Indian Institute of Technology Jodhpur

Probability, Statistics and Random Processes- MA221

Semester II (2016 - 2017)

Assignment II

1. Two dice are rolled. Let X be the larger of the two numbers shown. Compute $P(X \in [2, 4])$.

2. Consider a group of five potential blood donors - a, b, c, d and e - of whom only a and b have O^+ blood. Five blood samples, one from each individual, will be typed in random order until an O^+ individual is identified. Let Y be the number of typing necessary to identify an O^+ individual. Find the probability mass function (pmf) of Y.

3. There are three boxes A, B and C. Box A contains 5 red and 4 white balls, box B contains 8 red and 5 white balls and box C contains 2 red and 6 white balls. A ball is taken out randomly from each of the boxes. Let X be the total number of white balls taken out. Calculate the probability mass function.

4. Let X be a random variable with pmf

$$P(X = r) = p(1 - p)^r, \quad r = 0, 1, \cdots$$

   Show that $P(X > m + n | X > m) = P(X > n)$.

5. Since it is more economical to limit long-distance telephone calls to three minutes or less, the CDF (cumulative distribution function) of X-the duration in minutes may be of the form

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x/3} & 0 \leq x < 3 \\ 1 - \frac{e^{-x/3}}{2} & x \geq 3 \end{cases}$$

   Determine the pdf (probability density function) of X, and hence find the probability that X is

   (a) more that 2 minutes

   (b) between 2 and 6 minutes

6. Let X be a continuous random variable with density

$$f_X(x) = \begin{cases} 0.2 & -1 < x \leq 0 \\ 0.2 + cx & 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Determine the value of c.

   (b) Obtain the distribution function of X.

(c) Calculate $P(0 \leq X < 0.5)$.

(d) Determine $P(X > 0.5 | X > 0.1)$.

(e) Calculate the distribution function and the density function of the random variable $Y = 2X + 3$.

7. Let X be a continuous random variable with density

$$f_X(x) = \begin{cases} 1/4 & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

Let $Y = (X-3)^2$. Obtain the density of Y.

8. Let X be a continuous random variable with density

$$f_X(x) = \begin{cases} 2/3\pi & -\pi/2 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

Let $Y = \sin(X)$. Obtain the density of Y.

9. Let X be a random variable which can values $-1, 0, 1$ and 2 with $P(X = 0) = 0.1, P(X = 1) = 0.4$ and $P(X = 2) = \frac{1}{2}P(X = -1)$. Find $E(X)$.

10. In each of the following cases, compute $E(X)$, $Var(X)$ and $E(X^r)$ whenever they exist:

(a) $f(x) = (k-1)/x^k,\ x \geq 1, k > 1$

(b) $f(x) = p(1-p)^x, x = 1, 2, \cdots$

(c) $f(x) = 6x(1-x),\ 0 < x < 1$