Functions

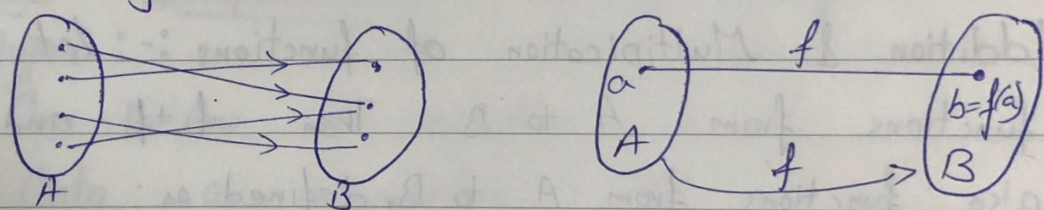
{Definition, Classification of functions, Composition of functions, Inverse function}

A function is a special case of relation. Let A & B be two non-empty sets. A function f from A to B ($f: A \to B$) is a set of ordered pairs with the property that $f \subseteq A \times B$: "for each element x in A, there is a unique element y in B such that $(x,y) \in f$".

$f \subseteq A \times B$

$f(a) = b$, If $a \in A$, then $f(a)$ denotes the unique element of B which f assigns to a.

The statement "f is a function from A to B" is usually represented by $f: A \to B$ or $A \to B$.



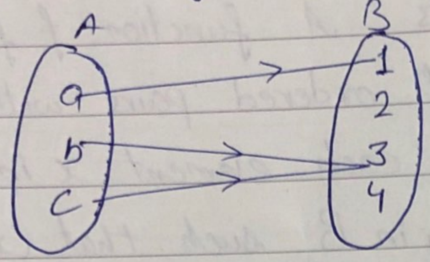
functions are also called mappings or transformations.

<u>Domain</u> & <u>Co-domain</u> :- If f is a function from A to B, we say that A is the domain of f and B is the codomain of f.

<u>Image</u> & <u>Pre-image</u> :- If $f(a) = b$, we say that b is the image of a & a is the pre-image of b.

Range :- The range of f is the set of all images of elements of A. The image of $f : A \to B$ is denoted by $Ran(f)$, $Im(f)$ or $f(A)$.

$$\text{Range} \subseteq \text{Co-domain}$$



Domain = $\{a, b, c\}$

Co-domain = $\{1, 2, 3, 4\}$

Range = $\{1, 3\}$

Equality of functions :- Two functions are equal when they have the same co-domain & map elements of their common domain to the same elements in their common co-domain. $f = g$, if $f(a) = g(a)$ $\forall a \in A$, $f : A \to B$ & $g : A \to B$

Addition & Multipication of functions :- Let f_1 & f_2 be functions from A to B. Then $f_1 + f_2$ and $f_1 \cdot f_2$ are also functions from A to B, defined as:

$$(f_1 + f_2)(x) = f_1(x) + f_2(x)$$

$$f_1 \cdot f_2 (x) = f_1(x) \cdot f_2(x)$$

A function can be expressed as a mathematical formula. When a function is given by a formula in terms of x, we assume unless it is otherwise stated, that the domain of the function is R & also the codomain is R.

Identity Function :- The function from A to A which assigns to each element that element itself is called identity function on A & is usually denoted by 1_A or simply 1.

$$1_A(a) = a \quad \forall\ a \in A.$$

Functions as Relations : Every function $f : A \to B$ gives rise to a relation from A to B called the 'graph of f' and defined by

$$\text{Graph of } f = \{(a,b) : a \in A,\ b = f(a)\}$$

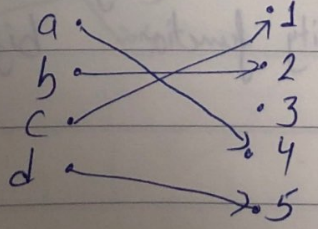
A function $f : A \to B$ is a relation from A to B (i.e. subset of $A \times B$) such that each $a \in A$ belongs to a unique ordered pair (a,b) in f.

Classification of functions : Broadly classified into three types :

- One-to-One or injective
- Onto or Surjective
- One-to-one correspondance or bijective or invertible

One-to-one function :- A function f is said to be one-to-one, or injective, iff $f(a) = f(b)$ implies that $a = b$, for all a and b in the domain of f.

$$\forall a \forall b\ (f(a) = f(b) \to a = b)$$



A function that is either strictly increasing or strictly decreasing must be one to one. A function that is increasing, but not strictly decreasing, is not necessarily one-to-one.

* One-to-one function never assign the same value to two different domain elements.

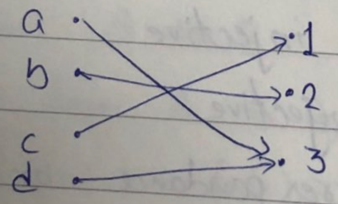
e.g. $f(x) = 3x - 1$ injective.

$f(x) = x^2$ not injective.

Onto function :- A function f from A to B is called onto, or surjective, if & only if for every element $b \in B$, there is an element $a \in A$ with $f(a) = b$.

$\forall b \, \exists a \, (f(a) = b)$ Range = Co-domain

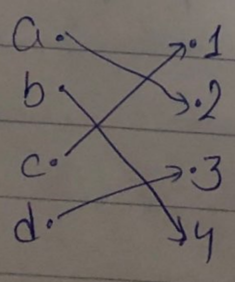
For onto functions, the range & the codomain are equal, i.e. every number of the codomain is the image of some element of the domain.



$f(x) = x^2$, not surjective.

$f(x) = x+1$, surjective.

One-to-one correspondance :- The function f is a one-to-one correspondance, or a bijection, if it is both one-to-one & onto. We also say that such a function is bijective.

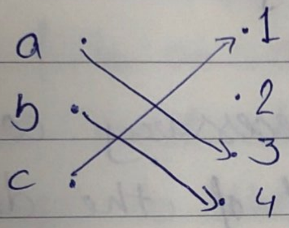


$f(x) = x^3$ bijective.

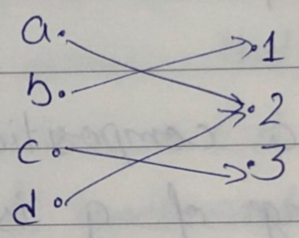
Identity function bijective

Suppose that $f: A \to B$

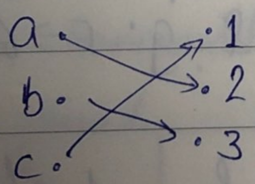
- To show that f is <u>injective</u>, show that if $f(x) = f(y)$ for arbitrary $x, y \in A$ with $x \neq y$, then $x = y$
- To show that f is <u>not injective</u>, Find particular elements $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$.
- To show that f is <u>surjective</u>, Consider an element $y \in B$ & find an element $x \in A$ such that $f(x) = y$.
- To show that f is <u>not surjective</u>, Find a particular $y \in B$, such that $f(x) \neq y$ for all $x \in A$.



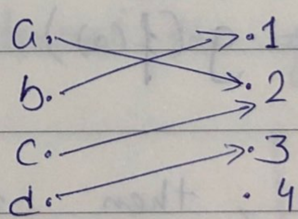
One-to-one & not onto



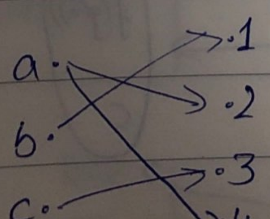
Onto, not one-to-one



One-to-one & onto



neither one-to-one, nor onto



Not a function.

Composition of function :- Let g be a function from the set A to the set B and let f be a function from the set B to set C. The composition of functions f and g, denoted for all $a \in A$ by $f \circ g$, is defined by

$$(f \circ g)(a) = f(g(a))$$

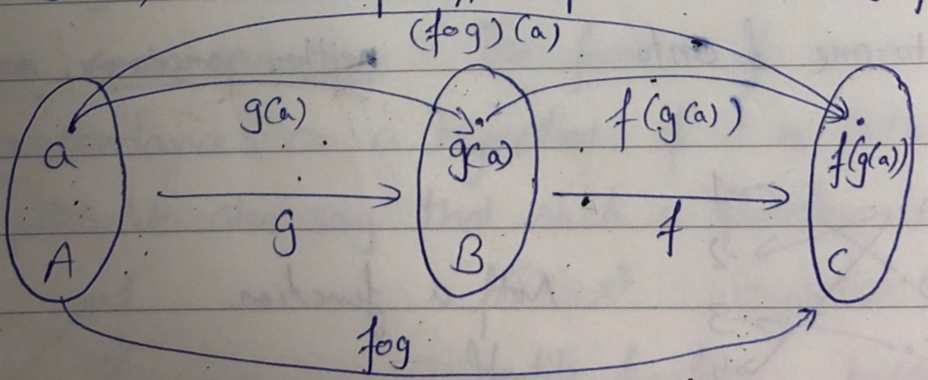
In other words, $f \circ g$ is the function that assigns to the element a of A the element assigned by f to $g(a)$. i.e., to find $(f \circ g)(a)$ we first apply the function g to a to obtain $g(a)$ & then we apply the function f to the result of $g(a)$ to obtain $(f \circ g)(a)$.

To define a composition, the necessary condition is that range of g is a subset of the domain of f. [Codomain of g is the domain of f.]

$(f \circ g)(a) = f(g(a))$ $\quad g : A \to B$ & $f : B \to C$

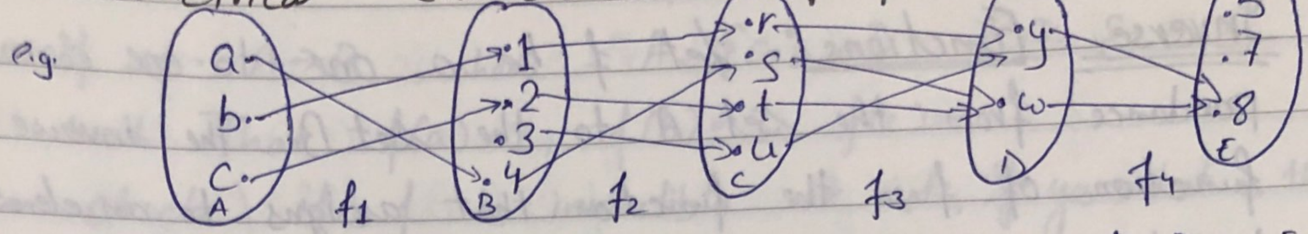
$(g \circ f)(a) = g(f(a))$ $\quad f : E \to F$ & $g : F \to G$

$f : A \to B$, then $f \circ 1_A = f$ & $1_B \circ f = f$



Geometrical Characterization of functions :-

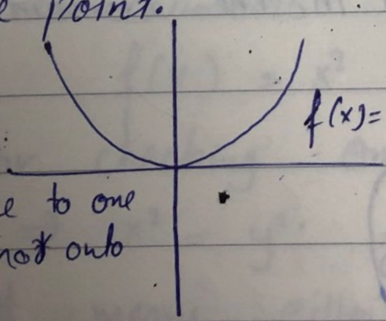
e.g.



$f_1 : A \to B$, $f_2 : B \to C$, $f_3 : C \to D$, $f_4 : D \to E$

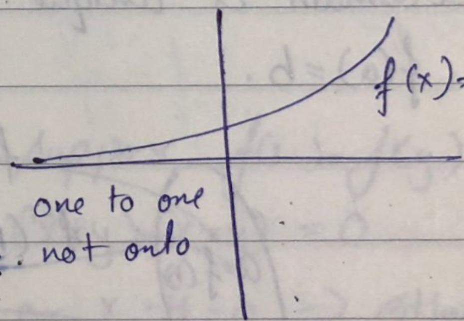
one-to-one, not onto | One to one, & onto | not one to one, onto | neither one-to-one nor onto.

- To say that a function $f : A \to B$ is one-to-one means that there are no two distinct pairs (a_1, b) & (a_2, b) in the graph of f; hence each horizontal line can intersect the graph of f in at most one point.
- If f is an onto function means that for every $b \in B$. there must be at least one $a \in A$ such that (a, b) belongs to the graph of f; hence each horizontal line must intersect the graph of A at least once.
- If f is both one to one. & onto i.e. invertible, then each horizontal line will intersect the graph at exactly one point.



$f(x) = x^2$

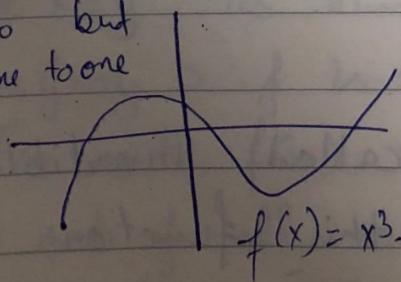
neither one to one not onto



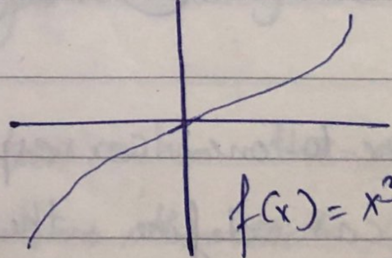
$f(x) = 2^x$

one to one ∴ not onto

onto but not one to one



$f(x) = x^3 - 2x^2 - 5x + 6$



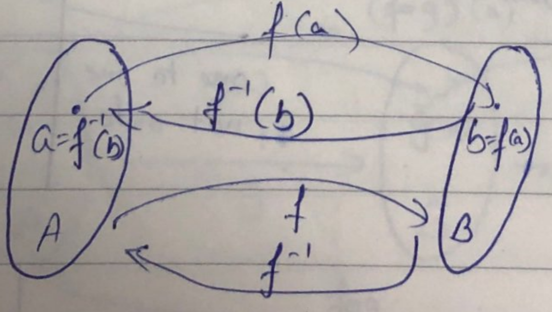
one to one & onto

$f(x) = x^3$

Inverse Functions :- Let f be a one-to-one correspondance from the set A to the set B. The inverse function of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$. The inverse function of f is denoted by f^{-1}. Hence, $f^{-1}(b) = a$, when $f(a) = b$.

* f^{-1} is not $1/f$.

If a function is not a one to one correspondance, we cannot define an ~~f~~ inverse function of f. When f is not a one-to-one, some element b in the codomain is the image of more than one element in the domain. & If f is not onto, for some element b in the codomain, no element a in the domain exists, for which $f(a) = b$. Consequently if f is not a one to one correspondance, we cannot assign to each element b in the codomain a unique element a in the domain such that $f(a) = b$.



A one-to-one correspondance is called invertible bcz we can define the inverse of this function.

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• If $f : A \to B$, where $A = \{a, b, c\}$ & $B = \{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, $f(c) = 1$.
Check if f is invertible or not, If yes, what is the inverse?

→ as the function f is both one to one & onto, it is inversible. $f^{-1} : B \to A$ such that $f^{-1}(1) = c$, $f^{-1}(2) = a$, $f^{-1}(3) = b$.

• $f : Z \to Z$, $f(x) = x+1$. Is f invertible, If yes, find the inverse

→ It is one to one correspondance.

$f(x) = x + 1 \Rightarrow y = x + 1 \Rightarrow x = y - 1 \Rightarrow y$

$f^{-1}(y) = y - 1$.

• Let $f : R \to R$, $f(x) = x^2$, whether invertible or not, if not make it invertible, by specifying some condition.

→ Rather than R, define the function on nonnegative real numbers.

$f(x) = x^2$

for checking one to one. Suppose $f(x) = f(y)$

$\Rightarrow x^2 = y^2 \Rightarrow x^2 - y^2 = (x+y)(x-y) = 0$

It means either $x + y = 0$ or $x - y = 0 \Rightarrow$ either $x = -y$ or $x = y$, as numbers or nonnegative $x = -y$ not possible

so, if $x = y \Rightarrow f$ is one to one.

For onto, $f(x) = x^2$, is onto bcz when the codomain is set of all non-negative numbers, then each such no. has a square root. $f^{-1}(y) = \sqrt{y}$.

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Q. Let $A = \{1,2,3,4\}$, $B = \{a,b,c,d\}$
$f: A \to B$, such that $f = \{(1,a),(2,a),(3,d),(4,c)\}$.
Show that f is a function but f^{-1} is not.

- We can define inverse of a function in one more way. Let $f: A \to B$. Then a map $g: B \to A$ is called inverse function of f,

$$\text{iff } g\circ f = I_A \ \& \ f\circ g = I_B$$

i.e. $g[f(x)] = x \quad \forall x \in A$

$f[g(y)] = y \quad \forall y \in B$

i.e. if $f(x) = y$, then $g(y) = g[f(x)] = x$

as the inverse is denoted is f^{-1}, $f(x) = y \Leftrightarrow x = f^{-1}(y)$

Q. Show that $f(x) = x^3$ & $g(x) = x^{1/3}$ $\forall x \in R$ are inverse of one another.

$\rightarrow$ $(f\circ g)(x) = f(g(x)) = \cancel{I_x}\ f(x^{1/3}) = (x^{1/3})^3 = x = I_x$

$(g\circ f)(x) = g(f(x)) = g(x^3) = (x^3)^{1/3} = x = I_x$

$\therefore f = g^{-1}$ & $g = f^{-1}$

Q. Show that the mapping $f: R \to R$ be defined by $f(x) = ax+b$, where $a, b, x \in R$, $a \neq 0$ is invertible. Define its inverse.

$\rightarrow$ for $x_1, x_2 \in R$, then

if $f(x_1) = f(x_2)$

$ax_1 + b = ax_2 + b$

$\Rightarrow ax_1 = ax_2 \Rightarrow x_1 = x_2$, proving that f is one to one.

If $y \in R$ &

$y = f(x)$

$y = ax+b$

$y-b = ax$

$\frac{y-b}{a} = x$

$\therefore$ for $x \in R$, there exist $\frac{y-b}{a} \in R$, so f is ~~inv~~ onto

As f is both one to one & onto, it is invertible.

& ~~g~~ f^{-1} is defined as

$$f^{-1}(y) = \frac{y-b}{a}$$

Characteristics of Composition :-

- $f \circ g(x) \neq g \circ f(x)$
- $f^{-1} \circ f(x) = f^{-1}(f(a)) = f^{-1}(b) = a$
- $f \circ f^{-1}(x) = f(f^{-1}(b)) = f(a) = b$
- $f(f^{-1}) = (f^{-1})^{-1} = f$

- If both f & g are one to one functions, then $f \circ g$ is also one to one.
- If both f & g are onto functions, then $f \circ g$ is also onto.
- If f & $f \circ g$ are onto onto, then g is also one to one.

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- If f & $f \circ g$ are onto, then it does not mean that g is also onto.
- If f is an invertible function from y to z & g is an invertible function from x to y. then $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$
- Let f be a function from A to B. Let S & T be subsets of A, then
  $$f(S \cup T) = f(S) \cup f(T)$$
  $$f(S \cap T) \subseteq f(S) \cap f(T)$$
- Number of functions, N_f on a set with n elements are, $N_f = n^n$.
- Number of functions, N_f from set A with n elements to set B with m elements are,
  $$N_f = m^n.$$

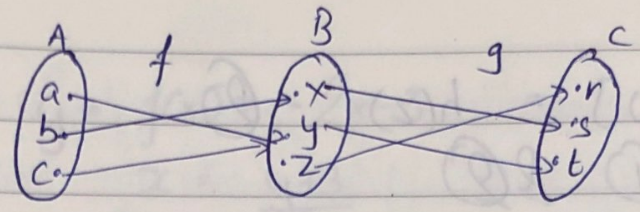
Q. If $f: R \to R$ & $g: R \to R$ are defined as $f(x) = x+2 \; \forall x \in R$ & $g(x) = x^2 \; \forall x \in R$. Find $f \circ g(x)$ & $g \circ f(x)$

$\Rightarrow$ $f \circ g(x) = f(g(x)) = f(x^2) = x^2+2$

$g \circ f(x) = g(f(x)) = g(x+2) = (x+2)^2$

- Composition of function is not commutative.

Q. Let the functions $f: A \to B$ & $g: B \to C$ be defined as



find the composition of $g \circ f : A \to C$

$\to$ $g \circ f(a) = g(f(a)) = g(y) = t$

$g \circ f(b) = g(f(b)) = g(x) = s$

$g \circ f(c) = g(f(c)) = g(y) = t$

Q. Let the function f & g be defined as $f(x) = 2x+1$ & $g(x) = x^2 - 2$. Find the mathematical formula for $g \circ f$.

$\to$ $g \circ f(x) = g(f(x)) = g(2x+1) = (2x+1)^2 - 2$

$g \circ f(x) = 4x^2 + 4x - 1.$

Associative law of function composition :

If $f : A \to B$, $g : B \to C$ & $h : C \to D$ then prove that $h \circ (g \circ f)(x) = (h \circ g) \circ f(x)$

Proof :- Since $f : A \to B$ & $g : B \to C$ & $h : C \to D$

$\therefore$ $g \circ f : A \to C$ & $h \circ g : B \to D$

$\therefore$ $h \circ g \circ f : A \to D$ & $(h \circ g) \circ f : A \to D$

$\Rightarrow$ $dom[h \circ (g \circ f)] = dom[(h \circ g) \circ f]$

Let $x \in A$, $y \in B$, $z \in C$. Such that $f(x) = y$, $g(y) = z$.

Then, $[(h \circ g) \circ f](x) = h \circ g[f(x)] = h \circ g(y)$

$= h(g(y)) = h(z)$ — 1

$[h \circ (g \circ f)](x) = h \circ [g \circ f(x)] = h[g(f(x))]$

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$= h[g(y)] = h(z)$ — ②

from ① & ②

$[(h \circ g) \circ f](x) = [h \circ (g \circ f)](x)$

Q. Consider a function $f: A \to B$ & $g: B \to C$. Prove the following statements.

a) If f & g are one to one, then composition of $g \circ f$ is one to one.

$\to$ ~~g(o~~ $g \circ f(x) = g \circ f(y)$

$g(f(x)) = g(f(y))$

$f(x) = f(y)$

$x = y$

$\Rightarrow$ $g \circ f$ is one-to-one.

b) If f & g are onto, then $g \circ f$ is also onto.

$\to$ let c be an arbitrary element of C. Since g is onto, there exist a $b \in B$ such that $g(b) = c$, since f is onto, there exist an $a \in A$, such that $f(a) = b$. But then, $g \circ f(a) = g(f(a)) = g(b) = c$. Hence, each $c \in C$ is the image of some element $a \in A$. So $g \circ f$ is onto.

Q. Let $f: R \to R$ be defined by $f(x) = 2x - 3$. Now, if f is one-to-one & onto, find f^{-1}.

→ $\therefore y = f(x) = 2x - 3$

$x = \frac{y+3}{2}$

$f^{-1}(y) = \frac{y+3}{2}$

Q. Which of the following is one to one.

a) To each person on the earth assign the number which corresponds to its age. No

b.) To each country in the world assign the lattitude & longitude of its capital. Yes

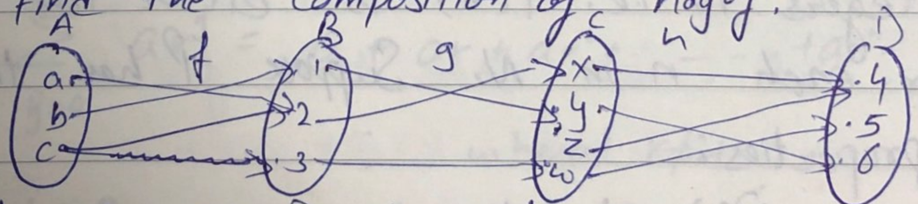
c.) To each book written by only one author assigns the author No

d.) To each country in the world which has a prime minister assigns its prime minister Yes

Q. Let the function $f: A \to B$, $g: B \to C$ & $h: C \to D$.

a) Determine if each f^n is onto.

b) Find the composition of $h \circ g \circ f$.



→ only $h: C \to D$ is onto.

$h(g \circ f)(x) = h(g(f(x))) = h(g(f(a)))$

$= h(g(2)) = h(x) = 4$

Similarly $\{(a,4), (b,6), \{c,4\}\}$

Q. find the composition $f \circ (g \circ h)(x)$, $f \circ g(x)$
where $f(x) = x+2$, $g(x) = x^2+9$, $h(x) = x^3 - 3$

$\rightarrow f \circ g(x) = f(g(x)) = f(x^2+9) = x^2+9+2 = x^2+11$

$f \circ g \circ h(x) = f(g(h(x))) = f(g(x^3-3)) = f((x^3-3)^2+9)$

$= (x^3-3)^2 + 9 + 2 = x^6 + 9 - 6x^3 + 9 + 2$

$= x^6 - 6x^3 + 20$

Mathematical Induction :- The structure of a proof by induction consists of four part.

1.) State the proof by induction.
2.) Specify the induction hypothesis. [let $P(n)$ is true]
3.) The basic step. [Prove $P(1)$ is true]
4.) The inductive step. [Prove $P(n+1)$ is true]

The principle of mathematical induction $N = \{1, 2, 3, \ldots\}$
Let P be a proposition defined on the positive integers N i.e. $P(n)$ is either true or false for each n in N. Suppose P has the following 2 properties:

(i) $P(1)$ is true.
(ii) $P(n+1)$ is true whenever $P(n)$ is true.

Then P is true for every integer.

$$(P(1) \wedge \forall k (P(k) \rightarrow P(k+1))) \rightarrow \forall n \, P(n)$$

Let P be the proposition that the sum of first n odd numbers is n^2,

i.e. $P(n) = 1+3+5+ \dots + (2n-1) = n^2$

n^{th} odd number is $2n-1$ &

$(n+1)^{th}$ odd number is $2n+1$

lets first check whether $P(n)$ is true for $n=1$,

i.e. $P(1) = 1 = 1^2$

if $P(n)$ is true, we add $2n+1$ to both sides of $P(n)$, obtaining.

$$1+3+5+ \dots + (2n-1) + (2n+1)$$
$$= n^2 + (2n+1)$$
$$= (n+1)^2 = P(n+1)$$

That is $P(n+1)$ is true whenever $P(n)$ is true.

$\therefore$ P is true for all n.

e.g. $\therefore 1+2+2^2+ \dots + 2^n = 2^{n+1} - 1$

- $\sum_{j=0}^{n} ar^j = a + ar + ar^2 + \dots + ar^n = \frac{ar^{n+1} - a}{r-1}$ where $r \neq 1$.
- $1+2+ \dots + n = \frac{n(n+1)}{2}$
- Proving inequalities, like $n < 2^n$
- $2^n < n!$
- Proving divisibility results, like $n^3 - n$ is divisible by 3.

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- $7^{n+2} + 8^{2n+1}$ is divisible by 57.
- $8^n - 3^n$ is ~~di~~ multiple of 5

- Show that if n is a positive integer, then
$$1 + 2 + \text{---} + n = \frac{n(n+1)}{2}$$

Solution :- Let P(n) be the proposition that the sum of first n positive integers, $1+2+\text{---}+n = \frac{n(n+1)}{2}$ is $\frac{n(n+1)}{2}$. We must do two things to prove that P(n) is true for n = 1, 2, 3, ---.
first, we must show that P(1) is true.
& second, that the conditional statement P(k) implies P(k+1) is true for k = 1, 2, 3, ---.

Basis Step : P(1) is true.

bcz $P(1) = \underset{\text{L.H.S}}{\underset{\downarrow}{1}} = \underset{\text{R.H.S}}{\underset{\downarrow}{\frac{1(1+1)}{2}}} = \underset{\text{RHS}}{\underset{\downarrow}{1}}$

Inductive Step :- For inductive hypothesis, we assume that P(k) holds true for an arbitrary positive integer k. i.e.

$$1+2+3+ \text{---} + k = \frac{k(k+1)}{2} \quad \text{— (1)}$$

Now to check if P(k+1) is also true add (k+1) on both sides of eqn. (1)

$$1+2+3+\text{---}+k+k+1 = \frac{k(k+1)}{2} + (k+1)$$

$1+2+3+ \_\_\_ +k+(k+1) = (k+1)\left(\frac{k}{2}+1\right)$

$= (k+1)\frac{(k+2)}{2}$

$P(k+1) = (k+1)\frac{((k+1)+1)}{2}$

This shows that $P(k+1)$ is true under the assumption that $P(k)$ is true.

By mathematical induction, $1+2+ \_\_ +n = \frac{n(n+1)}{2}$ for $n \in$ set of Positive integers

Q. Use mathematical induction to show that,

$1+2+2^2+ \_\_\_ +2^n = 2^{n+1}-1$

for all non-negative integers n.

Solution: let $P(n) = 1+2+2^2+ \_\_ +2^n = 2^{n+1}-1$

basis step: $P(0)$ is true

as L.H.S. ~~as~~ $2^0 = 1$

R.H.S $2^{0+1}-1 = 2^1-1 = 2-1 = 1$

as L.H.S = R.H.S., $P(0)$ holds true.

Inductive step: Assume $P(k)$ is true.

$1+2+2^2+ \_\_ +2^k = 2^{k+1}-1$

add 2^{k+1} on both sides

$1+2+2^2+ \_\_ +2^k+2^{k+1} = 2^{k+1}-1+2^{k+1}$

$= 2.2^{k+1}-1 = 2^{k+2}-1$

$\Rightarrow P(k+1)$ also holds true if $P(k)$ is true.

Q. Use mathematical Induction to prove the inequality $n < 2^n$ for all positive integers n.

Sol. $P(n) : n < 2^n$

basis Step: $P(1) \Rightarrow 1 < 2^1$ true.

Inductive Step: Suppose $P(k)$ is true $\Rightarrow k < 2^k$

for $P(k+1)$ ~~add~~ i.e. $k+1 < 2^{k+1}$

Add 1 to both sides of $P(k)$

$k+1 < \boxed{2^k + 1}$

$(k+1) < 2^{k+1}$

$= 2^k \cdot 2$

as 1 is less than 2^k

$k+1 < 2^k + 2^k$.

$k+1 < 2 \cdot 2^k \Rightarrow k+1 < 2^{k+1}$

$\Rightarrow P(k+1)$ is true.

Q. Use mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57.

Sol. $P(n)$: $7^{n+2} + 8^{2n+1}$ is divisible by 57.

Basis Step: $P(0)$

$7^{0+2} + 8^{0+1} = 7^2 + 8 = 49 + 8$

$= 57$

$P(0)$ holds true.

Inductive Step: assume $P(k)$ is true.

$57y$ = $7^{k+2} + 8^{2k+1}$ is divisible by 57.

To show $P(k+1)$ $7^{(k+1)+2} + 8^{2(k+1)+1}$ is also divisible by 57.

$7^{(k+1)+2} + 8^{2(k+1)+1} = 7^{k+3} + 8^{2k+3}$

$= 7 \cdot 7^{k+2} + 8^2 \cdot 8^{2k+1}$

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$= 7 \cdot 7^{k+2} + 64 \cdot 8^{2k+1}$

$= 7\underbrace{(7^{k+2} + 8^{2k+1})}_{P(k)} + 57 \cdot 8^{2k+1}$

$= 7 \cdot \underbrace{P(k)}_{\text{it is divisible by 57}} + \underbrace{57 \cdot 8^{2k+1}}_{\text{divisible by 57.}}$

$\Rightarrow 7^{k+3} + 8^{2k+3}$ is divisible by 57.

$\Rightarrow P(k+1)$ is true, if $P(k)$ is true.

Q. Show that $U_n = 3^n - 2^n$ for all $n \in N$ for $n \geq 2$, where $U_1 = 1$ & $U_2 = 5$. & $U_{n+1} = 5U_n - 6U_{n-1}$.

Sol. Induction basis are $P(1)$ & $P(2)$

$P(1) = 3^1 - 2^1 = 1 = U_1$

$P(2) = 3^2 - 2^2 = 9 - 4 = 5 = U_2$

Assume U_n & U_{n-1} are true.

$U_{n+1} = 5U_n - 6U_{n-1}$

$= 5(3^n - 2^n) - 6(3^{n-1} - 2^{n-1})$

$= (5 \times 3^n) - (5 \times 2^n) - (6 \times 3^{n-1}) + (6 \times 2^{n-1})$

$= 5 \times 3 \times 3^{n-1} - 6 \times 3^{n-1} - 5 \times 2 \times 2^{n-1} + 6 \times 2^{n-1}$

$= 3^{n-1}(15 - 6) - 2^{n-1}(10 - 6)$

$= 3^{n-1}(9) - 2^{n-1}(4)$

$= 3^{n-1}(3^2) - 2^{n-1}(2^2)$

$= 3^{n+1} - 2^{n+1}$

Q. Given that $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$, show that $n^3 + 2n$ is divisible by 3 $\forall n \geq 1$.

Relations :- Definition, operations on relations, Properties of Relations, Composite Relations, Equality of Relations, Order of relations, representation of relations, closures of Relations, Functions & relations Relations on a set, Equivalence relations equivalence classes, Partial order relations.

Q. Let R & S be two relations on set of positive integers I and $R = \{(a, 3a) \mid a \in I\}$

$S = \{(a, a+1) \mid a \in I\}$

$R \circ S = \{(a, 3a+1) \mid a \in I\}$

$R \circ R = \{(a, 9a) \mid a \in I\}$

$R \circ R \circ R = \{(a, 27a) \mid a \in I\}$

$R \circ S \circ R = \{(a, 9a+3) \mid a \in I\}$

* A relation R is transitive iff $R^n \subseteq R$ for $n \geq 1$.

Q. Given $A = \{1, 2, 3, 4\}$, Consider the following relation in A:

$R = \{(1,1), (2,2), (2,3), (3,2), (4,2), (4,4)\}$

a) Draw its directed graph.

b) Is R (i) reflexive Not, (ii) Symmetric Not, (iii) transistive Not (iv) antisymmetric. Not

c) Find $R^2 = R \circ R$.

$R^2 = \{(1,1), (2,2), (2,3), (3,2), (4,2), (4,3), (4,4)\}$

Q. Give examples of relations R on $A = \{1, 2, 3\}$, having the stated property.

a) R is both symmetric & antisymmetric.

b) R is neither symmetric nor antisymmetric.

c) R is transitive but $R \cup R'$ is not transitive.

Q. Consider a set $A = \{a, b, c\}$ & the relation R on A defined by.

$R = \{(a,a), (a,b), (b,a), (c,c)\}$

Find a) reflexive (R), b) Symmetric (R) & c) transistive(R)

Sol. a) $\{(a,a), (a,b), (b,c), (b,b), (c,c)\}$

b) $\{(a,a), (a,b), (b,a), (b,c), (c,b), (c,c)\}$

c) $R^2 = \{(a,a), (a,b), (a,c), (b,c), (c,c)\}$

$R3 = \{(a,a)(a,b), (a,c), (b,c), (c,c)\}$

transitive $(R) = R \cup R^2 \cup R^3 = \{(a,a), (a,b), (a,c), (b,$
$(c,c)\}$

Q. $A = \{x \mid x$ is Odd positive number$\}$

$R = \{(a,b) \mid a, b \in A$ & $a - b =$ odd positive nu

Sol. even - odd = odd

odd - even = odd.

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