INCOMPLETE

AID-521 Mathematics for Data Science

Module: Probability | Lecture: 4

SOME FREQUENTLY USED PROB. DISTRIBUTIONS

discrete distributions, continuous distributions, examples, properties

Why learn about these probability distributions?

- Underlying processes of such distributions appear in many practical cases.
- Theoretical properties of these distributions are well known.
- Modelling and analyzing an unknown distribution? One among these may be a good starting point.

Before we begin

P: S – [0,1]

X: S – RealL
D: X(S) – [0,1]

Discrete Uniform Distribution

$$\frac{n_A}{N}$$

Binomial Distribution

Bernoulli distribution

X is a discrete r.v. with values 0 or 1. The Bernoulli probability function of X is

$$p(x) = P(X = x) = \begin{cases} p^x(1-p)^{1-x}, & x = 0, 1 \\ 0, & \text{otherwise.} \end{cases}$$

- In a Bernoulli trial, 1 is usually called *success* and 0 is called *failure*.
- The Bernoulli distribution is fully characterized by one parameter, i.e., p.

Binomial Distribution

Fix a number n. The process of Bernoulli trial with parameter p is repeated n times.

- → The sample space here is all possible sequences of 1 and 0 of length n.
- → The probabilities of outcomes will depend on p.

Let X be the binomial r.v., i.e., it counts the number of successes x for each outcome of the process.

The probability distribution of X, called binomial probability distribution with parameters (n, p), is as below.

$$p(x) \;=\; P(X = x) \;=\; \binom{n}{x} p^x (1-p)^{n-x};$$

$$x \in \{0, 1, \dots, n\}, \;\; p \in [0, 1]$$

Binomial Distribution -- Properties

$\rightarrow \; E[X] = \mu = n \cdot p$

$\rightarrow \; Var(X) = \sigma^2 = n \cdot p \cdot (1 - p)$

Poisson Distribution

Pronounced *Pwa–sson*. Its French.

A is a random event of an underlying process (or experiment).

X is a r.v. that counts the occurrences of A.
It is assumed that values of X in disjoint intervals are independent and its mean value λ is constant.

The discrete r.v. X follows Poisson probability distribution with parameter $\lambda > 0$ if

$$P(X = x) \;=\; \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}; \;\; x \in \{0, 1, 2, ...\}$$

$\rightarrow$ Helpful for modeling rare events.

Poisson Distribution -- Properties

$\rightarrow\ E[X] = \mu = \lambda$

$\rightarrow\ Var(X) = \sigma^2 = \lambda$

$\rightarrow\ \lim_{n \to \infty,\ p \to 0,\ np = \lambda} \binom{n}{x} p^x (1-p)^{n-x} = \frac{e^{-\lambda}\lambda^x}{x!}$

Continuous Uniform Distribution

A r.v. X has uniform prob. distribution on (a, b) if X has the density function

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \in [a, b], \\ 0, & \text{otherwise.} \end{cases}$$

The cdf of X

$$F(x) = \int_{-\infty}^{x} \frac{1}{b-a} dx = = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b, \\ 1, & x \geq b. \end{cases}$$

Cont. Uniform Distribution -- Properties

$\rightarrow$ $E[X] = \mu = \frac{a+b}{2}$

$\rightarrow$ $Var(X) = \sigma^2 = \frac{(b-a)^2}{12}$

Normal or Gaussian Distribution

A r.v. X has a normal probability distribution with parameters μ and σ^2 if X has the density function

$$f(x) \;=\; \frac{1}{\sqrt{2\pi}\cdot\sigma}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}};\; x \in \mathbb{R}.$$

→ If $\mu = 0$ and $\sigma = 1$, X is called standard normal r.v.

→ $X \sim N(\mu, \sigma^2)$

Gaussian Distribution -- Properties

$\rightarrow$ $E[X] = \mu$

$\rightarrow$ $Var(X) = \sigma^2$

Log-normal Distribution

Let Y be a r.v. with normal distribution. Then $X = e^Y$ has a log-normal distribution.

$\rightarrow$ $E[X] = e^{\mu_Y + \frac{\sigma_Y^2}{2}}$

$\rightarrow$ $Var(X) = (e^{\sigma_Y^2} - 1) \cdot e^{2\mu_Y + \sigma_Y^2}$

So, when do we use log-normal to model a distribution?

Gamma Distribution

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Gamma Distribution -- Properties

$\rightarrow$ $E[X] = \alpha\beta$

$\rightarrow$ $Var(X) = \alpha\beta^2$

Exponential Distribution

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Exponential Distribution -- Properties

$\rightarrow$ $E[X] = \beta$

$\rightarrow$ $Var(X) = \beta^2$

Chi-square Distribution

Chi-square Distribution -- Properties

$\rightarrow$ $E[X] = n$

$\rightarrow$ $Var(X) = 2n$

The list of distributions we discussed here is incomplete.

An exhaustive list.. probably.

List of probability distributions

From Wikipedia, the free encyclopedia

Many probability distributions that are important in theory or applications have been given specific names.

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appendix, beginning of chapter