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Posets, Hasse Diagram and Lattices:

Introduction:-

In these topic we discussed various types of relations that can be defined on a set. Now, we narrow down our intrest to partial order relation which is defined on a set called a partial ordered set. This would be finaly leed to the consepct of latices.

Partial orderd Relation:-

A Relation 'R' on a set 'S' is called a partial order if it is reflexive, antisymetric and transitive.
That is:

① $(a,a) \in R$ forall $a \in S$ (reflexive)
② $(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b$ for $a, b \in S$ (antisymmetric)
③ $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ for $a, b, c \in S$ (transitive)

A set 'S' together with a partial order relation 'R' is called a partial order set for a, poset. It is denoted by (S, R). relation 'R' is often denoted by this symbol $\preceq$. Which is diffrent from the usual less then or equal to symbol $\leq$. Hence a posets is denoted by $(S, \succeq)$

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Q→ Consider a set of integers 'Z'. A relation 'R' defined as "a is ≤ b". Show that the relation is a partial order relation and set 'Z' is a poset.

sol→ To show that 'Z' with defined relation 'R' is a poset, we need to show that 'R' is:

① Reflexive
② Antisymmetric
③ Transitive

① R is Reflexive:
Since for any $a \in Z$, $a \leq a$ exists, therefore $(a, a) \in R$.
Thus 'R' is Reflexive.

② R is antisymmetric :-
for any $a, b \in Z$, if $(a, b) \in R$ and $(b, a) \in R$
$\Rightarrow a \leq b$ and $b \leq a$
$\Rightarrow a = b$
Hence 'R' is antisymmetric.

③ R is transitive :-
for any $a, b, c \in Z$, if $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow a \leq b$ and $b \leq c$
$\Rightarrow a \leq b \leq c$
$\Rightarrow a \leq c$
$\Rightarrow (a, c) \in R$
Thus R is transitive.

Since 'R' defined as "a is ≤ b" is reflexive, antisymmetric and transitive. and thus partial order relation and set 'Z' together with this partial order relation is a poset.

Comparable element :-

If $(A, \leq)$ is a poset then the elements $a, b \in A$ are said to comparable if $a \leq b$ or $b \leq a$ and if neighter $a \leq b$ nor $b \leq a$ then a, b are said to be non-comparable.

Hasse Diagram :-

$(A, \leq)$ we a Poset. A partial order on a set 'A' can be represented by a diagram non-as Hass diagram of $(A, \leq)$. In such a diagram of each element is represented by a dot or by a small circuit and two comparable element are joints by ~~tim~~ lines. In such a way that $a \leq b$ then a lies below 'b' in the diagram. In the case of transitive property, that is if $a \leq b$ and $b \leq c$ it fallows that $a \leq c$. We omit the line joining to 'a' and 'c'. How ever, we draw the line from 'a' to 'b' and from 'b' to 'c'. Non comparable elements are not joint.

For example -

Let $A = \{1, 2, 3, 4, 6, 12\}$ consider the partial order relation of divisibility on the given set 'A', that is: if $a, b \in A$ and $A \leq b$ iff a/b.

Draw the Hasse diagram.

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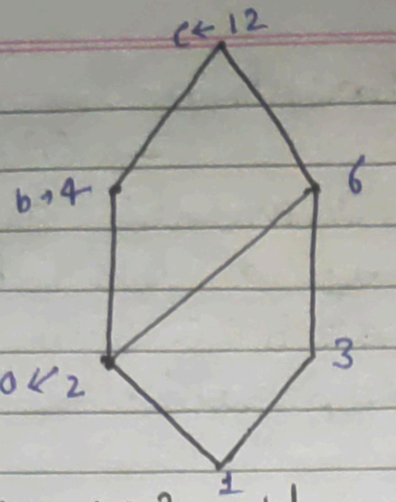


$a \leq b$ and $b \leq c$

$\Rightarrow a \leq c$

2021 Q.

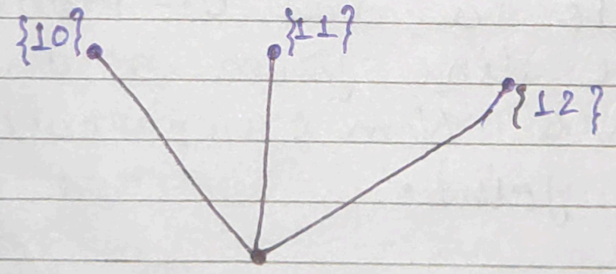
If $S = \{10, 11, 12\}$. determine the power set of 'S'. Draw the Hasse diagram of Poset$(P(S), \subseteq)$

sol:-

Since $S = \{10, 11, 12\}$

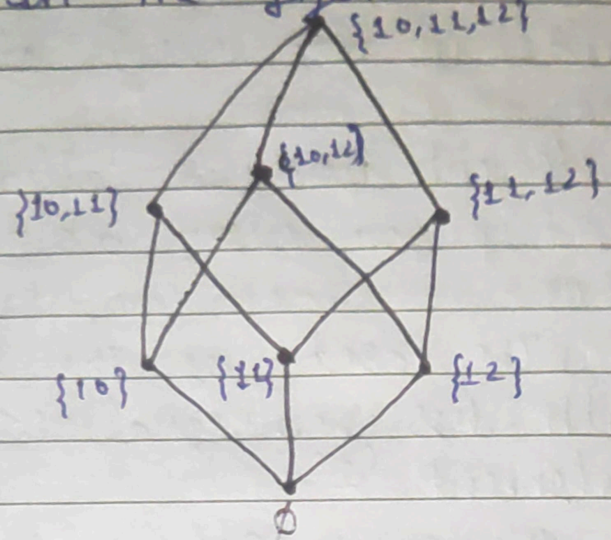
$\therefore P(S) = \{\phi, \{10\}, \{11\}, \{12\}, \{10, 11\}, \{11, 12\}, \{10, 12\}, \{10, 11, 12\}\}$.

since the null set 'ϕ' is the subset of all sets, it is the lowest point of the diagram. Now, $\{10\}$, $\{11\}$ and $\{12\}$ are the immediate successor of 'ϕ'. They are placed at higher level then 'ϕ' and connected with 'ϕ'.



Then $\{10, 11\}$ is the immediate succesor of $\{10\}$ and $\{11\}$, so $\{10, 11\}$ is placed at higher level then $(10, 11)$ and connected with $\{10\}$ and $\{11\}$

similarly, The Other paints are drawn. ~~tt~~
The Hasse diagram of the poset $(P(S), \subseteq)$ is shown in the figer-



Minimal and Maximal elements :→

Let $(A, \leq)$ be a Poset, where '$\leq$' represents an arvitrary partial order. Then an element $b \in A$ is a minimal element of 'A' if there is No element $a \in A$ that setisfie $a \leq b$. S

Similarly,

An elements $b \in A$ is a Maximal element of 'A' if there is no element $a \in A$ that satisfie $b \leq a$.

For example-

The Set of $\{\{1\}, \{2\}, \{1,2\}\}$ with Subset has to Minimal elements $\{1\}$ and $\{2\}$. Not that $\{1\}$ and $\{2\}$ are nat related to each other in Subset. Hence we can not say which is 'smaller than' which, that is they are nat compareble.

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Least and Greatest elements :-

Let '(A, ≤)' be a Poset. Then an element '$b \in A$' is the least element of 'A' if for every element $a \in A$, $b \leq a$. Similarly,

An element $b \in A$ is called greatest element of 'A' if for every $a \in A$, $a \leq b$.

For examples :- ①

The Poset of the set of Natural numbers with less than equal to ($\leq$) relation has the least element '1'.

② The Poset of the power set of {1, 2} with '$\subseteq$' has the least element 'ϕ'.

Totally ordered (Linearly ordered) Set :-

An ordered Set 'A' is said to be totally order, if every pair of element in 'A' are comparable. We also say that 'A' is a chain.

For example :-

The usual '$\leq$' (less than or equal to) is a partial order on 'I^+'. Thus (I^+, $\leq$) is a poset and this poset is a totally order set, that is a chain since every pair of elements (I^+, $\leq$) are comparable.

2021

Q→ Define the well-ordered set. Give an example of well order set.

Well-Ordered Set :-

A Poset (A, $\leq$) is called well ordered set if it is totally ordered and every non-empty subset of 'A' has a least element.

Example:-

The set of natural nos with order less then equal to is well-ordered set. The Posets $(Z, \leq)$ is not well-ordered since the set of negative intizer, which is a subset of 'Z', has no least element.

Upper and Lower bounds:-

Let 'B' be a subset of a Poset $(A, \leq)$. An element $u \in A$ is called an upper bound of 'B' if u succeeds every element of 'B', that is $x \leq = u$ for all $x \in B$.

An element $l \in A$ is called a lower bound of 'B' if l precedes every element of 'B', that is:

$l \leq x$ for all $x \in B$

$A = \{1, 2, 3, 5, 7, 9\}$

$B = \{2, 3, 5, 7\}$

Let $u = 7 \in A$

$2 \leq 7$

$3 \leq 7$

$5 \leq 7$

$7 \leq 7$

Let $l = 2 \in A$

$2 \leq 2$

$2 \leq 3$

$2 \leq 5$

$2 \leq 7$

least upper and greatest lower bounds:-

An element '$a \in A$' is called a least upper bound (lub) of 'B' if (i) a is an upper bound of 'B' and
(ii) $a \leq a'$, for any upper bound a' of 'B'. Thus $a = lub(B)$. A least upper bound is also called supremum and written as
$a = sup(B)$.
Similarly,
an element $a \in A$ is called the greatest the lower bound $glb(B)$ if -
(i) 'a' is a lower bound of 'B'
(ii) $a' \leq a$, whenever a' is a lower bound of 'B'.
Thus $a = glb(B)$. A greatest lower bound is also called infimum and written as
$a = inf(B)$.
Example -
In the Poset $A = (\{1, 2, 3, 4, \dots 10\}, /)$, the subset $\{2, 7\}$ has no upper bound since there is no intiger in 'A' which is divisival by both 2 and 7. The lower bound of the Subset is 1 since 2 and 7 are divisival by only 1. Hence, 1 is the greatest lower bound for $\{2, 7\}$ i.e -
$glb\{2, 7\} = 1$.
The Subset $\{1, 2, 3\}$ has 6 and 1 as unique upper and lower bounds. Hence 'lub' of $\{1, 2, 3\} = 6$ and $glb\{1, 2, 3\} = 1$.
The Subset $\{1, 2, 4\}$ has 4 and 8 as upper bounds. Hence $lub\{1, 2, 4\} = 4$.

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Lattice :-

A Posets $(A, \leq)$ is called a lattice if every 2-element subset of 'A' has both a least upper bound and a greatest lower bound, that is if $lub\{x, y\}$ and $glb\{x, y\}$ exist for all $x, y \in A$. In this case, we denote
$x \vee y = lub\{x, y\}$ (read as join of x & y).
$x \wedge y = glb\{x y\}$ (read as meet of x & y).

Therefore,
Lattice is a mathematical structured equipped with two binary operation join and meet.

Q⇒ Let $D_{20} = \{1, 2, 4, 5, 10, 20\}$ with the set of all positive divisors of 20. Show that D_{20} forms a lattice under the relation divisibility that for $a, b \in D_{20}$, $a \leq b \Rightarrow a/b$.

Sol Here $D_{20} = \{1, 2, 4, 5, 10, 20\}$

① Divisibility is reflexive:
i.e. $\forall a \in D_{20}$
$a \leq a$ exist iff a/a.

② Divisibility is antisymetric.
i.e. for $a, b \in D_{20}$
If a/b and $b/a \Rightarrow a = b$.

③ Divisibility is transitive:
i.e. $a, b, c \in D_{20}$
If a/b and b/c the a/c.

Therefore $(D_{20}, /)$ is a poset.
Now, for any $a, b \in D_{20}$
Join of a and b is
$a \vee b = lub\{a, b\}$ exist in D_{20}.
For example:-
$$2 \vee 5 = lub\{2, 5\}$$
$$= 10 \in D_{20}$$
And meet of 'a' and 'b' is $a * b = glb\{a, b\}$ exist in D_{20}.
For example:- $2 * 5 = glb\{2, 5\}$
$= 1 \in D_{20}$
Hence $(D_{20}, /)$ is a lattice.

2022
Q→ Let 'L' be the set of all factor of '12' and let '/' ~~will~~ the divisibility relation on 'L'. Show that $(L, /)$ is a lattice.

Sol - Hence
$L = \{1, 2, 3, 4, 6, 12\}$ for any $a, b \in L$, $a \leq b$ if a/b

① Divisibility is Reflexive:
~~i.e. ∀ a ∈ L~~ i.e, $\forall a \in L$
$a \leq a$ exist iff a/a

② Divisiblity is antisymmetric —
i.e, for $a, b \in L$
~~a ≤ a iff a/a~~ if a/b and $b/a \Rightarrow a = b$

③ Divisiblity is transitive —
i.e, for $a, b, c \in L$
if a/b and $b/c = a/c$

Therefor $(L, /)$ is poset.

Q3- Let L we the set of all factor of '30' and let '/' be the divisibilty relation on 'L'. Then show that (L, '/') is a lattice.

Sol:- Hence $L = \{1, 2, 3, 5, 6, 10, 15, 30\}$

for any $a, b \in L$, $a \leq b$ if a/b

(1) Divisiblity is antisymmetric-

i.e; for $a, b \in L$

if a/b and $b/a \Rightarrow a = b$

(2) Divisiblity is Reflexive -

i.e, $\forall\, a \in L$

$a \leq a$ iff a/a

(3) Divisibilty is transitive -

i.e; for $a, b, c \in L$

if a/b and $b/c = a/L$

Therefore $(L, /)$ is ~~latice~~.

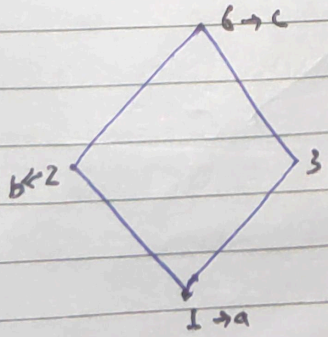
2021

Q> Prove the Hase diagram of the lattice of $(D_6, /)$.

Sol:- $D_6 = \{1, 2, 3, 6\}$

for any $a, b \in D_6$, $a \leq b \Rightarrow a/b$

we draw the Hasse diagram:



$(D_6, \cdot/)$

Q→ Let D_m denote the positive integer of integers of m ordered by divisibility. Draw the hasse diagram of :

(a) D_{24} , (b) D_{15}

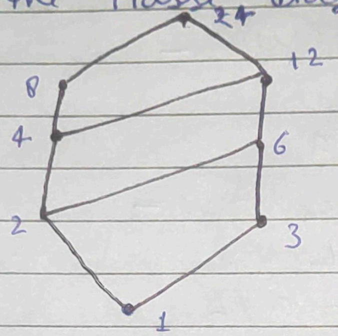
Sol:→

(a) D_{24} -

Here $D_{24} = \{1, 2, 4, 6, 12, 24\}$

for any $a, b \in D_{24}$, $a \leq b \Rightarrow a/b$

we draw the Hasse diagram.

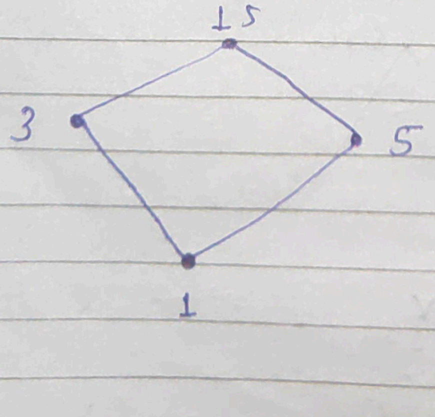


(b) D_{15}

Here $D_{15} = \{1, 3, 5, 15\}$

for any $a, b \in D_{15}$, $a \leq b \Rightarrow a/b$

we draw the Hasse diagram.



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2018 Q) Consider the poset $S = (\{1, 2, 3, 4, 6, 9, 12, 18, 36\}, /)$ Find the greatest lower bound and least upper bound ~~and~~ of this set $\{6, 18\}$ and $\{4, 6, 9\}$.

Sol) The set $\{6, 18\}$ has 18 and 36 as upper bounds.
Hence lub $\{6, 18\} = 18$
And the set $\{6, 18\}$ has 1, 2, 3 and 6 as lower bonds. Hence glb $\{6, 18\} = 6$
Now, the set $\{4, 6, 9\}$ has 36 and 1 as unique upper and lower bounds.
Hence, lub $\{4, 6, 9\} = 36$ and
glb $\{4, 6, 9\} = 1$

Properties of Lattices :-

(1) If $a \leq a \vee b$ and $b \leq a \vee b$ then $a \vee b$ is an upper bound of a and b.

(2) If $a \leq c$ and $b \leq c \Rightarrow a \vee b \leq c$ then $a \vee b$ is a least upper bound of a and b.

(3) If $a \wedge b \leq a$ and $a \wedge b \leq b$ then $a \wedge b$ is a lower bound of a and b.

(4) If $c \leq a$ and $c \leq b \Rightarrow c \leq a \wedge b$ then $a \wedge b$ is a greatest lower bound of a and b.

Complete Lattice :-

A Lattice is called complete if each of its non-empty subsets has a least upper bound and a greatest lower bound.
Clearly, Every finite lattice is complete

because every subset here is finite. Also every complete lattice must have a least element and a greatest element. The least and the greatest elements of a lattice are called bounds of the lattice and are denoted by 0 and 1 respectively.

Bounded lattice :-

A lattice 'L' is said to be bounded lattice if it has a greatest element '1' and a least element '0'.

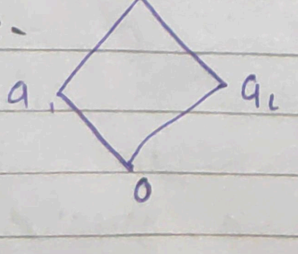
Example –

The lattice P(S) of all subsets of a set 'S' under the operations ~~Sub~~ '$\subseteq$' is a bounded lattice. Since in this lattice, its greatest element is the set 'S' and its least element is the set 'ϕ'.

Complemented lattice :-

Let 'L' be a bounded lattice with greatest element '1' and least element '0' and let $a \in L$. An element $a' \in L$ is called a complement of 'a'. If $a \vee a' = 1$ and $a \wedge a' = 0$.

Example :-



In this figur, complement of a_1 is a_2.

Since, $a_1 \vee a_2 = \text{lub}\{a_1, a_2\} = 1$

and $a_1 \wedge a_2 = \text{glb}\{a_1, a_2\} = 0$

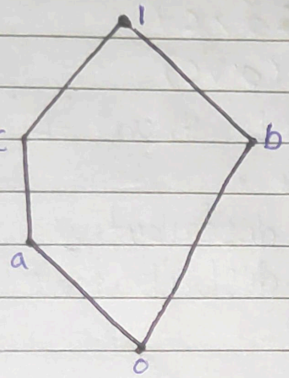
Modular lattice :-

A lattice 'L' is said to be madular lattice if for every $a, b, c \in L$ and $a \leq c$.

$\Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge c$.

Show that

Q: The pentagonal lattice is not madular.



Sol:- We have

$$a \vee (b \wedge c) = a \vee \text{glb}\{b, c\}$$
$$= a \vee 0$$
$$= \text{lub}\{a, 0\}$$
$$= a$$

$\Rightarrow a \vee (b \wedge c)$

and $(a \vee b) \wedge c = \text{lub}\{a, b\} \wedge c$
$$= 1 \wedge c$$
$$= \text{glb}\{1, c\}$$

$\Rightarrow (a \vee b) \wedge c = c$

$\because a \vee (b \wedge c) \neq (a \vee b) \wedge c$, though $a \leq c$.

Hence even pentagonal lattice is not madular.

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2019

Q:- Define the "Distributive lattice". Prove that in a distributive lattice, if an element has a complement then this compliment is unique.

Sol - Distributive lattice :→

Let 'L' be a lattice and let a, b, c be any element of 'L'. If following distributive property -

(1) $A \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

(2) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

hold for $a, b, c \in L$ then 'L' is said to be distributive lattice.

If 'L' is not distributive lattice then it is called non-distributive lattice.

Proof :→

Supose that an element a has two compliment 'b' and 'c'. Then

$a \vee b = 1$, $a \wedge b = 0$

and $a \vee c = 1$, $a \wedge c = 0$

we have,

$b = b \wedge 1$ $\{\because b \wedge 1 = glb\{b, 1\}\} = b$

$= b \wedge (a \vee c)$ $\{\because a \vee c = 1\}$

$= (b \wedge a) \vee (b \wedge c)$ {By distributive property.}

$= (a \wedge b) \vee (b \wedge c)$ {By commutative property}

$= 0 \vee (b \wedge c)$ $\{\because a \wedge b = 0\}$

$= (a \wedge c) \vee (b \wedge c)$ $\{\because a \wedge c = 0\}$

$= (a \vee b) \wedge c$

$= 1 \wedge c$ $\{\because a \vee b = 1\}$

$= glb\{1, c\}$

$\Rightarrow b = c$

2022

Q:-

Sol:

Hence in a distributive lattice, If an element has a compliment then this compliment is unique.

2022 Q. Define modular lattice. Also proof that every distributive lattice is modular.

Sol: Modular lattice :-

A lattice 'L' is said to be modular lattice if for every $a, b, c \in L$ and $a \leq c$.

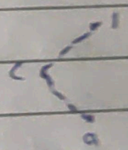
$\Rightarrow a \vee (b \wedge c) = (a \vee b) \wedge c$.

Proof -

Let $(L, \leq)$ be a distributive lattice and $a, b, c \in L$ be such that $a \leq c$.

Thus if $a \leq c$ then $a \vee c = lub\{a, c\}$

$\Rightarrow a \vee c = c$



Now, $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \vee (b \wedge c) = (a \vee b) \wedge c$

Hence Every distributive lattice is modular.

Boolean Algebra :-

An order 6 - tuples $(S, 0, 1, +, *, ')$ in which S is a set of elements of 0 and 1 are elements of S '+' and * are two binary operation on S and ' and a unary operation is a boolean algebra is following axiom hold.

axioms Properties :-

(1) Commutative laws :- For all $a, b \in S$

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$a+b = b+a$

and $a * b = b * a$

(2) Distributive laws :-

For $a, b, c \in S$

$a+(b*c) = (a+b)*(a+c)$

and $a*(b+c) = (a*b)+(a*c)$

(3) Identity Laws :-

For all $a \in S$

$a+0 = a$

and $a*1 = a$

(4) Complement laws :-

For each $a \in S$ there exist an element $a' \in S$ such that

$a+a' = 1$

and $a*a' = 0$

Thearems of Boolean Algebra :-

Let $a, b, c \in S$, then

1- Idempotent Laws :

(a) $a+a = a$

(b) $a*a = a$

2- Boundidness laws :-

(a) $a+1 = 1$

(b) $a*0 = 0$

3- Absorbtion Laws :

(a) $a+(a*b) = a$

(b) $a*(a+b) = a$

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4- <u>Associative Laws:</u>

(a) $(a+b)+c = a+(b+c)$

(b) $(a*b)*c = a*(b*c)$

5- <u>Uniqueness of compliment:</u>

$a+x=1$ and $a*x=0$, then

$x = a'$

6- <u>Involution Law:</u>

$(a')' = a$

7- <u>De-Morgan's Laws:</u>

(a) $(a+b)' = a' * b'$

(b) $(a*b)' = a' + b'$

Q→ Let $S = \{1, 2, 5, 7, 10, 14, 35, 70\}$ be a set for any $a, b \in S$. '+', '*' and unary operation(') are defined as:

$a+b = \text{lcm}(a,b)$

$a*b = \gcd(a,b)$

and $a' = \frac{70}{a}$

Show that 'S' is boolean algebra with 1 as 0 and 70 as 1.

Sol→ To show 'S' is a boolean algebra. we construct the composition table for '+', '*' operations.

+	1	2	5	7	10	14	35	70
1	1	2	5	7	10	14	35	70
2	2	2	10	14	10	14	70	70
5	5	10	5	35	10	70	35	70
7	7	14	35	7	70	14	35	70
10	10	10	10	70	10	70	70	70
14	14	14	70	14	70	14	70	70
35	35	70	35	35	70	70	35	70
70	70	70	70	70	70	70	70	70

Therefore, This operation is commutative.

*	1	2	5	7	10	14	35	70
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
5	1	1	5	1	5	1	5	5
7	1	1	1	7	1	7	7	7
10	1	2	5	1	10	2	5	10
14	1	2	1	7	2	14	7	14
35	1	1	5	7	5	7	35	35
70	1	2	5	7	10	14	35	70

Therefore, This operation is commutative.

(1) **Commutative laws** –

Since composition table for '+' and '*' are symetrical with respect to diagonals therefore operations '+' and '*' are commutative in the set 'S'.

(2) **Distributive laws hold.**

For $a, b, c \in S$

$$a*(b+c) = (a*b)+(a*c)$$

and $$a+(b*c) = (a+b)*(a+c)$$

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Example→ for. $2, 5, 7 \in S$

Now, $2 * (5 + 7) = 2 * 35 = 1$

and $(2 * 5) + (2 * 7) = 1 + 1 = 1$

$\therefore 2 * (5+7) = (2*5) + (2*7)$

Again, $2 + (5 * 7) = 2 + 1 = 2$

and $(2+5) * (2+7) = 10 * 14 = 2$

$\therefore 2 + (5 * 7) = (2+5) * (2+7)$

③ Identity laws hold:

Taking 1 as 0 and 70 as 1

we have from the composition table.

$a + 1 = a$

and $a * 70 = a$

Hence 1 and 70 are identity elements of 'S'.

④ Complement laws hold:

For each $a \in S$

$$a' = \frac{70}{a}$$

such that $a + a' = 70$

and $a * a' = 1$

Example→ $2 + 35 = 70$

and $2 * 35 = 1$

Hence each element in 'S' has its complement in 'S'.

Thus 'S' is a boolean algebra.

2021 Q3 If $B = \{1, 3, 5, 15\}$, then show that $(B, +, \cdot, ')$ is a boolean algebra, where $a+b = lcm(a, b)$, $a \cdot b = gcd(a, b)$ and $a' = \frac{15}{a}$.

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Boolean function :-

A boolean function is a function whose domain is a set of n-tuples of 0's and 1's and whose range is the basic boolean algebra on $\{0, 1\}$.

Boolean expressions :-

A boolean expressions on the variables $x_1, x_2, ---, x_n$ is a polynomial expression using those variable and the operations of a boolean algebra.
The variable are exlomed to have as theirs domain. This set $\{0, 1\}$ and the operations on the variables are the boolean expression defined on the set.

Boolean Polynomials :-

Let $x_1, x_2 - - - -, x_n$ be a set of n variables. A boolean polynomial $p(x_1, x_2, - - - x_n)$ in the variables x_K is defined recursively as follows:-

① $x_1, x_2 - - - - x_n$ are all boolean polynomials.
② The symbol '0' and '1' are boolean polynomial
③ If $p(x_1, x_2 - - - x_n)$ and $q(x_1, x_2 - - - x_n)$ are two boolean polynomial, then so are-
$p(x_1, x_2 - - - - x_n) \vee q(x_1, x_2 - - , x_n)$
and $p(x_1, x_2 - - - - x_n) \wedge q(x_1, x_2 - - - - x_n)$.
④ If $p(x_1, x_2 - - - x_n)$ is a boolean polynomial, then so is.
$(p(x_1, x_2 - - - x_n))'$

By tradition, $(0)'$ is denoted $0'$, $(1)'$ is

denoted 1' and $(x_k)'$ is denoted x_k'.
Boolean polynomial are also called boolean expression.

Example:→
Every boolean polynomial, that is boolean expression involving n-variables produces a boolean function. The boolean expression on $\{x_1, x_2\}$ is given by $x_1 * x_2 + x_1 * x_2'$ defines a boolean function. If we just subsitude for x_1 and x_2 if -
$f(x_1, x_2) = x_1 * x_2 + x_1 * x_2'$ then
$f(x_1, x_2)$ is completely defined by-

x_1	x_2	$f(x_1, x_2)$
0	0	0
0	1	0
1	0	1
1	1	1

Just as boolean function can be describe by a boolean expression.
Simlarly Every boolean expression can be describe by a special type of boolean function.

Boolean variables:→
Boolean variable is a variable whose domain is a set of {0, 1} and a literal is a boolean variable or its complement.

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Minterm :-

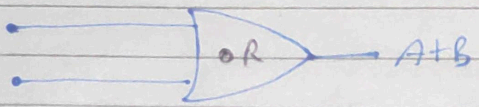
A minterm on n variables is a product of n-literals in which each variable is represented either by the variable or its complement.

Logic gates -

There are three basic logic gates-

(i) OR gate-

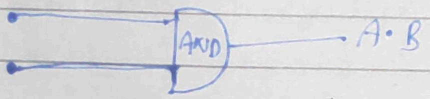
OR gate with inputs A and B then output $y = A + B$



A	B	$Y = A+B$
0	0	0
0	1	1
1	0	1
1	1	1

(2) **AND gate-**

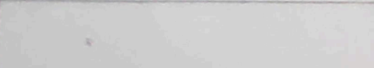
AND gate with inputs A and B then output $Y = A \cdot B$.



A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

(3) **Not gate-**

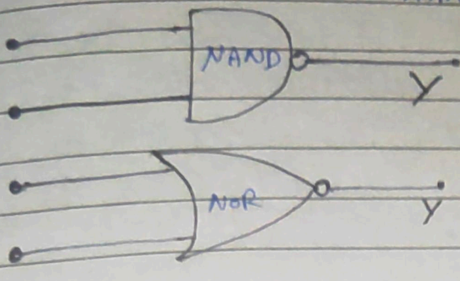
Not gate, also called a inverter, with input A then output $Y = A'$.



A	$Y = A'$
0	1
1	0

other logic gates :-

NAND and NOR gates :



A	B	NAND	NOR
0	0	1	1
0	1	1	0
1	0	1	0
1	1	0	0

Karnaugh maps (K-map) :-

The Karnaugh map method is a graphical technique which provids a simpl straight forward procedure for simplification of Boolean expression of two, three or ~~m~~ four variables. It can also we extended to fundions of five, six or more variables.

A Karnaugh map (K-map) is a diagram made ~~with~~ up of a number of squares. If the expression contains n variables, the map will have 2^n squares.

2- variable Karnaugh-maps-

	A'	A
B'	A'B'	AB'
B	A'B	AB

B \ A	0	1
0	A'B'	AB'
1	A'B	A B

Q» Find K-map and simplify the expression.

$AB' + A'B + A'B'$

Sol»

B \ A	0	1
0	1	~~0~~ 1
1	1	0

There are two pairs of 1s and they can be combine as shown in figur. Notice that the first column and 1 Row has been enclosed twise, as it is permissible to use the same 1 more than 1s. Looping of horizontal 1-squares gives the result B' and vertical 1-squares gives A'.

Now,

$$AB' + A'B + A'B' = AB' + A'(B+B')$$
$$= AB' + A' \quad \{\because B+B'=1\}$$

$$\boxed{A'B + A'B + A'B' = A'B'}$$

3-Variable Karnaugh map :-

	A'B'	A'B	AB	AB'
C'	A'B'C'	A'BC'	ABC'	AB'C'
C	A'B'C	A'BC	ABC	AB'C

	00	01	11	
0	A'B'C'	A'BC'	ABC'	AB'C'
1	A'B'C	A'BC	ABC	AB'C

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