Pacific Science Review A: Natural Science and Engineering 18 (2016) 190–202

HOSTED BY

ELSEVIER

Contents lists available at ScienceDirect

Pacific Science Review A: Natural Science and Engineering

journal homepage: www.journals.elsevier.com/pacific-science-review-a-natural-science-and-engineering/

Analysis of pricing decision for substitutable and complementary products with a common retailer

Raghu Nandan Giri*, Shyamal Kumar Mondal, Manoranjan Maiti

Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore 721102, India

ARTICLE INFO

Article history:
Received 2 September 2016
Accepted 24 September 2016
Available online 26 December 2016

Keywords:
Supply chain
Substitutable products
Complementary products
Stackelberg game

ABSTRACT

In this paper, a competition of selling two substitutable products and one complementary product has been studied in two-echelon supply chain systems in which one of these three products is produced by one manufacturer separately, and all produced items are sold through a common retailer in the market. The demand of each product depends linearly on prices of these three products as per their nature. In this study, four different decision scenarios are developed mathematically under the game theory framework to maximize the profit function of each participant of the supply chain, and a number of pricing strategies are subsequently worked out for manufacturers and retailer. Finally, the model under different scenarios is illustrated with numerical data to study the feasibility of the model exploring the managerial insights, as well. Different marketing policies are predicted for maximum individual and system profits.

1. Introduction

The concept of integrating business activities beyond the boundary of markets has led to develop the theory of supply chain management. In recent decades, many powerful retailers have appeared in the world and in size and capacity, and they are often much larger than the manufacturers and usually retail multiple substitutable and/or complementary products. Additionally, many manufactures have increased product varieties by differentiating one or several attributes of the products, or manufacturers have produced negative cross elastic products to get full utility of other products in order to compete for market share and profit gain. Substitute products means that a consumer considers the product to be similar or comparable; for example, the consumer might compare one brand of smart phone with another, or may compare something slightly different, such as coffee and tea or a laptop and a desktop. The complementary product is a product with a negative cross elasticity on demand in contrast to a substitute product, such as toothpaste and toothbrush or a desktop and operating system. A consumer has to buy a complementary item to get full utility of the corresponding main item. This case has recently gained interest by the researchers. In this scenario, the firms supplying the products to the market are coupled in the sense that their demands are interrelated. One firm's marketing decision can affect the other firm's market performance, and vice versa. There is a large body of literature in the area of substitute products or complementary products for cooperative and non-cooperative markets. In this area, McGillivray and Silver [13] first derived the optimal policy for substitute products during stock-out. Huang et al. [7] developed an algorithm for multi-product competitive newsboy shortage problem and partial product substitution. Parlar [16] analysed the inventory problem with two substitutable products having random demands using game theory concepts, and concluded that the players always gain if they cooperate and maximize a joint objective function. Zhang et al. [25] developed a one-manufacturer and one-retailer supply chain model for deteriorating items with controllable deterioration rate and price-dependent demand. Pineyro and Viera [17] maximized re-manufactured quantities for an Economic Order Quantity (EOQ) problem with product returns allowing one-way substitution. Maiti and Maiti [12], Stavrulaki [19], and Krommyda et al. [9] presented inventory models for substitutable products, and the items were substituted based on their inventory levels.

* Corresponding author.
E-mail addresses: raghunandan.giri86@gmail.com (R.N. Giri), shyamal_260180@yahoo.co.in (S.K. Mondal), mmaiti2005@yahoo.co.in (M. Maiti).

Peer review under responsibility of Far Eastern Federal University, Kangnam University, Dalian University of Technology, Kokushikan University.

http://dx.doi.org/10.1016/j.psra.2016.09.012

The most common assumption on the pricing/ordering decision process under game theory framework is that the manufacturer is a Stackelberg leader and the retailer is a Stackelberg follower in a two-echelon supply chain. McGuire and Staelin [14], and Gupta and Loulou [5] investigated the effect of product substitutability on non-cooperative distribution structures in a duopoly where each manufacturer distributes its goods through a single exclusive retailer. Choi's [3] model of product differentiation with two manufacturers selling to a common retailer showed that being a price leader is beneficial when demand is linear, but detrimental when demand is multiplicative. Tsay and Agrawal [20] studied a distribution system for a manufacturer and two retailers in which competition depends on price and service in a game theory framework. Lau and Lau [10] investigated a joint pricing model for a two-echelon system in the manufacturer-Stackelberg process, but in the absence of setup costs, they determined that under a downward-sloping price-versus-demand relationship, the manufacturer's profit is double that of the retailer's profit. Abad and Jaggi [1], and Ho et al. [6] formulated an integrated supplier–buyer inventory model with price sensitive demand, and where the supplier adopts a trade credit policy to determine the optimal pricing, shipment, and payment policy. Zhao et al. [26,27] formulated and analysed pricing strategies of substitute products in a supply chain with one manufacturer and two competitive retailers in crisp and fuzzy environments. Here, substitution was made based on retail prices of the products. Jiang et al. [8] considered information sharing of two firms that sell two substitute products under price competition, and showed that private signals are not perfectly correlated, and firms can benefit from sharing signals with each other. Zhang et al. [25] investigated the impact of consumer environmental awareness (CEA) on order quantities of one manufacturer and one retailer's supply chain with production capacity constraints, and showed that firms benefit from product customization and consumer segmentation based on CEA in the market. Li et al. [11] developed the joint ordering inventory games of multiple retailers who buy the same commodities from a supplier and are offered a permissible delay in payment. Their results showed that formation of a grand coalition of retailers is socially beneficial. Esmaeili et al. [4] proposed several cooperative and non-cooperative games for the seller–buyer coordination to optimize pricing and lot sizing decisions, while non-linearly demand depends on selling price and marketing expenditure. Raju and Roy [18] developed game theory models to understand how the intensity of competition affects the value of market information. They demonstrated that information is more beneficial in industries with fiercer competition, and has greater value for larger firms. Yue et al. [23] and Mukhopadhyay et al. [15] considered two separate firms for game theory models, which had private forecast information about market uncertainties and offered complementary goods. Yan and Bandyopadhyay [21] investigated the bundling of complementary products and showed that a firm can benefit from complementary bundling conditionally. Yan et al. [22] further investigated the strategic influence of product complementarity and advertising on the success of bundling products, and showed that when a firm sells bundled products, both the product complementarity and advertising have significantly impact the performance of bundled products. Yue et al. [23] showed that it is beneficial to share information in a Bertrand game, whereas Mukhopadhyay et al. [15] demonstrated that, in a Stackelberg game, information sharing could benefit the leader, but hurt the follower as well as the total profit. Bian et al. [2] found that information sharing significantly affects supply chain performance. If information is not shared correctly and accurately between manufacturers and retailers, it can make a firm's supply chain less reliable than that of the firm's competitors.

Thus, the applications of game theory to the supply chain, especially coordination, economic stability and the supply chain efficiency, have been discussed by a number of workers for either two substitutable products or two complementary products. This study investigates the cooperation and competition in a two-echelon supply-chain system where three manufacturers compete to sell two substitutable products and one complementary product using a common retailer in the market. The demands of the products generated by the consumers linearly depend on the prices of the products. Based on common sense, it has been seen that the market potentials (i.e., base demands) and manufacturing costs for different products are different. In this instance, one manufacturer separately manufactures one item. These concepts are introduced in the study of pricing strategies for four different marketing scenarios. We investigate the pricing strategies for different scenarios under asymmetric market potential and cost structure in the channel of the products both numerically and analytically. Additionally, we investigate the impact of price elasticity (i.e., price responses) in the pricing strategies of different scenarios, and explore the managerial insights. Finally, some discussions are drawn based on the numerical and analytical analyses.

Till now, no researcher has considered two substitutable items and one complementary item in the supply chain of four members in the context of game theories. The above literature survey and new features of the present investigation are presented in Table 1.

Table 1
Research papers on substitutable/complementary items.

Authors	No. of items	Nature of items	Substitute/complementary parameters	Solution method
McGillivray (1978)	2	Substitution during stock-out	Crisp	Traditional
Maity et al. (2009)	3	Complementary/substitutable	Crisp	Pontryagin principle
Yan et al. (2011)	2	Price-dependent complementary	Crisp	Traditional
Stavrulaki (2011)	2	Stock-dependent substitution	Crisp	Traditional
Yan et al. (2014)	2	Price-dependent complementary	Crisp	Traditional
Krommyda et al. (2015)	2	Stock-dependent substitution	Crisp	Traditional
Choi et al. (1991)	2	Price-dependent substitution	Crisp	Game theory
Gupta et al. (1998)	2	Price-dependent substitution	Crisp	Game theory
Raju et al. (2000)	2	Price-dependent substitution	Crisp	Game theory
Yue et al. (2006)	2	Price-dependent complementary	Crisp	Game theory
Mukhopadhyay (2011)	2	Price-dependent complementary	Crisp	Game theory
Zhao et al. (2012)	2	Price-dependent substitution	Fuzzy	Game theory
Jiang et al. (2014)	2	Price-dependent substitution	Stochastic	Game theory
Zhao et al. (2014)	2	Price-dependent substitution	Crisp	Game theory
Bian et al. (2014)	1	Independent	Crisp	Game theory
Zhang et al. (2015)	2	Price, environmental quality dependent substitution	Crisp	Game theory
Present investigation	3	Price-dependent substitutable and complementary	Crisp	Game theory

The rest of the paper is organized as follows. Section 2 contains some notations and assumptions, which are used to formulate the problems under different scenarios as discussed in Section 3. Section 4 presents a numerical experiment and the impact of parameters on pricing strategies as well as observations. The last section presents the study's conclusions and possible future research.

2. Model

We consider a two-echelon supply-chain system where three manufacturers compete to sell two substitutable products and one complementary product using a common retailer in the market. The demands of the products generated by the consumers linearly depend on the prices of the products. Each manufacture produces only one product. Let i-th manufacturer $M_i(i = 1, 2, 3)$ produces product i and sells to the common retailer, retailer sells to the consumer in order to derive their own pricing strategies so that the total chain profit and individual (i.e., manufacturers and retailer) profits are at a maximum. In this model, we consider products that 1 and 2 are substitutable for each other and product 3 is complementary for both products 1 and 2, and the consumers' demands are based on retail prices.

Lau and Lau (2003) showed that in many cases a small change in demand may result in a major change in decision. As a result, a manager has to investigate the factors that influence the demand because customers' purchasing behaviour may be affected by a variety of factors, such as selling price, inventory level, and seasonality. Here, the demand of each product depends on its own price, and the prices of the other products (cf. McGuire and Staelin (1983), Gupta and Loulou (1998)). The main objective in this investigation is to decide the pricing strategies of products 1, 2, and 3 so that members of the system may get the optimal profit under different marketing scenarios.

The following notations and assumptions have been used in the model formulation:

2.1. Notations

i: $(i = 1,2,3)$ the product index.
c_i: Unit manufacturing cost of product i.
p_i: Manufacturers' unit selling price (i.e., decision variable) of product i.
s_i: Retailer unit selling price (i.e., decision variable) of product i.
m_i: Price margin enjoyed by retailer for product i.
d_i: Customers' demand for product i.
π_{M_i}: Profit function of i-th (i.e., $i = 1, 2, 3$) manufacturer.
π_r: Profit function of the retailer.
π_c: Profit function of the whole chain.

2.2. Assumptions

(i) All activities are performed within a single period.
(ii) The supply chain system consists of three manufacturers and one retailer. Among the three manufacturers, each of them will produce one item among three items with two substitutable products and one complementary product of two substitutable items.
(iii) The demand of each product is interrelated by the attributed price. Here, it is assumed that the demand functions depend on the retail prices of three products linearly. It is noted that the demand functions of substitutable products decrease with respect to its own price and the price of complementary product, but increase with respect to the price of another substitutable product, which is its competitor, and the demand function of complementary product decreases with respect to its own and other products' prices. Therefore, the demands of two substitutable products that is product-1 and product-2, and one complementary product that is product-3 are assumed in the following forms:

$$d_1(s_1, s_2, s_3) = a_1 - \beta s_1 + \gamma s_2 - \delta s_3 \tag{2.1}$$

$$d_2(s_1, s_2, s_3) = a_2 + \gamma s_1 - \beta s_2 - \delta s_3 \tag{2.2}$$

$$d_3(s_1, s_2, s_3) = a_3 - \delta s_1 - \delta s_2 - \beta s_3 \tag{2.3}$$

where a_1, a_2 and a_3 are base (i.e., market potential) consumer demands (i.e., when free from prices) of the products- 1, 2 and 3, respectively, β is demand sensitivity with respect to (w. r. t.) its own price, γ is demand sensitivity w. r. t. other substitutable product price, and δ is the demand sensitivity with respect to the complementary product. The price sensitivity w. r. t. own price is greater then the other substitutable product price, and both demands' sensitivities w. r. t. own price and w. r. t. other substitutable product price are much greater then that of the complementary product price i.e., $\beta \geq \gamma \geq \delta$.

(iv) In the real world of business, it is seen that the inventory related costs and logistical costs have less effect on profit function. In this sense, profit has been considered, including only revenue and manufacturing/purchasing costs of the product.

3. Mathematical formulations

Lemma-1: *The price of product 3 is more sensitive than the prices of products 1 and 2 in the market.*

Prof. *The total demand of three products in the market is*

$$D = a_1 + a_2 + a_3 - (\beta - \gamma + \delta)(s_1 + s_2) - (\beta + 2\delta)s_3.$$

here, $(\beta - \gamma + \delta) < (\beta + 2\delta)$ as $\beta \geq \gamma \geq \delta \geq 0$. *Therefore, the price of product 3 has more effect on the total demand in the market than the other product prices as* $(\beta + 2\delta) > (\beta - \gamma + \delta)$.

The profit functions of manufacturers M_1, M_2 and M_3 are, respectively, denoted by $\pi_{M_1}(p_1, p_2, p_3)$, $\pi_{M_2}(p_1, p_2, p_3)$ and $\pi_{M_3}(p_1, p_2, p_3)$ and are given by

$$\pi_{M_1}(p_1, p_2, p_3) = (p_1 - c_1)\{a_1 - \beta(p_1 + m_1) + \gamma(p_2 + m_2) - \delta(p_3 + m_3)\} \tag{3.1}$$

$$\pi_{M_2}(p_1, p_2, p_3) = (p_2 - c_2)\{a_2 + \gamma(p_1 + m_1) - \beta(p_2 + m_2) - \delta(p_3 + m_3)\} \tag{3.2}$$

$$\pi_{M_3}(p_1, p_2, p_3) = (p_3 - c_3)\{a_3 - \delta(p_1 + m_1) - \delta(p_2 + m_2) - \beta(p_3 + m_3)\} \tag{3.3}$$

where m_i (i = 1, 2, 3) are the price margin enjoyed by retailer i.e., $m_i = s_i - p_i$. The retailer's profit function, denoted by $\pi_r(s_1, s_2, s_3)$ is given as

$$\pi_r(s_1,s_2,s_3) = (s_1-p_1)(a_1-\beta s_1+\gamma s_2-\delta s_3)+(s_2-p_2)(a_2+\gamma s_1-\beta s_2-\delta s_3)+(s_3-p_3)(a_3-\delta s_1-\delta s_2-\beta s_3) \tag{3.4}$$

The profit function of the supply chain, denoted by $\pi_c(s_1,s_2,s_3)$ is given as

$$\pi_c(s_1,s_2,s_3) = (s_1-c_1)(a_1-\beta s_1+\gamma s_2-\delta s_3)+(s_2-c_2)(a_2+\gamma s_1-\beta s_2-\delta s_3)+(s_3-c_3)(a_3-\delta s_1-\delta s_2-\beta s_3) \tag{3.5}$$

To derive the equilibrium (i.e., optimal) pricing strategies of manufacturers and retailer, we consider four different decision models such as the Centralized Decision (CD) model, the Manufacturer-leadership Stackelberg (MS) game model, the Retailer-leadership Stackelberg (RS) game model, the Nash game model so that they maximize their own profit function.

3.1. Centralized decision (CD) model

From an operational perspective, researchers focus on coordination mechanisms that can align the objective of individual channel members. Consider a prototype of two-echelon supply chain where a group of suppliers sells the products to a buyer or retailer. Here, three manufacturers' and one retailer in cooperation conduct the supply chain operation. Manufacturer M_i sells the product i to the retailer with wholesale price $p_i (i=1,2,3)$, and then retailer sells to the end customers' with retail price $s_i (i=1,2,3)$. In this supply chain, profit expression does not involve wholesale prices $p_i (i=1,2,3)$, so it does not affect the supply chain profit; instead, it only influences the profit of the individual participant. The integrated system tries to maximize the total profit expression $\pi_c(s_1,s_2,s_3)$ of the chain. The corresponding model can be formulated as follows:

$$\begin{cases} \max_{(s_1,s_2,s_3)} \pi_c(s_1,s_2,s_3) \\ \textit{subject to} \\ d_i > 0 \\ s_i > c_i, \quad i=1,2,3. \end{cases} \tag{3.6}$$

Theorem 1. *The profit function $\pi_c(s_1,s_2,s_3)$ of the supply chain given in* Eq. (3.5) *is concave in (s_1,s_2,s_3) if $2\delta^2+\beta\gamma<\beta^2$.*

The proof is given in Appendix A.

Theorem 2. *If the conditions*

$$\begin{gathered} a_1+\gamma c_2>\beta c_1+\delta c_3, a_2+\gamma c_1>\beta c_2+\delta c_3, a_3>\delta(c_1+c_2)+\beta c_3, \\ a_1\left(\beta^2-\delta^2\right)+a_2\left(\beta\gamma+\delta^2\right)>a_3(\beta+\gamma)\delta+c_1(\beta+\gamma)\left(\beta^2-\beta\gamma-2\delta^2\right), \\ a_1\left(\beta\gamma+\delta^2\right)+a_2\left(\beta^2-\delta^2\right)>a_3(\beta+\gamma)\delta+c_2(\beta+\gamma)\left(\beta^2-\beta\gamma-2\delta^2\right), \\ \textit{and } a_3(\beta-\gamma)>(a_1+a_2)\delta+c_3\left(\beta^2-\beta\gamma-2\delta^2\right). \end{gathered}$$

hold, then the optimal retail prices s_1^*, s_2^* and s_3^* are given in Eqs. (A.3)–(A.5), and the optimal chain profit is obtained by the substitution of s_1^*, s_2^* and s_3^* *in* Eq. (3.5).

The proof is given in Appendix A.

3.2. Manufacturer-leadership Stackelberg(MS) game model

In this case, we consider the retailers' size (i.e., capacity) is very small in comparison to the manufacturers so the three manufacturers are the leaders and the common retailer is the follower in the market. Each manufacturer first announces his wholesale price without knowing each other's prices. The retailer observes the manufacturers' wholesale prices and reacts rationally to the leader's decisions. We assume that every participant in the chain knows the complete information about the demands of the customers', which are generated at the retailer's level. The objective is to maximize individual participant's profit expression. The problem is formulated for MS game as

$$\begin{cases} \max_{p_i} \pi_{M_i}\left(p_i, s_1^*(p_1), s_2^*(p_2), s_3^*(p_3)\right), \; i=1,2,3. \\ \textit{subject to} \\ p_i > c_i, \; i=1,2,3. \\ \textit{and } s_1^*(p_1), s_2^*(p_2), s_3^*(p_3) \textit{ are obtained from solving the problem} \\ \begin{cases} \max_{(s_1,s_2,s_3)} \pi_r(s_1,s_2,s_3) \\ \qquad \textit{subject to} \\ \qquad\quad d_i>0 \\ s_i > p_i, \quad i=1,2,3. \end{cases} \end{cases} \tag{3.7}$$

Theorem 3.

(i) *The retailer profit expression $\pi_r(s_1,s_2,s_3)$ in MS game of the chain is concave in (s_1,s_2,s_3) if $2\delta^2+\beta\gamma<\beta^2$.*

(ii) *On the basis of individual announcement of manufactures' selling prices (p_1, p_2, p_3), the retailer fixes the optimal selling prices (s_1^*, s_2^*, s_3^*). Thus, the optimal selling prices s_1^*, s_2^* and s_3^* in MS game are given in* Eqs. (B.2)–(B.4), *provided that the following conditions hold*

$$\begin{gathered} a_1+\gamma p_2>\beta p_1+\delta p_3, a_2+\gamma p_1>\beta p_2+\delta p_3, a_3>\delta(p_1+p_2)+\beta p_3, \\ a_1\left(\beta^2-\delta^2\right)+a_2\left(\beta\gamma+\delta^2\right)>a_3(\beta+\gamma)\delta+p_1(\beta+\gamma)\left(\beta^2-\beta\gamma-2\delta^2\right), \\ a_1\left(\beta\gamma+\delta^2\right)+a_2\left(\beta^2-\delta^2\right)>a_3(\beta+\gamma)\delta+p_2(\beta+\gamma)\left(\beta^2-\beta\gamma-2\delta^2\right), \\ \textit{and } a_3(\beta-\gamma)>(a_1+a_2)\delta+p_3\left(\beta^2-\beta\gamma-2\delta^2\right) \end{gathered}$$

(iii) *After knowing the retailer's optimal prices (s_1^*, s_2^*, s_3^*), manufacturers then take the decision on p_1^*, p_2^* and p_3^* and the optimal wholesale prices (p_1^*, p_2^*, p_3^*) of the manufacturers' in MS game are given by* Eqs. (B.9)–(B.11)

The proof is given in Appendix B.

3.3. Retailer-leadership Stackelberg (RS) game model

In this case, we consider that the retailer's size and capacity is very large in comparison to the manufacturers so the retailer first announces his retail prices, then the manufacturer M_i chooses the wholesale price of product *i* based on the announced retail prices of the retailer. We assume that every participant in the chain knows the complete information about the demands of the customers. The retailer wants to maximize his/her own profit after learning about the information of the three manufacturers' strategies. The problem formulation for this case is

$$\begin{cases} \max_{(s_1,s_2,s_3)} \ \pi_r(s_1,s_2,s_3,p_1^*,p_2^*,p_3^*) \\ \textit{subject to} \\ d_i > 0 \\ s_i > p_i^*, \ i=1,2,3. \\ \textit{and } p_1^*(s_1,s_2,s_3), p_2^*(s_1,s_2,s_3), p_3^*(s_1,s_2,s_3) \textit{ are obtained from solving the problem} \\ \begin{cases} \max_{p_i} \ \pi_{M_i}(p_1,p_2,p_3), \ i=1,2,3. \\ \qquad \textit{subject to} \\ \qquad p_i > c_i, \ i=1,2,3. \end{cases} \end{cases} \tag{3.8}$$

Theorem 4. *In the RS game model*

(i) *The retailer's profit expression is concave if* $2\beta^2+\gamma^2+2\delta^2>3\beta\gamma, \ 2\delta^2+\beta\gamma<\beta^2$ *and* $\delta^2+\beta\gamma<2\beta^2$.
(ii) *The optimal manufacturers' pricing strategies* (p_1^*, p_2^*, p_3^*) *are given in* Eqs. (C.2)–(C.4).
(iii) *The retailer optimal price strategies* (s_1^*, s_2^*, s_3^*) *are given in* Eq. (C.7) *if the following conditions are hold*

$$a_1+\gamma s_2^*>\beta s_1^*+\delta s_3^*, a_2+\gamma s_1^*>\beta s_2^*+\delta s_3^*, a_3>(s_1^*+s_2^*)\delta+\beta s_3^*$$

The proof is provided in Appendix C.

3.4. Nash game model

In this case, we consider that the manufacturers and retailer are of same size (i.e., medium) in the market, meaning that they have the same bargaining power and thus make their decisions simultaneously. As a result, the manufacturers cannot dominate the retailer, and the retailer cannot dominate the manufacturers in the market. Therefore, Nash-equilibrium strategies can be employed to derive the optimal prices and profit for each of the participants of the market. We assume that every participant in the chain knows the complete information about the demands of the customers. Therefore, the problem is formulated as

$$\begin{cases} \max_{(s_1,s_2,s_3)} \pi_r(s_1,s_2,s_3) \\ \max_{p_i} \pi_{M_i}(p_i,s_1,s_2,s_3), i=1,2,3. \\ \textit{subject to} \\ \quad d_i>0 \\ \quad s_i>p_i \\ \quad p_i>c_i, \ \ i=1,2,3. \end{cases} \tag{3.9}$$

Theorem 5. *In the Nash game model,*

(i) *The retailer's profit expression is concave if* $2\delta^2+\beta\gamma<\beta^2$.
(ii) *The optimal manufacturers' pricing strategies* (p_1^*, p_2^*, p_3^*) *are given in* Eqs. (D.1)–(D.3).
(iii) *The retailer optimal prices strategies* (s_1^*, s_2^*, s_3^*) *are given in* Eq. (D.4)–(D.6).

The proof is provided in Appendix D.

3.5. Particular case (manufacturer M_3 absent)

We obtained the particular scenarios for the problems (3.6)–(3.9), by putting $\delta=0$ and omitted the third product P_3 from the supply chain. Then, the corresponding pricing Eqs. (A.3), (A.4), (B.2), (B.3), (B.9), (B.10), (C.2), (C.3), (C.7), (D.1), (D.2), (D.4), and (D.5) are reduced to the equations presented in Tables 2 and 3. The results for the asymmetric information of the two substitute products is shown in Table 2, and Table 3 contains the results for symmetric information about the two substitute products. The results presented in Tables 2 and 3 are the same as in J. Zhao et al. (2014).

4. Numerical illustrations

To explore more managerial insights in different scenarios, we consider numerical and parametric studies under different scenarios.

In CD model, $\frac{\partial s_i^*}{\partial c_j}=\frac{1}{2}$ *for* $i=j$ and 0*for* $i\neq j(i,j=1,2,3)$.

Observation-1. *In CD scenario, the optimal selling price* (s_i^*) *of product i increases if its respective costs* (c_i) *increases, but not affected by the other products' costs.*

Table 2
Results for particular scenarios of asymmetric information.

Models	Retailer prices	Manufacturers prices
CD model	$s_1^*=\frac{c_1}{2}+\frac{a_1\beta+a_2\gamma}{2(\beta^2-\gamma^2)}$	–
	$s_2^*=\frac{c_2}{2}+\frac{a_1\gamma+a_2\beta}{2(\beta^2-\gamma^2)}$	–
MS game	$s_1^*=\frac{p_1^*}{2}+\frac{a_1\beta+a_2\gamma}{2(\beta^2-\gamma^2)}$	$p_1^*=\frac{2\beta(a_1+c_1\beta)+\gamma(a_2+c_2\beta)}{4\beta^2-\gamma^2}$
	$s_2^*=\frac{p_2^*}{2}+\frac{a_1\gamma+a_2\beta}{2(\beta^2-\gamma^2)}$	$p_2^*=\frac{\gamma(a_1+c_1\beta)+2\beta(a_2+c_2\beta)}{4\beta^2-\gamma^2}$
RS game	$s_1^*=\frac{3\beta\gamma A_1+(2\beta^2+\gamma^2)A_2}{2(4\beta^4-5\beta^2\gamma^2+\gamma^4)}$	$p_1^*=\frac{1}{\beta}(\beta c_1+a_1-\beta s_1^*+\gamma s_2^*)$
	$s_2^*=\frac{(2\beta^2+\gamma^2)A_1+3\beta\gamma A_2}{2(4\beta^4-5\beta^2\gamma^2+\gamma^4)}$	$p_2^*=\frac{1}{\beta}(\beta c_2+a_2+\gamma s_1^*-\beta s_2^*)$
Nash game	$s_1^*=\frac{p_1^*}{2}+\frac{a_1\beta+a_2\gamma}{2(\beta^2-\gamma^2)}$	$p_1^*=\frac{3\beta(a_1+2c_1\beta)+\gamma(a_2+2c_2\beta)}{9\beta^2-\gamma^2}$
	$s_2^*=\frac{p_2^*}{2}+\frac{a_1\gamma+a_2\beta}{2(\beta^2-\gamma^2)}$	$p_2^*=\frac{\gamma(a_1+2c_1\beta)+3\beta(a_2+2c_2\beta)}{9\beta^2-\gamma^2}$
	* $A_1=-2\gamma a_1+3\beta a_2-\beta(\gamma c_1+\beta c_2)$	* $A_2=3\beta a_1-2\gamma a_2+\beta(\beta c_1-\gamma c_2)$

Table 3
Results for particular scenarios of symmetric information.

Models	Retailer prices	Manufacturers prices
CD model	$s_1^* = \frac{c}{2} + \frac{a}{2(\beta-\gamma)}$	–
	$s_2^* = \frac{c}{2} + \frac{a}{2(\beta-\gamma)}$	–
MS game	$s_1^* = \frac{a+c\beta}{2(2\beta-\gamma)} + \frac{a}{2(\beta-\gamma)}$	$p_1^* = \frac{a+c\beta}{2\beta-\gamma}$
	$s_2^* = \frac{a+c\beta}{2(2\beta-\gamma)} + \frac{a}{2(\beta-\gamma)}$	$p_2^* = \frac{a+c\beta}{2\beta-\gamma}$
RS game	$s_1^* = \frac{c\beta}{2(2\beta-\gamma)} + \frac{a(3\beta-2\gamma)}{2(\beta-\gamma)(2\beta-\gamma)}$	$p_1^* = \frac{a}{2(2\beta-\gamma)} + \frac{c(3\beta-\gamma)}{2(2\beta-\gamma)}$
	$s_2^* = \frac{c\beta}{2(2\beta-\gamma)} + \frac{a(3\beta-2\gamma)}{2(\beta-\gamma)(2\beta-\gamma)}$	$p_2^* = \frac{a}{2(2\beta-\gamma)} + \frac{c(3\beta-\gamma)}{2(2\beta-\gamma)}$
Nash game	$s_1^* = \frac{a+2c\beta}{2(3\beta-\gamma)} + \frac{a}{2(\beta-\gamma)}$	$p_1^* = \frac{a+2c\beta}{3\beta-\gamma}$
	$s_2^* = \frac{a+2c\beta}{2(3\beta-\gamma)} + \frac{a}{2(\beta-\gamma)}$	$p_2^* = \frac{a+2c\beta}{3\beta-\gamma}$

The effects of a_i in the optimal pricing strategies of CD model, we have $\frac{\partial s_i^*}{\partial a_i} > 0\, for\ i = 1, 2, 3$; $\frac{\partial s_i^*}{\partial a_j} > 0\ when\ i \neq j, i, j = 1, 2$; $\frac{\partial s_i^*}{\partial a_3} < 0\ and$ $\frac{\partial s_3}{\partial a_i} < 0 for\ i = 1, 2$.

Observation-2. *In CD scenario,*

(a) *The optimal selling price(s_i^*) of product i, (=1, 2, 3) increases if its respective base market demands (a_i) increases.*
(b) *The optimal selling prices and based market demands of two substitute products are positively correlated.*
(c) *The optimal selling prices of two substitute products and one complementary product are negatively correlated with the complementary product's base market demand and substitute products' base market demands, respectively.*

In the MS game, we have the following effects of c_i and a_i on the optimal wholesale prices: $\frac{\partial p_i^*}{\partial c_i} > 0\, for\ i = 1, 2, 3$; $\frac{\partial p_i^*}{\partial c_j} > 0$ $when\ i \neq j, i, j = 1, 2$; $\frac{\partial p_i^*}{\partial c_3} < 0\ and\ \frac{\partial p_3^*}{\partial c_i} < 0 for\ i = 1, 2$. Additionally, $\frac{\partial p_i^*}{\partial a_j} = \frac{1}{\beta} \frac{\partial p_i^*}{\partial c_j}\ for\ i, j = 1, 2, 3$. and the effects on retail prices: $\frac{\partial s_i^*}{\partial c_j} = \frac{1}{2} \frac{\partial p_i^*}{\partial c_j}\ for\ i, j = 1, 2, 3.$; $\frac{\partial s_i^*}{\partial a_i} > 0, i = 1, 2, 3$; $\frac{\partial s_i^*}{\partial a_j} > 0\ when$ $i \neq j, i, j = 1, 2$; $\frac{\partial s_i^*}{\partial a_3} < 0\ and\ \frac{\partial s_3^*}{\partial a_i} < 0\ for\ i = 1, 2$.

Observation-3. *In MS game scenario,*

(a) *The optimal wholesale prices (p_i^*) of product i (=1, 2, 3) increases if its respective costs (c_i) and base market demand (a_i) increases.*
(b) *The optimal wholesale prices of two substitute products are positively correlated with their costs and base market demands.*
(c) *The optimal wholesale prices of two substitute products and one complementary product are negatively correlated with the complementary product costs and their base market demands, and substitute products' costs and base market demands, respectively.*
(d) *The effects of costs and base market demands on the optimal retail pricing strategies are the same as the effects on the optimal wholesale prices of the manufacturers.*

Experiment: Three manufacturers' M_1, M_2 and M_3 produce three products, including a desktop, a laptop, and a printer with production costs \$30, \$35, and \$25, respectively, and they sell to a common retailer. The retailer sells to the end customers at the rate of demand with the following parametric values: $a_1 = 100,\ a_2 = 110,\ a_3 = 150,\ \beta = 0.4,\ \gamma = 0.15,\ and\ \delta = 0.1$ in (2.1)–(2.3). Ignoring transportation and inventory costs, find the optimal wholesale prices of the manufacturers and optimal prices of the retailer in the cooperative and non-cooperative manner.

With these input data, we solve and obtain the optimal pricing strategies for different scenarios and those are presented in Table 4.

Observation-4. *Based on the results in Table 4, we observe the following:*

(a) *The profit of the whole chain under CD scenario is higher than the other decision scenarios, including Nash, RS and MS games, respectively. This is because of the increased total demand, resulting from lower retail prices dominating over the loss of revenue per unit due to the lower retail price. Hence, the whole chain and the consumers are better off when no channel member in a dominant position.*
(b) *The profits of the whole chain in two Stackelberg game scenarios are not equal. This is because the retail prices and demands in MS and RS game scenarios are different.*
(c) *For the common retailer, lowest retail prices are obtained in Nash game scenario among MS, RS, and Nash games, and the price margin enjoyed by retailer is highest in the RS game than the Nash and MS games. The results achieve the highest profit in the RS game than Nash and MS games.*
(d) *For the manufactures, lowest wholesale price is obtained in the RS game scenario among MS, RS, and Nash games, and the wholesale price margin enjoyed by manufactures is highest in the MS game than the Nash and RS games. The results achieved the highest profit in the MS game than Nash and RS games.*
(e) *In the non-cooperative market, the party who has more power plays as a leader in the supply chain, and has the advantage to get the higher profit.*

The expressions of pricing strategies in the RS game are very complex. Hence, the effects of a_i's and c_i's on the optimal pricing strategies are explored graphically in Figs. 1–6, respectively, using the data sets $a_1 = \{80, 90, 100, 110, 120\}$, $c_1 = \{20, 25, 30, 35, 40\}$, $a_2 = \{90, 100, 110, 120, 130\}$, $c_2 = \{25, 30, 35, 40, 45\}$, $a_3 = \{130, 140, 150, 160, 170\}$ and $c_3 = \{15, 20, 25, 30, 35\}$. The other parameters' values remain unchanged as in the Experiment.

Observation-5. *From the Figs.1, 3 and 5, we observe the RS game as,*

(a) *The optimal wholesale prices (p_i^*) and selling prices (s_i^*) of the product i (=1, 2, 3) increase if their respective base market demands (a_i) increase and, as a result, the whole chain profit quadratically increases.*
(b) *The optimal wholesale prices and selling prices of two substitute products are positively correlated with their base market demands.*

Table 4
Optimal results and pricing strategies under different scenarios.

Scenarios	s_1^*	p_1^*	$s_1^* - p_1^*$	s_2^*	p_2^*	$s_2^* - p_2^*$	s_3^*	p_3^*	$s_3^* - p_3^*$	$TP_{M_1}^*$	$TP_{M_2}^*$	$TP_{M_3}^*$	TP_{Ms}^*	TP_r^*	TP_c^*
CD	179.2	_	_	190.8	_	_	115.6	_	_	_	_	_	_	_	20453.3
MS	239.4	150.5	89.0	254.8	163.1	91.7	183.5	160.8	22.7	2903.3	3282.7	3688.3	9874.3	5111.5	14985.9
RS	226.4	92.7	133.7	241.9	101.5	140.3	206.1	76.8	129.3	1573.4	1770.6	1073.8	4417.8	9769.2	14187.0
Nash	218.1	107.9	110.3	232.4	118.2	114.2	164.5	122.8	41.7	2406.0	2750.8	3702.5	8859.3	8758.3	17617.6

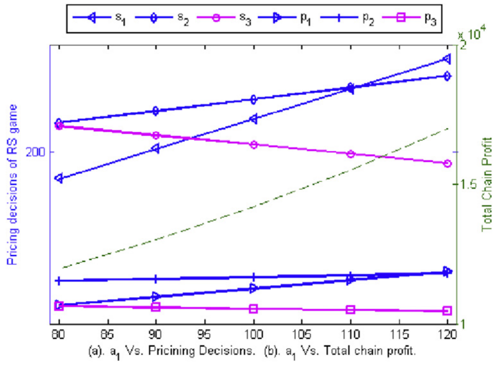

Fig. 1. Pricing decisions and total chain profit vs. a_1.

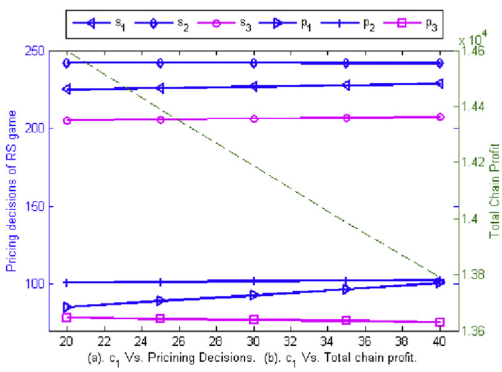

Fig. 2. Pricing decisions and total chain profit vs. c_1.

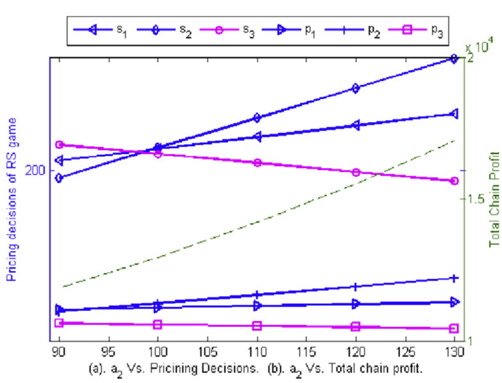

Fig. 3. Pricing decisions and total chain profit vs. a_2.

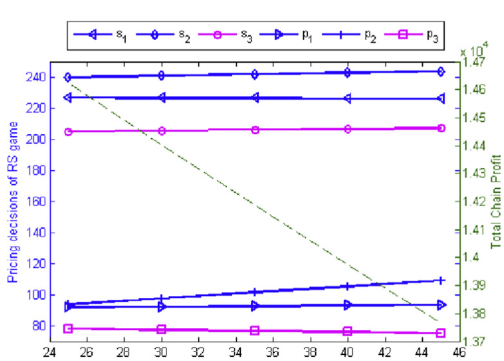

Fig. 4. Pricing decisions and total chain profit vs. c_2.

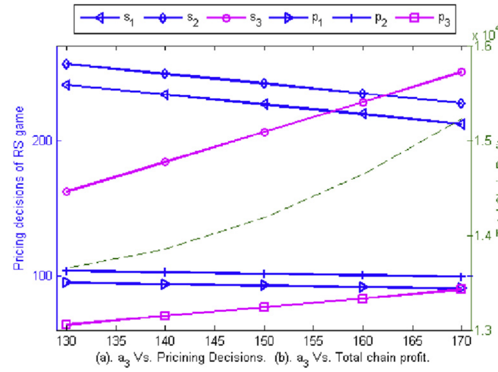

Fig. 5. Pricing decisions and total chain profit vs. a_3.

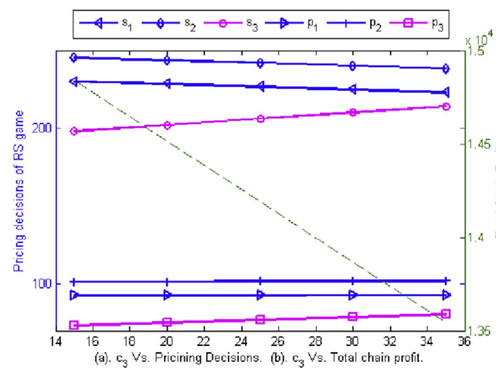

Fig. 6. Pricing decisions and total chain profit vs. c_3.

(c) *The optimal wholesale prices and selling prices of two substitute products and one complementary product are negatively correlated with the complementary product's base market demand and substitute products' base market demands, respectively.*

Observation-6. *From the Figs. 2, 4 and 6, we observe in RS game as,*

(a) *The optimal wholesale prices* (p_1^*, p_2^*) *and selling prices* (s_i^*, s_3^*) *increase and wholesale price* (p_3^*) *and selling prices* $(s_{(3-i)}^*)$ *decrease as* $c_i, (i = 1, 2)$ *increases.*
(b) *The optimal wholesale prices* (p_1^*, p_2^*, p_3^*) *and selling prices* (s_3^*) *increase and selling prices* (s_1^*, s_2^*) *decrease as* c_3 *increases.*
(c) *If costs* (c_i) *increases, the optimal chain profit linearly decreases.*

Insight: *The effects of base market demands in the pricing strategies of the RS and MS games are same, but the effects of the costs of products are not same for the RS and MS games.*

To study the effects of price elasticity parameters (β, γ and δ) in the pricing strategies of the MS and RS games, which are presented in the Tables 5–7, we consider the parametric values to be the same as in the Experiment. The only changes include $\beta = 0.6$ for Table 6 and $\beta = 0.6, \gamma = 0.2$ for Table 7.

Observation-7. *The effects of price elasticity in the optimal pricing strategies of the MS and RS games presented in Tables 5–7, are as follows:*

(a) *The optimal wholesale prices* (p_1^*, p_2^*, p_3^*) *and selling prices* (s_1^*, s_2^*, s_3^*) *decrease with increase of self-pricing responsibility* (β) *and, as a result, the chain profit decreases.*

Table 5
Effects of β in optimal pricing strategies of the MS and RS games.

β	MS game							RS game						
	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*
0.35	298.9	315.8	177.7	172.8	186.6	175.5	18189.3	274.5	291.4	213.0	105.3	114.7	78.9	17135.5
0.40	239.5	254.9	183.5	150.5	163.1	160.8	14985.9	226.5	241.9	206.1	92.7	101.5	76.8	14187.1
0.45	203.0	217.2	178.4	133.9	145.6	148.1	12827.4	195.2	209.4	194.3	83.6	92.0	74.1	12198.8
0.50	177.7	190.8	170.2	121.1	131.9	137.2	11210.5	172.7	185.8	181.9	76.7	84.6	71.4	10709.5
0.55	158.7	170.9	161.4	110.8	121.0	127.8	9929.6	155.4	167.6	170.3	71.2	78.8	68.7	9529.2
0.60	143.9	155.3	152.8	102.4	112.0	119.6	8879.7	141.6	153.0	159.7	66.8	74.1	66.3	8560.5

Table 6
Effects of γ in optimal pricing strategies of the MS and RS games.

γ	MS game							RS game						
	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*
0.10	130.8	142.9	156.7	97.2	107.2	120.5	8140.9	128.8	140.9	163.4	64.2	71.7	66.7	7813.8
0.15	143.9	155.3	152.8	102.4	112.0	119.6	8879.7	141.6	153.0	159.7	66.8	74.1	66.3	8560.5
0.20	160.0	170.9	148.0	108.0	117.3	118.7	9852.0	157.3	168.2	155.0	69.6	76.7	65.8	9541.5
0.25	180.5	190.8	141.8	114.2	123.1	117.7	11168.1	177.3	187.7	148.9	72.7	79.7	65.3	10867.7
0.30	207.7	217.6	133.3	121.0	129.7	116.6	13020.2	203.9	213.8	140.7	76.2	83.0	64.7	12731.2

Table 7
Effects of δ in optimal pricing strategies of the MS and RS games.

δ	MS game							RS game						
	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*
0.00	188.1	199.0	193.8	119.9	129.1	137.5	14586.3	188.1	199.0	197.9	74.9	82.1	77.1	14294.6
0.05	171.7	182.6	169.2	113.5	122.8	127.7	11873.5	170.7	181.6	174.4	72.0	79.1	71.2	11569.7
0.10	160.0	170.9	148.0	108.0	117.3	118.7	9852.0	157.3	168.2	155.0	69.6	76.7	65.8	9541.5
0.15	152.4	163.3	128.3	103.3	112.6	110.5	8325.5	147.1	158.0	138.1	67.7	74.9	60.6	8007.3
0.20	149.6	160.5	107.7	99.3	108.6	102.9	7188.8	139.6	150.5	122.7	66.3	73.5	55.6	6847.3

(b) *The optimal wholesale prices* (p_1^*, p_2^*) *and selling prices* (s_1^*, s_2^*) *increase and wholesale price* (p_3^*) *and selling price* (s_3^*) *decrease as the cross price responsibility of substitute product* (γ) *increases and, as a result, the chain profit increases.*
(c) *The optimal wholesale prices* (p_1^*, p_2^*, p_3^*) *and selling prices* (s_1^*, s_2^*, s_3^*) *decrease as the cross price responsibility of complementary product* (δ) *increases and, as a result, the chain profit decreases.*

To study the effects of the base demands (a_i), costs (c_i) and price elasticity parameters (β, γ and δ) in the optimal pricing strategies of the Nash game, which are presented in Tables 8–13, we consider the parametric values to be the same as in the Experiment, but the only change is $\beta = 0.6$ for Table 12 and $\beta = 0.6, \gamma = 0.2$ for Table 13.

Observation-8. *From* Tables 8–10, *we observe the Nash game as follows,*

(a) *The optimal wholesale prices* (p_i^*) *and selling prices* (s_i^*) *of the product i (=1, 2, 3) increase if their respective base market demands* (a_i) *and costs* (c_i) *increase.*
(b) *The optimal wholesale prices and selling prices of the two substitute products are positively correlated with their base market demands and costs.*
(c) *The optimal wholesale prices and selling prices of the two substitute products and one complementary product are negatively correlated with the complementary product's base market demand and cost and substitute products' base market demands and costs, respectively.*
(d) *If base market demands* (a_i) *and costs* (c_i) *increase, the optimal chain profit increases and decreases, respectively.*

Observation-9. *The effects of price elasticity in the optimal pricing strategies of the Nash games presented in* Tables 11–13 *are as follows:*

(a) *The optimal wholesale prices* (p_1^*, p_2^*, p_3^*) *and selling prices* (s_1^*, s_2^*, s_3^*) *decrease with the increase of self-pricing responsibility* (β) *and, as a result, each member's profit of the chain and the whole chain profit decrease.*
(b) *The optimal wholesale prices* (p_1^*, p_2^*) *and selling prices* (s_1^*, s_2^*) *increase and wholesale price* (p_3^*) *and selling price* (s_3^*) *decrease*

Table 8
Effects of a_1, c_1 in optimal pricing strategies of Nash game.

a_1	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*	c_1	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*
80	175.5	215.4	177.8	90.8	116.0	124.4	15460.4	20	214.7	232.0	164.9	101.1	117.3	123.5	18308.8
90	196.8	223.9	171.2	99.3	117.1	123.6	16598.9	25	216.4	232.2	164.7	104.5	117.8	123.2	18099.7
100	218.1	232.4	164.5	107.9	118.3	122.8	17895.0	30	218.1	232.4	164.5	107.9	118.3	122.8	17895.0
110	239.5	240.9	157.9	116.4	119.4	122.0	19348.9	35	219.9	232.7	164.4	111.3	118.7	122.5	17694.6
120	260.8	249.5	151.2	125.0	120.5	121.2	20960.4	40	221.6	232.9	164.2	114.7	119.2	122.2	17498.4

Table 9
Effects of a_2, c_2 in optimal pricing strategies of the Nash game.

a_2	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*	c_2	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*
90	201.1	189.8	177.8	105.6	101.2	124.4	15350.5	25	217.7	229.0	164.9	107.0	111.4	123.5	18339.0
100	209.6	211.1	171.2	106.7	109.7	123.6	16543.9	30	217.9	230.7	164.7	107.4	114.8	123.2	18114.9
110	218.1	232.4	164.5	107.9	118.3	122.8	17895.0	35	218.1	232.4	164.5	107.9	118.3	122.8	17895.0
120	226.7	253.7	157.9	109.0	126.8	122.0	19403.8	40	218.4	234.1	164.4	108.3	121.7	122.5	17679.5
130	235.2	275.1	151.2	110.2	135.3	121.2	21070.4	45	218.6	235.8	164.2	108.8	125.1	122.2	17468.2

Table 10
Effects of a_3, c_3 in optimal pricing strategies of the Nash game.

a_3	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*	c_3	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	TP_c^*
130	231.5	245.7	124.8	109.5	119.9	105.9	16788.3	15	218.5	232.7	161.2	108.5	118.9	116.1	18462.8
140	224.8	239.1	144.7	108.7	119.1	114.4	17270.1	20	218.3	232.6	162.8	108.2	118.6	119.4	18176.7
150	218.1	232.4	164.5	107.9	118.3	122.8	17895.0	25	218.1	232.4	164.5	107.9	118.3	122.8	17895.0
160	211.5	225.8	184.4	107.1	117.4	131.3	18663.1	30	218.0	232.3	166.2	107.6	117.9	126.2	17617.6
170	204.8	219.1	204.3	106.3	116.6	139.8	19574.2	35	217.8	232.1	167.9	107.2	117.6	129.6	17344.6

Table 11
Effects of β in optimal pricing strategies of the Nash game.

β	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	$TP_{M_1}^*$	$TP_{M_2}^*$	$TP_{M_3}^*$	TP_r^*	TP_c^*
0.35	273.1	288.8	157.7	121.3	132.5	135.4	2916.2	3329.3	4262.1	11047.1	21554.6
0.40	218.1	232.4	164.5	107.9	118.2	122.8	2426.1	2772.2	3827.7	8869.0	17895.0
0.45	184.9	198.1	160.6	97.7	107.3	112.6	2060.0	2353.9	3452.7	7502.3	15368.9
0.50	161.9	174.2	153.7	89.6	98.7	104.1	1775.7	2027.9	3129.6	6518.6	13451.8
0.55	144.8	156.3	146.0	83.1	91.7	97.0	1548.6	1766.5	2850.1	5757.0	11922.1
0.60	131.5	142.3	138.5	77.7	85.9	90.9	1362.9	1552.4	2606.9	5141.1	10663.2

Table 12
Effects of γ in optimal pricing strategies of the Nash game.

γ	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	$TP_{M_1}^*$	$TP_{M_2}^*$	$TP_{M_3}^*$	TP_r^*	TP_c^*
0.10	119.7	131.1	142.0	75.1	83.6	91.2	1222.1	1414.4	2628.2	4609.9	9874.6
0.15	131.5	142.3	138.5	77.7	85.9	90.9	1362.9	1552.4	2606.9	5141.1	10663.2
0.20	146.1	156.4	134.0	80.3	88.3	90.6	1520.2	1706.8	2584.3	5870.7	11682.1
0.25	165.0	174.8	128.1	83.2	91.0	90.3	1696.4	1880.1	2560.4	6901.8	13038.8
0.30	190.3	199.7	120.0	86.2	93.8	90.0	1894.4	2075.1	2535.0	8416.4	14920.9

Table 13
Effects of δ in optimal pricing strategies of the Nash game.

δ	s_1^*	s_2^*	s_3^*	p_1^*	p_2^*	p_3^*	$TP_{M_1}^*$	$TP_{M_2}^*$	$TP_{M_3}^*$	TP_r^*	TP_c^*
0.00	171.1	181.4	175.0	86.0	94.0	100.0	1881.6	2088.6	3375.0	9328.3	16673.5
0.05	156.5	166.7	152.9	83.0	91.0	95.2	1687.1	1883.4	2953.9	7312.0	13836.3
0.10	146.1	156.4	134.0	80.3	88.3	90.6	1520.2	1706.8	2584.3	5870.7	11682.1
0.15	139.7	149.9	116.3	77.9	85.9	86.4	1376.9	1554.8	2258.2	4831.4	10021.4
0.20	137.9	148.1	97.4	75.7	83.7	82.3	1253.9	1423.9	1969.0	4105.9	8752.6

as the cross price responsibility of substitute product (γ) increases and, as a result, each member profit of the chain and the whole chain profit increases.

(c) *The optimal wholesale prices (p_1^*, p_2^*, p_3^*) and selling prices (s_1^*, s_2^*, s_3^*) decrease as the cross price responsibility of complementary product (δ) increase and, as a result, each member profit of the chain and the whole chain profit decreases.*

5. Conclusions and future research

In the present paper, we have considered a realistic model for manufacturing three items, of which two are substitutable and one is complementary, by three manufacturers in terms of one item each, and the items are sold using a common retailer. In this study, we have predicted the marketing strategies evaluating the selling prices of manufacturers (p_1, p_2, p_3) and retailer (s_1, s_2, s_3) such that individual profits and system profits are maximized separately under the non-cooperative and cooperative marketing scenarios. Demands of the items are price dependent, and demand of an item is negatively influenced by the prices of complementary items but positively by the substitutable item. All of the above investigations and managerial insights have been derived formulating the models under game theory frameworks using MS, RS, and Nash games and the CD model.

In this study, the model formulation is highly general, but it can be extended to include n manufacturers producing n items separately, of which a portion are substitutable and others are complementary, and selling through a common retailer such that their individual profits or joint profit is maximized. Moreover, demand sensitivities to prices can be taken as imprecise (i.e., fuzzy), and using the fuzzy expectation and fuzzy measures (i.e., possibility, necessity and credibility), the models can be solved. Limitation of the present investigation is that in this instance, demand sensibilities of items' to prices are deterministic. Moreover, these findings are for two-stage supply chains only.

Acknowledgements

The first author is grateful to the University Grants Commission (UGC, India) for partial financial support to continue this research by the Innovative Research Project grants (Ref. No. VU/Innovative/Sc/03/2015).

Appendix A

The profit of the supply chain is

$$\pi_c(s_1,s_2,s_3)=(s_1-c_1)(a_1-\beta s_1+\gamma s_2-\delta s_3)+(s_2-c_2)(a_2+\gamma s_1-\beta s_2-\delta s_3)+(s_3-c_3)(a_3-\delta s_1-\delta s_2-\beta s_3) \tag{A.1}$$

The first and second order partial differentiations of Eq. (A.1) w. r. to s_1, s_2 and s_3 are as follows

$$\frac{\partial\pi_c}{\partial s_1}=-2\beta s_1+2\gamma s_2-2\delta s_3+a_1+\beta c_1-\gamma c_2+\delta c_3,\ \frac{\partial\pi_c}{\partial s_2}=2\gamma s_1-2\beta s_2-2\delta s_3+a_2-\gamma c_1+\beta c_2+\delta c_3,$$
$$\frac{\partial\pi_c}{\partial s_3}=-2\delta s_1-2\delta s_2-2\beta s_3+a_3+\delta c_1+\delta c_2+\beta c_3,\ \frac{\partial^2\pi_c}{\partial s_1^2}=-2\beta,\ \frac{\partial^2\pi_c}{\partial s_2\partial s_1}=2\gamma,\ \frac{\partial^2\pi_c}{\partial s_3\partial s_1}=-2\delta,$$
$$\frac{\partial^2\pi_c}{\partial s_1\partial s_2}=2\gamma,\frac{\partial^2\pi_c}{\partial s_2^2}=-2\beta,\frac{\partial^2\pi_c}{\partial s_3\partial s_2}=-2\delta,\frac{\partial^2\pi_c}{\partial s_1\partial s_3}=-2\delta,\frac{\partial^2\pi_c}{\partial s_2\partial s_3}=-2\delta, and\ \frac{\partial^2\pi_c}{\partial s_3^2}=-2\beta$$

Determinate value of the Hessian matrix $\left|H\right|=\begin{vmatrix}-2\beta & 2\gamma & -2\delta\\ 2\gamma & -2\beta & -2\delta\\ -2\delta & -2\delta & -2\beta\end{vmatrix}=8(\beta+\gamma)(2\delta^2-\beta^2+\beta\gamma)$

Therefore, the above profit expression is concave if $2\delta^2-\beta^2+\beta\gamma<0$ i.e., $2\delta^2+\beta\gamma<\beta^2$.

At an extreme point, we have

$$\begin{array}{l}-2\beta s_1^*+2\gamma s_2^*-2\delta s_3^*=-(a_1+\beta c_1-\gamma c_2+\delta c_3)\\ 2\gamma s_1^*-2\beta s_2^*-2\delta s_3^*=-(a_2-\gamma c_1+\beta c_2+\delta c_3)\\ -2\delta s_1^*-2\delta s_2^*-2\beta s_3^*=-(a_3+\delta c_1+\delta c_2+\beta c_3)\end{array} \tag{A.2}$$

Solving the system of linear eqs. given in (A.2), we have

$$s_1^*=\frac{c_1}{2}-\frac{a_1\left(\beta^2-\delta^2\right)+a_2\left(\beta\gamma+\delta^2\right)-a_3(\beta+\gamma)\delta}{2(\beta+\gamma)\left(2\delta^2-\beta^2+\beta\gamma\right)} \tag{A.3}$$

$$s_2^*=\frac{c_2}{2}-\frac{a_1\left(\beta\gamma+\delta^2\right)+a_2\left(\beta^2-\delta^2\right)-a_3(\beta+\gamma)\delta}{2(\beta+\gamma)\left(2\delta^2-\beta^2+\beta\gamma\right)} \tag{A.4}$$

$$s_3^*=\frac{c_3}{2}+\frac{(a_1+a_2)\delta-a_3(\beta-\gamma)}{2\left(2\delta^2-\beta^2+\beta\gamma\right)} \tag{A.5}$$

From the constraints $d_i>0$ (i = 1,2,3) with the help of Eq. (A.2), we have

$$a_1+\gamma c_2>\beta c_1+\delta c_3,\ a_2+\gamma c_1>\beta c_2+\delta c_3,\ a_3>\delta(c_1+c_2)+\beta c_3$$

From the constraints $s_i>c_i$ (i = 1,2,3) with the help of Eqs. (A.3)–(A.5), we have

$$a_1\left(\beta^2-\delta^2\right)+a_2\left(\beta\gamma+\delta^2\right)>a_3(\beta+\gamma)\delta+c_1(\beta+\gamma)\left(\beta^2-\beta\gamma-2\delta^2\right),$$

$$a_1\left(\beta\gamma+\delta^2\right)+a_2\left(\beta^2-\delta^2\right)>a_3(\beta+\gamma)\delta+c_2(\beta+\gamma)\left(\beta^2-\beta\gamma-2\delta^2\right)$$

$$and\ \ a_3(\beta-\gamma)>(a_1+a_2)\delta+c_3\left(\beta^2-\beta\gamma-2\delta^2\right)$$

Appendix B

The profit of the retailer in the MS game is

$$\pi_r(s_1,s_2,s_3)=(s_1-p_1)(a_1-\beta s_1+\gamma s_2-\delta s_3)+(s_2-p_2)(a_2+\gamma s_1-\beta s_2-\delta s_3)+(s_3-p_3)(a_3-\delta s_1-\delta s_2-\beta s_3) \tag{B.1}$$

Similar to Appendix A, retailer profit expression in the MS game is concave if $2\delta^2+\beta\gamma<\beta^2$, and the optimal retailer pricing strategies become

$$s_1^*=\frac{p_1}{2}-\frac{a_1\left(\beta^2-\delta^2\right)+a_2\left(\beta\gamma+\delta^2\right)-a_3(\beta+\gamma)\delta}{2(\beta+\gamma)\left(2\delta^2-\beta^2+\beta\gamma\right)} \tag{B.2}$$

$$s_2^* = \frac{p_2}{2} - \frac{a_1\left(\beta\gamma + \delta^2\right) + a_2\left(\beta^2 - \delta^2\right) - a_3(\beta+\gamma)\delta}{2(\beta+\gamma)\left(2\delta^2 - \beta^2 + \beta\gamma\right)} \tag{B.3}$$

$$p_1^* = -\frac{(a_1+\beta c_1)\left(4\beta^2 - \delta^2\right) + (a_2+\beta c_2)\left(2\beta\gamma + \delta^2\right) - (a_3+\beta c_3)(2\beta+\gamma)\delta}{(2\beta+\gamma)\left(2\delta^2 - 4\beta^2 + 2\beta\gamma\right)} \tag{B.9}$$

$$p_2^* = -\frac{(a_1+\beta c_1)\left(2\beta\gamma + \delta^2\right) + (a_2+\beta c_2)\left(4\beta^2 - \delta^2\right) - (a_3+\beta c_3)(2\beta+\gamma)\delta}{(2\beta+\gamma)\left(2\delta^2 - 4\beta^2 + 2\beta\gamma\right)} \tag{B.10}$$

$$s_3^* = \frac{p_3}{2} + \frac{(a_1+a_2)\delta - a_3(\beta-\gamma)}{2\left(2\delta^2 - \beta^2 + \beta\gamma\right)} \tag{B.4}$$

From the constraints $d_i > 0$ and $s_i > p_i$ (i = 1,2,3) that is similar to Appendix A, we have

$$a_1 + \gamma p_2 > \beta p_1 + \delta p_3,\ a_2 + \gamma p_1 > \beta p_2 + \delta p_3, a_3 > \delta(p_1+p_2) + \beta p_3$$

$$a_1\left(\beta^2 - \delta^2\right) + a_2\left(\beta\gamma + \delta^2\right) > a_3(\beta+\gamma)\delta + p_1(\beta+\gamma)\left(\beta^2 - \beta\gamma - 2\delta^2\right),$$

$$a_1\left(\beta\gamma + \delta^2\right) + a_2\left(\beta^2 - \delta^2\right) > a_3(\beta+\gamma)\delta + p_2(\beta+\gamma)\left(\beta^2 - \beta\gamma - 2\delta^2\right)$$

$$\textit{and}\ \ a_3(\beta-\gamma) > (a_1+a_2)\delta + p_3\left(\beta^2 - \beta\gamma - 2\delta^2\right)$$

At the retailer optimal pricing strategies s_1^*, s_2^* and s_3^*, the manufacturer M_1's profit in the MS game is

$$\pi_{M_1}\left(p_1, s_1^*(p_1), s_2^*(p_2), s_3^*(p_3)\right) = (p_1 - c_1)\left(a_1 - \beta s_1^* + \gamma s_2^* - \delta s_3^*\right) \tag{B.5}$$

The first and second order partial differentiations of Eq. (B.5) w. r. to p_1 are as follows

$$\frac{\partial \pi_{M_1}}{\partial p_1} = -\beta p_1 + \frac{\gamma}{2}p_2 - \frac{\delta}{2}p_3 + \frac{1}{2}(a_1+\beta c_1), \textit{and} \frac{\partial^2 \pi_{M_1}}{\partial p_1^2} = -\beta < 0$$

Therefore, the manufacturer's M_1 profit expression in MS game is concave as $\beta > 0$ and at an extreme point, we have

$$-\beta p_1^* + \frac{\gamma}{2}p_2^* - \frac{\delta}{2}p_3^* = -\frac{1}{2}(a_1+\beta c_1) \tag{B.6}$$

Similarly, we can show that manufacturers M_2's and M_3's profit expressions in MS game are also concave, and at an extreme point it becomes

$$\frac{\gamma}{2}p_1^* - \beta p_2^* - \frac{\delta}{2}p_3^* = -\frac{1}{2}(a_2+\beta c_2) \tag{B.7}$$

$$-\frac{\delta}{2}p_1^* - \frac{\delta}{2}p_2^* - \beta p_3^* = -\frac{1}{2}(a_3+\beta c_3) \tag{B.8}$$

Solving Eqs. (B.6)–(B.8), we have

$$p_3^* = \frac{(a_1+\beta c_1 + a_2 + \beta c_2)\delta - (a_3+\beta c_3)(2\beta-\gamma)}{\left(2\delta^2 - 4\beta^2 + 2\beta\gamma\right)} \tag{B.11}$$

Appendix C

The manufacturer's M_1's profit function in the RS game is

$$\pi_{M_1}(p_1,p_2,p_3) = (p_1-c_1)\{a_1 - \beta(p_1+m_1) + \gamma(p_2+m_2) - \delta(p_3+m_3)\} \tag{C.1}$$

The first and second order partial differentiations of Eq. (C.1) w. r. to p_1 are as follows:

$$\frac{\partial \pi_{M_1}}{\partial p_1} = a_1 - \beta(p_1+m_1) + \gamma(p_2+m_2) - \delta(p_3+m_3) - \beta(p_1-c_1),\ \textit{and}\ \frac{\partial^2 \pi_{M_1}}{\partial p_1^2} = -2\beta < 0$$

Therefore, the manufacturer's M_1 profit expression in RS game is concave as $\beta > 0$ and at an extreme point, optimum pricing strategy of M_1 is

$$p_1^* = c_1 + \frac{1}{\beta}(a_1 - \beta s_1 + \gamma s_2 - \delta s_3) \tag{C.2}$$

Similarly, we can show that manufacturers' M_2's and M_3's profit expressions in RS game are concave as $\beta > 0$ and optimum pricing strategies for M_2 and M_3 are, respectively,

$$p_2^* = c_2 + \frac{1}{\beta}(a_2 + \gamma s_1 - \beta s_2 - \delta s_3) \tag{C.3}$$

$$p_3^* = c_3 + \frac{1}{\beta}(a_3 - \delta s_1 - \delta s_2 - \beta s_3) \tag{C.4}$$

The retailer profit in RS game is

$$\pi_r(s_1,s_2,s_3,p_1^*,p_2^*,p_3^*) = (s_1-p_1^*)(a_1-\beta s_1+\gamma s_2-\delta s_3) + (s_2-p_2^*)(a_2+\gamma s_1-\beta s_2-\delta s_3) + (s_3-p_3^*)(a_3-\delta s_1-\delta s_2-\beta s_3) \quad (C.5)$$

The first order and second order partial differentiation of Eq. (C.5) w. r. to s_1, s_2 and s_3 are as follows

$$\frac{\partial\pi_r}{\partial s_1} = -Ps_1+Qs_2-Rs_3+B_1 \quad \frac{\partial\pi_r}{\partial s_2} = Qs_1-Ps_2-Rs_3+B_2$$
$$\frac{\partial\pi_r}{\partial s_3} = -Rs_1-Rs_2-Ss_3+B_3, \frac{\partial^2\pi_r}{\partial s_1^2} = -P, \frac{\partial^2\pi_r}{\partial s_2\partial s_1} = Q, \frac{\partial^2\pi_r}{\partial s_3\partial s_1} = -R,$$
$$\frac{\partial^2\pi_r}{\partial s_1\partial s_2} = Q, \frac{\partial^2\pi_r}{\partial s_2^2} = -P, \frac{\partial^2\pi_r}{\partial s_3\partial s_2} = -R, \frac{\partial^2\pi_r}{\partial s_1\partial s_3} = -R, \frac{\partial^2\pi_r}{\partial s_2\partial s_3} = -R, and \frac{\partial^2\pi_r}{\partial s_3^2} = -S$$

$$Where,\ P=\frac{2}{\beta}\left(2\beta^2+\gamma^2+\delta^2\right), Q=\frac{2}{\beta}\left(3\beta\gamma-\delta^2\right), R=\frac{2}{\beta}(3\beta\delta-\gamma\delta),$$
$$S=\frac{4}{\beta}\left(\beta^2+\delta^2\right), B_1=\frac{1}{\beta}\left(3\beta a_1-2\gamma a_2+2\delta a_3+\beta^2c_1-\beta\gamma c_2+\beta\delta c_3\right),$$
$$B_2=\frac{1}{\beta}\left(-2\gamma a_1+3\beta a_2+2\delta a_3-\beta\gamma c_1+\beta^2c_2+\beta\delta c_3\right),$$
$$B_3=\frac{1}{\beta}(2\delta a_1+2\delta a_2+3\beta a_3+\delta c_1+\delta c_2+\beta c_3)$$

The above retailer profit expression in RS game is concave if

$$2\beta^2+\gamma^2+2\delta^2>3\beta\gamma,\ \ 2\delta^2+\beta\gamma<\beta^2 and\ \ \delta^2+\beta\gamma<2\beta^2$$

$$p_1^* = -\frac{(a_1+2\beta c_1)\left(9\beta^2-\delta^2\right)+(a_2+2\beta c_2)\left(3\beta\gamma+\delta^2\right)-(a_3+2\beta c_3)(3\beta+\gamma)\delta}{(3\beta+\gamma)\left(2\delta^2-9\beta^2+3\beta\gamma\right)} \quad (D.1)$$

$$p_2^* = -\frac{(a_1+2\beta c_1)\left(3\beta\gamma+\delta^2\right)+(a_2+2\beta c_2)\left(9\beta^2-\delta^2\right)-(a_3+2\beta c_3)(3\beta+\gamma)\delta}{(3\beta+\gamma)\left(2\delta^2-9\beta^2+3\beta\gamma\right)} \quad (D.2)$$

At an extreme point, we have

$$Ps_1^*-Qs_2^*+Rs_3^*=B_1,\ \ -Qs_1^*+Ps_2^*+Rs_3^* = B_2,\ \ and\ \ Rs_1^*+Rs_2^*+Ss_3^*=B_3 \quad (C.6)$$

Solving the system of linear eq. given in (C.6), we have

$$s_1^* = \frac{B_1\left(PS-R^2\right)+B_2\left(QS+R^2\right)-B_3(P+Q)R}{(P+Q)\left(PS-QS-2R^2\right)}$$
$$s_2^* = \frac{B_1\left(QS+R^2\right)+B_2\left(PS-R^2\right)-B_3(P+Q)R}{(P+Q)\left(PS-QS-2R^2\right)} \quad (C.7)$$
$$s_3^* = -\frac{(B_1+B_2)R-B_3(P-Q)}{\left(PS-QS-2R^2\right)}$$

Obviously, the optimal retail pricing strategies satisfy the conditions $d_i>0,\ (i=1,\ 2,\ 3)$

$i.e.\ a_1+\gamma s_2^*>\beta s_1^*+\delta s_3^*,\ a_2+\gamma s_1^*>\beta s_2^*+\delta s_3^*,\ a_3>(s_1^*+s_2^*)\delta+\beta s_3^*$

Additionally, form the Eqs. (C.2)–(C.4), the conditions $p_i>c_i,\ (i=1,\ 2,\ 3)$ evidently satisfied as $d_i>0,\ (i=1,\ 2,\ 3)$

Appendix D

The optimum decisions of the three manufacturers' and one retailer are derived in the RS and MS game in Eqs. (C.2)–(C.4) and (B.2)–(B.4), respectively. For Nash-equilibrium strategies, solving these six equations, we have

$$p_3^* = \frac{(a_1+2\beta c_1+a_2+2\beta c_2)\delta-(a_3+2\beta c_3)(3\beta-\gamma)}{\left(2\delta^2-9\beta^2+3\beta\gamma\right)} \quad (D.3)$$

$$s_1^* = \frac{p_1^*}{2}-\frac{a_1\left(\beta^2-\delta^2\right)+a_2\left(\beta\gamma+\delta^2\right)-a_3((\beta+\gamma)\delta)}{2(\beta+\gamma)\left(2\delta^2-\beta^2+\beta\gamma\right)} \quad (D.4)$$

$$s_2^* = \frac{p_2^*}{2} - \frac{a_1\left(\beta\gamma + \delta^2\right) + a_2\left(\beta^2 - \delta^2\right) - a_3((\beta+\gamma)\delta)}{2(\beta+\gamma)\left(2\delta^2 - \beta^2 + \beta\gamma\right)} \tag{D.5}$$

$$s_3^* = \frac{p_3^*}{2} + \frac{(a_1 + a_2)\delta - a_3(\beta - \gamma)}{2\left(2\delta^2 - \beta^2 + \beta\gamma\right)} \tag{D.6}$$

References

[1] P.L. Abad, C.K. Jaggi, A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive, Int. J. Prod. Econ. 83 (2003) 115–122.
[2] J. Bian, X. Guo, K.K. Lai, Z. Hua, The strategic peril of information sharing in a vertical-Nash supply chain: a note, Int. J. Prod. Econ. 158 (2014) 37–43.
[3] S.C. Choi, Price competition in a channel structure with a common retailer, Mark. Sci. 10 (4) (1991) 271–296.
[4] M. Esmaeili, M.B. Aryanezhad, P. Zeephongsekul, A game theory approach in seller-buyer supply chain, Eur. J. Oper. Res. 195 (2009) 442–448.
[5] S. Gupta, R. Loulou, Process innovation, product differentiation, and channel structure: strategic incentives in a duopoly, Mark. Sci. 17 (4) (1998) 301–316.
[6] C.H. Ho, L.Y. Ouyang, C.H. Su, Optimal pricing, shipment and payment policy for an integrated supplier buyer inventory model with two-part trade credit, Eur. J. Oper. Res. 187 (2008) 496–510.
[7] D. Huang, H. Zhou, Q.H. Zhao, A competitive multiple-product newsboy problem with partial product substitution, Omega 39 (2011) 302–312.
[8] L. Jiang, Z. Hao, On the value of information sharing and cooperative price setting, Oper. Res. Lett. 42 (2014) 399–403.
[9] I.P. Krommyda, K. Skouri, I. Konstantaras, Optimal ordering quantities for substitutable products with stock dependent demand, Appl. Math. Model 39 (2015) 147–164.
[10] A.H.L. Lau, H.S. Lau, Effects of a demand-curves shape on the optimal solutions of a multi-echelon inventory/pricing model, Eur. J. Oper. Res. 147 (2003) 530–548.
[11] J. Li, h. Fen H, Y. Zeng, Inventory games with permissible delay in payments, Eur. J. Oper. Res. 234 (3) (2014) 694–700.
[12] K. Maity, M. Maiti, Optimal inventory policies for deteriorating complementary and substitute items, Int. J. Syst. Sci. 40 (2009) 267–276.
[13] A.R. McGillivray, E.A. Silver, Some concepts for inventory control under substitutable demand, INFOR 16 (1978) 47–63.
[14] T.W. McGuire, R. Staelin, An industry equilibrium analyses of down stream vertical integration, Mark. Sci. 2 (2) (1983) 161–191.
[15] S.K. Mukhopadhyay, X. Yue, X. Zhu, A Stackelberg model of pricing of complementary goods under information asymmetry, Int. J. Prod. Econ. 134 (2) (2011) 424–433.
[16] M. Parlar, Game theoretic analysis of the substitutable product inventory problem with random demands, Nav. Res. Log. 35 (1988) 397–409.
[17] P. Pineyro, O. Viera, The economic lot sizing problem with remanufacturing and one way substitution, Int. J. Prod. Econ. 24 (2010) 482–488.
[18] J. Raju, A. Roy, Market information and firm performance, Manag. Sci. 46 (8) (2000) 1075–1084.
[19] E. Stavrulaki, Inventory decisions for substitutable products with stock-dependent demand, Int. J. Prod. Econ. 129 (2011) 65–78.
[20] A.A. Tsay, N. Agrawal, Channel dynamics under price and service competition, Manuf. Serv. Oper. Manag. 2 (2000) 372–391.
[21] R. Yan, S. Bandyopadhyay, The profit benefits of bundle pricing of complementary products, J. Retail. Consum. Serv. 18 (4) (2011) 355–361.
[22] R. Yan, C. Myers, J. Wang, S. Ghose, Bundling products to success: the influence of complementarity and advertising, J. Retail. Consum. Serv. 21 (1) (2014) 48–53.
[23] X. Yue, S.K. Mukhopadhyay, X. Zhu, A Bertrand model of pricing of complementary goods under information asymmetry, J. Bus. Res. 59 (2006) 1182–1192.
[25] L. Zhang, J. Wang, J. You, Consumer environmental awareness and channel coordination with two substitutable products, Eur. J. Oper. Res. 241 (2015) 63–73.
[26] J. Zhao, W. Tang, R. Zhao, J. Wei, Pricing decisions for substitutable products with a common retailer in fuzzy environments, Eur. J. Oper. Res. 216 (2012) 409–419.
[27] J. Zhao, J. Wei, Y. Li, Pricing decisions for substitutable products in a two-echelon supply chain with firms' different channel powers, Int. J. Prod. Econ. 153 (2014) 243–252.