Division, remainder

Modular arithmatic

Problems

Modular operation: %

$$\frac{73}{11} \longrightarrow \begin{matrix} 6, 7 \\ q, r \end{matrix}$$

$$\begin{array}{r} 6 \leftarrow q \\ \text{Divisor} \rightarrow 11 \overline{)\,73} \leftarrow \text{dividend} \\ \underline{66} \\ 7 \leftarrow \text{remaind} \end{array}$$

$$\underset{D}{\boxed{73}} = \underset{d}{\boxed{11}} \times \underset{q}{\boxed{6}} + \underset{r}{\boxed{7}}$$

Dividend = divisor × quotient + remainder

$$\boxed{D = dq + r}$$

$$r = D - d \times q$$

$$r = 73 - \boxed{66}$$

largest multiple of divisor ≤ D

$$r = 7$$

Division :

$\frac{150}{11}$

$11 \times 10 = 110$
$11 \times 11 = 121$
$11 \times 12 = 132$
$11 \times 13 = 143$

$r = D - \underbrace{d \times q}_{\boxed{\text{largest multiple of divisor} \le D}}$

$r = 150 - 143$

$r = 7$

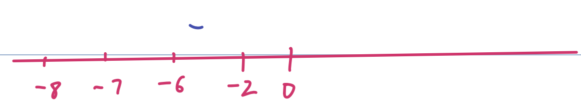
$\frac{100}{7}$

$r = D - \underbrace{d \times q}_{\boxed{\text{largest multiple of divisor} \le D}}$

$r = 100 - 98$

$= 2$

$\boxed{-2 > -6 > -7 > -8}$



$\frac{-40}{7}$

-7

-14

-21

-35

-42

$r = D - \underbrace{d \times q}_{\boxed{\text{largest multiple of divisor} \le D}}$

$= -40 - (-42)$

$= -40 + 42 = 2$

$\frac{-60}{9}$

$r = D - \underbrace{d \times q}_{\boxed{\text{largest multiple of divisor} \le D}}$

$r = -60 - (-63)$

$= 3$

python	C++, Java, C#, JS
$-60 \% 9 = 3$	$-60 \% 9 = -6$
Correct	Not
	$60 \% 9 = 6 \times -1 = -6 + \overset{d}{9} = 3$

$$\boxed{\begin{aligned} & r = A \% n \\ & \text{if } (r < 0) \\ & \quad r = r + n \end{aligned}}$$

$\% 5$

1 → 1	2 → 2	3 → 3	4 → 4	5 → 0	6 → 1	7 → 2	8 → 3	9 4	10 0
11 1	12 2	13 3	14 4	15 0	16 1	17 2	18 3	19 4	20 0

$A \% 5 \longrightarrow 0, 1, 2, 3, 4 \ : \ [0, 4]$

$-\infty$

$$\boxed{A \% N \longrightarrow [0, N-1]}$$

∞

1) Hashmap
2) Consistent hashing
3) Encryption

Modular Arithmatic

$$\boxed{(a+b)\% m = (a\% m + b\% m)\% m}$$

a	b	m
8	6	10

LHS: $(8+6)\% 10 = 4$

RHS: $8\% 10 + 6\% 10 = 14$

$$\boxed{(a\times b)\% m = (a\% m \times b\% m)\% m}$$

$(a-b)\% m$

$(a/b)\% m$

$\rightarrow$ Advance

Fermat's little theorem

Q1) Given a, n, calculate value of a^n?

$$\boxed{a = 2 \qquad n = 5 \qquad 2^5 = 32}$$

```
pow(a, n){
    ans = 1
    for (i=1; i<=N; i++){
        ans = ans * a
    }
    return ans
}
```

int : $2^{31} \approx 10^9$

long : $2^{63} \approx 10^{18}$

Q) Given 3 numbers a, n & p

Implement a power function $(a^n)\%p \rightarrow [0, p-1]$

$1 \le a \le 10^{18}$

$1 \le n \le 10^5$

$1 \le p \le 10^9$

$a = 2 \quad n = 5 \quad p = 7$

$(2^5)\%7 = 4$

$a = 2$

$n = 1000$

2^{1000}

```
pow(a, n, p)
    ans = 1
    for (i=1; i<=N; i++){
        ans = ans * a
    }
    return ans%p
}
```

why incorrect? due to overflow

$\boxed{a\%m = (a\%m)\%m}$

```
int pow(a, n, p)
    long ans = 1
    for (i=1; i<=N; i++){
        ans = (ans * a)%p
    }
    return (int) ans %p
}
```

$\boxed{TC: O(n)}$

$(ans\%p * a\%p)\%p$

$7\%5 = 2$

$((7\%5)\%5)\%5 = 2$

$(a\ p)^{r}$.

4 3 r 7 6 2 2 1

Divisibility rules

Class 9^{th}

Rule for 3 $\longrightarrow$ (Find sum of digits) $\% 3 = 0$

(2 3 2) $7 \% 3 != 0$ ✗ 4560 $15 \% 3 = 0$ ✓

238 $13 \% 3 != 0$ ✗ 433 $10 \% 3 != 0$ ✗

$(8\ 9\ 1\ 3) \% 3 = (8000 + 900 + 10 + 3) \% 3$

$= (8 \times 10^3 + 9 \times 10^2 + 10^1 + 3 \times 10^0) \% 3$

$= [(8 \times 10^3) \% 3 + (9 \times 10^2) \% 3 + (10^1) \% 3 + (3 \% 3)] \% 3$

$1 \% 3 = 1$ $\quad [8 \% 3 + 9 \% 3 + 1 \% 3 + 3 \% 3] \% 3$

$10 \% 3 = 1$ $\quad \downarrow$

$100 \% 3 = 1$ $\quad (a + b + c + d) \% 3 = (a \% 3 + b \% 3 + c \% 3 + d \% 3) \% 3$

$1000 \% 3 = 1$ $\quad \downarrow$

$\vdots$

$10^n \% 3 = 1$ $\quad (8 + 9 + 1 + 3) \% 3$

$$792 = (7\times10^2\%3 + 9\times10^2\%3 + 2\times10^0\%3)\%3$$

$$= (7\%3 + 9\%3 + 2\%3)\%3$$

$$= (7+9+2)\%3$$

Divisibility of 4

$$\underbrace{7953\underline{28}}_{\text{last 2digits}}$$

$$795328 = (7\times10^5 + 9\times10^4 + 5\times10^3 + 3\times10^2 + 2\times10^1 + 8\times10^0)\%4$$

$$7\times10^5\%4 \rightarrow 0 \qquad 9\times10^4\%4 \rightarrow 0 \qquad 5\times10^3\%4 \rightarrow 0 \qquad 3\times10^2\%4 \rightarrow 0$$

$$((2\times10)\%4 + 8\%4)\%4$$

$$1\%4 = 1$$

$$10\%4 = 2$$

$$(20\%4 + 8\%4)\%4$$

$$100\%4 = 0$$

$$1000\%4 = 0$$

$$(28)\%4$$

$$10000\%4 = 0$$

$$10^n\%4 = 0$$

$$n>2$$

divisibility by 5 $\longrightarrow$ last digit 0/5

HW

" " 11

Imp

$$\boxed{\begin{array}{l} \text{1) open the number using powers of 10} \\ \text{2) Apply mod arithmatic} \end{array}}$$

Break (10:35- 10:45)

Google

Q) Given an array of size N representing a number A. Given another number p. Calculate A % p

$1 \le p \le 10^5$

$1 \le N \le 100$

N = 8

0	1	2	3	4	5	6	7
8	9	3	2	6	4	1	9

$\boxed{10^{100}} \propto$ $\overline{100}$

$1 \le A[i] \le 9$

p = 45

num : 8 9 3 2 6 4 1 9 % 45

$$89326419 = \left(8\times10^7 + 9\times10^6 + 3\times10^5 + 2\times10^4 + 6\times10^3 + 4\times10^2 + 1\times10^1 + 9\right) \% 45$$

a_0	a_1	a_2	a_3	. . . .	a_i	a_{N-2}	a_{N-1}

$$\left(a_0\times10^{N-1} + a_1\times10^{N-2} + a_2\times10^{N-3} \ldots\ldots a_{N-2}\times10^1 + a_{N-1}\times10^0\right) \% p$$

$a_i \times 10^{N-i-1}$

$$\boxed{\left(\sum_{i=0}^{N-1} a_i \times 10^{N-i-1}\right) \% p}$$

✓✓

$0 \longrightarrow N-1$

$1 \longrightarrow N-2$

$2 \longrightarrow N-3$

$i \rightarrow N-1-i$

$N-1 \longrightarrow 0$

$$\left(\sum_{i=0}^{N-1} a_i \times 10^{N-i-1}\right) \% p$$

$$\left(\sum_{i=0}^{N-1} a_i \% p \times \left(10^{N-i-1}\right) \% p\right) \% p$$

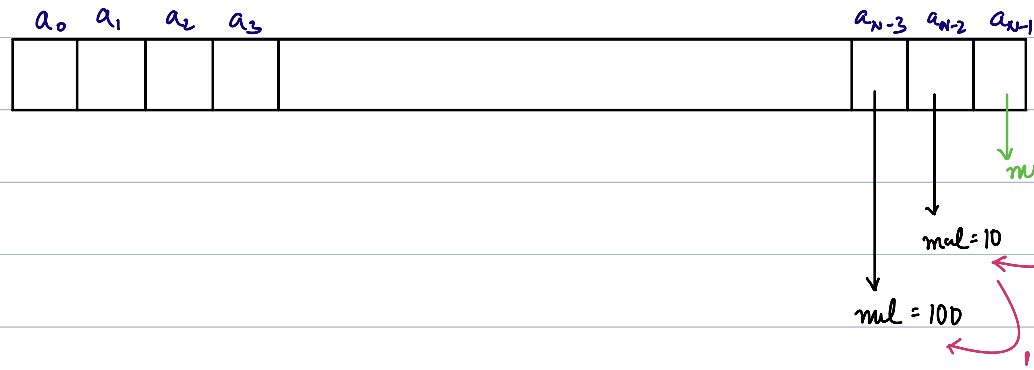
```
sum = 0

for (i=0; i<N; i++){
    sum += (A[i]%p * pow(10, N-i-1, p))%p)%p
                     (10^9 %7)
}
return sum
```

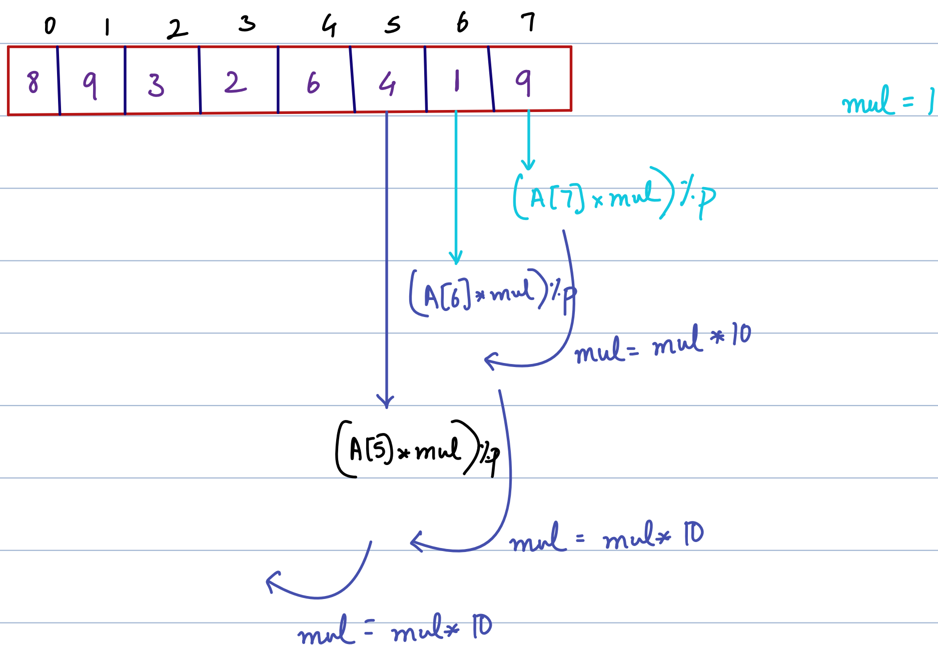
```
int pow(a, n, p)
    long ans = 1
    for (i=1; i<=N; i++){
        ans = (ans * a)%p
    }
    return (int) ans %p
}
```

TC : $O(N^2)$

SC : $O(1)$



Carry forward



Tips: 1) Break using prod, sum formula

2) Use long

$1 \le A[i] \le 9$

```
sum = 0                                   TC: O(N)
long mul = 1
for(i = N-1; i>=0; i--){
    temp = (A[i] * mul)%p
    mul = (mul * 10)%p
    sum = (sum + temp)%p
}

return sum
```

$(10^9)\% \; 7 = [0-6]$

a = 2   b = 5   p = 7

```
int pow(a, n, p)
{
    long ans = 1
    for(i=1; i<=N; i++){
        ans = (ans * a)%p
    }
    return (int) ans %p
}
```

i	ans
1	2
2	4
3	1
4	2
5	4

$(2^5)\%7 = 4$

Prime number $\boxed{10^9+7}$ / 10^9+11

fermat's little theorem

Magic number

Reverse of bits for 1

000...   1

↓

1....   0 2^{31}

2^2 2^1 2^0

100101

←

unsigned

Input 0 .... 0 0 1 0 1 1 9

1 1

2^{31} 2^{30}

ans = 0 , i = 31

```
while (A > 0) {
    if (A & 1) {
        ans = ans + 2^i
    }
    A = A >> 1
    i--
}
```

i

1 1 1 0 0 1 0 0 1

1 1 1 1 0 0 0 0 0

i ← 0

1 1 1 1 1 1 1