Class Test-1

MCAC102: Discrete Mathematics

Date: 29/12/21   Time: 1:20-3:00(Wednesday)

All the questions are compulsory.

1. Simplified the given statement in Exclusive OR and Exclusive AND operator form:

$$(\sim P \leftrightarrow \sim Q) \leftrightarrow (P \leftrightarrow Q)$$ Note: ~ sign is used for negation

2. Show that $\sim (m \oplus n)$ and $m \leftrightarrow n$ are logically equivalent.

3. Determine whether $(\sim P \wedge (P \rightarrow Q)) \rightarrow \sim Q$ is a tautology.

4. Determine whether $(\sim Q \wedge (P \rightarrow Q)) \rightarrow \sim P$ is a tautology.

5. Let $R(a, b)$ be the statements "$a + b = a - b$". The domain in each case consists of all integers, what will the truth values for given predicates?

   I. $\exists a \exists b R(a, b)$

   II. $\exists b \forall b R(a, b)$

   III. $\forall a \exists b R(a, b)$

   IV. $\forall a \forall b R(a, b)$

6. Find the truth values for the given statements if domain consists of all integers.

   I. $\exists x \forall y (x < y^2)$

   II. $\forall x \exists y (x + y = 0)$

   III. $\exists x \forall y (xy = y)$

7. Translate the nested quantifications into English sentences that express the mathematical fact.

   The domain for all the variables consists of all real integers.

I. $\forall m \forall n(((m < 0) \land (n < 0) \rightarrow (mn > 0))$

II. $\exists m \exists n((m^2 > n) \land (m < n))$

8. Rewrite the given statements so that negations appear only within predicates.

I. $\sim \forall n \forall m(P(m,n) \lor Q(m,n))$

II. $\sim (\exists m \exists n \sim (P(m,n) \land \forall m \forall n \, Q(m,n))$