

# Indian Institute of Technology Jodhpur

Probability, Statistics and Random Processes- MA221

Semester II (2016 - 2017)

## Assignment V

1. Suppose the number of customers arriving at a shop follows Poisson distribution with an average of  $\lambda$  customers per unit time, that is, if  $X$  is the number of customers arriving in an interval of length  $t$ , then  $X \sim \text{Poisson}(\lambda t)$ . Suppose that the store opens at time  $t = 0$ . What is the distribution of the arrival time ( $Y$ ) of first customer?
2. Let  $X$  has normal distribution with mean 2 and standard deviation 2. Define  $Y = 3 - 2X$ .
  - (a) Find  $P(X > 1)$ .
  - (b) Find  $P(-2 < Y < 1)$ .
  - (c) Find  $P(X > 2|Y < 1)$ .
3. Let  $X$  and  $Y$  be independent discrete random variables with same CDFs  $F_X$  and  $F_Y$ . Define  $U = \max(X, Y)$  and  $V = \min(X, Y)$ . Find the CDFs of  $U$  and  $V$ .
4. Let  $X$  and  $Y$  be two discrete random variables, with support

$$S_{XY} = \{(i, j) \in \mathbb{Z}^2 | i, j \geq 0, |i - j| \leq 1\}$$

and the joint probability mass function given by

$$P_{XY}(i, j) = \frac{1}{k \cdot 2^{\min(i, j)}}, \quad \text{for } (i, j) \in S_{XY}$$

- (a) Draw the graph of  $S_{XY}$ .
  - (b) For what value of  $k$  is  $P_{XY}(i, j)$  a joint PMF?
  - (c) Find the marginal PMFs  $P_X(i), P_Y(j)$ .
  - (d) Find  $P(X = Y | X < 2)$ .
  - (e) Find  $P(X = Y)$ .
  - (f) Find  $P(1 \leq X^2 + Y^2 \leq 5)$ .
5. Let  $X$  and  $Y$  be two continuous random variables with joint pdf  $f_{XY}(x, y) = x + cy^2$ ,  $0 \leq x \leq 1, 0 \leq y \leq 1$ .
    - (a) Find the constant  $c$ .
    - (b) Find  $P(0 \leq X \leq 1/2, 0 \leq Y \leq 3/4)$ .
    - (c) Find the marginal PDFs  $f_X(x)$  and  $f_Y(y)$ .
  6. Let  $X$  follows exponential distribution with mean 1. Find

(a) the conditional PDF of  $X$  given  $X > 1$ .



(b)  $E(X|X > 1)$ .

(c)  $Var(X|X > 1)$ .

7. Consider the unit disc  $D = \{(x, y) | x^2 + y^2 \leq 1\}$ . Suppose that the joint density of  $(X, Y)$  is given by  $f_{XY}(x, y) = c$ ,  $(x, y) \in D$ .

(a) Find the constant  $c$ .

(b) Find the marginal PDFs  $f_X(x)$  and  $f_Y(y)$ .

(c) Find the conditional PDF of  $X$  given  $Y = y$ .

(d) Are  $X$  and  $Y$  independent?

8. Let  $X$  and  $Y$  be two continuous random variables with joint pdf  $f_{XY}(x, y) = cx + 1$ ,  $x, y \geq 0, x + y < 1$ .

(a) Find the constant  $c$ .

(b) Find the marginal PDFs  $f_X(x)$  and  $f_Y(y)$ .

(c) Find  $P(Y < 2X^2)$ .

9. Let  $X$  and  $Y$  be independent standard normal random variables and

$$U = 1 + X + XY^2, \quad V = 1 + X$$

Find the covariance between  $U$  and  $V$ .

10. Let  $X$  and  $Y$  be two continuous random variables with joint pdf  $f_{XY}(x, y) = 2$ ,  $x, y \geq 0, x + y \leq 1$ . Find  $Cov(X, Y)$  and correlation coefficient  $\rho_{XY}$ .