

Master of Computer Application (MCA)

MCAC-102: Discrete Mathematics

Unique Paper Code: 223401102

Semester: I

March 2022

Year of admission: 2021

Time: 3 Hours

Max. Marks: 70

Instructions for the Students:

Attempt any 4 out of 6 questions. All questions carry equal marks.

1. a. Find the coefficient of $e_1 + e_2 + e_3 = 16$ where e_1, e_2 and e_3 are non – negative integers with $2 \leq e_1 \leq 5, 3 \leq e_2 \leq 6$ and $4 \leq e_3 \leq 7$.
 - b. Prove that if x is real number, then $[3x] = [x] + \left[x + \frac{2}{5}\right] + \left[x + \frac{3}{5}\right]$
 - c. Find the value of ‘c’ and ‘n’ such that $n^2 + 3n - 4 = O(n^2 - 2n + 3)$
2. a. In a “Discrete Mathematics” class having 30 students, the teacher wants to analyze the result at the end of the semester. She/he prepares the following tally of marks obtained by the students:

< 40	8
>= 40 and < 50	5
>= 50 and < 60	7
>=60 and < 70	6
>=70 and < 80	4

While sending the data over the network, the tally is compressed so that minimum number of bits are sent over the network. Use a compression technique to generate the optimal binary prefix codes.

With the help of above table decode the message for the pattern 10110110000011011.

- b. Let f be a function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a) = 4, f(b) =$

1, $f(c) = 3$ and $f(d) = 2$. Is f an injective/surjective/bijective function? Justify your answer.

- c. Find the solution to the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with the initial conditions $a_0 = 1, a_1 = -2$ and $a_2 = -1$.

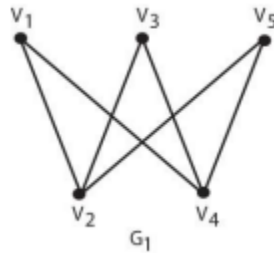
3. a. Arrange the following functions in increasing order of their rate of growth

i) $n \lg n$ ii) $n^{\sqrt{n}}$ iii) $n^{\lg n}$ iv) $(\lg n)^n$

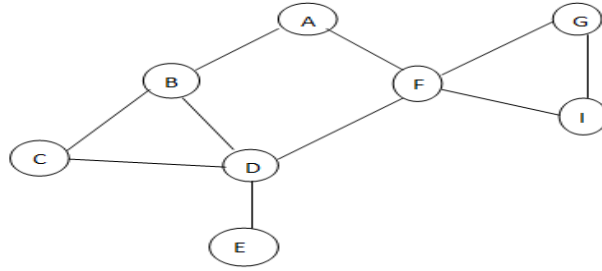
- b. Define Lower bound and greatest lower bound. Let $P = \langle \{2, 4, 5, 10, 12, 20, 25\}, | \rangle$ be a POSET, where $a|b$ denotes a divides b . Draw the Hasse diagram. Find

- a maximal element & a minimal element.
- Greatest Lower Bound (GLB) and Least Upper Bound (LUB) of $\{10, 20\}$.
- Find a lower bound different from GLB and an upper bound different from the LUB of $\{10, 20\}$, if any

- c. State the condition for Euler path (EP), Euler circuit (EC), Hamiltonian path (HP) and Hamiltonian circuit (HC). Determine whether the graph G_1 has EP, EC, and HP.



4. a. Run the Depth first search (DFS) algorithm and Breadth first search (BFS) algorithms on the network given in the figure starting from vertex $s=A$. Also assume that both the BFS and DFS algorithms will choose the left-most node first.



- i. Determine if the graph is bipartite. If yes, give the two partitions else justify your answer.
 - ii. Find the tree edges, cross edges and back edges using DFS.
 - iii. Explain how DFS can be used to find a cycle in the graph
 - iv. Given a pair of vertices s and t, explain how BFS can be used to find a shortest path between s-t in graph.
- b. Negate the following statement $\forall x \exists y \forall z (P(x, y) \wedge Q(y, z))$
- c. Show that $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology using predicate logic.
5. a. Consider the following district map of Tripura state. Is it four colorable? If yes, assign the color to each region else give the chromatic number and its coloring.



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- b. Obtain the Principal conjunctive normal form and Principal disjunctive normal form for the following statement $(\sim P \vee \sim Q) \rightarrow (P \leftrightarrow \sim Q)$
- c. Assume $f(x) = x+1$, $g(x) = x-1$ and $h(x) = 2x$, $x \in \mathbb{R}$. Find the composition (i) $g \circ f$ (ii) $h \circ f$ and (iii) $f \circ h$.

6. a. Show that the complete graph K_4 is a planar graph.
- b. Represent the following argument symbolically and determine whether the argument is valid?
“if today is Mahatma Gandhi Jayanti then today is Mahatma Gandhi’s birthday”. “if today is Mahatma Gandhi’s birthday then today is 1st October”.
Hence “if today is Mahatma Gandhi Jayanti then today is 1st October”.
- c. Consider a variation of binary search algorithm; The instructor wants to search a number in a sorted array of size n by dividing it into two parts of size $2n/3$ and $n/3$. Write down the recurrence for the running time for the best and worst case scenario.