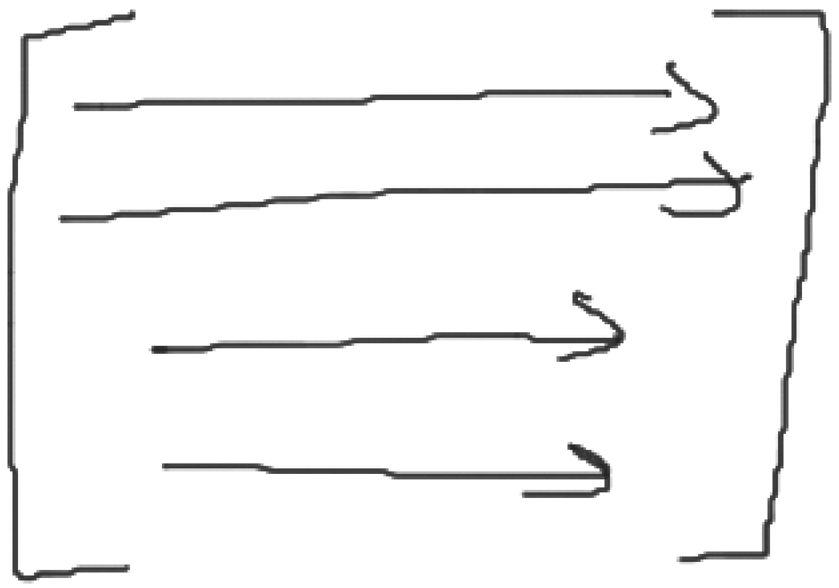
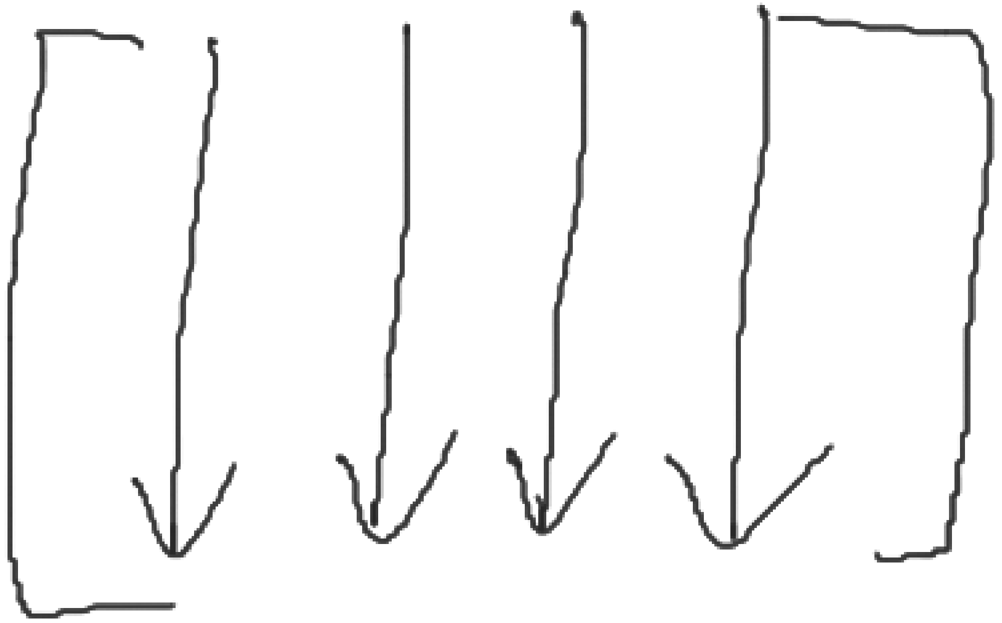
8 Mar 22

Matrices — Abstract mathematical object, which can be interpreted as collection of vectors



Collection of Row vectors



Collection of Column vectors

In CS → a discipline to store related data.

Applications in CS

Image Representation $\begin{bmatrix} \boxed{} & \\ & \end{bmatrix}_{m \times n}$ → Pixel value (Gray | RGB)

m & n determine resolution

Repository of text documents $\begin{bmatrix} & \\ & \end{bmatrix}_{m \times n}$

m – total no. of unique tokens in the repo

n – # docs.

Graphics - Transformation of images

Neural Networks → Weight of connections

Solving system of linear Equations

Graphs - Adjacency matrix

Special Matrices

1. Zero Matrix // Unit Matrix
2. Diagonal Matrix // Scalar matrices // Identity
3. Triangular Matrix (Upper / Lower)

Transpose operation on Matrix

4. Symmetric Matrix ($A = A^T$)
5. Skew-Symmetric Matrix ($A = -A^T$)

Transpose gives alternate view of data

$$
\begin{array}{c|cccc}
\underline{\text{Temp}} & S_1 & S_2 & S_3 & S_4 \\
t_1 & & & & \\
t_2 & & & & \\
\vdots & & & & \\
t_K & & & &
\end{array}
$$

$K \times 4$

Temp readings of sensors S_1 for all times

$$
\begin{array}{c|cccc}
\text{Temp}^T & t_1 & t_2 & \cdots & t_K \\
S_1 & & & & \\
S_2 & & & & \\
S_3 & & & & \\
S_4 & & & &
\end{array}
$$

$4 \times K$

Temp readings of all sensors at time t_1

Rules for Transpose (unary operator)

- $(\mathbf{A}^T)^T = \mathbf{A}$
- $(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
- $\mathbf{c}(\mathbf{A})^T = \mathbf{c}\mathbf{A}^T$
- $(\mathbf{A}\mathbf{B})^T = \mathbf{B}^T\mathbf{A}^T$

Useful for making matrices compatible for multiplication.

If A is the term-document matrix then AA^T and A^TA have interesting interpretations.

Matrix Addition & Scalar Multiplication

- $A + B = B + A$
- $(A + B) + C = A + (B + C)$ (written $A + B + C$)
- $A + 0 = A$
- $A + (-A) = 0$, here 0 denotes the zero matrix (of size m × n), that is, the m × n matrix with all entries zero.
- $c(A + B) = cA + cB$
- $(c + k)A = cA + kA$
- $c(kA) = (ck)A$ (written ckA)
- $1A = A$

Matrix Multiplication — Compatible

NOT Commutative

- $(k\mathbf{A})\mathbf{B} = k(\mathbf{AB}) = \mathbf{A}(k\mathbf{B})$ **written kAB or Ak**
- $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$ **written ABC** (associative law)
- $(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$ (distributive law)
- $\mathbf{C}(\mathbf{A} + \mathbf{B}) = \mathbf{CA} + \mathbf{CB}$ (distributive law)

Equality of a Matrix

Inverse of a Matrix

$A * A^{-1} = I$

Inverse of a matrix is unique

Let B be inverse of matrix A.

$AB = BA = I$ — (1)

Suppose C is also the inverse of A

$AC = CA = I$ — (2)

Multiply (1) by C $\quad BAC = IC = C$

But $BAC = B \cdot I$ (using 2)

Thus $BAC = B = C$ or $B = C$

i.e. Inverse is unique

Transpose of inverse is the inverse of transpose.

$$(A^{-1})^{T} = (A^{T})^{-1}$$

$I = AA^{-1} = A^{-1}A$ & $I^T = I$

$(AA^{-1})^T = (A^{-1})^T \cdot A^T$ — 1

$(A^{-1}A)^T = A^T (A^{-1})^T$ — 2

Equate 1 & 2

Matrices are used as Transformations

$$\underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{n\times n} \underbrace{\begin{bmatrix} \quad \\ \quad \end{bmatrix}}_{n\times 1} = \underbrace{\begin{bmatrix} \quad \\ \quad \end{bmatrix}}_{n\times n} \longrightarrow \text{transformed Vector}$$

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \longrightarrow \text{Rotation matrix}$$

for $\theta = 90, 180, 270$

Rotation in 3D

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Stretching

$$\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$$

Shearing

$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$$

Matrix multiplication is transformation of a matrix

$A \times B = \underline{C}$ Matrix B

↳ Transformed to C