

Indian Institute of Technology Jodhpur

Probability, Statistics and Random Processes- MA221

Semester II (2016 - 2017)

Assignment VII

1. Let X and Y be jointly continuous random variables with density function $f_{XY}(x, y) = \frac{3}{2}x^2 + y, 0 < x < 1, 0 < y < 1$. Find the covariance matrix of (X, Y) .
2. Let X_1 and X_2 be independent standard normal random variables. Define $Y_1 = 2X_1 + X_2$ and $Y_2 = X_1 - X_2$. Find $E(Y_1), E(Y_2), Cov(Y_1, Y_2)$ and the joint density function of (Y_1, Y_2) .
3. Let X denotes Mid-Term I score of a randomly selected student. Let Y denotes the Mid-Term II score of a randomly selected student. History suggests that $X \sim N(22.7, 17.64), Y \sim N(22.7, 12.25)$ and $Corr(X, Y) = 0.78$. What is the probability that a randomly selected student's Mid-Term II score is between 18.5 and 25.5 given that his Mid-Term I score was 23?
4. Let X and Y be two random variables with joint density

$$f(x, y) = \begin{cases} \frac{1}{x} & 0 < x < 1, 0 < y \leq x \\ 0 & \text{otherwise} \end{cases}$$

Obtain the covariance matrix and show that it is positive semi-definite.

5. Let random variables X_1, X_2 , and X_3 be independent and distributed according to $N(0, 1), N(1, 1)$, and $N(2, 1)$, respectively. Determine probability $P(X_1 + X_2 + X_3 > 1)$.
6. A random sample of size $n = 18$ is taken from the distribution whose pdf

$$f(x) = 1 - x/2, \quad -0 \leq x \leq 2$$

- (a) Find μ and σ^2 .
 - (b) Find, by using central limit theorem, approximately, $P(2/3 \leq \bar{X} \leq 5/6)$.
7. Let $X \sim B(n, p)$. Use the central limit theorem to find n such that $P(X > n/2) = 1 - \alpha$. In particular, calculate n with $\alpha = 0.1$ and $p = 0.45$.
 8. Let X_1, \dots, X_{100} be iid $P(\lambda)$, where $\lambda = 0.02$. Let $S = S_{100} = \sum_{i=1}^{100} X_i$. Use central limit theorem to evaluate $P(S \geq 3)$. Also, compare the result with exact probability.