Date 29/03/17

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⟨1⟩ $f_{XY}(x,y) = \frac{3x^2+y}{2}$; $x \in (0,1)$; $y \in (0,1)$

$\Rightarrow f_X(x) \Rightarrow \int_0^1 \frac{3x^2}{2} \cdot dy + \int_0^1 y \cdot dy$

$\Rightarrow \frac{3x^2}{2}(1) + \frac{1}{2} \Rightarrow \frac{3x^2+1}{2}$

$\Rightarrow E(X) \Rightarrow \int_0^1 \frac{3x^3}{2} \cdot dx + \int_0^1 \frac{x}{2} \cdot dx = \frac{3x^4}{8}\Big|_0^1 + \frac{x^2}{4}\Big|_0^1 \Rightarrow \frac{3}{8} + \frac{1}{4} = \frac{5}{8}$

$\Rightarrow E(X^2) \Rightarrow \int_0^1 \frac{3x^4}{2} \cdot dx + \int \frac{x^2}{2} \cdot dx \Rightarrow \frac{3x^5}{10}\Big|_0^1 + \frac{x^3}{6}\Big|_0^1 \Rightarrow \frac{3}{10} + \frac{1}{6} = \frac{7}{15}$

$\Rightarrow Var(X) \Rightarrow \frac{7}{15} - \frac{25}{64} \Rightarrow \frac{448-375}{960} = \frac{73}{960}$

$\Rightarrow f_Y(y) \Rightarrow \int_0^1 \frac{3x^2}{2}(dx) + \int_0^1 y \cdot dx \Rightarrow \frac{x^3}{2}\Big|_0^1 + xy\Big|_0^1 = y + 1/2$

$\Rightarrow E(Y) \Rightarrow \int_0^1 y^2 dy + \int_0^1 \frac{y}{2} \cdot dy \Rightarrow \frac{y^3}{3}\Big|_0^1 + \frac{y^2}{4}\Big|_0^1 \Rightarrow \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

$\Rightarrow E(Y^2) \Rightarrow \int_0^1 y^3 dy + \int_0^1 \frac{y^2}{2} \cdot dy \Rightarrow \frac{y^4}{4}\Big|_0^1 + \frac{y^3}{6}\Big|_0^1 \Rightarrow \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

$\Rightarrow Var(Y) \Rightarrow \frac{5}{12} - \frac{49}{144} \Rightarrow \frac{11}{144}$

$\Rightarrow \cancel{Cov(X,Y) \Rightarrow}$

$\Rightarrow E(XY) \Rightarrow \iint \frac{3x^3y}{2} \cdot dx \cdot dy + \iint xy^2 \cdot dx \cdot dy \Rightarrow \frac{3x^4}{16} y^2 \Big|_0^1\Big|_0^1 + \frac{x^2}{6} y^3 \Big|_0^1$

$\Rightarrow \frac{3}{16} + \frac{1}{6} \Rightarrow \frac{9+8}{48} = \frac{17}{48}$

$\Rightarrow Cov(X,Y) \Rightarrow \frac{5}{8} \times \frac{7}{12} - \frac{17}{48} \Rightarrow \frac{35}{96} - \frac{34}{96} = 1/96$

$$\Sigma = \begin{bmatrix} 73/960 & 1/96 \\ 1/96 & 11/144 \end{bmatrix}$$

$\langle 2 \rangle$ $Y_1 \sim N(0,5)$; $Y_2 \sim N(0,2)$

ρ $\text{cov}(Y_1, Y_2) = -1$.

$f_{X_1}(x_1) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} = f(x_2)$

$\Rightarrow X_1 = \frac{Y_1+Y_2}{3}$; $X_2 = Y_1 - 2X_1 \Rightarrow Y_1 - \frac{2Y_1}{3} - \frac{2Y_2}{3}$

$= \frac{Y_1 - 2Y_2}{3}$.

$\Rightarrow |J| = \begin{vmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{vmatrix} \Rightarrow \left| -\frac{2}{9} - \frac{1}{9} \right| = +1/3$.

$$\Rightarrow \boxed{f_{Y_1Y_2}(y_1, y_2) = \frac{1}{6\pi^2} e^{-\frac{1}{2}\left(\left(\frac{y_1+y_2}{3}\right)^2 + \left(\frac{y_1 - 2y_2}{3}\right)^2\right)}}$$

$\langle 3 \rangle$ $X \sim N(22.7, 17.64)$, $Y \sim N(22.7, 12.25)$

$\rho = 0.78$.

$P(25.5 > Y > 18.5 \mid X = 23) = ?$

$\Rightarrow \mu' \Rightarrow \mu_2 + \frac{\rho\sigma_2}{\sigma_1}(x - \mu_1) \Rightarrow 22.7 + 0.78\sqrt{\frac{12.25}{17.64}}(23 - 22.7)$

$\Rightarrow 22.7 + 0.195 = 22.895$.

$\Rightarrow \sigma'^2 \Rightarrow \sigma_2^2(1-\rho^2) \Rightarrow 12.25(0.3916) = 4.7971$.

$\Rightarrow P(25.5 > Y > 18.5) \Rightarrow P\left(\frac{25.5 - 22.895}{\sqrt{4.7971}} > Z > \frac{18.5 - 22.895}{\sqrt{4.7971}}\right)$

$\Rightarrow P(0.543 > Z > -0.916) = \underline{0.63}$.

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$$f_{XY}(x,y) = \begin{cases} \frac{1}{x} & 0<y\le x<1 \\ 0 & \text{o.w.} \end{cases}$$

(4) $f_X(x) \Rightarrow \int_0^x \frac{1}{x}\,dy = 1$

$\Rightarrow f_Y(y) \Rightarrow \int_y^1 \frac{dx}{x} = 0 - \ln(y)$.

$\Rightarrow E(X) \Rightarrow \int_0^1 x\,dx \Rightarrow 1/2$

$\Rightarrow E(X^2) \Rightarrow \int_0^1 x^2\,dx = 1/3$

$\Rightarrow Var(X) = 1/12$

$\boxed{E(XY) \Rightarrow \int_0^1\int_0^y \frac{xy}{x}\,dx\,dy \Rightarrow \int_0^1 (y-1)(y)\,dy \Rightarrow \frac{y^3}{3}\Big|_0^1 - \frac{y^2}{2}\Big|_0^1 = -1/6}$

$Cov(X,Y)$

$\Rightarrow E(Y) \Rightarrow \int_0^1 -y\ln y\,dy$

$\Rightarrow \frac{y^2}{4}\Big|_0^1 - \frac{y^2}{2}\ln y\Big|_0^1 \Rightarrow \frac{1}{4} - \frac{1}{2}(0) - 0 = 1/4$.

$\Rightarrow E(Y^2) \Rightarrow \int_0^1 -y^2 \ln y\,dy = \frac{y^3}{9}\Big|_0^1 - \frac{y^3}{3}\ln y\Big|_0^1 = 1/9$.

$\Rightarrow Var(Y) \Rightarrow 1/9 - 1/16 = \frac{7}{144}$.

$\boxed{\Rightarrow Cov(X,Y) \Rightarrow -1/6 - 1/8 = -\frac{7}{24}}$

$$\Sigma = \begin{bmatrix} 1/3 & -7/24 \\ -7/24 & 7/144 \end{bmatrix}.$$

(5) $\mu' \Rightarrow \mu_1 + \mu_2 + \mu_3 = 3$

$(\sigma')^2 \Rightarrow 1 + 1 + 1 = 3$.

$X_1 + X_2 + X_3 \sim N(3,3)$
(X')

$\Rightarrow P(X' > 1) \Rightarrow P\left(Z > \frac{1-3}{\sqrt{3}}\right) \Rightarrow P\left(Z > \frac{-2}{\sqrt{3}}\right)$

$= 0.79$.

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(6) $E(X) \Rightarrow \int_0^2 x\cdot dx - \int_0^2 \frac{x^2}{2}\cdot dx \Rightarrow \frac{x^2}{2}\Big|_0^2 - \frac{x^3}{6}\Big|_0^2$

$\Rightarrow 2 - \frac{4}{3} = 2/3.$

(a) $\mu = 2/3$

$E(X^2) \Rightarrow \int_0^2 x^2\cdot dx - \int_0^2 \frac{x^3}{2} dx \Rightarrow \frac{x^3}{3}\Big|_0^2 - \frac{x^4}{8}\Big|_0^2$

$\Rightarrow \frac{8}{3} - 2 = 2/3.$

$\Rightarrow \sigma^2 \Rightarrow 2/3 (\times \frac{1}{n}) = 1/27.$

(b) $P\left(\left(2/3 - 2/3\right)\sqrt{27} \leq \bar{z} \leq (5/6 - 2/3)\sqrt{27}\right)$

$= P(0 \leq \bar{z} \leq \sqrt{3}/2) =$

(7) $\mu' = np, \quad \sigma' = \sqrt{p(1-p)}$

$\Rightarrow P\left(z < \frac{n/2 - np}{\sqrt{p(1-p)}}\right) = 1-\alpha$

$\Rightarrow P\left(z < \frac{0.5n - 0.45n}{\sqrt{0.45(0.55)}}\right) = 1-\alpha \Rightarrow P\left(z < \frac{n}{10}\right) = 0.9$

$\Rightarrow n = 12.9 \approx 13$

(8) $\lambda = 0.02 ; \quad S = \sum^{100} X_i \quad ; \quad E(S) = 2.$

$Var(S) = 2.$

$\Rightarrow P(S \geq 3) = P\left(z \geq \frac{1}{\sqrt{2}}\right)$

Applying continuity correction;

$\Rightarrow P(S \geq 3) \Rightarrow P(S \geq 2.5) \Rightarrow P\left(z \geq \frac{1}{2\sqrt{2}}\right) = 0.36.$