Department of Computer Science
University of Delhi
Master of Computer Application
MCAC 103: Mathematical Techniques for Computer Science Applications
UPC: 223401103
Semester I - March-2022
Year of Admission: 2021

Max marks: 70
Max time: 3 Hours + 1 Hour for Scanning and Uploading answer scripts

Note1: Attempt four questions. All questions carry equal marks.
Note2: For full credit use proper notation, follow systematic approach towards the problem and show all intermediate steps neatly.

1. (a) Solve the following system of linear equations using Gauss elimination method. (6)
$$3x + 4y - z = -6$$
$$-2y + 10z = -8$$
$$4y - 2z = -2$$

   (b) Find the eigenvalues and eigenvectors for the given matrix. (7.5)
$$\begin{pmatrix} 7/2 & 0 & 3 \\ -3/2 & 2 & -3 \\ -3/2 & 0 & -1 \end{pmatrix}$$

   (c) Check if the following vectors in $\mathbf{R}^3$ are linearly independent. (4)
   $v_1 = (0\,2\,1), v_2 = (2\,2\,1), v_3 = (1\,2\,3)$

2. (a) A biased coin is flipped four times with probability of H as 0.3. Let X be the number of H. Find the probability mass function of X. Find E[X] and Var[X]. Find the cumulative distribution function of X and plot it neatly. (10)

   (b) Inquiries to an on-line computer system arrive on five communication lines. The percentage of messages received from lines 1, 2, 3, 4, 5, are 20, 30, 10, 15, and 25, respectively. The corresponding probabilities that the length of an inquiry will exceed 100 characters are 0.4, 0.6, 0.2, 0.8, and 0.9. What is the probability that a randomly selected inquiry will be longer than 100 characters? (3)

   (c) Consider the joint distribution over two discrete variables X and Y, which take on possible values $\{x_1, ..., x_l\}$ and $\{y_1, ..., y_m\}$ respectively.
   i. In general, how many numbers are required to specify the joint probability mass function $p(x, y) = P(X = x, Y = y)$?
   ii. Suppose we are given the table of values for $p(x_i, y_j)$. Write down an equation that specifies the computation of marginal distribution $p(x)$ from the table.
   iii. How to calculate $p(y|x)$ from the joint pmf given as the table. (4.5)

3. (a) The probability that an "accident prone" policy holder has an accident within a year is 0.8, while the probability that a "non-accident prone" policyholder has an accident within a year is 0.2. Assume that 30% of the policyholders are "accident prone". If a policyholder had an accident, what is the probability that this policyholder is "non-accident prone"? (4)

   (b) Suppose it has been determined that the average rate of inquiries that arrive per second at the reservation counter is 10 per second. What is the probability that no inquiries arrive in a one second period? What is the probability that exactly 10 inquiries arrive in a one-second period? More than 10 inquiries arrive ?(4.5)

   (c) Scores on an examination are normally distributed with mean 78 and standard deviation 6. What is the probability that a student's score exceeds 84, given that it exceeds 72? (4)

   (d) Suppose you are taking a multiple-choice test with c choices for each question. In answering a question on this test, the probability that you know the answer is p. If you don't know the answer, you choose one at random. What is the probability that you knew the answer to a question, given that you answered it correctly? (5)

4. (a) Of all the foodies in a particular district, 60% prefer Asian (A) cusine and 40% prefer Italian (B). Of those, who prefer A, 20% are females, and of those who prefer B, 10% are female. What is the probability that a randomly selected person prefers A, given that the person selected is a female? (3)

   (b) Let C and D be two events with $P(C) = 0.25, P(D) = 0.45$, and $P(C \cap D) = 0.1$. What is $P(C' \cap D)$? (2)

   (c) Two dice are rolled simultaneously. Let A and B be two events defined as follows. $A = Sum - is - 3, B = First - dice - lands - 2$. Are the events independent? (2)

   (d) A study of the response time of a certain computer system yields that the response time in seconds has an exponentially distributed time with parameter 0.25. What is the probability that the response time exceeds 5 seconds? (4)

   (e) Find the determinant of the following matrix.
   $$\begin{pmatrix} 7 & 2 & 0 & 1 \\ 0 & 0 & 6 & 1 \\ 1 & 2 & 3 & 1 \\ 3 & 5 & 4 & 0 \end{pmatrix}$$ (6.5)

5. (a) The pdf of a random variable X is given as follows.
   $f(x) = \frac{a}{x^2}$ for $1 \le x \le 2$, and 0 elsewhere.
   Find $a, E[X], Var[X]$. Find the probability that X will take a value within two standard deviations of the mean and compare it with Chebychev's bound. (10)

(b) Let A and B be two events. Suppose the probability that neither A or B occurs is 2/3. What is the probability that one or both occur?. (2)

(c) Find the value of $\cos\theta$ for vectors $v = (3, 5, 6)$ and $w = (1, 6, 8)$, and check Cauchy-Schwarz and Triangle Inequalities for v and w. (5.5)

6. (a) Consider a roullete wheel consisting of 50 numbers 1 through 50, 0, and 00. If you always bet that the outcome will be one of the numbers 1 through 20, what is the probability that (a) you will lose first 7 bets, (b) first win will occur on 9th attempt? (6)

(b) In a recent production, 15% of certain electronic components are defective. We need to find 5 non-defective components. What is the probability that more than 15 components will have to be tested? (3)

(c) Given the following matrix, find its rank and inverse. (8.5)

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$