# Indian Institute of Technology Jodhpur

Probability, Statistics and Random Processes- MA221

Semester II (2016 - 2017)

Assignment VI

1. Let $X$ and $Y$ be independent random variables with density functions $f(x) = \frac{x^2}{9}, 0 < x < 3$ and $g(y) = \frac{1}{y^2}, 0 < y < 1$. Find $P(XY > 1)$.

2. Let $X$ and $Y$ be independent random variables with $X \sim B(2, 1/2)$ and $Y \sim B(3, 1/3)$. Calculate $P(X = Y)$.

3. Suppose $X$ has uniform distribution on the interval $(-\pi, \pi)$. Define $Y = \cos X$. Show that $Cov(X, Y) = 0$ though $X$ and $Y$ are dependent.

4. Let $X$ and $Y$ be independent uniform random variables in $(0, 1)$. Define $U = max(X, Y)$ and $V = min(X, Y)$. Find $Cov(U, V)$.

5. Let $X$ and $Y$ be two discrete random variables, with support

$$S_{XY} = \{(x, y) \in Z^2 | x^2 + |y| \leq 2\}$$

and the joint probability mass function given by

$$P_{XY}(x, y) = a, \quad \text{for } (x, y) \in S_{XY}$$

   **(a)** Construct a joint probability distribution table.

   **(b)** Find $E(X|Y = 1)$.

   **(c)** Find $Var(X|Y = 1)$.

   **(d)** Find $E(X||Y| \leq 1)$.

   **(e)** Find $E(X^2||Y| \leq 1)$.

6. Let $X$ and $Y$ be two continuous random variables. Suppose $\sigma_X^2 = 4$ and $\sigma_Y^2 = 9$. If we know that the two random variables $U = 2X - Y$ and $V = X + Y$ are independent, find $Cov(X, Y)$ and $\rho(X, Y)$.

7. Let $X$ follows exponential distribution with mean 1. Find

   **(a)** the conditional PDF of $X$ given $X > 1$.

   **(b)** $E(X|X > 1)$.

   **(c)** $Var(X|X > 1)$.

8. Let $X$ and $Y$ be jointly continuous random variables with joint density

$$f_{XY}(x, y) = \frac{x^2}{4} + \frac{y^2}{4} + \frac{xy}{6}, \; 0 \leq x \leq 1, 0 \leq y \leq 2$$

**(a)** Find the conditional PDF of $X$ given $Y = y$.

**(b)** Find $P(X < 1/2|Y = y)$.

**(c)** Find $E(X|Y = 1)$.

**(d)** Find $Var(X|Y = 1)$.

9. Let $X$ and $Y$ be jointly continuous random variables with joint density

$$f_{XY}(x, y) = \frac{1}{x}, \; 0 \leq x \leq 1, 0 \leq y \leq x$$

Compute the covariance matrix.

10. Let $X$ and $Y$ be independent standard normal random variables. Let also, $U = 2X - Y$ and $V = -X + Y$. Find the joint density of $U$ and $V$.

11. Assume that $X$ and $Y$ are independent and uniformly distributed over $(0, 1)$ and $(0, 2)$ respectively. Find the joint density for $(U, V)$, where $U = 2X + Y$ and $V = X + 3Y$.