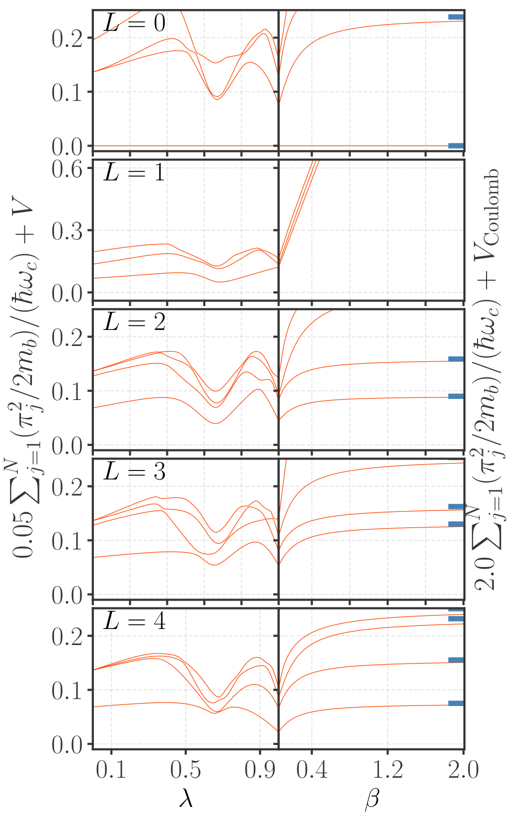
We have been studying different adiabatic paths for

$$H_{model} \to H_{Coulomb}$$

In the presence of lowest 3 LLs. These notes are meant to document all our findings.

Apart from the usual path we have taken in the paper, where after introducing a small cyclotron energy cost β, we simply increase the $\lambda : 0 \to 1$. Here we will probe some different regimes.

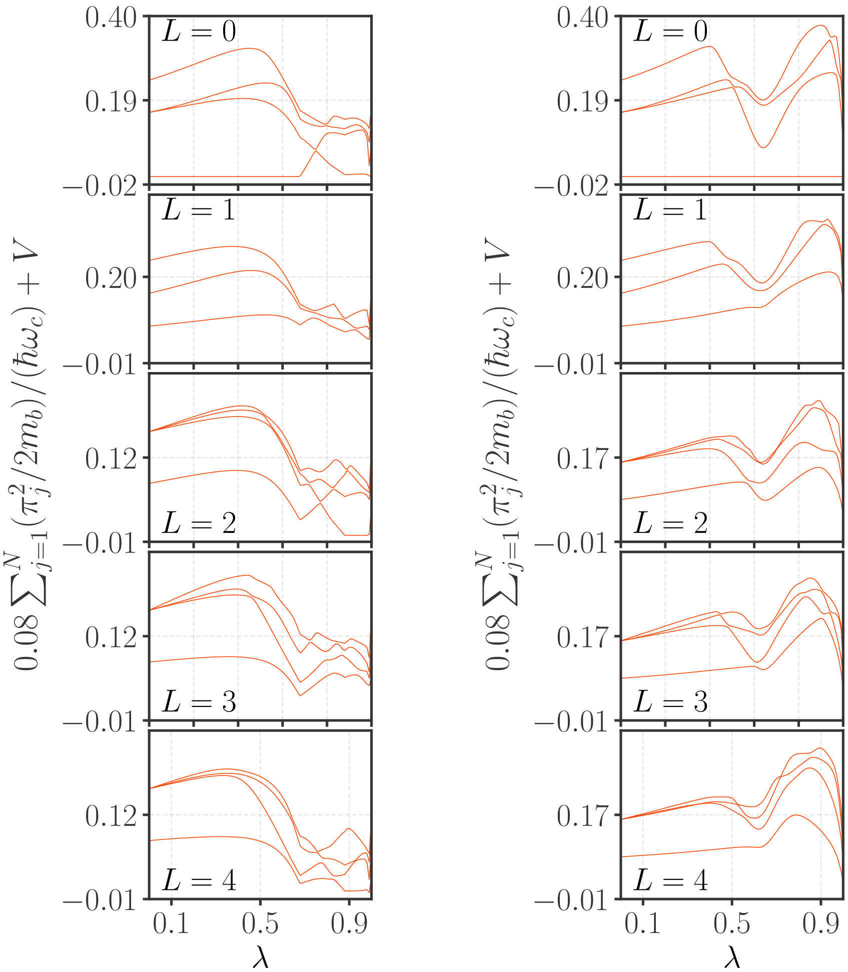
Just to make sense of what we were doing, first lets take a look at the adiabatic plot for system at $\nu = 2/5$ with config $(11, 6)$



For $L = 4$ sector, the lowest energy states seem to come really close to each other. We started off with checking different paths where the gap might be larger than this path.

1. Changing the relative strengths of Pseudo-pots $V^{l}_{n_1,n_2}$

Our first try was to tune the strengths of all non-zero PPs systematically to choose different parameter paths for adiabatic transformation. By increasing the relative strengths for PPs between particles of different LLs results in preference of different fillings of LLs over the path.

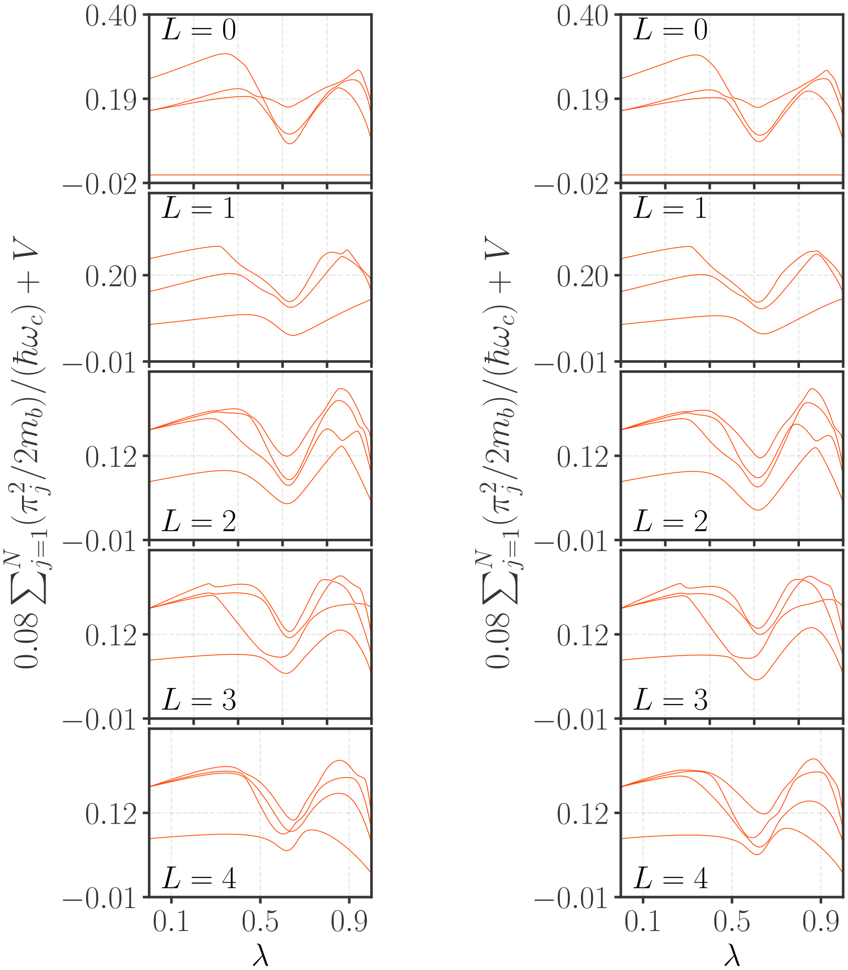


(a) $V^{\alpha}_{0,0} \to 20$

(b) $V^{\alpha}_{1,0} \to 20$

Figure 1: Adiabatic transformation $H_{Model} \to H_{Coulomb}$

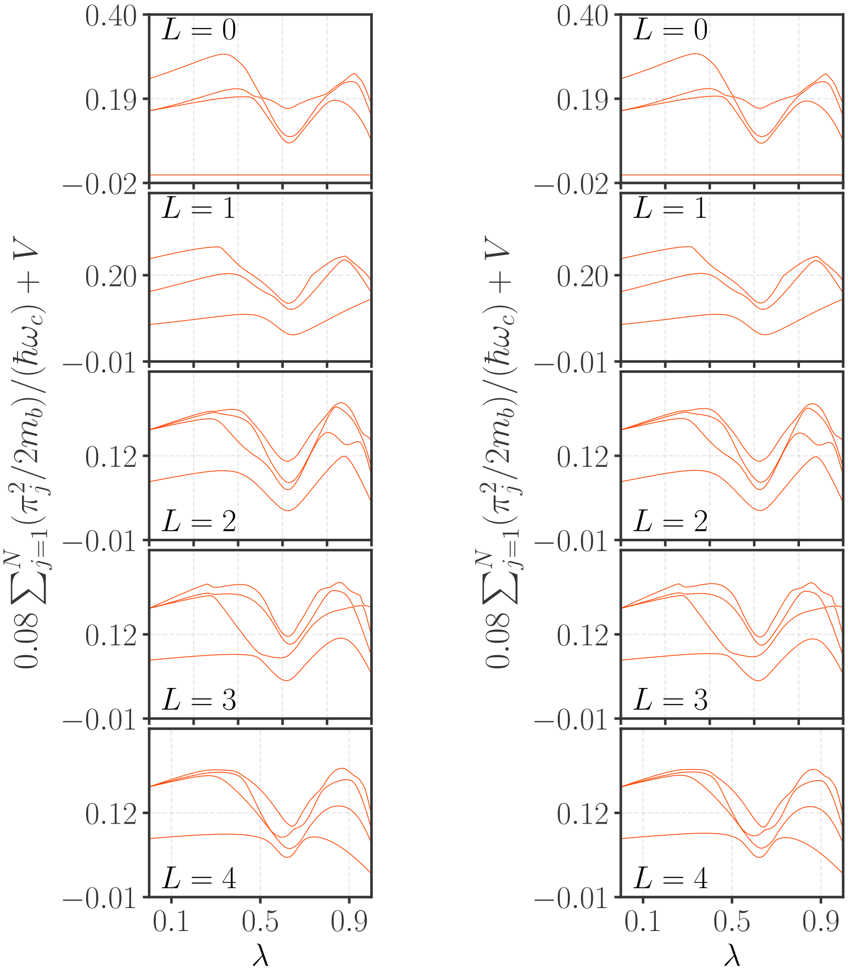
Here, around $\lambda = 0.68$, the energy of gs of $l > 0$ goes below that of $l = 0$. Hence the enegy of gs for $l = 0$ goes above 0 after that λ. Increasing $V^{\alpha}_{0,0}$ means it will be harder for $LL = 0$ to accomodate more particles.



(a) $V_{2,0}^{\alpha} \to 20$

(b) $V_{1,1}^{\alpha} \to 20$

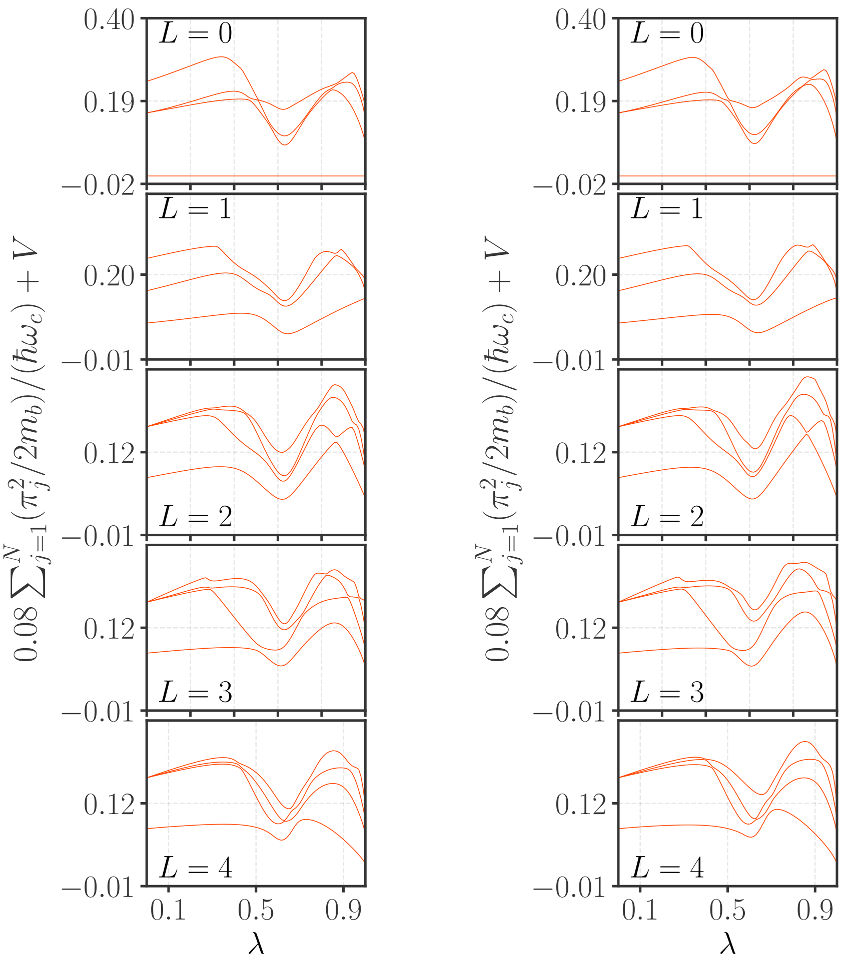
Figure 2: Adiabatic transformation $H_{Model} \to H_{Coulomb}$



(a) $V_{2,1}^{\alpha} \to 20$

(b) $V_{2,2}^{\alpha} \to 20$

Figure 3: Adiabatic transformation $H_{Model} \to H_{Coulomb}$

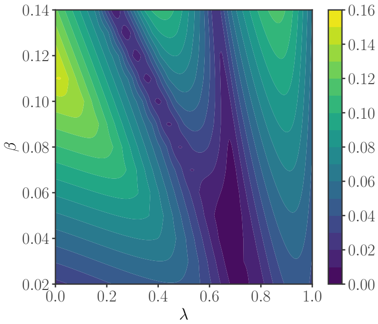


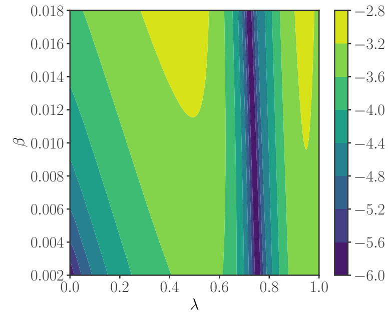
(a) $V^{\alpha}_{2,1}, V^{\alpha}_{2,0} \to 20$

(b) $V^{\alpha}_{2,1}, V^{\alpha}_{2,0}, V^{\alpha}_{1,1} \to 20$

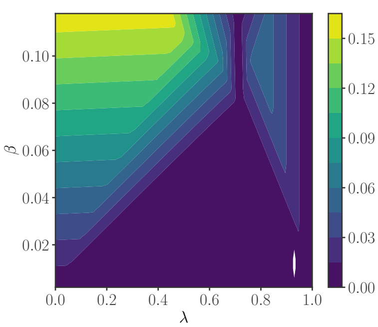
Figure 4: Adiabatic transformation $H_{Model} \to H_{Coulomb}$

We also made color plots measuring the gap in $L = 4$ over the paramter space of (β, λ) to scan different paths which might avoid the gap closing. But as the plots suggest, there is no such path for $L = 4$.





We also checked for the case where we only take the LLL components of $H_{coulomb}$ as that is what we achieve later by $\beta \rightarrow 2$. But as we see, this doesn't help much.



Level crossing check using overlap :

We wanted to check if there are points of level crossing when the levels comes really close to each other. For this, the overlap of $\psi_i^L(\lambda_0 + \mathrm{d}\lambda)$, where $i = 0, 1, 2, 3$ were taken with $\psi_0^L(\lambda_0)$, where $i = 0$ means the state with lowest energy in given L sector.

NOTE : The overlap checking is not a conclusive check for level crossing!!!!

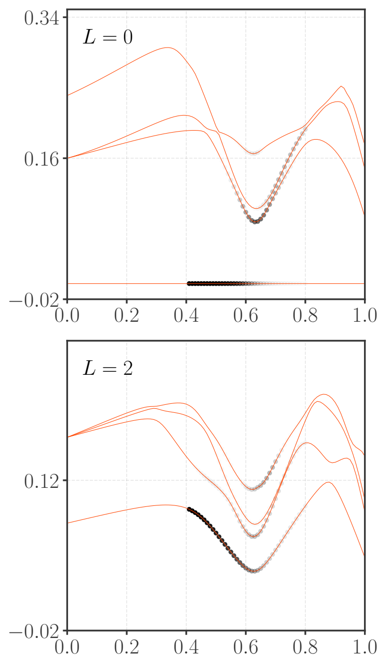


Figure 7: Energies and overlap of $\psi_0^L(\lambda_0)$ with $\psi_i^L(\lambda_0 + \mathrm{d}\lambda)$ for default H_{Model} in $\lambda \in [0.4, 0.8]$

For QH and QP in $\nu = 2/5$ and $1/3$:

Remember

$$2Q^* = 2Q - 2p(N - 1) \tag{1}$$

in spherical geometry, where $2Q^*$ is the effective flux seen by CF particles after $2p$ vortices are attached to each of N electrons.

For $\nu = 2/5$, CFs fill $n = 2$ Landau-like levels. Hence to find $(2Q, N)$ configuration corresponding to 1-QP, condition is

$$\begin{aligned}(2Q*+1) + (2Q*+3) &= N - 1\\ 4Q^* + 4 &= N - 1\end{aligned}$$

and for 1-QH case,

$$4Q^* + 4 = N + 1$$

in both the cases, putting it back in eq (1), gives back us the relation, which helps us generate the required $(2Q, N)$ configurations.

1-QH at $\nu = 2/5$ **:** $(9, 5)$

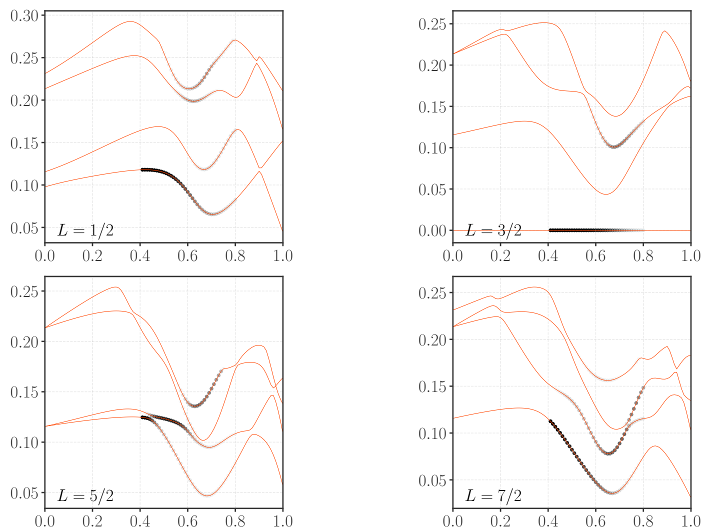


Figure 8: Energies and overlap in $\lambda \in [0.4, 0.8]$

1-QP at $\nu = 1/3$ **:** $(11, 5)$

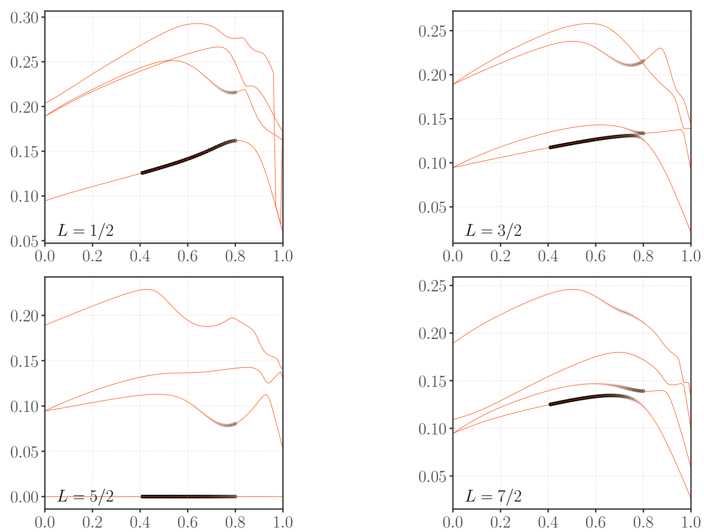


Figure 9: Energies and overlap in $\lambda \in [0.4, 0.8]$

1-QP at $\nu = 2/5$ **:** $(8, 5)$

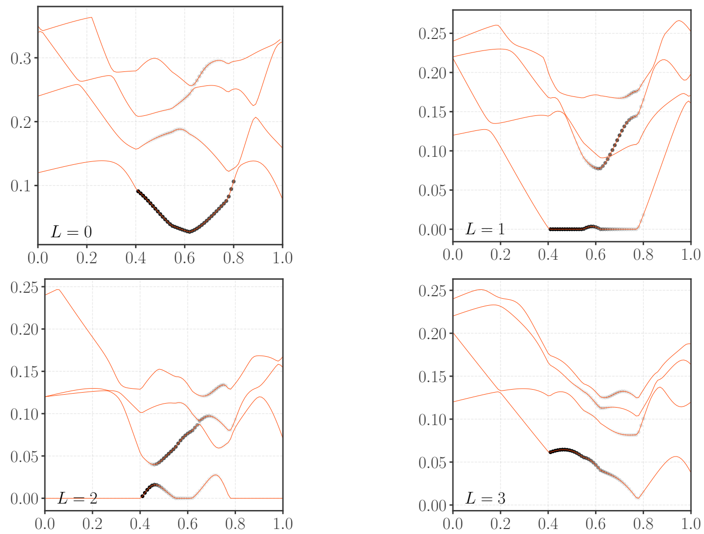


Figure 10: Energies and overlap in $\lambda \in [0.4, 0.8]$

2-QP at $\nu = 2/5$ **:** $(10, 6)$

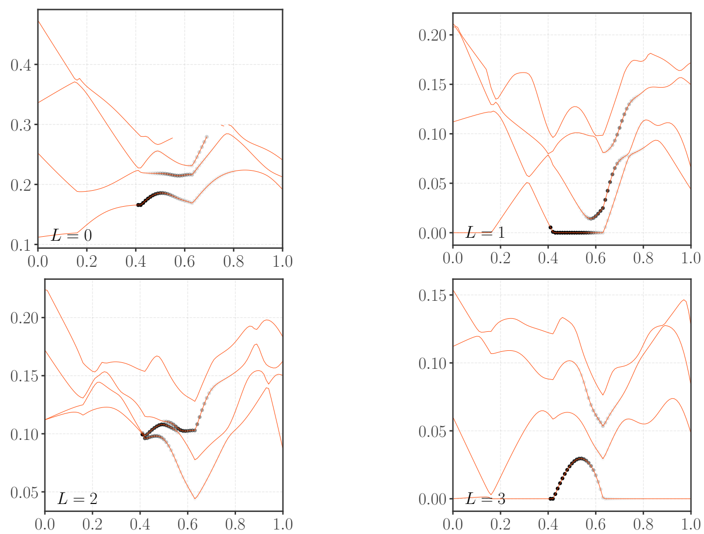


Figure 11: Energies and overlap in $\lambda \in [0.4, 0.8]$

2-QP at $\nu = 1/3$ **:** $(10, 5)$

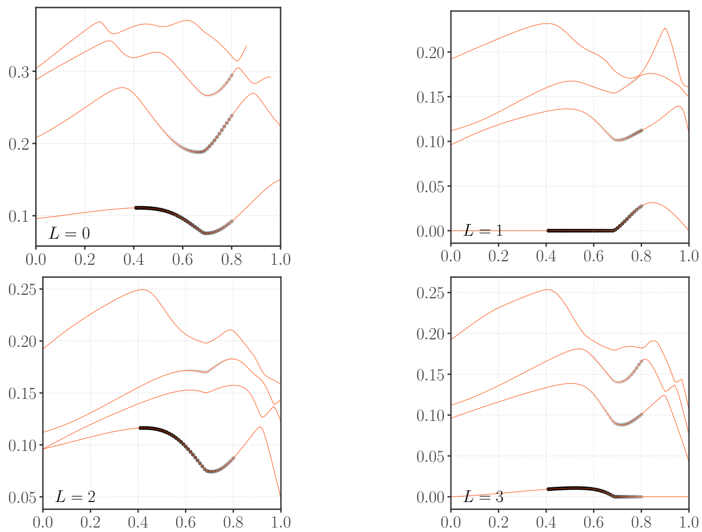


Figure 12: Energies and overlap in $\lambda \in [0.4, 0.8]$

2-QH at $\nu = 2/5$ **:** $(12, 6)$

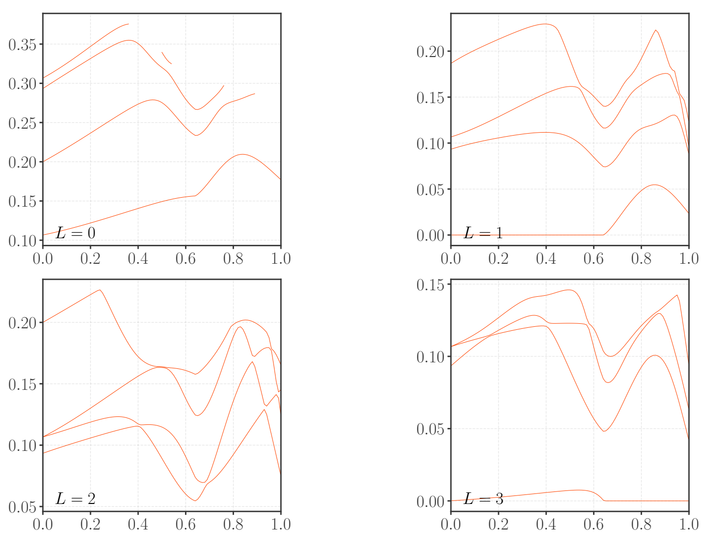


Figure 13: Energies and overlap in $\lambda \in [0.4, 0.8]$

2-QH at $\nu = 1/3$ **:** $(14, 5)$

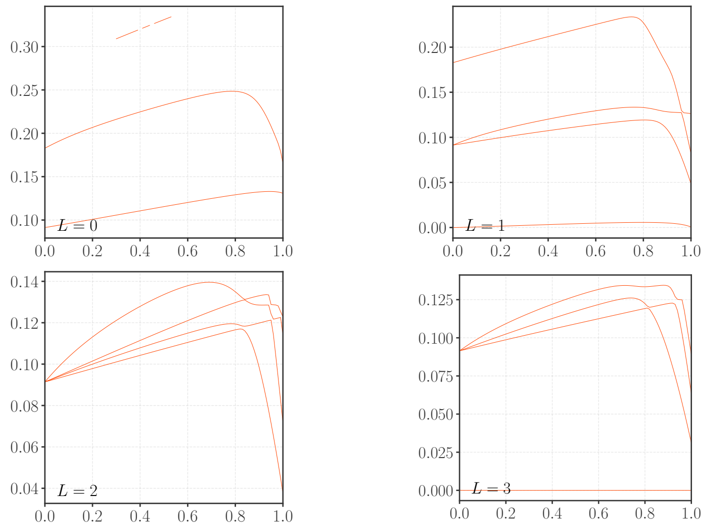
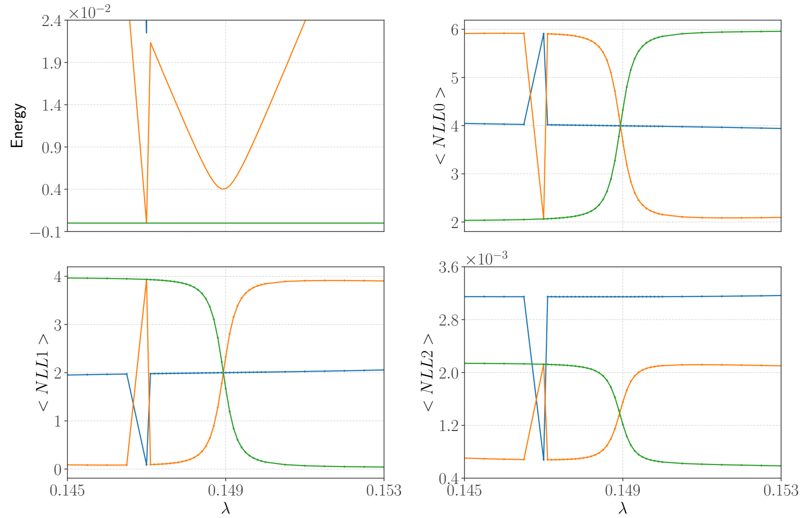


Figure 14: Energies and overlap in $\lambda \in [0.4, 0.8]$

Increasing β and λ linearly :

In this check, we test two system sizes $(20, 8)$ and $(11, 6)$. From the results gathered from $(11, 6)$ system, we see a gap closing as shown below :



Also, we compared the gap for $L = 0$ sector in these two system to check the effect of size, and found the gap to be lesser in larger system. Although this can be attributed to increasing Q, which reduces the factors adding to cyclotron energy cost;r but the gap is smaller than that of $(11, 6)$ even when taken the planar terms for cyclotron energy. The plots comparing $L = 0$ gaps for $(11, 6)$ (left) and $(20, 8)$ (right) :