

# Indian Institute of Technology Jodhpur

Probability, Statistics and Random Processes- MA221

Semester II (2016 - 2017)

## Assignment VIII

1. Let  $X_1, \dots, X_n$  be a random sample from a Uniform  $(0, \theta)$  distribution, with  $\theta > 0$  as unknown parameter. Let  $\bar{X}$  denote the sample mean
  - (a) Is  $\bar{X}$  unbiased for  $\theta$ ?
  - (b) Find an unbiased estimator of  $\theta$ .
  - (c) Find the variance of the unbiased estimator.
  - (d) Suggest an alternative estimator for  $\theta$ .
2. Let  $(X_1, Y_1), \dots, (X_n, Y_n)$  be a random sample from bivariate population with variances  $\sigma_1^2, \sigma_2^2$  and correlation  $\rho$ . If

$$S_{11} = \frac{1}{n-1} \sum_j (X_j - \bar{X})(Y_j - \bar{Y})$$

Find  $E(S_{11})$ .

3. Let  $X_1, \dots, X_n$  represent a random sample from a distribution with pdf

$$f(x) = \frac{x}{\theta} e^{-x^2/(2\theta)} \quad x > 0$$

Show that  $E(X^2) = 2\theta$ . Use this to construct an unbiased estimator of  $\theta$  based on  $\sum X_i^2$ . Also, estimate  $\theta$  from the following  $n = 10$  observations on vibratory stress of a turbine blade under specified conditions:

16.88 10.23 4.59 6.66 13.68 14.23 19.87 9.40 6.51 10.95

4. Let  $X_1, X_2, X_3, X_4, X_5$  be a sample from  $N(0, 4)$ . Find  $P(\sum_{i=1}^5 X_i^2 \geq 5.75)$ .
5. A random sample of 5 is taken from a normal population with mean 2.5 and variance 36.
  - (a) Find the probability that the sample variance lies between 30 and 40.
  - (b) Find the probability that the sample mean lies between 1.3 and 3.5, while sample variance lies between 30 and 44.
6. Let  $X_1, X_2, \dots$  be iid  $N(\mu, \sigma^2)$  random variables. Find  $Var(S^2)$  and show that  $S^2$  is consistent for  $\sigma^2$ .

7. A sample of 3 observations ( $X_1 = 0.4, X_2 = 0.7, X_3 = 0.9$ ) is collected from a continuous distribution with density  $f(x) = \theta x^{\theta-1}, 0 < x < 1$ . Estimate  $\theta$  by the method of maximum likelihood.
8. Let  $X$  denote the proportion of allotted time that a randomly selected student spends working a certain aptitude test. Suppose the pdf of  $X$  is  $f(x) = (\theta + 1)x^\theta, 0 < x < 1, \theta > -1$ . A random sample of ten students yields data

$$x_1 = 0.92, \quad x_2 = 0.79, \quad x_3 = 0.90, \quad x_4 = 0.65, \quad x_5 = 0.86$$

$$x_6 = 0.47, \quad x_7 = 0.73, \quad x_8 = 0.97, \quad x_9 = 0.94, \quad x_{10} = 0.77$$

Obtain the maximum likelihood estimator of  $\theta$ , and then compute the estimate for the given data.

9. Consider a random variable  $X$  with density  $f(x) = \frac{2x}{b}e^{-x^2/b}, x \geq 0$ . Let  $X_1, \dots, X_n$  be a random sample from a population with this density. Find the maximum likelihood estimator of  $b$ . Is the MLE unbiased?
10. Let  $X \sim \text{Bernoulli}(p), p \in [1/4, 3/4]$ . Find the MLE of  $p$  and check if it is unbiased.