1 Notes of Spherical Ansatz

1.1 ED results

After establishing that the ansatz we have is the zero-energy eigenstate of model Hamiltonian, we want to check if it produces right excitation properties. First, we checked the density and charge of single quasi-particle and quasi-hole (for $\nu = 1/3, 2/5$) using exact digonalization:

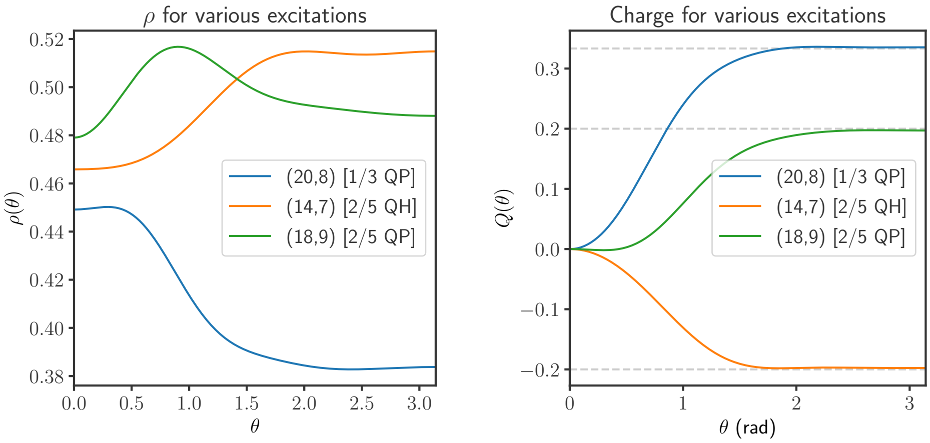

Figure 1: Density ρ and charge for 2/5 q-particle and q-hole ((18, 9) and (14, 7)) and 1/3 q-particle (20, 8)

NOTE: In spherical geometry, $\nu = \frac{N}{2Q}$ only when $N \to \infty$. Hence, instead of getting the right densities, we get a shifted density away from the quasi-particles. For example, in the above figures, (20, 8) which is a 1-qp system for $\nu = 1/3$, we get $\rho = \frac{N-(1/3)}{2Q} \sim 0.3833$ instead of 1/3.

1.2 Writing ansatz for sphere

It produces right density profile and charge for the q-particle and q-holes. This gave us motivation to write the ansatz wavefuntion for spherical-geometry. We tried to do the stereographic projection to get a conversion of coordinates $(x, y) \to (\theta, \phi)$. We then reached to the following form of wf in spherical geometry :

$$\text{planar} \quad \Psi_\nu^\alpha = \prod_{j<k} (\hat{Z}_j - \hat{Z}_k)^{2p} \times \Phi_\nu^\alpha \tag{1}$$

$$\text{spherical} \quad \Psi_\nu^\alpha = \Phi_\nu^\alpha \; (Y_{Q^*,n,m} \to (Y_{Q^*,n,m} - \hat{Y}^{Q-Q^*}_{Q^*,n,m})) \quad \forall n \geq 1 \tag{2}$$

Where in eq(2), we replace single particle wf $Y_{Q^*,n,m}$ with $(Y_{Q^*,n,m} - \hat{Y}^{Q-Q^*}_{Q^*,n,m})$, where $\hat{Y}^{Q-Q^*}_{Q^*,n,m}$ is an LLL projector type operator (Jain, J.15) for spherical geometry.

By doing a proportionality check for 1, 2 and 3 quasi-particles with the corresponding ED ground state wf, we have established that this spherical ansatz is equal to ED ground state upto a multiplicative constant. We also reproduce correct charges for 1 and 2 q-particles for $\nu = 1/3$ using the spherical ansatz.

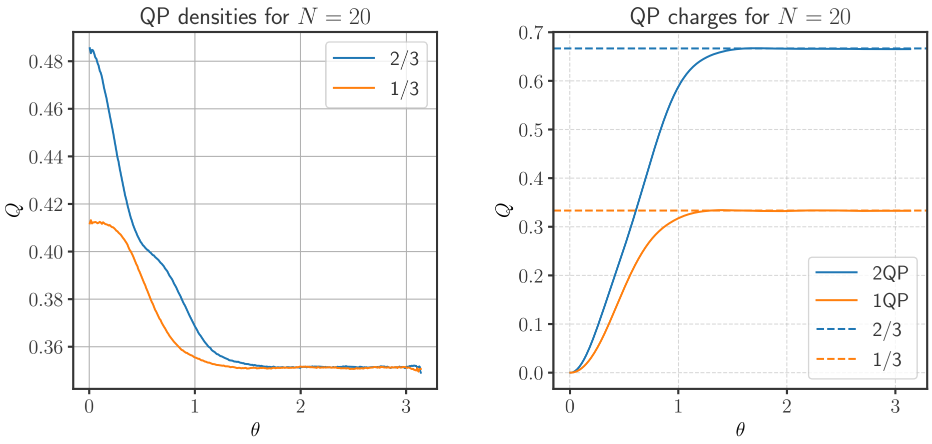

Figure 2: Density ρ and charge for 1/3 1 and 2 q-particles in a system of $N = 20$

1.3 Making ansatz numerics friendly

After establishing the proportionality, we wanted to check if we get $\nu = 2/5$ ground state by completely filling 2^{nd} LL in our ansatz wf. But the problem arises because of derivatives ∂_u, ∂_u's present in the $\hat{Y}^{Q-Q^*}_{Q^*,n,m}$. They make the wf analytically intractible with each successive addtion of particle in highter LLs. Hence we wanted to check if we could drop the cross-terms in derivatives without losing important features of physics. This essentially means, replacing

$$\partial_{u_i} \rightarrow \sum_{k \neq i} \frac{2v_k}{u_i v_k - u_k v_i}$$
$$\partial_{v_i} \rightarrow \sum_{k \neq i} \frac{2u_k}{v_i u_k - v_k u_i}$$

and it allows us to write ansatz for higher LL very easily. Here is the overlap plot of *full* ansatz with one when the cross-terms in the derivatives are ignored (for 2 and 3 qp's respectively).

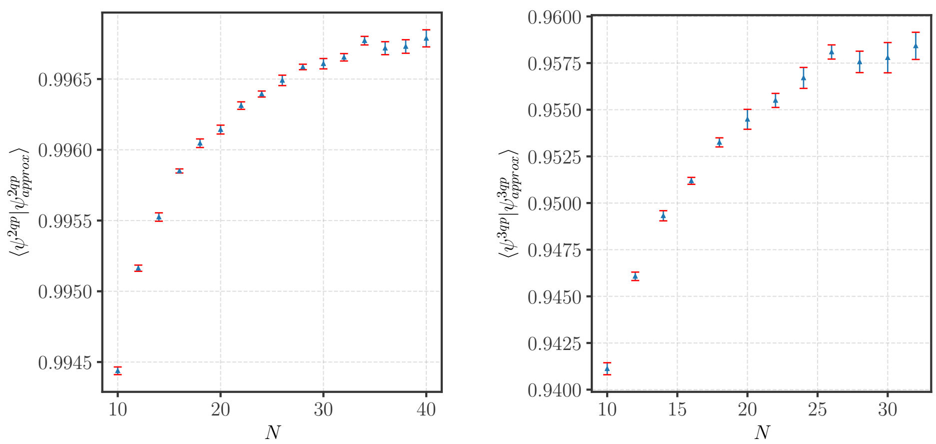

Figure 3: Overlaps of *full* ansatz with modified one with N. Left 2 qps and right is for 3 qps

Finally, we have density and charge of 2/5 quasi-hole, using the modified ansatz wf.

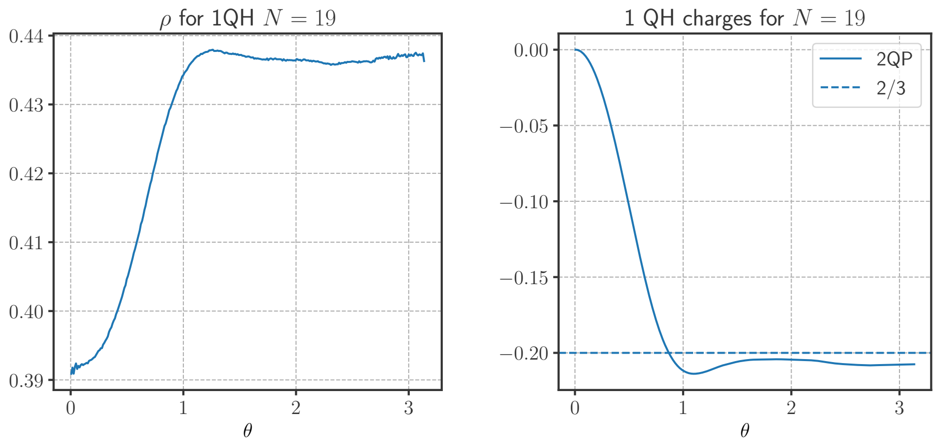

Figure 4: Density and charge for 2/5 q-hole using modified ansatz wf

2 ToFix

1. [TODO] Change the x-axis from θ to $\sqrt{Q}\theta$
2. [DONE] Fix ED density plots
3. [TODO] Compare 1/3 2qp case for Full and approx wf
4. [TODO] Compare 1/3 1qp case for ED and Full, different system sizes comparison
5. [TODO] Compare densities of 1QH for ED, full and approx 2/5 case

3 Todo

This is the list of calculations which are yet to be done :

1. overlap b/w ED and approximate 2/5 state

2. pair correlation function for 2/5 ED and Approx state