VECTORS

Why should computer scientists study?

- use it as data type to represent multiple attributes of an object.
- Flexible interpretation & representation

class marks.

→ vector of marks for a student

→ vector of marks for a subject

Vector is an ordered list of numbers.

Integers | Reals | Strings?

List of size 2 → vector in 2-Dimension

List of size 3 → Vector in 3-D

List of size n → Vector in n-D

Geometrical Interpretation

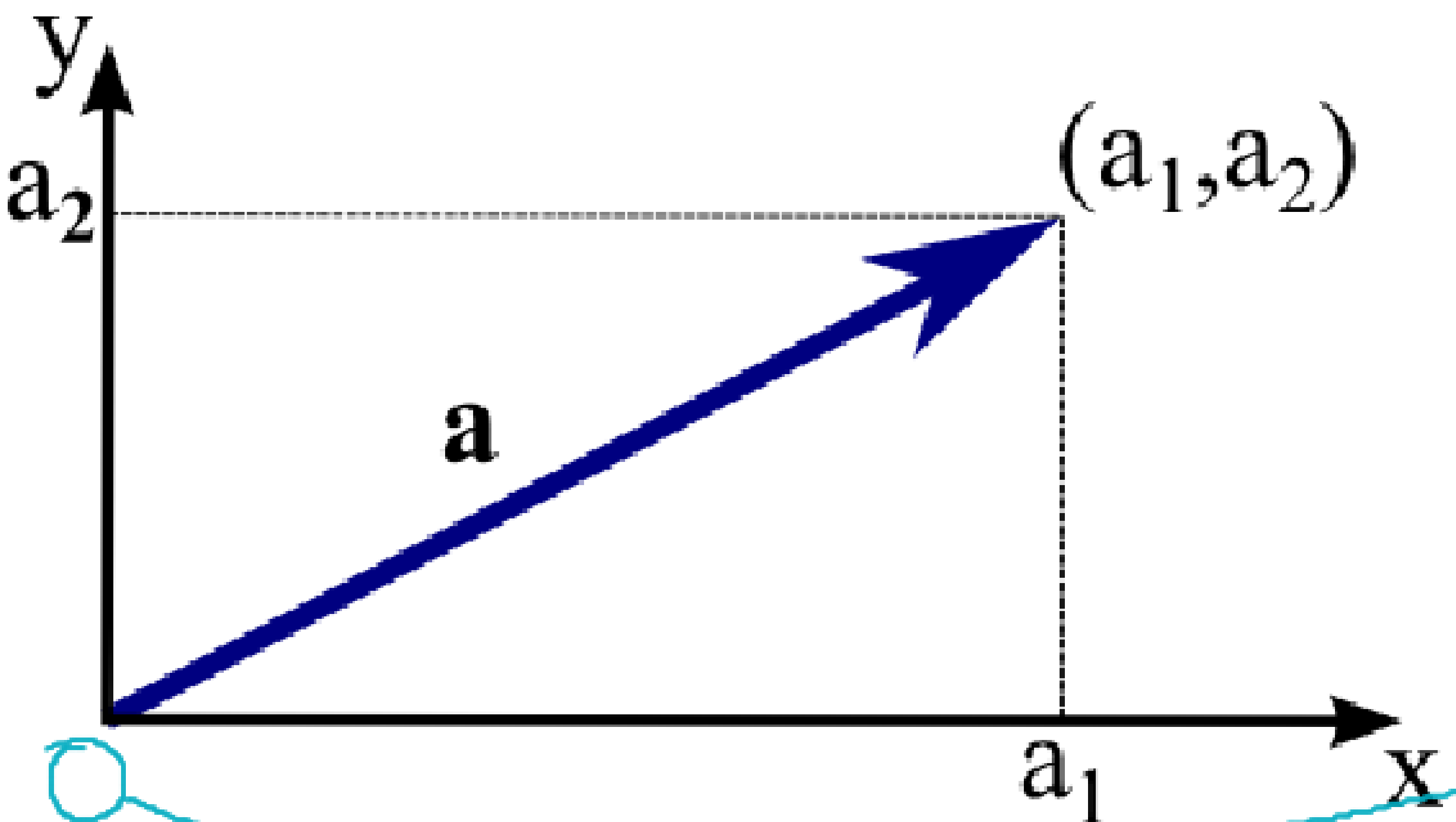
Vector of size 2 → point in 2D space

Vector of size 3 → point in 3D space

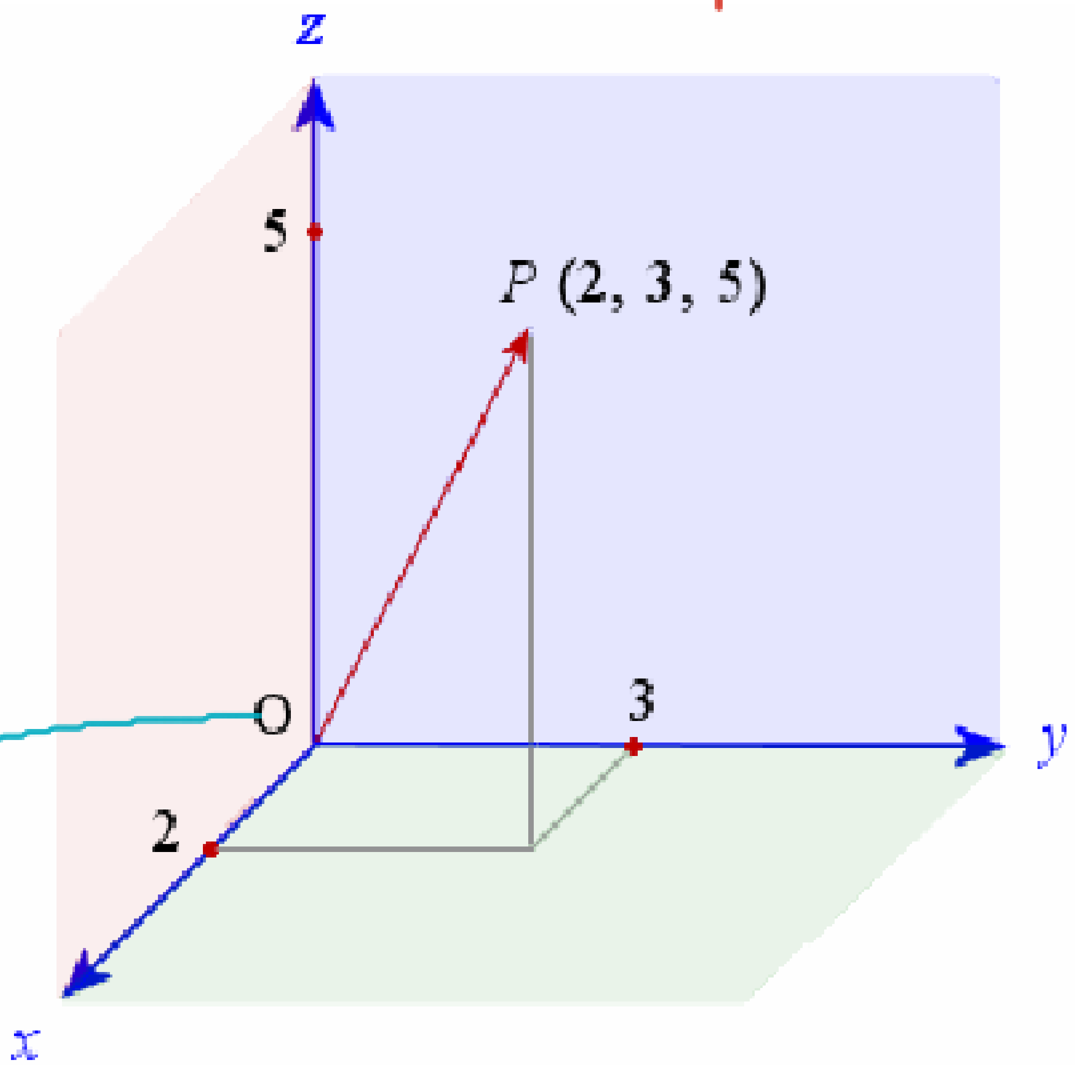
Row Vector Vs Column Vector

Vector has magnitude & direction

$\vec{a}$



$\vec{p}$



Origin is the reference point

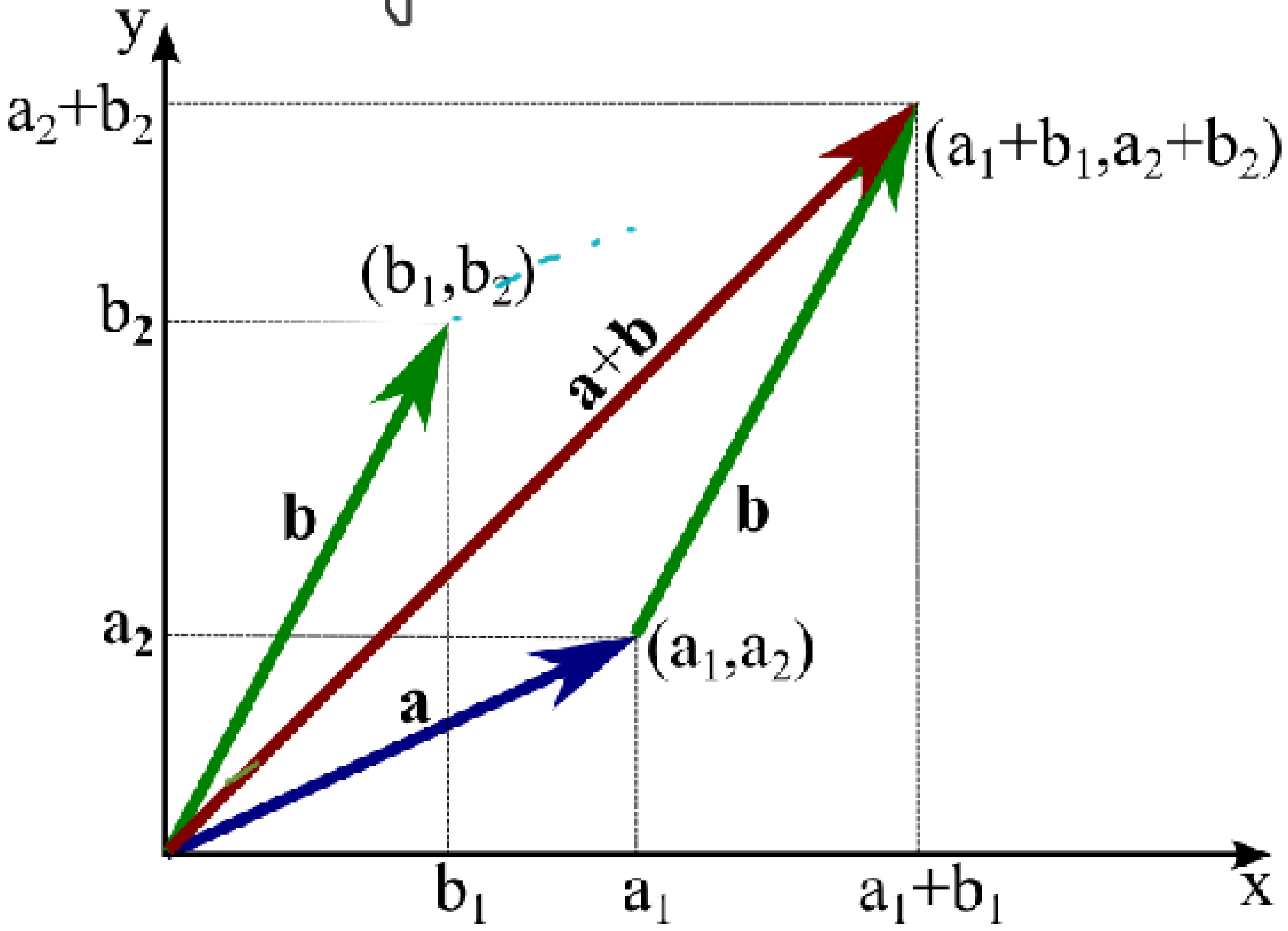
Higher dimensions?

- Abstract representation of an object with n attributes
- Origin $(\underbrace{0, 0, \ldots\ 0}_{n})$ — n dimensional coordinate system

Algebra (set of operations) for vectors.

1) Vector Addition

(geometrically in 2Dim)



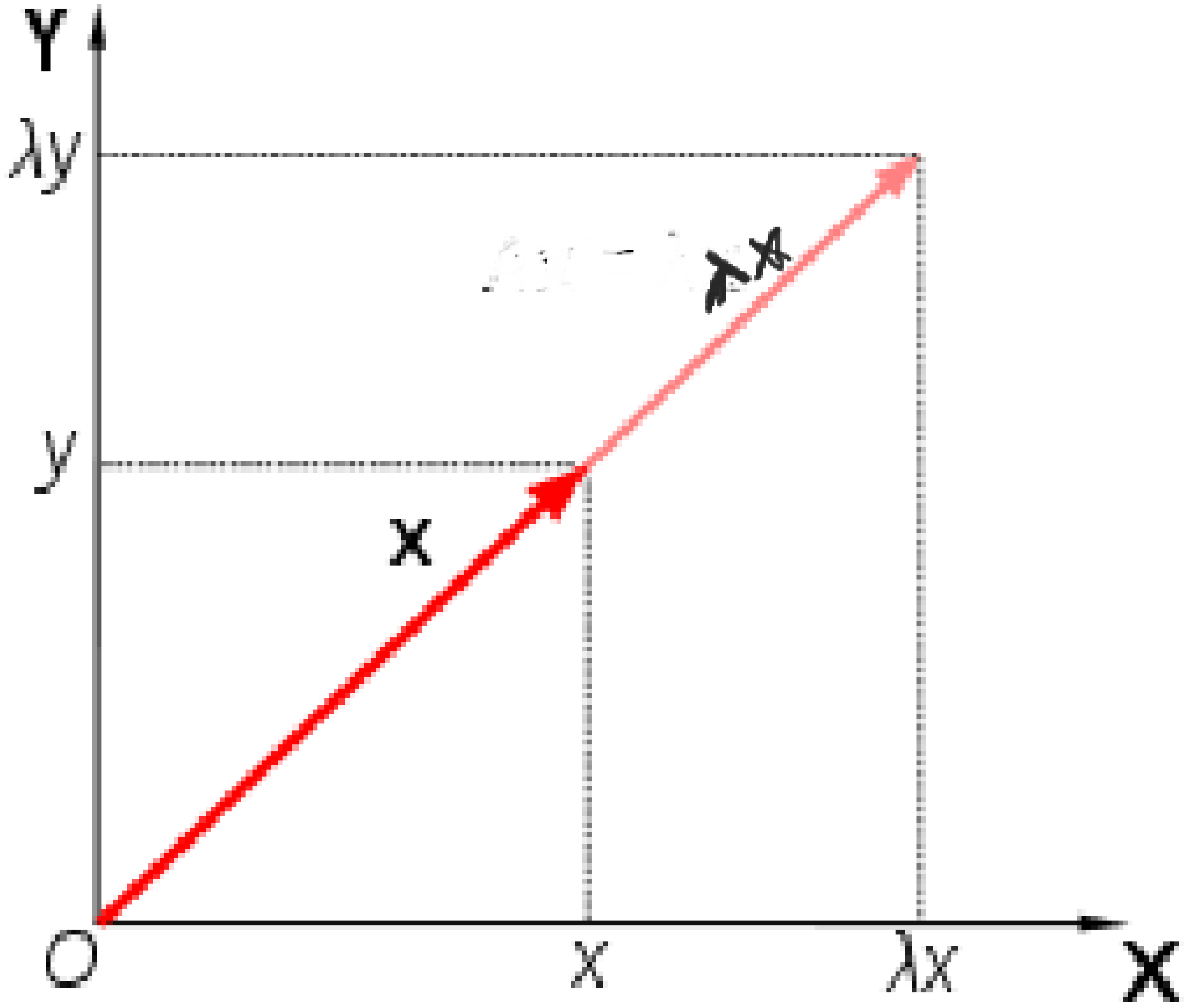
(in n dimensions)

$$\vec{A} + \vec{B} = \overrightarrow{A+B}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} a_1+b_1 \\ \\ \\ a_n+b_n \end{pmatrix}$$

Scaling of Vectors

→ No change in the direction
→ Magnitude changes



If $\lambda < 1$?
$\lambda < 0$?

Properties of Vector addition an scalar multiplication

- $\bar{a}+\bar{b}=\bar{b}+\bar{a}$ vector addition is commutative
- $\bar{a}+(\bar{b}+\bar{c})=(\bar{a}+\bar{b})+\bar{c}$ vector addition is associative
- $\bar{a}+\bar{0}=\bar{a}$
- $\bar{a}+(-\bar{a})=\bar{0}$
- $(k_1+k_2)\bar{a}=k_1\bar{a}+k_2\bar{a}$
- $k(\bar{a}+\bar{b})=k\bar{a}+k\bar{b}$

Vector Multiplication

1. Hadamard Product – element by element

Applications

$$\begin{pmatrix} \\ \\ \end{pmatrix} \circledast \begin{pmatrix} \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

→ Total cost of each item

→ Cost/unit

→ Quantity of items

2. Inner Product $\vec{a} \cdot \vec{b}$ (compatibility)

$$\begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix} \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \sum_{i=1}^{n} a_i b_i$$

Also called dot product

Related to angle between two vectors

$$\vec{a} \cdot \vec{b} = \|a\| \, \|b\| \cos\theta$$

where $\|a\|$ is the magnitude of $\vec{a}$ $\left(\sqrt{\sum a_i^2}\right)$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|a\|\,\|b\|}$$

What if $\vec{a}\,\vec{b} = 0$

$(3, -5)\ (-5, 3)$

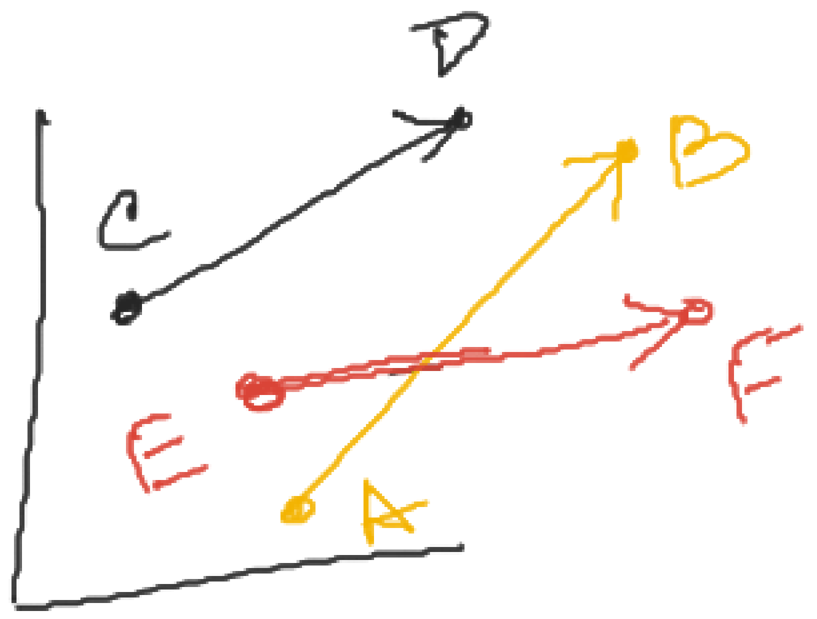
$(2, 2, -6)\ (3, 3\ 2)$

Application of Dot Product

Numerous in physics, chemistry, engineering

Extensively used in Data Science - (N.N, similarity)

Located Vector — Vector with end point NOT at origin



$\overrightarrow{AB}$ $\overrightarrow{CD}$ $\overrightarrow{EF}$

Length (Magnitude) of a located vector

$$\|AB\| = \sqrt{(A-B)(A-B)}$$

Parallel Vectors $\overrightarrow{AB}$ & $\overrightarrow{PQ}$ are parallel if

$$(A-B) = c(P-Q) \ \& \ c \neq 0$$

If A & B are perpendicular, then

$$\|A+B\|^2 = \|A\|^2 + \|B\|^2 \quad \text{(Why?)}$$

Normalized vector.

$$\hat{A} = \frac{A}{\|A\|} = \frac{A}{\sqrt{\sum a_i^2}} = \left\langle \frac{a_1}{\sqrt{\sum a_i^2}}, \frac{a_2}{\sqrt{a_i^2}}, \cdots \frac{a_n}{\sqrt{a_i^2}} \right\rangle$$

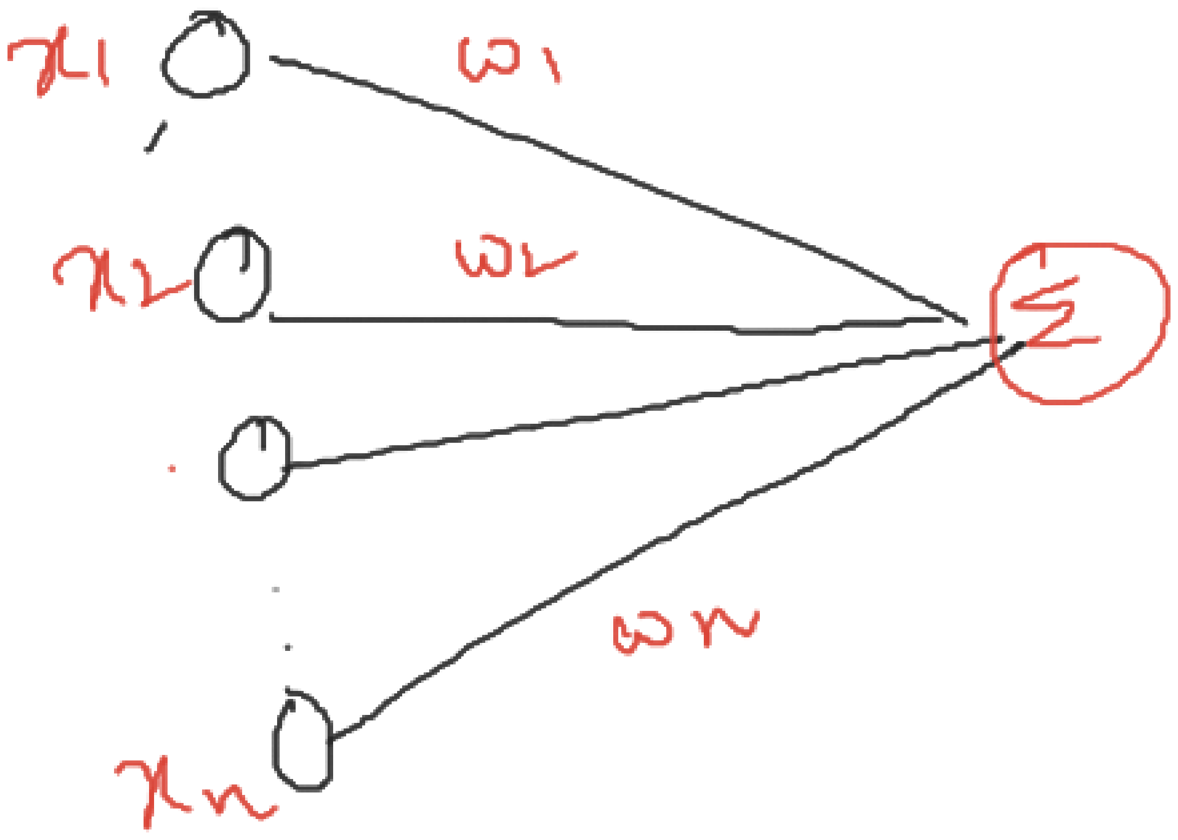
Orthonormal vectors

Perpendicular and normalized.

CHECK – What is $\|\hat{A}\|$?

Application of dot product
inner

① Neural Network



Fire if

$\sum x_i w_i > \delta$

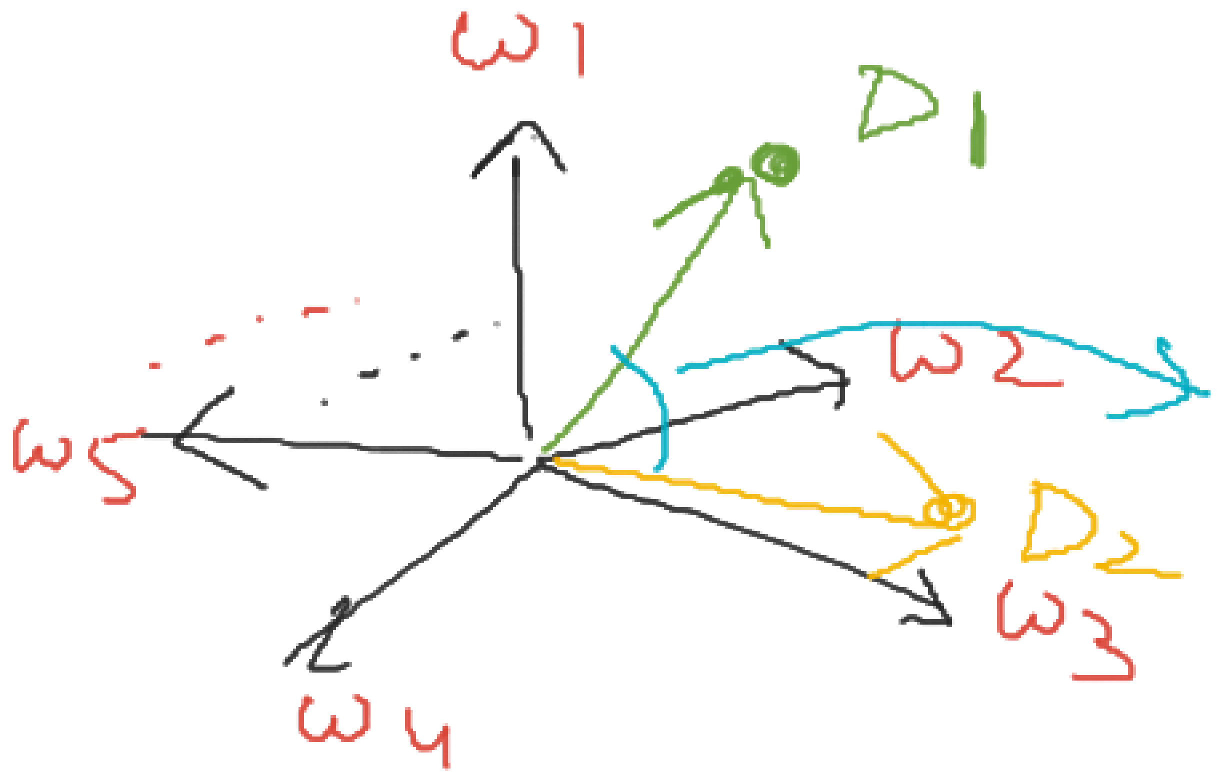
$(\quad X \quad)\,(W)$

2. Similarity computation in Documents.

Each word is a dimension

Each doc. is a vector

Similarity between the documents D_1 & D_2 is $\dfrac{D_1 \cdot D_2}{\|D_1\| \, \|D_2\|}$



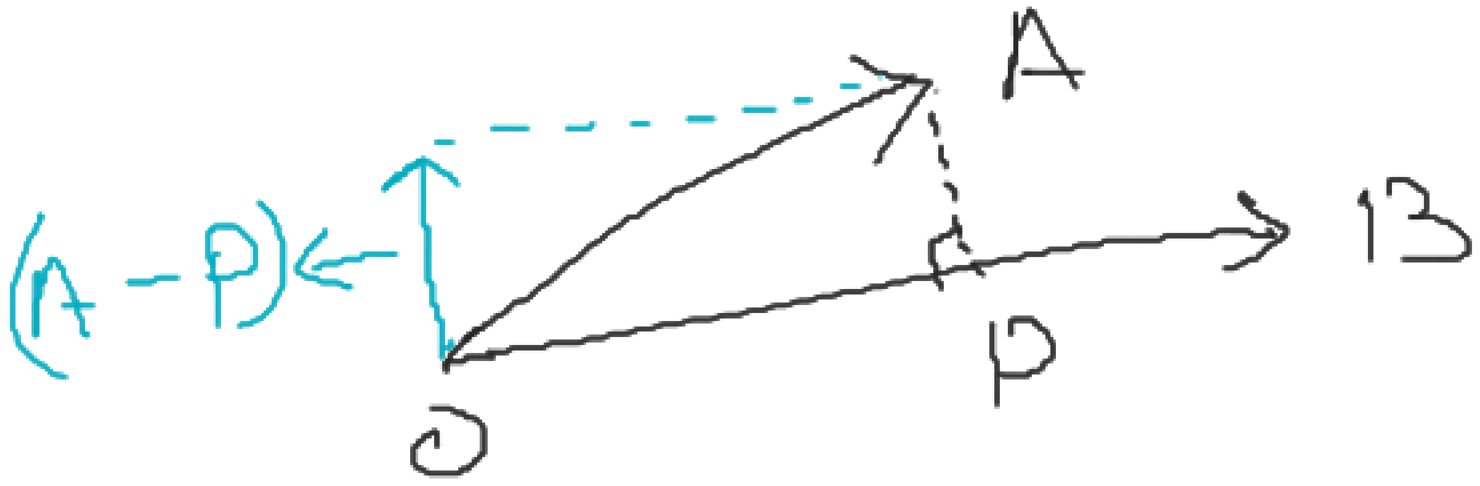
$\theta = \cos^{-1}\left(\dfrac{D_1 \cdot D_2}{\|D_1\| \, \|D_2\|} \right)$

similarity

7 Marks Outer Product of Vectors. $\vec{a}$ and $\vec{b}$

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \begin{pmatrix} b_1 & b_2 & b_n \end{pmatrix} = \begin{pmatrix} a_1b_1 & a_1b_2 & a_1b_n \\ a_2b_1 & a_2b_2 & \cdots a_2b_n \\ \vdots & & \\ a_nb_1 & \cdots & a_nb_n \end{pmatrix}$$

Projection of a vector



P is projection of A on B.

$P = cB.$

$(A-P) \perp B$

$(A-P) \cdot B = 0$

$(A - cB) B = 0$

$$c = \frac{A \cdot B}{B \cdot B}.$$

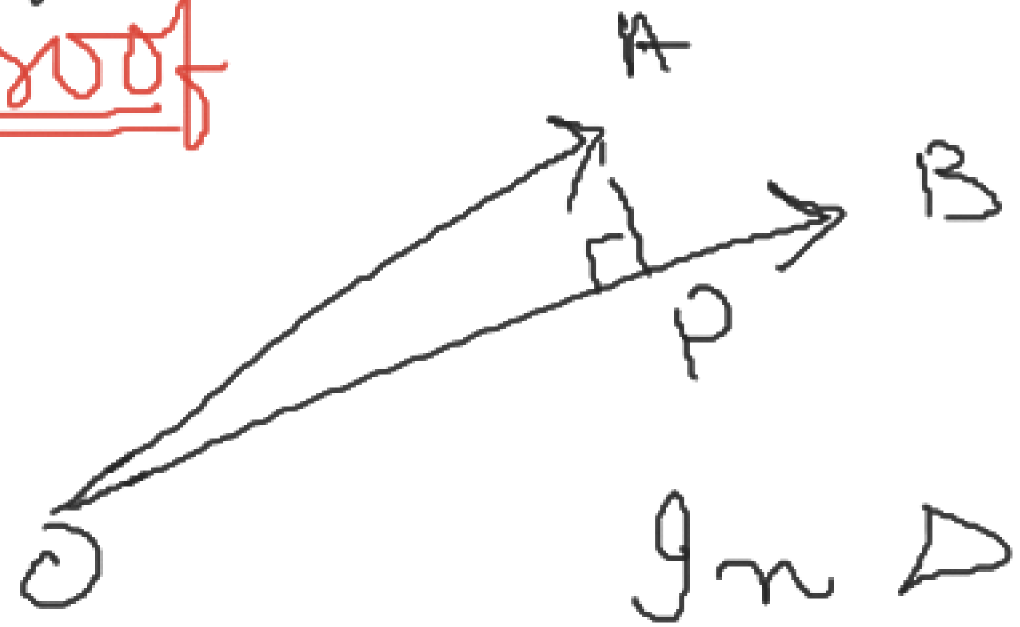
Show that

$$\cos\theta = \frac{A \cdot B}{\|A\| \cdot \|B\|}.$$

Cauchy-Schwartz Inequality

If A & B are two vectors then $|A\cdot B| \le \|A\|\cdot\|B\|$

Proof



$P = cB$ where $c = \dfrac{A\cdot B}{B\cdot B}$

In $\triangle OAP$, $OA^2 = OP^2 + AP^2$

$$\|A\|^2 = \|cB\|^2 + \|A - cB\|^2$$

$$\|A\|^2 = c^2\|B\|^2 + \|A - cB\|^2$$

$$c^2\|B\|^2 \le \|A\|^2$$

$$\left(\frac{A\cdot B}{B\cdot B}\right)^2 \|B\|^2 \le \|A\|^2$$

(geometric)

$$\frac{|A\cdot B|^2}{\|B\|^4}\cdot\|B\|^2 \leq \|A\|^2$$

$$|A\cdot B|^2 \leq \|A\|^2\|B\|^2$$ (Equality arises when?)

Intuitive?

$$A\cdot B = \|A\|\,\|B\|\cos\theta$$

In words?

$$E|X\cdot Y| \leq \sqrt{E(X^2)\,E(Y^2)}$$

Bounds expected values which are difficult to compute

Triangle Inequality

$$\|A+B\| \leq \|A\| + \|B\|$$

Proof

$$\|A+B\|^2 \leq (\|A\| + \|B\|)^2$$

$$(A+B)(A+B) \leq (\|A\| + \|B\|)^2$$

Consider LHS

$$= A\cdot A + B\cdot B + 2A\cdot B$$

$$\|A\|^2 + \|B\|^2 + 2A\cdot B. \quad \text{— (1)}$$

From Cauchy Schwartz inequality

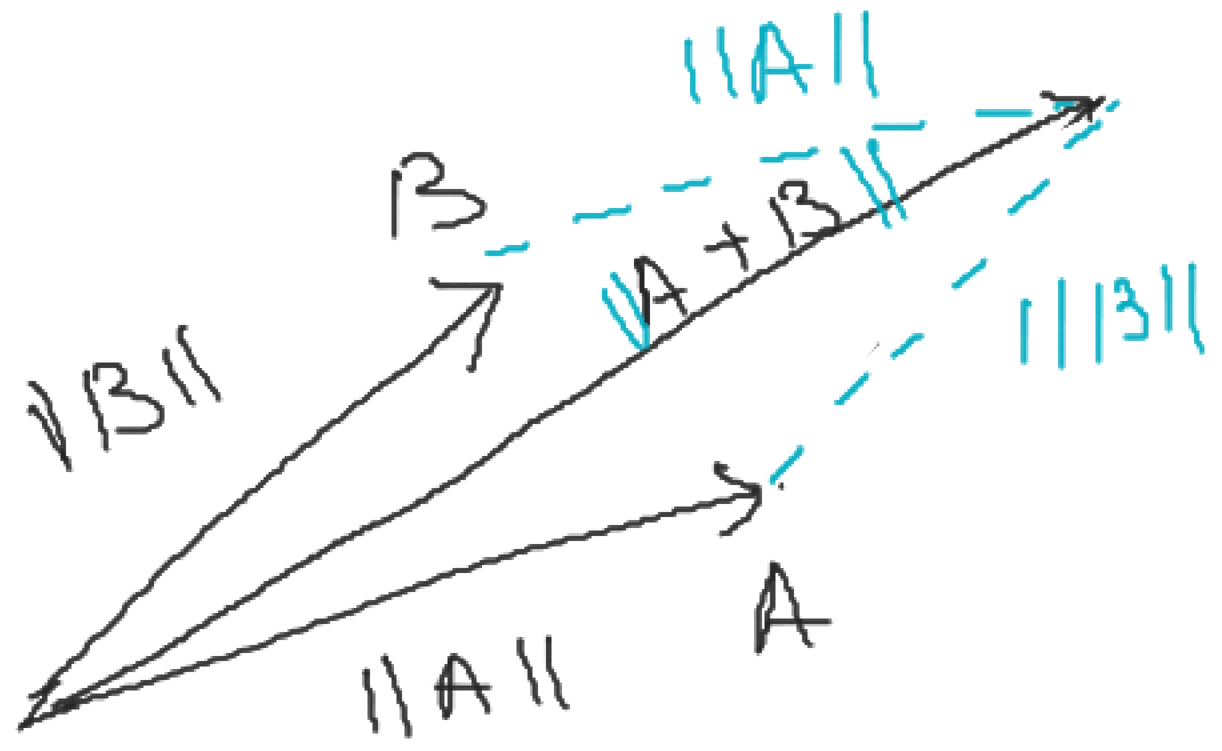
$$|A\cdot B| \leq \|A\|\,\|B\| \quad \text{— (2)}$$

Substitute (2) in (1)

$$(A+B)\cdot(A+B) \leq \|A\|^2 + \|B\|^2 + 2\|A\|\|B\|$$

$$\|A+B\| \leq \|A\| + \|B\|$$

Why $\boxed{\text{triangle}}$ inequality?



||A||
B
	A + B	
	B	
	B	
A
	A	