Indian Institute of Technology Jodhpur

Probability, Statistics and Random Processes- MA221

Semester II (2016 - 2017)

Assignment IX

1. A random sample $X_1, \cdots, X_{100}$ is given from a distribution with known variance 16. For the observed sample, the sample mean is 23.5. Find an approximate 95% confidence interval for population mean.

2. A theoretical model suggests that the time to breakdown of an insulating fluid between electrodes at a particular voltage has an exponential distribution with mean $1/\lambda$. A random sample of $n = 10$ breakdown times yields the following sample data (in minutes)

$$x_1 = 41.53, x_2 = 18.73, x_3 = 2.99, x_4 = 30.34, x_5 = 12.33$$

$$x_6 = 117.52, x_7 = 73.02, x_8 = 223.63, x_9 = 4.00, x_{10} = 26.78$$

   (a) Show that $2\lambda \sum X_i$ follows Chi-square distribution.

   (b) Obtain a 95% confidence interval for λ.

3. Assume that the helium porosity of coal samples taken from any particular seam is normally distributed with the standard deviation 0.75

   (a) Compute a 95% confidence interval for the true average porosity of a certain seam if the average porosity for 20 specimens from the seam was 4.85.

   (b) Compute a 98% confidence interval for the true average porosity of a certain seam if the average porosity for 16 specimens from the seam was 4.56.

   (c) How large a sample is necessary if the width of the 95% confidence interval is to be 0.40?

   (d) How large a sample is necessary if the width of the 99% confidence interval is to be 0.20?

4. A random sample $X_1, \cdots, X_{16}$ is given from a normal distribution with unknown mean μ and unknown variance σ^2. For the observed sample, the sample mean is $\bar{X} = 16.7$, and the sample variance is $S^2 = 7.5$. Find a 95% confidence interval for μ and σ^2.

5. Let X be a random variable lies in interval $[a, b]$. Show that $Var(X) \leq \frac{(b-a)^2}{4}$. Further, let $X_1, \cdots, X_n$ be a random sample from an unknown distribution with CDF $F_X(x)$. Also, $E(X)$ and $Var(X)$ are unknown. Find a $(1 - \alpha)100\%$ confidence interval for $\theta = E(X)$. Assume n is large.

6. To estimate p, the proportion of all newborn babies that are male, the gender of 10,000 newborn babies was noted. If 5,106 of them were male, determine 90% and 99% confidence intervals for p.

7. An airline is interested in determining the proportion of its customers who are flying for reasons of business. If they want to be 90 percent certain that their estimate will be correct within 2 percent, how large the sample should they select?

8. A random sample $X_1, \cdots, X_{81}$ is given from unknown distribution. The sample parameters are given as $\bar{X} = 8.25$ and $S^2 = 14.6$. Design a test to decide between $\mu = 9$ and $\mu < 9$. Also, calculate the p-value for the observed data.

9. A random sample $X_1, \cdots, X_4$ is given from $N(\mu, 1)$ distribution. Suppose that the four observed values are

$$x_1 = 2.82, x_2 = 2.71, x_3 = 3.22, x_4 = 2.67$$

We would like to decide between $\mu = 2$ and $\mu \neq 2$.

(a) Assuming that $\alpha = 0.1$, which one do you accept?

(b) If we require significance level α, find β as a function of μ and α.

10. Suppose we would like to test the hypothesis that at least 10% of students suffer from allergies. We collect a random sample of 225 students and 21 of them suffer from allergies.

(a) State the null and alternative hypotheses.

(b) Obtain a test statistic and a p-value.

(c) State the conclusion at the $\alpha = 0.05$ level.