order relation if R is:

① Reflexive, i.e for any $a \in A$ then $(a,a) \in R$
② Antisymetric, i.e if $(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b$, for $a, b \in A$

③ Transitive, i.e. if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$, for $a, b, c \in A$.

2024
Q→ Prove that the relation $(x,y) \in R$, if $x \geq y$ defined on the set of positive intizer is a partial order relation.

Sol:→ To show that set of positive intigers (I^+) with defined relation R is a partial order relation, we need to show that 'R' is-
① Reflexive
② Antisymmetric
③ transitive

① R is reflexive:
Since for any $a \in I^+$, $a \geq a$ exists, therefore $(a,a) \in R$.
Thus R is reflexive.

② R is Antisymmetric:

For any $a, b \in I^+$, if $(a,b) \in R$ and $(b,a) \in R$

$\Rightarrow a \geq b$ and $b \geq a$

$\Rightarrow a = b$

Hence R is antisymmetric.

③ R is transitive:

For any $a, b, c \in I^+$. if $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow a \geq b$ and $b \geq c$

$\Rightarrow a \geq b \geq c$

$\Rightarrow a \geq c$

$\Rightarrow (a,c) \in R$

Thus R is Transitive

Since 'R' is Reflexive, Antisymmetric and transitive. Therefore 'R' is a partial order Relation.

UNIT = II

30/10/2023

Posets, Hasse Diagram and Lattices:

Introduction:-

In these topic we discused various types of relations that can be defined on a set. Now, we narrow down our intrest to partial order relation which is defined on a set called a partial ordered set. This would be finaly leed to the consept of latices.

Partial ordered Relation:-

A Relation 'R' on a set 'S' is called a partial order if it is reflexive, antisymetric and transitive. That is:

1. $(a,a) \in R$ forall $a \in S$ (reflexive)
2. $(a,b) \in R$ and $(b,a) \in R \Rightarrow a=b$ for $a,b \in S$ (antisymetric)
3. $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ for $a,b,c \in S$ (transitive)

A set 'S' together with a partial order relation 'R' is called a partial order set for a poset. It is denoted by (S,R). This relation 'R' is often denoted by the symbol $\preceq$. Which is diffrent from the usual less then or equal to symbol $\leq$. Hence a posets is denoted by $(S, \preceq)$

Q→ Consider a set of integers 'Z'. A relation 'R' defined as "a is $\leq b$". Show that the relation is a partial order relation and set 'Z' is a poset.

sol→ To show that 'Z' with defined relation 'R' is a poset, we need to show that 'R' is:

① Reflexive
② Antisymmetric
③ Transitive

① R is Reflexive:
Since for any $a \in Z$, $a \leq a$ exists, therefore $(a, a) \in R$.
Thus 'R' is Reflexive.

② R is antisymmetric :-
for any $a, b \in Z$, if $(a, b) \in R$ and $(b, a) \in R$
$\Rightarrow a \leq b$ and $b \leq a$
$\Rightarrow a = b$
Hence 'R' is antisymmetric.

③ R is transitive :-
for any $a, b, c \in Z$, if $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow a \leq b$ and $b \leq c$
$\Rightarrow a \leq b \leq c$
$\Rightarrow a \leq c$
$\Rightarrow (a, c) \in R$
Thus R is transitive.
Since 'R' defined as "a is $\leq b$" is reflexive, antisymmetric and transitive and thus partial order relation and set 'Z' together with this partial order relation is a poset.

Comparable element :-

If $(A, \leq)$ is a poset then the elements $a, b \in A$ are said to comparable if $a \leq b$ or $b \leq a$ and if neighter $a \leq b$ nor $b \leq a$ then a, b are said to be non-comparable.

Hasse Diagram :-

$(A, \leq)$ we a Poset. A partial order on a set 'A' can be represented by a diagram non-as Hass diagram of $(A, \leq)$. In such a diagram of each element is represented by a dot or by a small circuit and two comparable element are joints by ~~tim~~ lines. In such a way that $a \leq b$ then a lies below 'b' in the diagram. In the case of transitive property, that is if $a \leq b$ and $b \leq c$ it fallowes that $a \leq c$. We omit the line joining to 'a' and 'c'. How ever, we draw the line from 'a' to 'b' and from 'b' to 'c'. Non comparable elements are not joint.

For example -

Let $A = \{1, 2, 3, 4, 6, 12\}$ consider the partial order ~~relation~~ of divisibility on the given set 'A', that is: if $a, b \in A$ and $A \leq b$ iff a/b.

Draw the Hasse diagram.

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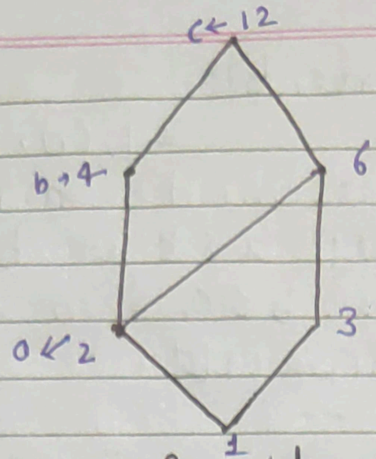


$a \leq b$ and $b \leq c$
$\Rightarrow a \leq c$

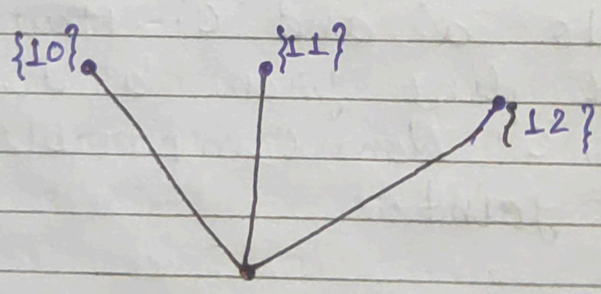
2021
Q7. If $S = \{10, 11, 12\}$. determine the power set of 'S'. Draw the Hasse diagram of Poset$(P(S), \subseteq)$

sol:-

Since $S = \{10, 11, 12\}$

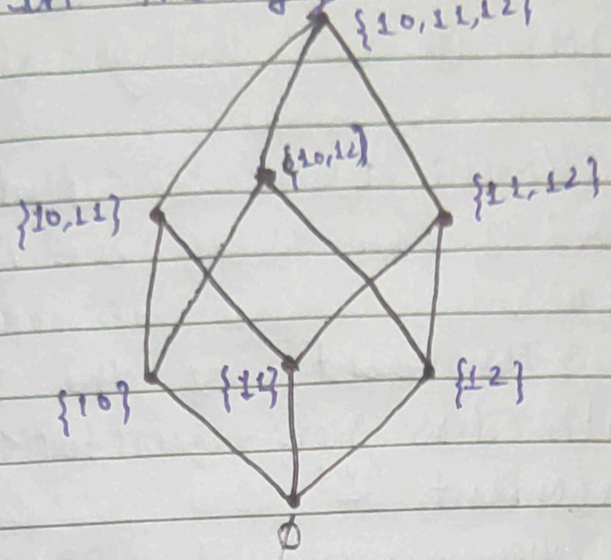
$\therefore P(S) = \{\phi, \{10\}, \{11\}, \{12\}, \{10, 11\}, \{11, 12\}, \{10, 12\},$
$\{10, 11, 12\}\}$.

Since the null set 'ϕ' is the subset of all sets, it is the lowest point of the diagram. Now, $\{10\}$, $\{11\}$ and $\{12\}$ are the immediate successor of 'ϕ'. They are placed at higher level then 'ϕ' and connected with 'ϕ'.



Then $\{10, 11\}$ is the immediate succesor of $\{10\}$ and $\{11\}$, so $\{10, 11\}$ is placed at higher level then $(10, 11)$ and connected with $\{10\}$ and $\{11\}$

similarly, The Other points are drawn. The Hasse diagram of the poset $(P(S), \subseteq)$ is shown in the figer-



Minimal and Maximal elements :→

Let $(A, \leq)$ be a Poset, where '$\leq$' represents an arbitrary partial order. Then an element $b \in A$ is a minimal element of 'A' if there is No element $a \in A$ that setisfie $a \leq b$. S

Similarly,
An elements $b \in A$ is a Maximal element of 'A' if there is no element $a \in A$ that satisfie $b \leq a$.

For example-
The Set of $\{\{1\}, \{2\}, \{1,2\}\}$ with Subset has to Minimal elements $\{1\}$ and $\{2\}$. Not that $\{1\}$ and $\{2\}$ are not related to each other in Subset. Hence we can not say which is 'smaller than' which, that is they are not compareble.

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Least and Greatest elements :-

Let '(A, ≤)' be a Poset. Then an element '$b \in A$' is the least element of 'A' if for every element $a \in A$, $b \leq a$. Similarly,

An element $b \in A$ is called greatest element of 'A' if for every $a \in A$, $a \leq b$.

For examples :- ①

The Poset of the set of Natural numbers with less then equal to ($\leq$) relation has the least element '1'.

② The Poset of the power set of {1, 2} with '$\subseteq$' has the least element 'ϕ'.

Totally ordered (Linearly ordered) Set :-

An ordered Set 'A' is said to be totally order, if every pair of element in 'A' are comparable. We also say that 'A' is a chain.

For example :-

The usual '$\leq$' (less than or equal to) is a partial order on 'I^+'. Thus (I^+, $\leq$) is a poset and this poset is a totally order set, that is a chain since every pair of elements (I^+, $\leq$) are comparable.

2021
Q.> Define the well-ordered set. Give an example of well order set.

Well-Ordered Set :-

A Poset (A, ≤) is called well ordered set if it is totally ordered

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and every non-empty subset of 'A' has a least element.

Example:-

The set of natural nos with order less then equal to is well-ordered set. The Posets $(Z, \leq)$ is not well-ordered since the set of negative intiger, which is a subset of 'Z', has no least element.

Upper and Lower bounds:-

Let 'B' be a subset of a Poset $(A, \leq)$. An element $u \in A$ is called an upper bound of 'B' if u succeeds every element of 'B', that is $x \leq = u$ for all $x \in B$.

An element $l \in A$ is called a lower bound of 'B' if l precedes every element of 'B', that is:

$l \leq x$ for all $x \in B$

$A = \{1, 2, 3, 5, 7, 9\}$
$B = \{2, 3, 5, 7\}$
Let $u = 7 \in A$
$2 \leq 7$
$3 \leq 7$
$5 \leq 7$
$7 \leq 7$

Let $l = 2 \in A$
$2 \leq 2$
$2 \leq 3$
$2 \leq 5$
$2 \leq 7$

Date 03/11/2

least upper and greatest lower bounds:-

An element '$a \in A$' is called a least upper bound (lub) of 'B' if (i) a is an upper bound of 'B' and

(ii) $a \leq a'$, for any upper bound a' of 'B'. Thus $a = lub(B)$. A least upper bound is also called supremum and written as

$a = sup(B)$.

Similarly,

an element $a \in A$ is called the greatest the lower bound $glb(B)$ if -

(i) 'a' is a lower bound of 'B'

(ii) $a' \leq a$, whenever a' is a lower bound of 'B'. Thus $a = glb(B)$. A greatest lower bound is also called infimum and written as

$a = inf(B)$.

Example -

In the Poset $A = (\{1, 2, 3, 4, \ldots 10\}, /)$, the subset $\{2, 7\}$ has no upper bound since there is no intizer in 'A' which is divisival by both 2 and 7. The lower bound of the Subset is 1 since 2 and 7 are divisival by only 1. Hence, 1 is the greatest lower bound for $\{2, 7\}$ i.e-

$glb\{2, 7\} = 1$.

The Subset $\{1, 2, 3\}$ has 6 and 1 as unique upper and lower bounds. Hence 'lub' of $\{1, 2, 3\} = 6$ and $glb\{1, 2, 3\} = 1$.

The Subset $\{1, 2, 4\}$ has 4 and 8 as upper bounds. Hence $lub\{1, 2, 4\} = 4$.

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Lattice :-
A Posets $(A, \leq)$ is called a lattice if every 2-element subset of 'A' has both a least upper bound and a greatest lower bound, that is if lub $\{x, y\}$ and glb $\{x, y\}$ exist for all $x, y \in A$. In this case, we denote
$x \vee y = $ lub $\{x, y\}$ (read as join of x & y).
$x \wedge y = $ glb $\{xy\}$ (read as meet of x & y).

Therefore,
Lattice is a mathematical structured equipped with two binary operation join and meet.

Q→ let $D_{20} = \{1, 2, 4, 5, 10, 20\}$ with the set of all positive divisors of 20. Show that D_{20} forms a lattice under the relation divisibility that for $a, b \in D_{20}$, $a \leq b \Rightarrow a/b$.

Sol Here $D_{20} = \{1, 2, 4, 5, 10, 20\}$
① Divisibility is reflexive:
i.e. $\forall a \in D_{20}$
$a \leq a$ exist iff a/a.

② Divisibility is antisymetric.
i.e- for $a, b \in D_{20}$
If a/b and $b/a \Rightarrow a = b$.

③ Divisibility is transitive:
i.e- $a, b, c \in D_{20}$
If a/b and b/c the a/c.

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Therefore $(D_{20}, /)$ is a poset.
Now, for any $a, b \in D_{20}$
Join of a and b is
$a \vee b = lub\{a, b\}$ exist in D_{20}.
For example:-
$$2 \vee 5 = lub\{2, 5\}$$
$$= 10 \in D_{20}$$
And meet of 'a' and 'b' is $a * b = glb\{a, b\}$ exist in D_{20}.
For example:- $2 * 5 = glb\{2, 5\}$
$$= 1 \in D_{20}$$
Hence $(D_{20}, /)$ is a lattice.

2022
Q→ Let 'L' be the set of all factor of '12' and let '/' with the divisibility relation on 'L'. Show that $(L, /)$ is a lattice.

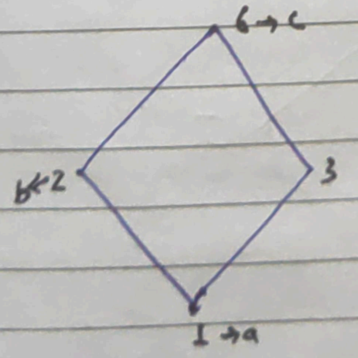
Sol - Hence
$$L = \{1, 2, 3, 4, 6, 12\}$$

Q. - Let L be the set of all factor of '30' and let '/' be the divisibilty relation on 'L'. Then show that (L, '/') is a lattice.

Sol:- Hence $L = \{1, 2, 3, 5, 6, 10, 15, 30\}$

2021
Q→ Prove the Hase diagram of the lattice of $(D_6, /)$.

Sol:- $D_6 = \{1, 2, 3, 6\}$
for any $a, b \in D_6$, $a \leq b \Rightarrow a/b$
we draw the Hasse diagram:



$(D_6, /)$

Q→ Let D_m denote the positive integer of integers of m ordered by divisibility. Draw the hasse diagram of :

(a) D_{24} , (b) D_{15}

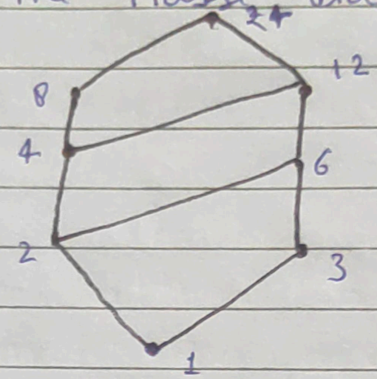
Sol→

(a) D_{24} -

Here $D_{24} = \{1, 2, 4, 6, 12, 24\}$

for any $a, b \in D_{24}$, $a \leq b \Rightarrow a/b$

we draw the Hasse diagram.

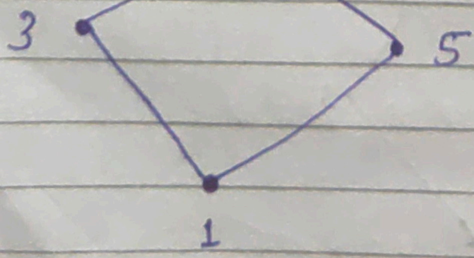


(b) D_{15}

Here $D_{15} = \{1, 3, 5, 15\}$

for any $a, b \in D_{15}$, $a \leq b \Rightarrow a/b$

we draw the Hasse diagram.



ntegers of
hasse diagram

Dated 06/12/23

2018
Q. Consider the poset $S = (\{1, 2, 3, 4, 6, 9, 12, 18, 36\}, /)$
Find the greatest lower bound and least upper bound ~~and~~ of this set $\{6, 18\}$ and $\{4, 6, 9\}$.

Sol. The set $\{6, 18\}$ has 18 and 36 as upper bounds
Hence $lub\{6, 18\} = 18$
And the set $\{6, 18\}$ has 1, 2, 3 and 6 as lower bonds. Hence $glb\{6, 18\} = 6$
Now, the set $\{4, 6, 9\}$ has 36 and 1 as unique upper and lower bounds.
Hence, $lub\{4, 6, 9\} = 36$ and
$glb\{4, 6, 9\} = 1$

Properties of Lattices :-

① If $a \leq a \vee b$ and $b \leq a \vee b$ then $a \vee b$ is an upper bound of a and b.

② If $a \leq c$ and $b \leq c \Rightarrow a \vee b \leq c$ then $a \vee b$ is a least upper bound of a and b.

③ If $a \wedge b \leq a$ and $a \wedge b \leq b$ then $a \wedge b$ is a lower bound of a and b.

④ If $c \leq a$ and $c \leq b \Rightarrow c \leq a \wedge b$ then $a \wedge b$ is a greatest lower bound of a and b.

Complete Lattice :-

• A Lattice is called complete if each of its non-empty subsets has a least upper bound and a greatest lower bound
Clearly, Every finite lattice is complete

because every subset here is finite. Also every complete lattice must have a least element and a greatest element. The least and the greatest elements of a lattice are called bounds of the lattice and are denoted by 0 and 1 respectively.

Bounded lattice :-

A lattice 'L' is said to be bounded lattice if it has a greatest element '1' and a least element '0'.

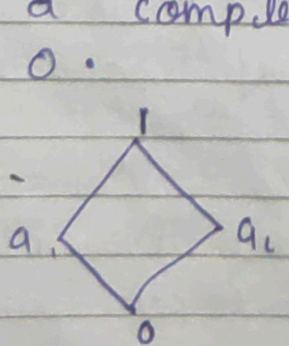
Example -

The lattice $P(S)$ of all subsets of a set 'S' under the operations ~~sub~~ '$\subseteq$' is a bounded lattice. Since in this lattice, its greatest element is the set 'S' and its least element is the set 'ϕ'.

Complemented lattice :-

Let 'L' be a bounded lattice with greatest element '1' and least element '0' and let $a \in L$. An element $a' \in L$ is called a complement of 'a'. If $a \vee a' = 1$ and $a \wedge a' = 0$.

Example :-



In this figure, complement of a_1 is a_2.

Since, $a_1 \vee a_2 = lub\{a_1, a_2\} = 1$

and $a_1 \wedge a_2 = glb\{a_1, a_2\} = 0$

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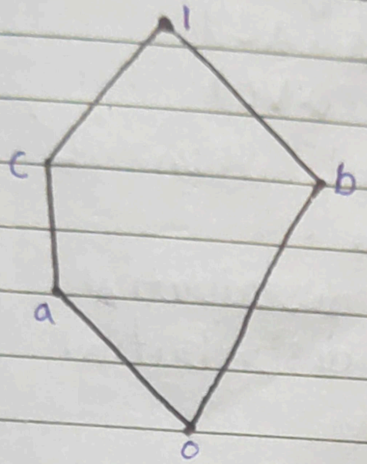
Madular lattice :-

A lattice 'L' is said to be madular lattice if for every $a, b, c \in L$ and $a \leq c$.

$\Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge c$.

Q: Show that The pentagonal lattice is not madular.



Sol:- We have

$a \vee (b \wedge c) = a \vee glb\{b, c\}$

$= a \vee 0$

$= lub\{a, 0\}$

$= a$

$\Rightarrow a \vee (b \wedge c)$

and $(a \vee b) \wedge c = lub\{a, b\} \wedge c$

$= 1 \wedge c$

$= glb\{1, c\}$

$\Rightarrow (a \vee b) \wedge c = c$

$\because a \vee (b \wedge c) \neq (a \vee b) \wedge c$, though $a \leq c$.

Hence iven pentagonal lattice is not madular.

07/11/23
2019

Q. Define the "Distributive lattice". Prove that in a distributive lattice, If an element has a complement then this compliment is unique.

Sol - Distributive lattice :→

Let 'L' be a lattice and let a, b, c be any element of 'L'. If following distributive property -

(1) $A \wedge (b \vee c) = a \wedge b) \vee (a \wedge c)$

(2) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

hold for $a, b, c \in L$ then 'L' is said to be distributive lattice.

If 'L' is not distributive lattice then it is called non-distributive lattice.

Proof :→

Supose that an element a has two compliment 'b' and 'c'. Then

$a \vee b = 1$, $a \wedge b = 0$

and $a \vee c = 1$, $a \wedge c = 0$

we have,

$b = b \wedge 1$ $\{\because b \wedge 1 = glb\{b, 1\}\} = b$

$= b \wedge (a \vee c)$ $\{\because a \vee c = 1\}$

$= (b \wedge a) \vee (b \wedge c)$ {By distributive property.}

$= (a \wedge b) \vee (b \wedge c)$ {By commutative property}

$= 0 \vee (b \wedge c)$ $\{\because a \wedge b = 0\}$

$= (a \wedge c) \vee (b \wedge c)$ $\{\because a \wedge c = 0\}$

$= (a \vee b) \wedge c$

$= 1 \wedge c$ $\{\because a \vee b = 1\}$

$= glb\{1, c\}$

$\Rightarrow b = c$

Hence in a distributive lattice, If an element has a compliment then this compliment is unique.

2022
Q. Define modular lattice. Also proof that every distributive lattice is modular.

Sol: Modular lattice :-

A lattice 'L' is said to be modular lattice if for every $a, b, c \in L$ and $a \leq c$.

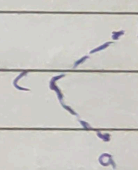
$\Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge c$.

Proof -

Let $(L, \leq)$ be a distributive lattice and $a, b, c \in L$ be such that $a \leq c$.

Thus if $a \leq c$ then $a \vee c = lub\{a, c\}$

$\Rightarrow a \vee c = c$



Now, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \vee (b \wedge c) = (a \vee b) \wedge c$

Hence Every distributive lattice is modular.

Boolean Algebra :-

An order 6- tuples $(S, 0, 1, +, *, ')$ in which S is a set of elements ~~of~~ 0 and 1 are elements of S '+' and * are two binary operation on S and ' and a unary operation is a boolean algebra is following axiom hold.

axioms Properties :-

① Commutative laws :- For all $a, b \in S$

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$$a+b = b+a$$

and $a * b = b * a$

(2) Distributive laws :-

For $a, b, c \in S$

$$a+(b*c) = (a+b)*(a+c)$$

and $a*(b+c) = (a*b)+(a*c)$

(3) Identity Laws :-

For all $a \in S$

$$a+0 = a$$

and $a*1 = a$

(4) Complement laws :-

For each $a \in S$ there exist an element $a' \in S$ such that

$$a+a' = 1$$

and $a*a' = 0$

Theorems of Boolean Algebra :-

Let $a, b, c \in S$, then

1- Idempotent Laws :

(a) $a+a = a$

(b) $a*a = a$

2- Boundedness laws :-

(a) $a+1 = 1$

(b) $a*0 = 0$

3- Absorption Laws :

(a) $a+(a*b) = a$

(b) $a*(a+b) = a$