Tutorial 3
AID-521: Mathematics for Data Science

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Qn. 1.

A process for refining sugar yields up to 1 ton of pure sugar per day, but the actual amount produced, Y, is a random variable because of machine breakdowns and other slowdowns. Suppose that Y has density function given by

$$f(y) = \begin{cases} 2y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

The company is paid at the rate of Rs. 300 per ton for the refined sugar, but it also has a fixed overhead cost of Rs. 100 per day. Thus the daily profit, in hundreds of rupees, is $U = 3Y - 1$.

(a) Find the probability density function for U, using method of distribution functions.

(b) Find the probability density function for U, using the transformation method.

(c) Find the density function of $V = -4Y + 3$.

Ans/Sol.

Qn. 2.

Y_1 and Y_2 have a joint density function

$$f(y_1, y_2) = \begin{cases} e^{-(y_1+y_2)}, & 0 \leq y_1, \ 0 \leq y_2, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the density function for $Y_1 + Y_2$.

Ans/Sol.

Qn. 3.

The fracture strength of tempered glass averages 14 (measured in thousands of pounds per square inch) and has standard deviation 2.

(a) What is the probability that the average fracture strength of 100 randomly selected pieces of this glass exceeds 14.5?

(b) Find an interval that includes, with probability 0.95, the average fracture strength of 100 randomly selected pieces of this glass.

Ans/Sol.