Unit - 4 : Propositional Logic

Proposition / Statement :- Any declarative statement is called proposition, which should be either true or false (but not both).

e.g. $2+2=28$ → proposition (false)

$2+x=42$ → not a proposition

Chandigarh is in India → Proposition

where are you going? → not a proposition

Do your homework. → not a proposition

Compound Proposition :- A proposition which is composed of subpropositions. A proposition is called primitive if it cannot be broken down into simpler propositions.

e.g. Roses are Red & Violets are blue.

John is intelligent or studies every night.

John is intelligent and studies every night.

We use letters to denote propositional variables like $p, q, r, s \ldots$. The truth value of a proposition is denoted by T or F, depending upon whether it is true or false.

Negation :- Let P be a proposition. The negation of P, denoted by $\neg P$ ($\overline{P}$), is the statement

"It is not the case that P."

The proposition is read as not p. The truth ~~table~~ value of ¬P, negation of P is the opposite of truth value of P.

P	¬P
T	F
F	T

e.g. Raman's PC runs Linux.

⇒ it is not the case that Raman's PC runs Linux

⇒ Raman's PC doesnot run Linux.

Law of contradiction: A proposition cannot be true or false simultaneously.

Law of Excluded Middle: If anything is not true, then it is false & If anything is not false, then it is true.

Not true ≅ False

Not False ≅ True.

Basic Logical Operations:

Conjunction:- Any two propositions can be combined by the word "and" to form a compound proposition called conjunction of the original propositions. Let p & q be propositions. The conjunction of p & q is denoted by $p \wedge q$, is the proposition "p and q".

truth table.

P	q	$p \wedge q$
T	T	T
T	F	F
F	F	F
F	T	F

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Conjunction Syllogism

$$\begin{array}{c} \sim(p \wedge q) \\ q \\ \hline \sim p \end{array} \qquad \begin{array}{c} \sim(p \wedge q) \\ p \\ \hline \sim q \end{array}$$

* "but" sometimes is used instead of "and" in a conjunction
  e.g. The sun is shining, but it is raining.
  The sun is shining and it is raining.

Disjunction: Any two propositions can be ~~defined~~ combined by the word "or" to form a compound proposition is called the disjunction of the original propositions. Let p & q be propositions. The disjunction of p & q, denoted by $p \vee q$, is the proposition "p or q".

Truth table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

e.g. Students who have taken calculus or computer can take this class.

disjunction Syllogism:

$$\begin{array}{c} p \vee q \\ \sim q \\ \hline p \end{array} \qquad \begin{array}{c} p \vee q \\ \sim p \\ \hline q \end{array}$$

Exclusive OR ($\oplus$) : when exactly one of p & q is true.

Conditional Operator :- Let p & q be the propositions. The conditional statement $p \to q$ is the proposition "If p then q". In the conditional statement $p \to q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion (or consequence).

$p \to q$ is false, when p is true & q is false.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

e.g. If you get 100% on the final, then you will get an A.

~~Modus~~ Modus Poneny

$$\frac{p \to q \quad p}{q} \text{ valid} \qquad \frac{p \to q \quad \sim p}{\sim q} \text{ invalid}$$

Modus Tollens

$$\frac{p \to q \quad \sim p}{\sim p} \text{ valid} \qquad \frac{p \to q \quad q}{p} \text{ invalid}$$

- If p then q
- p implies q
- p only if q
- a sufficient condition for q is p.
- q whenever p.
- q is necessary for p.
- q follows from p.
- q when p
- q unless $\neg p$.
- a necessary condition for p is q.
- If p, q
- p is sufficent for q.
- q if p.

p	q	$p \to q$	inverse form $\neg p \to \neg q$	contrapositive form $\neg q \to \neg p$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	F
T	T	T	T	T

$$p \to q \equiv \neg q \to \neg p$$

$$p \to q = \neg p \vee q$$

e.g. Let P be the statement "Maria learns discrete Mathematics" and q be the statement "Maria will find a good job". Express the statement $p \to q$ as a statement in English.

⇒ If Maria learns discrete mathematics, then she will find a good job.

⇒ Maria will find a good job when she learns discrete mathematics.

⇒ For Maria to find a good job, it is sufficent for her to learn discrete mathematics.

⇒ Maria will find a good job unless she does not learn discrete mathematics.

$q \rightarrow p$ is called converse of $p \rightarrow q$

$\neg q \rightarrow \neg p$ is called contrapositive of $p \rightarrow q$

$\neg p \rightarrow -q$ is called inverse of $p \rightarrow q$.

when two compound propositions always have the same truth table, we call them equivalent.

<u>Biconditional</u> <u>Operation</u> :- Let p & q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q". The biconditional statement $p \leftrightarrow q$ is true when p & q have the same truth values, and is false otherwise. Biconditional statements are also called Bi-implications.

⇒ If p then q, but if q exists then p should also be there.

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

- p is necessary & sufficient for q.
- If p then q, & conversely
- p iff q.

p : you can take the flight.

q : you ~~by~~ buy a ticket

$p \leftrightarrow q$ you can take the flight if and only if you buy the ticket.

Biconditionals are not always explicit in nature, 'if and only if' is rarely used in common language, instead it is more often expressed using "if, then" or an "only if" construction.

e.g: If you finish your meal, then you can have a dessert.

what it really meant is,

you can have dessert only if you finish your meal.

Contradiction: When the final column in resultant table contains all false values, it is called contradiction.

i.e. If a compound statement is always false then it is called contradiction.

e.g.

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Tautology: When resultant compound proposition always contain the true as the truth value, it is called tautology.

e.g.

p	$\neg p$	$p \vee \neg p$
F	T	T
T	F	T

Contingency : When some values are true & some values are false, then it is called Contingency.

e.g

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

Satisfiable : A compound proposition is said to be satisfiable if at least one truth ~~or~~ value ~~or va~~ is true in the final column.

Tautological Implication ($\Rightarrow$) **Logical Implication**

$p \rightarrow p \cup q$

$p \Rightarrow p \vee q$

A proposition $P(p, q \_\_)$ is said to be logically imply a proposition $Q(p, q, --)$, if Q is true whenever P is true.

Laws of algebra of propositions.

a) $p \vee p \equiv p$ ~~b)~~ $p \wedge p \equiv p$ Idempotent

b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ Associative

c) $p \vee q \equiv q \vee p$ commutative

d) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ distributive

e) $p \vee T \equiv T$ $p \vee F = p$ identity

$p \wedge T \equiv p$ $p \wedge F \equiv F$

f) $p \vee \neg p \equiv T$ Complement.

g) $\neg T = F$ $\neg \neg p \equiv p$ Involution.

h) $\neg (p \vee q) = \neg p \wedge \neg q$ Demorgan's law

Arguments :- An argument is an assertion that a given set of propositions $P_1, P_2, \ldots P_n$, called premises, yields another proposition Q, called the conclusion.
Such an argument is denoted by

$$P_1, P_2, \ldots P_n \vdash Q$$

An argument $P_1, P_2, \ldots P_n \vdash Q$ is said to be valid if Q is true whenever all the premises $P_1, P_2, \ldots P_n$ are true.

An argument which is not true is called a fallacy.

e.g. $p, p \to q \vdash q$

p	q	p→q	q
T	T	T	T
T	F	F	F
F	F	T	F
F	T	T	T

valid

P_2 $p \to q, q \vdash p$

p→q	p	q	p
T	T	T	T
F	T	F	T
T	F	T	$\boxed{F}$
T	F	F	F

fallacy.

The argument $P_1, P_2, ---, P_n \vdash Q$ is valid if and only if the proposition $(P_1 \wedge P_2 \wedge \_\_ \wedge P_n) \to Q$ is a tautology.

<u>Law of Syllogism</u> $p \to q, q \to r \vdash p \to r$

i.e. If p implies q & q implies r, then p implies r.

Q. Consider following arguments & find which of them are valid.

(i) $\dfrac{P \cap Q}{P}$ valid

(ii) $\dfrac{P}{P \cup Q}$ valid

(iii) $\dfrac{\neg P}{P \to Q}$ valid

(iv) $\dfrac{\neg P}{Q \to P}$ Invalid

(v) $\dfrac{Q}{P \to Q}$ valid

(vi) $\dfrac{\neg Q}{P \to Q}$ Invalid

(vii) $\dfrac{\neg (P \to Q)}{\neg Q}$ valid

(viii) $\dfrac{\neg (P \to Q)}{P}$ valid

(ix) $\dfrac{\neg (P \to Q)}{\neg P}$ invalid

(x) $\dfrac{P \quad Q}{P \cap Q}$ valid

(xi) $\dfrac{P \cup Q \quad \neg P}{Q}$ Valid

(xii) $\dfrac{P \cup Q \quad P \to R \quad Q \to R}{R}$ Valid

(xiii) $\dfrac{P \quad Q}{S}$ Invalid

(xiv) $\dfrac{P \to R \quad Q \to S \quad \neg R \cup \neg S}{\neg P \cup \neg Q}$ valid

(xv) $\dfrac{P \cap Q \quad \neg P \cap \neg Q}{F}$ Valid

(xvi) $\dfrac{P \to Q \quad Q \to R \quad \neg R}{\neg P}$ valid

(xvii) $\dfrac{R \to S \quad P \to Q \quad R \cup P}{S \cup Q}$ valid

(xviii) $\dfrac{\neg P \to \neg R \quad \neg S \quad P \to W \quad R \cup S}{W}$ Valid.

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Q. P: If today is Gandhiji's birthday, then today is 2nd Oct.

Q : Today is 2nd Oct. invalid

Conclusion : Today is Gandhiji's birthday.

Q. P: If Canada is a country then London is a city.

Q: London is not a city.

Conclusion : Canada is not a country. valid.

Q. Check whether following arguments are valid or not.

(i) $\neg(P \vee Q) \vee (\neg P \wedge Q) \vee P = T$ valid

(ii) $(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ valid.

Q. P : If it rains then the cricket match will not be played.

Q_1: The cricket match was played.

Conclusion 1: There were no rain. valid

Q_2 : It did not rain.

Conclusion 2: Cricket match was played. invalid

$p \to$ it rains, $q \to$ match played.

$$\begin{array}{l} p \to \neg q \\ \quad q \\ \hline \neg p \end{array} \Rightarrow \text{conclusion 1 is true}$$

$$\begin{array}{l} p \to \neg q \\ \quad \neg p \\ \hline \neg q \end{array} \text{ invalid} \Rightarrow \text{conclusion 2 is false}$$

Q. Write contrapositive, converse & inverse form of the following statement.

" If I stay if you go.

Sol. $p \rightarrow$ you go

$q \rightarrow$ I stay

$\Rightarrow p \rightarrow q$

Contrapositive : $\neg q \rightarrow \neg p$

If i will not stay then you should not go.

$\Rightarrow$ I don't stay if u dont go.

Converse : $q \rightarrow p$

u go if i stay.

$\Rightarrow$ If i stay then you go.

Inverse : $\neg p \rightarrow \neg q$

u don't go if i don't stay.

Precedence of logical Operators :

operator	precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Logical Equivalence (⇔)

- He is either not informed, or he is not honest. $\neg P \vee \neg Q$
- It is not true that he is informed & honest. $\neg(P \wedge Q)$

These two statements are logically equivalent.

- If the goods were not delivered, the coustmer cannot have paid.
- If the coustmer have paid, the goods must have been delivered.

Removing Conditional & Bi-conditional Operator

- $P \rightarrow Q \equiv \neg P \vee Q$
- $P \leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$
- $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

$$\equiv (\neg P \vee Q) \wedge (P \vee \neg Q)$$

Q. Remove Conditional & Biconditional operators from the following expression:

$$(P \rightarrow Q \wedge R) \vee ((R \leftrightarrow S) \wedge (Q \vee S))$$

Sol. $(\neg P \vee Q \wedge R) \vee (((\neg R \vee S)) \wedge (Q \vee S))$

Normal Forms : Standard forms for expressions, so that, they can be identified & compared easily.

Disjunctive Normal form (dnf) :- A logical expression is said to be in disjunctive normal form, if it is written as a disjunction, in which all the terms are conjunction of literals. e.g. $(P \land Q) \lor (P \land \neg Q)$,

$P \lor (Q \land R)$, & $\neg P \lor T$

$\neg (P \land Q) \lor R$ is not a disjunction.

Conjuctive Normal form (cnf) :- A logical expression is said to be in conjunctive normal form if it is written as a conjunction of disjunction of literals.

e.g. $P \land (Q \lor R)$, $P \land F$.

$P \land (R \lor (P \land Q))$ is not in cnf, bcz a disjunction contains a conjunction.

* Remove all conditional & bi-conditional operators.
* If the expression contains any negated compound subexpression, either remove the negation by using the double negation law or use De-Morgan's law to reduce the scope of the negation.
* Once an expression with no negated compound subexpression is found, use the following two laws to reduce the scope of $\lor$.

$$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$$
$$(A \wedge B) \vee C = (A \vee C) \wedge (B \vee C)$$

Q. Convert the following expression into normal form

- $\neg((P \vee \neg Q) \wedge \neg R)$

$$\Rightarrow \neg(P \vee \neg Q) \vee \neg\neg R$$
$$\Rightarrow (\neg P \wedge \neg\neg Q) \vee R$$
$$\Rightarrow (\neg P \wedge Q) \vee R$$
$$\Rightarrow (\neg P \vee R) \wedge (Q \vee R)$$

- $(P \vee Q) \wedge P \wedge (Q \vee R) \wedge (\underbrace{P \vee \neg P}_{T} \vee R) \wedge (\neg Q \vee R)$

R

$$\underbrace{(P \vee Q) \wedge P}_{\Downarrow} \wedge R \wedge T$$

Absorption Law. $\underline{P \wedge R}$

~~a∧b~~
(a∨b)∧a
a

Absorption Law

$$P \vee (P \wedge Q) = (P \wedge T) \vee (P \wedge Q)$$
$$= P \wedge (T \vee Q)$$
$$= P \wedge T$$
$$= P$$

- Obtain the dnf form of $(p \to q) \wedge (\neg p \wedge q)$

$\Rightarrow (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

- Obtain the dnf form of $\neg (p \to (q \wedge r))$

$\Rightarrow (p \wedge \neg q) \vee (p \wedge \neg r)$

- Obtain the cnf form of $(p \wedge q) \vee (\neg p \wedge q \wedge r)$

$\Rightarrow (p \vee q) \wedge (p \vee r) \wedge (q \vee \neg p) \wedge q \wedge (q \vee r)$

- Obtain the cnf form of $(\neg p \to r) \wedge (p \leftrightarrow q)$

$\Rightarrow (p \vee r) \wedge (\neg p \vee q) \wedge (\neg q \vee p)$

Constructing dnf from truth table

Let p be a statement containing n variables $p_1, p_2 \ldots p_n$. Its dnf can be obtained by: the truth table. For each row in which p assumes value T, form the conjuncta $p_1 \wedge p_2 \wedge \ldots p_j \wedge \ldots p_n$. Such a term is minterm. The disjunction of the minterm is the dnf.

Predicate Logic :- Propositions are either true or false, but some mathematical statements does not fall under this category, e.g. $x > 3$.. where x is assigned some value, the resulting statement is proposition with certain truth value. The statement $x<3$ has 2 parts. x is the variable & part of the statement & 'is less than 3' is a predicate.

"Predicate refers to a property that the subject of the statement can have.

This type of statement is represented as $p(x)$ where p denotes predicate & x denotes the variable

$p(2)$ is false, $p(4)$ is true

Predicate can have more than one variable.

e.g. $a+b=7$

$p(a,b)$ is true for $p(5,2)$

& false for $p(2,3)$

Any predicate function can have any number of variables & it will become a proposition once all the values of the variables are defined.

Q. Let $Q(x,y)$ denote the statement "$x = y+3$". What are the truth values of the proposition $Q(1,2)$ and $Q(3,0)$?

$Q(1,2) \downarrow$ false $Q(3,0) \downarrow$ true.

Quantifiers: Quantification expresses the extent to which a predicate is true over a range of elements. In english, the words all, some, many, none, & few are used in quantifications.

When we apply quantifiers on predicate functions then it becomes predicate calculus.

Universal Quantifier ($\forall$): $\forall x\, P(x)$, It means function $P(x)$ will give true for all the values of x.

Existensial Quantifier ($\exists$) $\exists x\, P(x)$, There exist atleast one value of x for which function will give the true value.

Negation of the Quantified Statements:

- All math majors are male.

$\Rightarrow$ Negation Operation:

$\Rightarrow$ It is not the case that all math majors are male.

$\Rightarrow$ There exists at least one math major who is a female.

- **Uniqueness Quantifier** ($\exists!$, $\exists_1$) : There exist a unique x such that $P(x)$ is true.
  e.g. $x-1=0$

$$\neg \forall x\, P(x) \equiv \exists x \neg P(x)$$
$$\neg \exists x\, P(x) \equiv \forall x \neg P(x)$$

- For all positive integers n we have $n+2>8$
  ⇒ There exists a positive integer n such that $n+2>8$.

- The following statements are also negatives of each other.
  ⇒ There exists a (living) person who is 150 years old.
  ⇒ Every living person is not 150 years old.

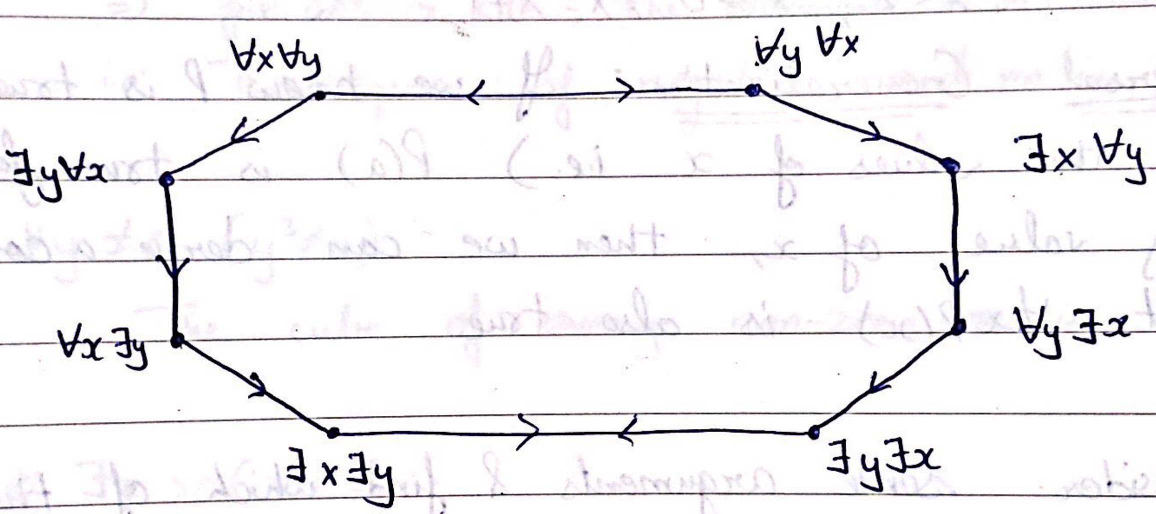
A propositional function preceded by a quantifier for each variable, for example,

$$\forall x \exists y, p(x,y) \quad \text{or} \quad \exists x \forall y \exists z, P(x,y,z)$$

Q. Consider a function $P(x,y)$: x loves y.
Consider some statement & identify the meaning

1. $\forall x \forall y \; P(x,y)$
2. $\forall y \forall x \; P(x,y)$
3. $\exists x \exists y \; P(x,y)$
4. $\exists y \exists x \; P(x,y)$
5. $\forall x \exists y \; P(x,y)$
6. $\exists y \forall x \; P(x,y)$
7. $\forall y \exists x \; P(x,y)$
8. $\exists x \forall y \; P(x,y)$

1. Everybody loves everybody.
2. Everybody is loved by Everybody.
3. There exists someone who loves someone.
4. There exist someone who is loved by someone.
5. Everybody loves someone.
6. Someone is loved by everybody.
7. Everybody is loved by someone.
8. Someone loves everybody.



(1) $\forall x\,(P(x) \rightarrow Q(x))$ $\qquad$ $P(a) \rightarrow Q(a)$

$$\frac{\exists x\,(P(x))}{\exists x\,(Q(x))} \quad \text{Valid} \qquad \frac{P(c)}{Q(c)}$$

<u>Existensial Specification</u>: If $\exists x P(x)$ is true then we can say there exist at least one constant c for which $P(c)$ is true. This process is called Existensial Specification.

$\forall x P(x)$ is false when there is an x for which $P(x)$ is false.
$\exists x P(x)$ is false when $P(x)$ is false for every x.

Universal Specification: If $\forall x P(x)$ is true then $P(a)$ will be true for any value of a.

$x+1 > x \quad , \quad x^2 \geq x$

Generalization :-

Existensial Generalization: If we know P is true, for some constant c, then we can derive a conclusion that $\exists x P(x)$ is also true.

$x < 2 \quad , \quad x^2 > 0 \quad , \quad x = x+1$

Universal Generalization: If we know P is true for all the values of x i.e.) $P(a)$ is true for any value of x, then we can derive a conclusion that $\forall x P(x)$ is also true.

Q. Consider some arguments & find which of them is valid.

① $\forall x \, (P(x) \rightarrow Q(x))$
$\overline{\forall x \, P(x) \rightarrow \forall x \, Q(x)}$ invalid

② $\exists x \, P(x)$
$\exists x \, Q(x)$
$\overline{\exists x \, (P(x) \wedge Q(x))}$ invalid

(5) $L(x,y)$: x likes y

'there is someone, whom no one likes'

a) $\forall x \exists y \{\sim L(x,y)\}$ $\Rightarrow$ Everybody is not loved by someone.

b) $\sim \forall x \exists y \{L(x,y)\}$

c) $\sim \{\forall y \exists x L(x,y)\}$ $\Rightarrow$ Someone is not loved by all.

✓d) $\sim \{\exists y \forall x L(x,y)\}$

- $\forall x < 0 \ (x^2 > 0)$

  $\Rightarrow$ for all $x < 0$, the statement $x^2 > 0$ is true

  i.e. The square of negative real no. is positive.

- $\forall y \neq 0 \ (y^3 \neq 0)$

  The cube of every non-zero no. is non-zero.

- $\exists z > 0 \ (z^2 = 2)$

  There is a positive square root of 2.

* The quantifiers $\forall$, $\exists$ has highest precedence.

  e.g. $\forall x \, P(x) \vee Q(x)$

  is disjunction of $\forall x \, P(x)$ and $Q(x)$.

  $(\forall x \, P(x)) \vee Q(x)$

* ~~b~~ $\exists x \, (x + y = 1)$ $\Rightarrow$ x is bounded, while y is free

$$\forall x\,(P(x) \land Q(x)) \equiv \forall x P(x) \land \forall x Q(x)$$

$$\forall x\,(P(x) \lor Q(x)) \not\equiv \forall x P(x) \lor \forall x\, Q(x)$$

$$\exists x\,(P(x) \lor Q(x)) \equiv \exists x\, P(x) \lor \exists x\, Q(x)$$

$$\exists x\,(P(x) \land Q(x)) \not\equiv \exists x\,(P(x)) \land \exists x\, Q(x)$$

Q. Write the negation of the statement.

"There is an honest politician".

$H(x)$: Honest Politician

$\exists x H(x)$

$\neg \exists x H(x) = \forall x \neg H(x)$

All politicians are not honest.

× equivalent ~~⟷~~ { Every politican is dishonest. } ✓

{ Not all politicans are honest. }

"All americans eat cheeseburgers".

$C(x)$: x eats cheesburgers.

$\forall x\, C(x)$, $x \in$ all Americans

$\neg \forall x\ C(x) = \exists x \neg C(x)$

There is an American who does not eat cheeseburgers.

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- $\forall x \, (x^2 > x)$

  $\neg \forall x \, (x^2 > x)$

  $\exists x \, \neg (x^2 > x)$

  $\exists x \, (x^2 \leq x)$

- $\exists x \, (x^2 = 2)$

  $\neg \exists x (x^2 = 2)$

  $\forall x \, \neg (x^2 = 2)$

  $\forall x \, (x^2 \neq 2)$

- $\neg \forall x \, (P(x) \rightarrow Q(x)) \equiv \exists x \, (P(x) \wedge \neg Q(x))$

  $\exists x \, \neg (P(x) \rightarrow Q(x))$

  $\exists x \, \neg (\neg P(x) \vee Q(x))$

  $\exists x \, (P(x) \wedge \neg Q(x))$

Q. Express the statement using predicates & quantifiers.
"Every student in this class has studied calculus".
=> For every student in this class, that student has studied calculus.
=> For every student x in this class, x has studied calculus.

$C(x)$: x has studied calculus.

$\forall x \, C(x)$, $x \in$ students in the class

Scanned with CamScanner

• Some student in this class has visited Mexico.

$\exists x\, M(x)$

x visited Mexico.

student is in this class.

$\exists x\, (S(x) \land M(x))$

• Every student in this class has visited either mexico ~~&~~ or canada.

$\forall x\, \cancel{\forall x}\, (C(x) \lor M(x))$

$\forall x\, (S(x) \rightarrow (C(x) \lor M(x)))$

Q. All lions are fierce.

Some lions do not drink coffee.

Some fierce lions do not drink coffee.

$\forall x\, (P(x) \rightarrow Q(x))$

$\exists x\, (P(x) \land \neg R(x))$

$\exists x\, (Q(x) \land \neg R(x))$ valid

Q. All humming birds are richly colored.

No large birds live on honey.

Birds that do not live on honey are dull in color

Hummingbirds are small.

$\forall x\, (P(x) \rightarrow S(x))$

$\neg \exists x\, (Q(x) \land R(x))$

$\forall x\, (\neg R(x) \rightarrow \neg S(x))$

$\forall x\, (P(x) \rightarrow \neg Q(x))$