Subarrays basics
Printing subarrays
Generating all subarrays sum
Printing all subarray sums

Announcement : 9 June 9:00 – 10:30
1·5 hours
3 questions
you should be able to solve
atleast 2
→ Watch all lectures if not already
→ complete HWs & assigments
Sylabus : Arrays & TC

Contest discussion on same day

Subarray basics

- Continous part of an array is called subarray
- Single element is subarray ? Yes
- Complete array is subarray ? Yes

  0 elements " " ? No.

0 1 2 3 4 5 6 7 8 9

Ar[10] : { -2 4 6 3 8 1 4 3 2 -10 }

{ 2 4 1 6 -3 7 8 4 }

{ 1 4 } ✗ { 4 1 } ✓

{ 1 6 8 } ✗

{ 6 1 4 2 } ✗

{ 7 8 4 } ✓

{ 4 5 1 9 0 2 3 5 }

{ 5 } ✓

{ 4 5 1 0 } ✗

{ 9 0 2 3 } ✓

{ 4 5 1 } ✓

a c d

Subarray can be defined by a start ind and end index

s e $\boxed{s \le e}$

0 1 2 3

{ 4 5 1 3 }

$s = 0$ $e = 2$ { 4 5 1 }

$s = 0$ $e = 0$ { 4 }

$s = 3$ $e = 1$ ∝ invalid

$\boxed{A\ [s : e] \qquad len : \ e - s + 1}$ ✓

Number of subarrays

$$\begin{array}{cccccc} & & 0 & 1 & 2 & 3 \\ A[4] & = \{ & -1 & 3 & 2 & 3 \; \} \\ & & \uparrow & & & \\ & & s & & & \end{array}$$

$s \le e$

s	e	
0	0	{-1}
0	1	{-1 3}
0	2	{-1 3 2}
0	3	{-1 3 2 3}

s	e	
1	1	{3}
1	2	{3 2}
1	3	{3 2 3}

s	e	
2	2	{2}
2	3	{2 3}

s	e	
3	3	{3}

Ans: 10

$A[N] = \{ \overset{0}{a_0} \quad \overset{1}{a_1} \quad \overset{2}{a_2} \ldots \overset{\cdot\ \cdot}{} \quad \overset{N-1}{a_{N-1}} \}$ $\quad s \leq e$

No. of subarrays:

s	e
0	0
0	1
⋮	
0	N-1
N	

s	e
1	1
1	2
⋮	
1	N-1
N-1	

s	e
N-1	N-1
1	

$$N + (N-1) + (N-2) \ldots 1 = \frac{N \times (N+1)}{2} \quad ✓$$

Q1) Given a subarray, print it.   A, s, e

s = 1   e = 3

	0	1	2	3	4	
{	2	3	4	5	7	}

```
printsub(A, s, e){
    for(i=s: i<=e: i++){
        print(A[i])
    }
}
```

$\boxed{\text{TC: } O(N)}$

Q2) Given N array elements, print start and end index of each and every subarray?

$s \leq e$

$$A = \{ \overset{0}{6} \quad \overset{1}{8} \quad \overset{2}{-1} \quad \overset{3}{7} \}$$

TC: $O(N^2)$

s	e
0	0
0	1
0	2
0	3
1	1
1	2
1	3
2	2
2	3
3	3

```
for(s=0 ; s <= N-1 ; s++){
    for(e=s ; e<=N-1 ; e++){
        print(s, e)
    }
}
```

Q2) Given N array elements, print each and every subarray?

$\quad\quad\ \ 0 \quad\ \ 1 \quad\ \ 2 \quad 3$

A: { 6 8 -1 7 }

CANNOT OPTIMISE

$\boxed{TC: O(N^3)}$

s	e	
0	0	{ 6 }
0	1	{ 6 8 }
0	2	{ 6 8 -1 }
0	3	{ 6 8 -1 7 }
1	1	{ 8 }
1	2	{ 8 -1 }
1	3	{ 8 -1 7 }
2	2	{ -1 }
2	3	{ -1 7 }
3	3	{ 7 }

```
for(s=0; s <= N-1; s++){
    for(e=s; e<=N-1; e++){
        for(k=s; k<=e; k++){
            print(A[k])
        }
        print(\n)
    }
}
```

TC to print a subarray : $O(N)$

Total No. of subarrays : $\frac{N \times (N+1)}{2} \simeq O(N^2)$

TC to print all subarray : $N \times N^2$

: $O(N^3)$

1 banana = 10Rs

N^2 bamas = $N^2 \times 10$

Q2) Given N array elements, print each subarray sum?

0 1 2 3

A: { 6 8 -1 7 }

s	e		
0	0	{ 6 }	→ 6
0	1	{ 6 8 }	→ 14
0	2	{ 6 8 -1 }	→ 13
0	3	{ 6 8 -1 7 }	→ 20
1	1	{ 8 }	→ 8
1	2	{ 8 -1 }	→ 7
1	3	{ 8 -1 7 }	→ 14
2	2	{ -1 }	→ -1
2	3	{ -1 7 }	→ 6
3	3	{ 7 }	→ 7

TC: $O(N^3)$

```
for(s=0 ; s <= N-1 ; s++){
    for(e=s ; e<= N-1 ; e++) {
        sum=0
        for(k=s ; k<=e ; k++){
            sum+= A[k]
        }
        print(sum)
    }
}
```

prefix sum

TC: $O(N^2)$

SC: $O(N)$

```
Build pf array → N
for(s=0; s<=N-1; s++){
    for(e=s; e<=N-1; e++){ → N^2
        if (s>0){ pf[e] - pf[s-1]}
        else{ pf[e] }
    }
}
```

sum(A[s:e]) =

Finding sum of subarray [s : e]

```
sum=0
for(k=s; k<=e; k++){
    sum+= A[k]
}
print(sum)
```

{ 1 2 3 4 }

pf: { 1 3 6 10 }

sum [A[s:e]] : pf[e] - pf[s-1]

Carry forward

Q Given an ar[N] print all subarray sums starting at index 3

	0	1	2	3	4	e 5	6	7	8	9
A[10] = {	3	8	4	7	9	4	3	2	7	6 }
sum = 0				7	16	20	23	25	32	38

print 7 16 20 23 25 32 38

TC: $O(N^2)$

SC: $O(1)$

```
for (s=0; s<=N-1; s++) {
    sum = 0
    for (e=s; e<=N-1; e++) {
        sum = sum + A[e]
        print (sum)
    }
}
```

print all subarray sum starting from index 0

print all subarray sum starting from index 1

⋮ 2

⋮

N-1

Break (10:22 - 10:32)

Google

Q) Given N array elements, return sum of {All subarray sums}

$$A = \{\overset{0}{6} \quad \overset{1}{8} \quad \overset{2}{-1} \quad \overset{3}{7}\}$$

s	e			
0	0	{6}	→	6
0	1	{6 8}	→	14
0	2	{6 8 -1}	→	13
0	3	{6 8 -1 7}	→	20
1	1	{8}	→	8
1	2	{8 -1}	→	7
1	3	{8 -1 7}	→	14
2	2	{-1}	→	-1
2	3	{-1 7}	→	6
3	3	{7}	→	7

$\boxed{\text{sum : 94}}$

$O(N^3)$

Find all subarray sums

using global var

$O(N^2)$

prefix sum

using global var

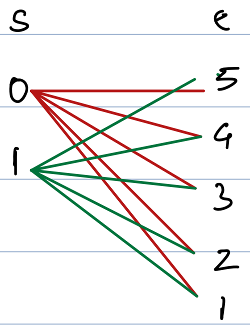
$O(N^2)$

carry forward

using global var

How many subarrays index 1 is present?

$$A = \{ \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 3 & \boxed{-2} & 4 & -1 & 2 & 6 \end{array} \}$$



5x2 = 10

$$A = \{ \begin{array}{cccccc} 0 & 1 & \overset{\downarrow}{2} & 3 & 4 & 5 \\ 3 & -2 & \boxed{4} & -1 & 2 & 6 \end{array} \}$$

s	e
0	5
1	4
2	3
	2

3 x 4 = 12

	0	1	2	3	4	5	
A = {	$\boxed{3}$	-2	4	-1	2	6	}

s	e
0	5
	4
	3
	2
	1
	0

1x6 = 6

$$\begin{array}{cccccccccc} & 0 & 1 & 2 & 3 & 4 & \cdot\cdot & & \cdot\cdot & N\text{-}1 \\ \{ & a_0 & a_1 & a_2 & a_3 & a_4 & \ldots\ldots & \boxed{\begin{array}{c} i \\ a_i \end{array}} & & a_{N-1} \ \} \end{array}$$

s	e	Contributi
0	N-1	
1	N-2	
2	⋮	
⋮		
i	i	

$[0 \ \ i] \qquad [i \ \ N\text{-}1]$

$(i+1) \qquad (N-1-i+1)$

$(i+1) \times (N-i)$

No. of subarrays having i^{th} index

	0	1	2	3
	{ 6	8	-1	7 }
$(i+1)\times(N-i)$	4	6	6	4
Contribution	6×4	6×8	-1×6	7×4
	24	48	-6	28

= 94

Contribution technique

TC: O(N)

SC: O(1)

```
sum = 0
for (i=0; i<N; i++){
    # freq of A[i]
    freq = (i+1) (N-i)
    contribution = A[i] × freq
    sum += contribution
}

return sum
```

Done!

1) Paper , pen