Master of Computer Applications (MCA)

MCAC-301: Design and Analysis of algorithms

Unique Paper Code: 223401301

Semester: III

December 2022

Year of admission: 2021

Time: 3 Hours **Max. Marks: 70**

Instructions for the Students:

Attempt all questions. Parts of a question should be answered together.

1. a. Arrange the following functions in the increasing order of their rate of growth: 2

$$n^{lgn}, \quad \sqrt{n}, \quad nlogn, \quad n, \quad \frac{100}{n}, \quad n^{\sqrt{n}}$$

b. Consider the algorithm for integer multiplication using divide and conquer approach. Fill the missing details correctly. 4

key = A[i]

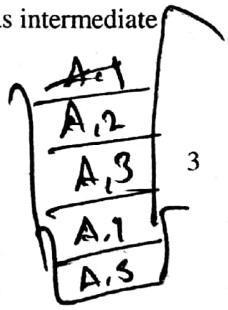
j ←

Multiply (a, b)

$$Assume\ n = length(a) = length(b)$$
$$if\ \ length(a)\_\_\_1 \quad then\ return \quad a * b$$
$$Partition\ a, b\ into\ a = a_1 * 10^{\frac{n}{2}} + a_2 \quad and \quad b = b_1 * 10^{\frac{n}{2}} + b_2$$
$$A = Multiply\ (a_1, \_\_\_)$$
$$\_\_\_\_ = Multiply\ (b\_, b\_\_)$$
$$C = Multiply\ (a_1 + \_\_\_, b_1 + \_\_\_)$$
$$Return\ A * 10^n + (\_\_\_ - A - \_\_\_\_) * 10^{\frac{n}{2}} + \_\_$$

c. Suppose you are given an array A having 8 integers {10, 20, 30, 40, 50, 60, 70, 80}. Draw a randomized binary search tree (BST) using the randomized Quicksort. Why ordinary quick sort is not good for drawing a randomized BST. Justify your answer. 4

2. a. Consider sorting algorithm A with run time O(n). What is the condition on A for it to be usable as the intermediate sort in Radix sort? Is it possible to use quick sort as intermediate sort? Justify your answer. 3



b. Write a recursive algorithm for insertion sort. 3

A,5

Scanned with CamScanner

c. Consider an instance of weightage interval scheduling with 6 intervals as specified below: 4

Interval Number (i)	Starting time (s_i)	Finishing time (f_i)	Weight (w_i)
1	0	2	2
2	1	3	4
3	2	4	4
4	1	5	7
5	4	5	2
6	4	6	1

With the help of the above example argue that the memoized recursive algorithm solves lesser number of subproblems than the corresponding iterative algorithm.

3. a. Consider a complete undirected graph with vertex set {1, 2, 3, 4, 5, 6, 7, 8, 9}. Entry Wij in the matrix W below is the weight of the edge {i, j}. 5

W =

	1	2	3	4	5	6	7	8	9
1	0	7	3	∞	12	∞	∞	∞	∞
2	7	0	∞	4	∞	13	∞	∞	∞
3	3	∞	0	8	9	5	∞	∞	∞
4	∞	4	8	0	∞	15	∞	∞	∞
5	12	∞	9	∞	0	1	∞	∞	∞
6	∞	13	5	15	1	0	∞	∞	∞
7	∞	∞	∞	∞	∞	∞	0	2	6
8	∞	∞	∞	∞	∞	∞	2	0	10
9	∞	∞	∞	∞	∞	∞	6	10	0

Find a minimum spanning tree (MST) T in the above graph. Suppose you increase the weight of each edge by five in the graph, will T still be an MST or not.

b. Consider (QA) be Quick sort algorithm to sort integers in non-decreasing order using last element as pivot. C1 and C2 be the number of comparisons made by QA for the given inputs {12, 8, 6, 7, 8, 10} and {4, 4, 4, 4, 4, 4} respectively. What will be the values of C1 and C2? 5

Scanned with CamScanner

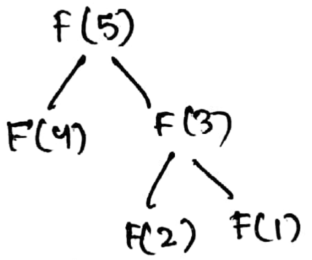
4 a. Which sorting algorithm is best if the list is almost sorted? Why? 2

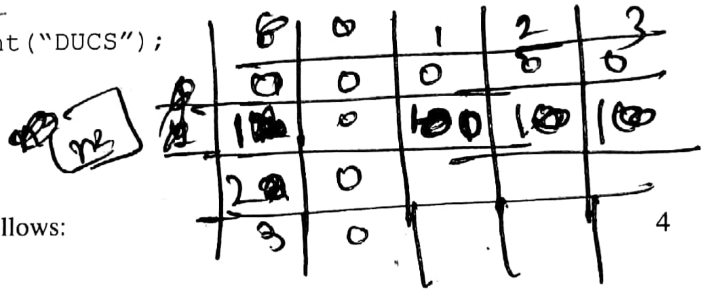
b. Analyze the time complexity of the following algorithm (written in Pseudo-code) by counting the number of steps. Assume $n=2^{2^k}$ for some positive integer k. 4

```
A(n)
{
        for (i=1; i≤n;  i++)
        {
            j=2
              while (j≤n)
                {
                    j*j
                    Print("DUCS");
                }
        }
}
```

$i=1$ 2^{2^k} (k^2)

2, 4, 16, 32





c. We define Fibonacci numbers as follows: 4

```
F(0)=0
F(1)=1
F(n)=F(n-1)+F(n-2)   for all n>1
```

Write a Fib_memoize(n) to find the n^{th} fibonacci number using memoization.

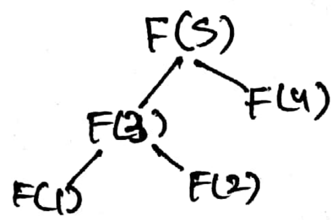
5. a. What are the two key factors that decide whether dynamic programming is applicable for an optimization problem or not. 2

b. Prove that the number of comparisons required by any comparison based sorting algorithm in the worst case is Ω (n log n). 3

c. Is the following recurrence for the knapsack problem, correct? If yes, then run it on a suitable example else give the correct recurrence. 5

if w < wi, then OPT(i,w)=opt(i-1,w)

Otherwise OPT(i,w)=max(opt(i-1,w), vi+opt(i-1,w))

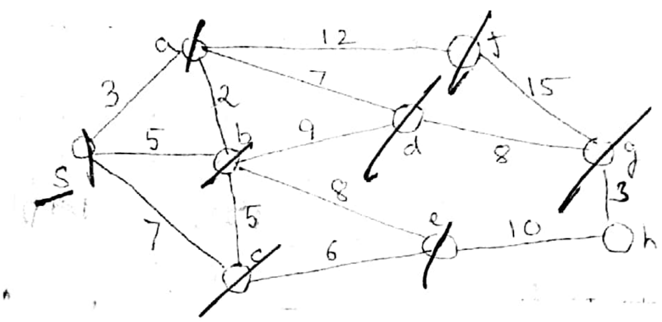


Scanned with CamScanner

6\. a Write an algorithm to find both the smallest and the largest element in a set of n entries. The algorithm should perform no more than 1.5*n comparisons of elements. 5

b. Write an algorithm to compute the Prefix function. Given a text "ababacbababacabcababacaa" and pattern "*ababaca*". Use this prefix function to find all the occurrences of the pattern. 5

7\. a. Run the Dijkstra's algorithm on the network given in the figure starting from vertex s. Show all the steps. You do not need to draw the graph repeatedly, just show the updated values and the next vertex picked. 5



$\frac{3n}{2}+1$ $3(n-1)$ $3n+2$ $3n-3$ $\frac{3(n-2)}{2}$ $\frac{3(n+1)}{2}$ $\frac{3n-6}{2}$ $\frac{3n}{2}-3+1$ $\frac{3n}{2}-2$

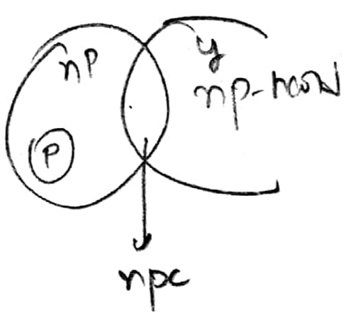
b. Which of the following is/are true/false about NP-Complete and NP-Hard problems? 2

1\. If we want to prove that a problem X is NP-Hard, we take a known NP-Hard problem Y and reduce Y to X. (True)

2\. NP-complete is a subset of NP Hard

$Y \le X$

c. Consider a variation of binary search algorithm; The instructor wants to search a number in a sorted array of size n by dividing it into two parts of size 2n/3 and n/3. Write the recurrence for the running time for the best and the worst case scenario. 3



$Y \le X$

$T(\frac{n}{2})$

$\log_2 n$

$\frac{3n}{2}+2$

Scanned with CamScanner