Digital image:

- An image may be defined as a two-dimensional function, $f(x,y)$ where x and y are spatial (plane) coordinate.
- The amplitude of f at any pair of coordinates (x,y) called **intensity** or gray level of the image at that point.
- When x, y and the intensity values of f are all finite, discrete quantities we call the image a **digital image**.

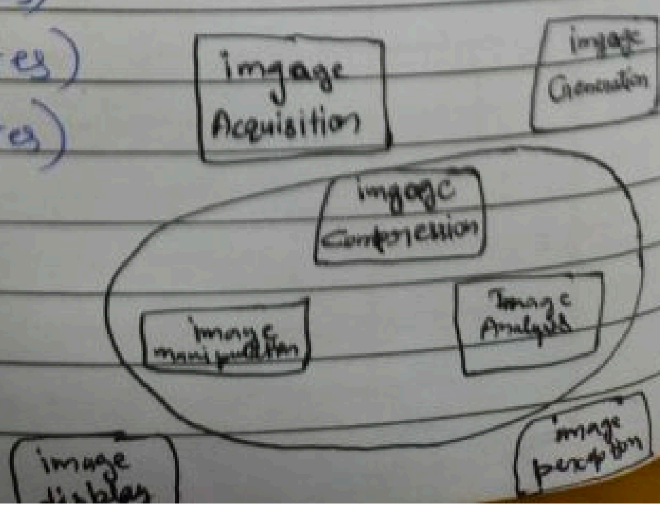
⇒ Elements of digital image:

- picture elements
- image elements
- pels
- pixels

⇒ Digital storage for image processing -

- short-term storage for use during processing.
- on-line storage for relatively fast recall.
- archival storage, characterized by infrequent access.

→ bytes (8 bits)
→ Kbytes (10^3 bytes)
→ Mbytes (10^6 bytes)
→ Gbytes (10^9 bytes)
→ Tbytes (10^{12} bytes)



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⇒ Simple image formation model-

- Let the intensity (gray level) of a monochrome image at any coordinates (x,y) -

$$l = f(x, y)$$

and l lies in the range-

$$L_{min} \leq l \leq L_{max}$$

here the requirement on L_{min} is that it be nonnegative, and on L_{max} that it be finite

- $L_{min} = i_{min} r_{min}$ $\left\{ i = \text{Illumination} \right\}$
  $L_{max} = i_{max} r_{max}$ $\left\{ r = \text{Reflactance} \right\}$
- The interval $[L_{min}, L_{max}]$ is called
- intensity (or gray) scale
- Common practice is to shift this interval numerically to the interval $[0, 1]$ or $[0, C]$ where, $l = 0$ is considered black,
  $l = C$ or 1 is considered white on scale

All the intermediate values are shades of gray varying from black to white.

—— x —— x —— x —— x ——

- Our objective is to generate digital image from sensed data. To create a digital image, we need to convert the continuous sensed data into a digital format.
- this required two processes-

(i) sampling (ii) quantization.

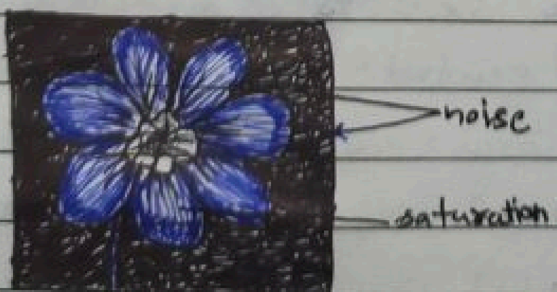
- An image may be continuous with respect to the x- and y-coordinate and also in amplitude.
- To digitize it, we have to sample the function in both coordinates and also in amplitude.
- Digitizing the coordinate values is called as sampling.
- Digitizing the amplitude value is called as quantization.

→ The coordinates of the image center are -

$$(x_c, y_c) = \left(\text{floor}\left(\frac{x}{2}\right), \text{floor}\left(\frac{y}{2}\right)\right)$$

—x————x————x————x—

→ Saturation is the highest value beyond which all intensity value are clipped.
- the entire saturated area has a high, constant intensity level.)



- Visible noise in this case appears as a grainy texture pattern. The dark background is noisier, but the noise is difficult to see.

⇒ dynamic range of an imaging system = $\frac{\text{maximum measurable intensity}}{\text{maximum detectable intensity}}$.

→ dynamic range establishes the lowest and highest itensity levels that a system can represent and consequently that an image can have.

⇒ image contrast = highest intensity level − lowest intensity level.

⇒ contrast ratio = $\frac{\text{highest intensity}}{\text{lowest intensity}}$

⇒ Bits required to store a image (digital):

$$b = M * N * k$$

$$b = M^2 k \quad \{\because M = N\}$$

• 256-level image is called an 8-bit image.

Linear vs Coordinate indexing:

• When the location of a pixel is given by its 2-D coordinates, it is refered as coordinate indexing or subscript indexing. while Linear indexing consists of a 1-D string of nonnegative integers based on computing offsets from coordinates (0,0).

LI • row scan

LI • Column scan

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Spatial resolution is a measure of the smallest discernible detail in an image.

discernible → प्रत्यक्ष [जो दिख रहा]

- line pairs per unit distance.
- dots (pixels) per unit distance

Ex newspaper = 75 dpi
magazines = 133 dpi
brochures = 175 dpi

dots per inch

Intensity resolution refers to the smallest discernible change in intensity level.

- Number of bits used to quantize intensity is known as the "Intensity resolution".

X ———— X ———— X ————

Interpolation: is used in tasks such as zooming, shrinking, rotating and geometrically correcting digital images.

- Interpolation is the process of using known data to estimate values at unknown locations.

★. Nearest neighbour interpolation: assign to each new location the intensity of its nearest neighbour in the original image.

★ Bilinear interpolation: uses four nearest neighbour to estimate the intensity at a given location.

$$v(x,y) = ax + by + cxy + d$$

★ Bicubic interpolation: uses sixteen nearest neighbours of a point. The intensity value assigned to point (x,y) is obtained using the equation-

$$v(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

Ex: Adobe photoshop, Corel photopoint.

→ bilinear gives much better result than nearest neighbour interpolation.

Neighbour of a pixel :-

- A pixel p at (x,y) has two vertical and two horizontal neighbours
  $(x+1, y), (x-1, y), (x, y+1), (x, y-1)$.
- This set of pixels are called 4-neighbour of p, denoted as $N_4(p)$.
- The four diagonal neighbours of p:
  $(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$.
- This denoted as $N_D(p)$. here together with the 4-neighbour are called the 8-neighbours of p, denoted as $N_8(p)$.

Adjacency : (पड़ोसी) 3 types -

- 4-adjacency : two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$
- 8-adjacency : two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.
- m-adjacency : (mixed adjacency) : two pixels p and q with values from V are m-adjacent if:
  (a) q is in $N_4(p)$ or
  (b) q is in $N_D(p)$ and the set $N_4(p)$ & $N_4(q)$ has no pixels whose values are from V.

digital path from pixel p (x_0, y_0) to pixel q (x_n, y_n) -
$(x_0, y_0), (x_1, y_1) \cdots\cdots (x_n, y_n)$.

- If $(x_0, y_0) = (x_n, y_n)$ then the path is closed path.

⇒ Let S represents the subset of pixels that are connected to it in S is call connected component of S.

- If it only has one component, and that component is connected, then S is called connected set.

* Let R represents a subset of pixels in an image. here R stands for Region.
- Two regions Ri & Rj are said to be adjacent if their union forms a connected set. Regions that are not adjacent are said to be disjoint.

Distance Measures :-

$D(p, q) = 0$ iff $p = q$.

- Euclidean distance b/w p & q -

$$D_e(p, q) = \left[(x-u)^2 + (y-v)^2\right]^{1/2}$$

if $p = (x, y)$ & $q = (u, v)$.

- city-block distance -

$$D_4(p, q) = |x-u| + |y-v|$$

- chessboard distance -

$$D_8(p, q) = \max(|x-u|, |y-v|)$$

Linear Operator.

→ Consider a general operator $\mathcal{H}$ generates an output $g(x, y)$ by input $f(x, y)$.

$$\mathcal{H}[f(x, y)] = g(x, y)$$

- $\mathcal{H}$ is the sum operator, Σ.

$$\mathcal{H}[a f_1(x, y) + b f_2(x, y)] = a \mathcal{H}[f_1(x, y)] + b\mathcal{H}[f_2(x, y)]$$
$$= a g_1(x, y) + b g_2(x, y).$$

$$\Rightarrow \Sigma[a f_1(x, y) + b f_2(x, y)] = \Sigma a f_1(x, y) + \Sigma b f_2(x, y)$$

$= a\Sigma f_1(x,y) + b\Sigma f_2(x,y)$

$= a g_1(x,y) + b g_2(x,y)$

Addition of Noisy images for reduction Noise reduction:

- Noisless image = $f(x,y)$
- Noice = $\eta(x,y)$
- Carrupted image = $g(x,y)$

Now,

$$g(x,y) = f(x,y) + \eta(x,y)$$

Shading correction: Image multiplication or division.

- Uses in masking, also called region of interest (ROI) operations.

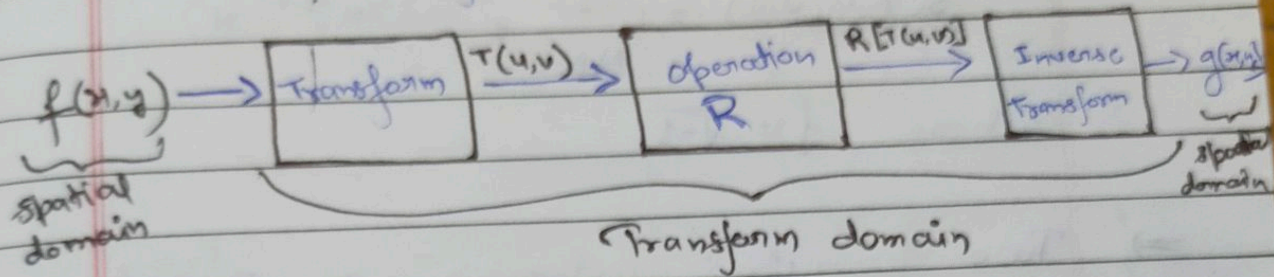
Image registration: is an application of digital image processing used to **align** two or more images of the same scene.

- we have available an input image and a reference image.

Image Transforms:

All the images operates directly on the pixels of input images; they work directly in spatial domain (भी शुरू का है)

- Some cases image processing done by transform domain; and applying the inverse transform to return the spatial domain



- 2-D linear transform-

$$T(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, r(x,y,u,v) \quad \text{—— (i)}$$

$f(x,y)$ = input image

$r(x,y,u,v)$ = forward transformation

$u = 0, 1, 2, \ldots, M-1$ (M = row)

$v = 0, 1, 2, \ldots, N-1$ (N = column)

- recover $f(x,y)$ by inverse transform of $T(u,v)$:-

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u,v)\, s(x,y,u,v) \quad \text{—— (ii)}$$

$x = 0, 1, 2, \ldots, M-1$

$y = 0, 1, 2, \ldots, N-1$

$s(x,y,u,v)$ = inverse transformation kernel.

- together (i) & (ii) is called transform pair.

Fourier Transform:

- forward kernel-

$$r(x,y,u,v) = e^{-j2\pi(ux/M + vy/N)}$$

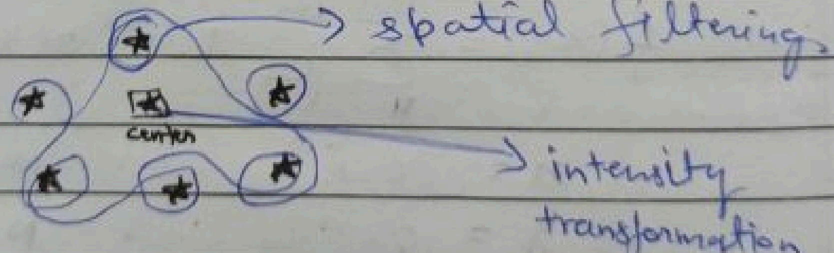
- inverse kernel-

$$s(x,y,u,v) = \frac{1}{MN} e^{j2\pi(ux/M + vy/N)}$$

where $j = \sqrt{-1}$.

=> Two principal categories of spatial processing are intensity transformation and spatial filtering.

- Intensity transformation operate on a single pixels of an image for task such as contrast manipulation and image thresholding.
- Spatial filtering performs operations on the neighborhood of every pixel in an image. Ex: image smoothing & sharpening



$$g(x,y) = T[f(x,y)]$$

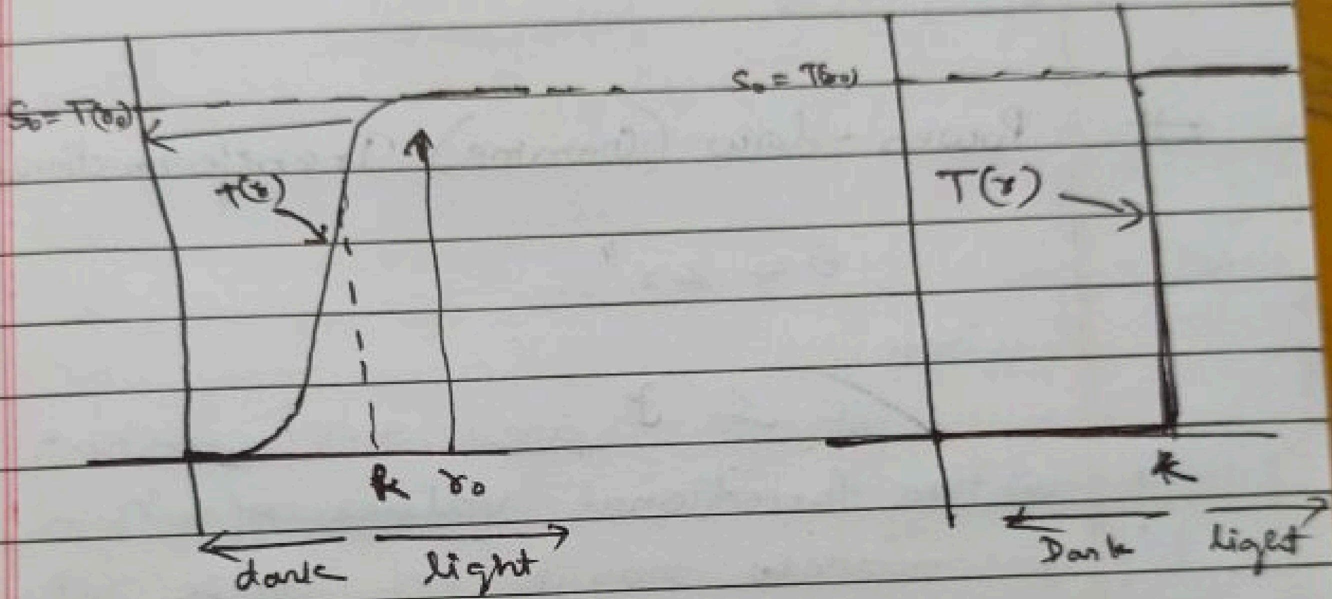
$f(x,y)$ is input image.
$g(x,y)$ is output image
T is an operator on f defined over a neighbourhood of point (x,y).

→ The smallest possible neighborhood is of size 1×1. Here T becomes intensity (also called gray-level or mapping) transformation function of the form-

$$s = T(r)$$

s intensity of g at (x,y)

r " " f " "



- r is lower than k reduces the value of s ⇒ black
- r is greater than k increases the value of s ⇒ light (whight)
- r is equal to k ⇒ thresholding function.

⇒ Negative Transformation function-

$$s = L - 1 - r$$

It is used in enhancing white or gray detail embedded in dark regions of an image.

Log Transformation :-

$$s = c\log(1+r)$$

where

c is constant

$r \geq 0$

- It compresses the dynamic range of pixels values.

Power - law (Gamma) Transformations-

$$s = cr^{\gamma}$$

where,

c & γ are +ve constant.

- with fractional values of γ map a narrow range of dark input values into a wider range of output value.

Ex image capture, printing.

Laplacian Filter: detects sudden intensity transition in the image and highlight the edge.

0	1	0
1	-4	1
0	1	0

Laplacian Kernel

$$\begin{matrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{matrix}$$

Kernel includes diagonals

Unsharp masking: a blurred image is subtracted from the original image to get edge only and then the result is added to the original image to get an enhanced image.

- Adding the subtracted image to the original image sharpens it by reducing the blur.

Gaussian blur: Gausian function is applied for gaussian smoothing on the input source image.

- Any hard ~~image~~ edges are smoothed.
- Image smoothing techniques helps in reducing the noise.

High boost Filtering:

Unsharp masking produces a mask $m(x,y)$ as –

$$m(x,y) = \underbrace{f_0(x,y)}_{\text{original image}} - \underbrace{f_b(x,y)}_{\text{blurred image.}}$$

Then that mask is added back to the orinal image which results in enhancing.

$g(x,y)$ the high-frequency components.

$$g(x,y) = f(x,y) + k\, m(x,y)$$

where

k specifies what position of mask is to be added. when $k=1$ we call this is high-boost filtering because we are boosting the high-frequency components by giving more weight to the masked image.

Q An image segment is shown below. Let v be the set of gray-level values used to define connectivity in the image. Compute D_4 & D_8 & D_m distances b/w pixel p and q for-

1. $V = \{2,3\}$
2. $V = \{2,6\}$

2(p)	3	2	6	1
6	2	3	6	2
5	3	2	3	5
2	4	3	5	2
4	5	2	3	6 (q)

coordinates of $p(x,y) = (0,0)$

" " $q(s,t) = (4,4)$

$$D_4(p,q) = |x-s| + |y-t|$$

$$= |0-4| + |0-4|$$

$= 8$ unit.

$D_8(p,q) = \max(|x-s|, |y-t|)$
$= \max(|0-4|, |0-4|)$
$= \max(4, 4)$
$\Rightarrow 4$ unit.

(i) $V = \{2, 3\}$
There is no path b/w p & q as q(6) is not included in the set V.

(ii) $V = \{2, 6\}$
There is no path b/w p & q.

Q Consider the image segment. Compute Dm, D4 & D8 distances b/w pixels 'p' and 'q' for $V = \{0, 1\}$ where V is the set of grey-level values.

3	1	2	1 (q) (3,3)
0	2	0	2
1	2	1	1
1 (p)	0	1	2

(0,0)

Coordinate of $p(x,y) = (0,0)$
" " $q(s,t) = (3,3)$

$D_4 = |x-s| + |y-t|$
$= |0-3| + |0-3|$
$= 6$ unit.

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$$D_8 = \max(|x-s|, |y-t|)$$
$$= \max(3, 3)$$
$$\Rightarrow 3 \text{ unit.}$$

for $V = \{0, 1\}$

$D_m = 5$ unit.

——x——x——x——

Q Thresholding with $T = 4$

$L = 8$

input image
$$\begin{bmatrix} 4 & 3 & 5 & 2 \\ 3 & 6 & 4 & 6 \\ 2 & 2 & 6 & 5 \\ 7 & 6 & 4 & 1 \end{bmatrix}$$

Hence,

$$L - 1 = 7$$

$$s = \begin{cases} L-1 & r \geq 4 \\ 0 & r < 4 \end{cases}$$

$r = 0, 1, 2, 3 \rightarrow s = 0$

$r = 4, 5, 6, 7 \rightarrow s = 7$

Output image will be -

$$\begin{bmatrix} 7 & 0 & 7 & 0 \\ 0 & 7 & 7 & 7 \\ 0 & 0 & 7 & 7 \\ 7 & 7 & 7 & 1 \end{bmatrix}$$

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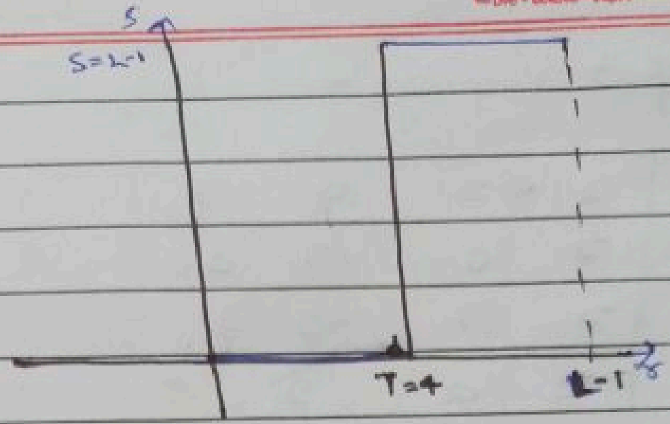


Cliping: refers to the process of removing or separating specific element within an image. As a result people, objects and background can be removed from the image.

Q clipping with $r_1 = 2$, & $r_2 = 5$
$L = 8$

input image:

$$\begin{bmatrix} 4 & 3 & 5 & 2 \\ 3 & 6 & 4 & 6 \\ 2 & 2 & 6 & 5 \\ 7 & 6 & 4 & 1 \end{bmatrix}$$

hence,

$L - 1 = 7$

$$S = \begin{cases} L-1 & 2 \leq r \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

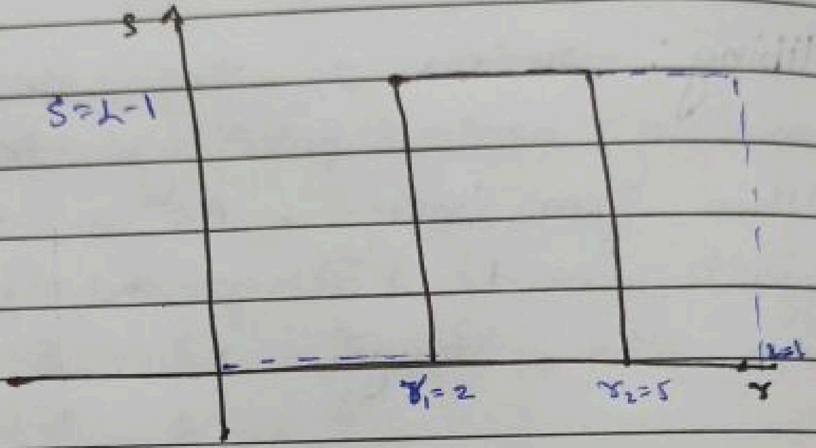
here,

$r_1 = 2, 3, 4, 5 \longrightarrow S = 7$

$r = 0, 1, 6, 7 \longrightarrow S = 0$

output image -

$$\begin{bmatrix} 7 & 7 & 7 & 7 \\ 7 & 0 & 7 & 0 \\ 7 & 7 & 0 & 7 \\ 0 & 0 & 7 & 0 \end{bmatrix}$$



Bit-plane slicing: is a method of representing an image with one or more bits of the byte used for each pixel. One can use only MSB to represent the pixel, which reduces the original grey-level to a binary image.

★. main goal of bit slicing:-

1. Converting an image with fewer bits and corresponding the image to a smaller size.
2. Converting a grey-level image to a binary image.
3. Enhancing the image by focusing.

Q input image -

$$\begin{bmatrix} 6 & 7 & 6 & 6 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 2 & 3 \\ 4 & 5 & 5 & 4 & 2 \\ 6 & 6 & 6 & 7 & 7 \end{bmatrix}$$

hence

the given image has a maximum grey level of 7, it is a 3-bit image.

Now,

we convert the image to binary ~~the~~ and separate the bit plane.

110	111	110	110	111
000	000	000	001	010
001	001	001	010	011
100	101	101	100	010
110	110	110	111	111

Now, sapating bit planes -

$$\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{matrix} \qquad \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix}$$

MSB plane

center bit plane

0	1	0	0	1
0	0	0	1	0
1	1	1	0	1
0	1	1	0	0
0	0	0	1	1

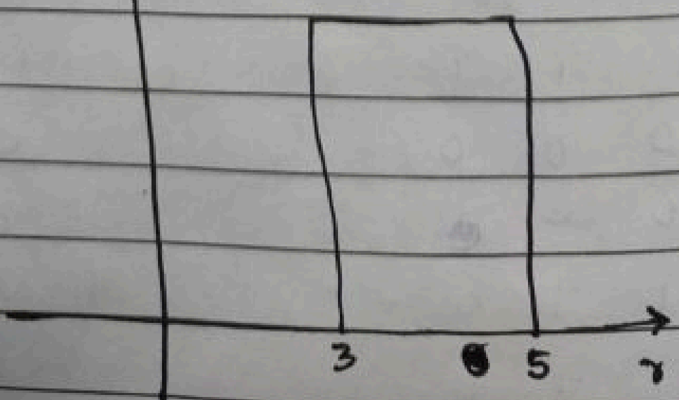
LSB plane.

Intensity level slicing: means highlighting a specific range of intensities in an image. Other words we segment certain gray level region from the image.

Q slicing with $r_1 = 3$ & $r_2 = 5$
(i) without background - clipping
(ii) with "

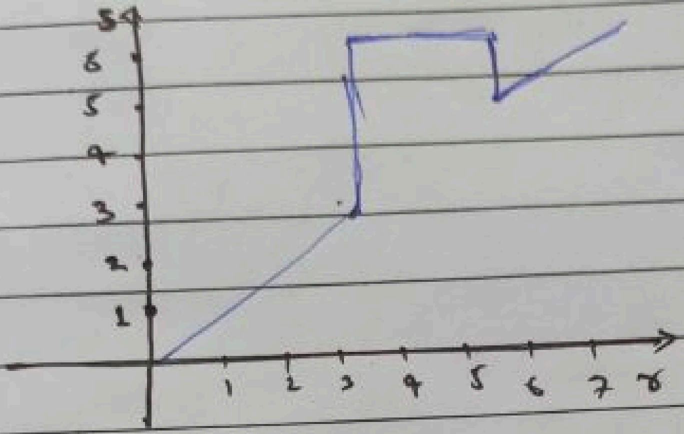
(i)

$$S = \begin{cases} L-1 & 3 \le r \le 5 \\ 0 & \text{otherwise} \end{cases}$$



(ii) with background-

$$s = \begin{cases} L-1 & 3 \le r \le 5 \\ 0 & \text{otherwise} \end{cases}$$



input image-

4	3	5	2
3	6	4	6
2	2	6	5
7	6	4	1

output image-

7	7	7	2
7	6	7	6
2	2	6	7
7	6	7	1