

- level order
- left / right view
- vertical order
- top / bottom view

- bin tree types
- review

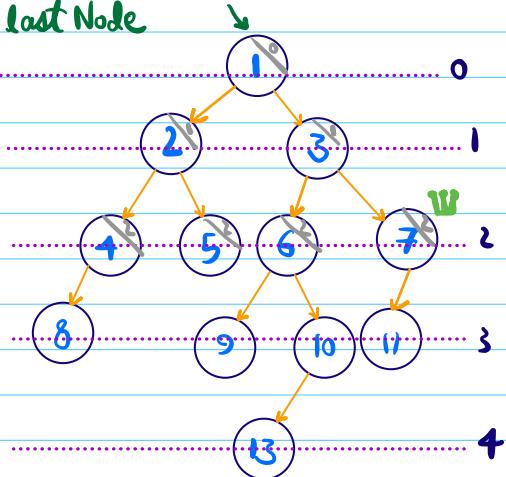
Level order

level by level
left to right

1, "
2, 3, "
4, 5, 6, 7, "
8, 9, 10, 11, "
12, "

output 8

Last Node



Queue

Fifo

output 8 ① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩ ⑪ ⑫ ⑬

```
void levelOrder(TNode r){
```

```
    if(r==null) ret;
```

```
    //Q initialize
```

```
    q.enq(r);
```

```
    last=r
```

`enq == enQueue`

`deq == deQueue`

TC: O(n)

SC: O(n)

```
while(!q.empty()) {
```

```
    n=q.deq();
```

```
    print(n.val);
```

```
    if(n.left!=null) q.enq(n.left);
```

```
    if(n.right!=null) q.enq(n.right);
```

```
    if(n==last){
```

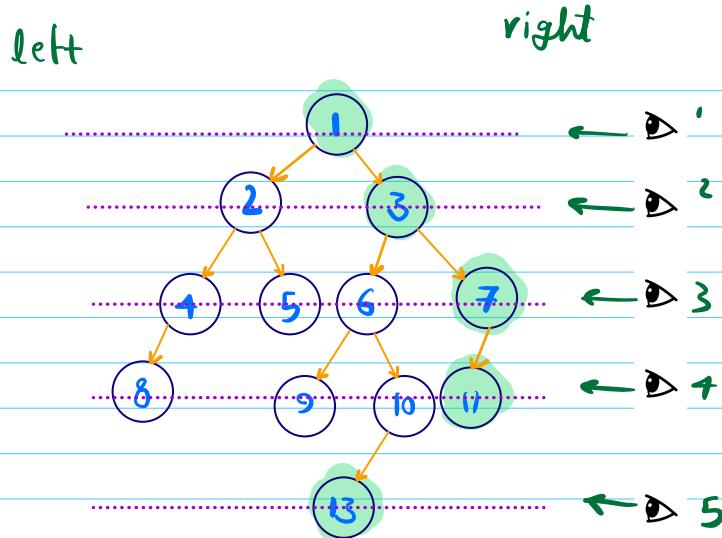
```
        print("\n")
```

```
        last=q.rear();
```

}

}

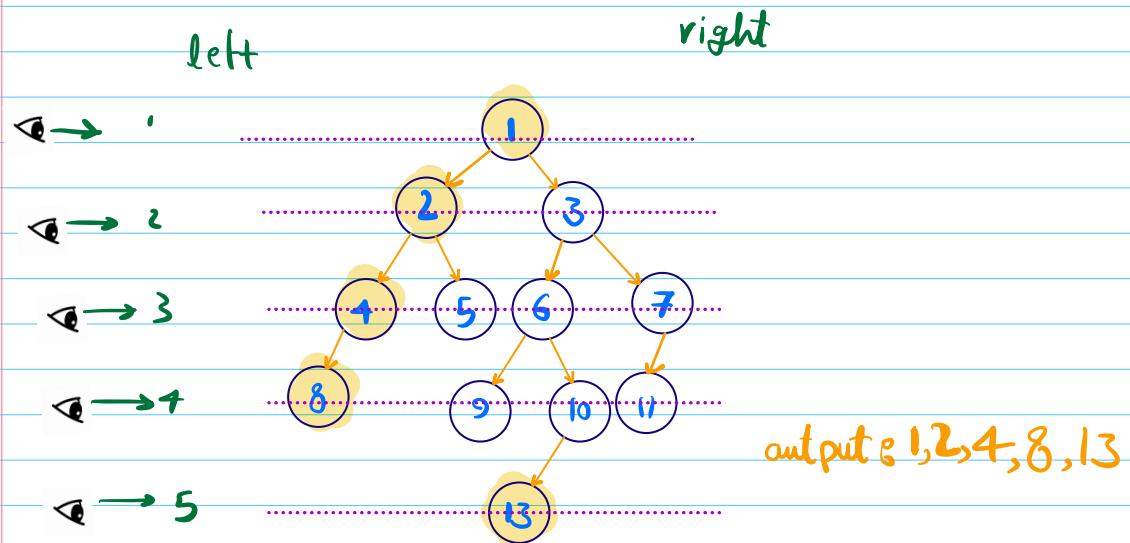
P1 Right View of tree?



output ①, ③, ⑦, ⑪, ⑬

```
void levelOrder(TNode r){  
    if(r==null) ret;  
    //Q initialize  
    q.enq(r);  
    last=r;  
    while(!q.empty()) {  
        n=q.deq();  
        if(n.left!=null) q.enq(n.left);  
        if(n.right!=null) q.enq(n.right);  
        if(n==last){  
            print(n.val);  
            last=q.rear();  
        }  
    }  
}
```

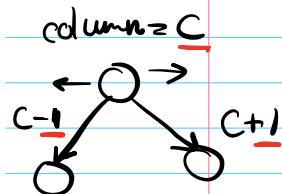
P2 left view of tree?



idea¹: add right child
then left child
assignments

idea²: print
the front
of queue
pass crown
accordingly

P3 Vertical order of tree?



- * lower level comes first
- * left nodes come first if both nodes same level

output 8, 4, 2, 9, 1, 5, 6, 13, 3, 10, 11, 7

Queue

$\langle 3, +1 \rangle \langle 4, -2 \rangle \langle 5, 0 \rangle \dots$

$\langle 1, 0 \rangle \langle 2, -1 \rangle$

Step 1: populate HM
assign columns

```

void levelOrder(TNode r) {
    if(r == null) ret;
    // HM<int, List<TNode>>
    // Q initialize
    q.enq(r, 0);
    while(!q.empty()) {
        TNode n = q.deq();
        nc = q.deg();
        if(n.left != null) q.enq(<n.left, nc.col+1>)
        if(n.right != null) q.enq(<n.right, nc.col+1>)
        // check if key exist
        HM[nc.col].add(n)
    }
}

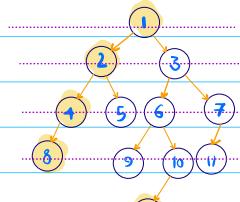
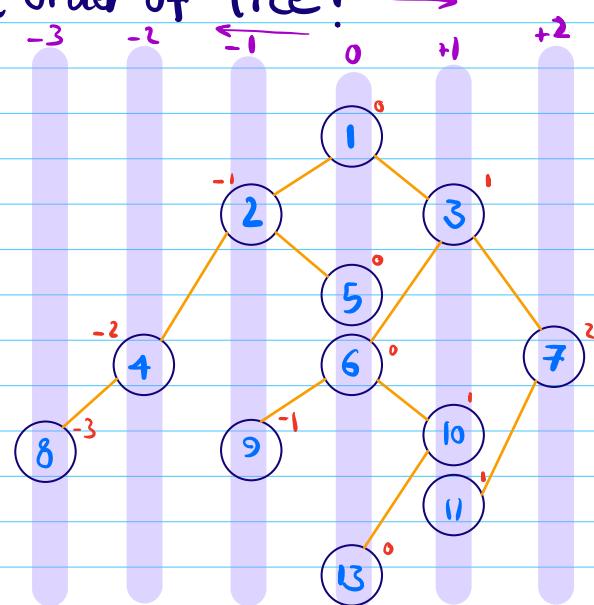
```

$\langle n.left, nc.col+1 \rangle$

enq => enQueue
deg => deQueue

$\langle n.left, nc.col+1 \rangle$

break



NC
Pair<node, int>
col

Q<NC>

Step 2:

#cols

KS = HM.keySet() →

minCol = MIN(KS)

maxCol = MAX(KS)

for(int i = minCol; i <= maxCol; i++) {

if(col i exist in HM)

for(n in HM[i]) {

print(n.val) list(Nodes)

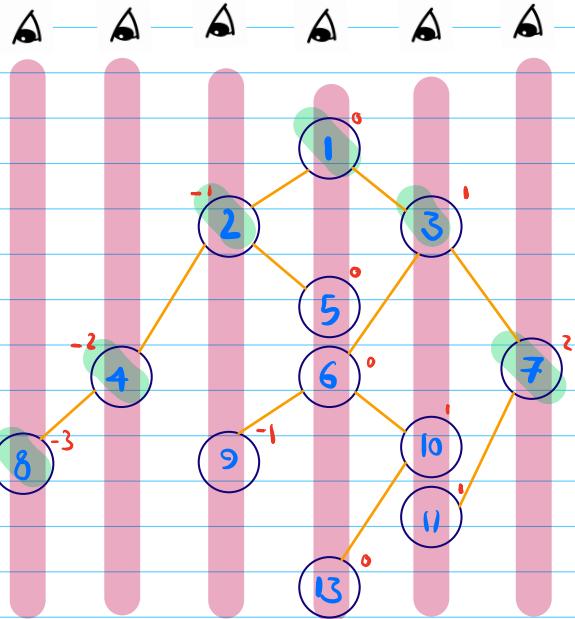
}

P4 top view of tree

8, 4, 2, 1, 3, 7

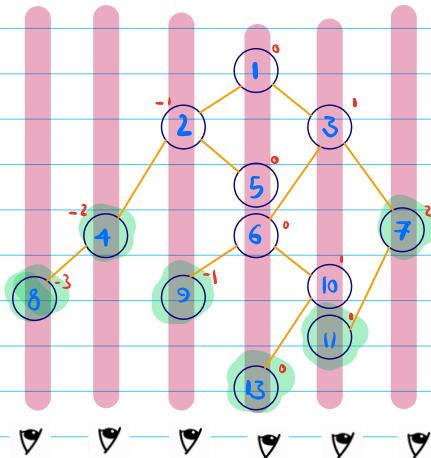
```

Step 2: #cols
kS = HM.keySet()
minCol = MIN(kS)
maxCol = MAX(kS)
for(int i = minCol; i <= maxCol; i++) {
    if(col i exist in HM)
        print(HM[i].first().val)
}
  
```



ideas

use last page code, print only first item
of list



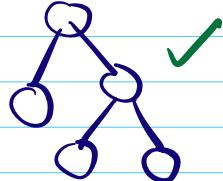
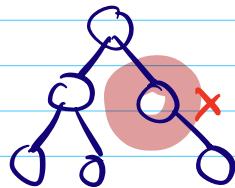
P5 bottom view of tree → assignment

types of bin trees

- proper
- complete
- Perfect

```
isProper(Tnode n) {
    if (n == null) ret true
    if (n.left != null && n.right == null) ret false
    if (n.right != null && n.left == null) ret false
    ret isProper(n.left) && isProper(n.right)
}
```

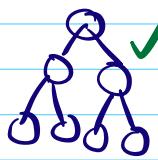
- proper either 0 or 2 children



hint 3 level order ← isComplete?

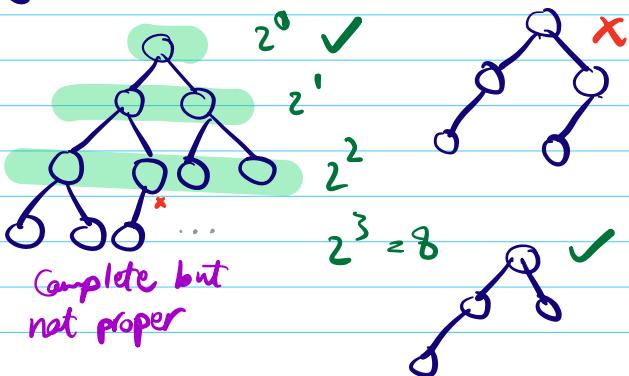
- Complete all levels are completely filled except than maybe

Quiz



the last level +

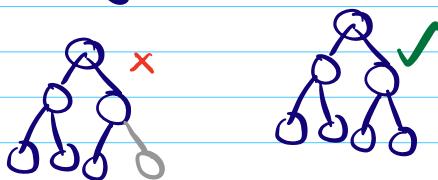
if last level not full, it should left to right in order



- Perfect all levels are completely filled

Quiz perfect ⇒ proper

Quiz



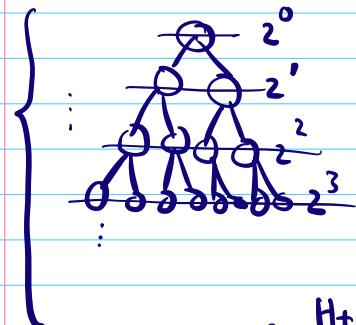
isPerfect?

Count nodes
with any traversal
and check

isPerfect? ← (#nodes == 2^{height} - 1)

P6 Find the height of a **perfect** bin. tree with n nodes

known n



$$H = l - 1$$

$$n = \frac{1(2^{H+1} - 1)}{2 - 1}$$

$$n = 2^{H+1} - 1 \Rightarrow H = \log_2(n+1) - 1$$

$$n \propto h$$

$$n = 2^0 + 2^1 + 2^2 + \dots + 2^H$$

$$q = 2$$

$$a = 2^0 = 1$$

$$\boxed{\frac{a(q^n - 1)}{q - 1}}$$

$$H+1$$

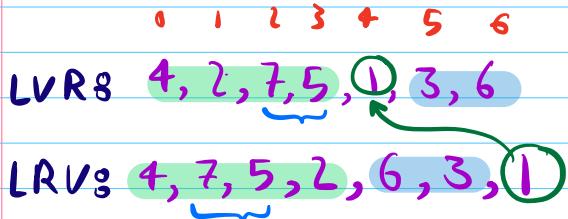
?

Last session P2 dry run

from last session

node vals are distinct

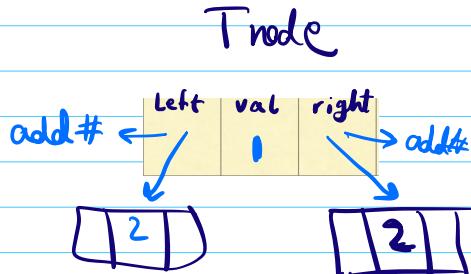
P2 Construct a tree from given inorder & Postorder



hint

① Last node of LRV
is root

② inorder
... root ...
size of L subtree index size of R subtree



tree[0, len-1] inclusive initial call
TC: O(n) SC: O(H)

```
tNode tree(in[], post[], sin, ein, sIndex, eIndex){  
    if(sin > ein || sIndex > eIndex)  
        return null  
    }
```

$O(1)$ $a-b+1$

```
{  
    root = new tNode(post[eIndex])  
    rIndex = find_index_of_root_in_inorder(in[], sin, ein, data)  
}
```

didn't use !!

```
{  
    lSize = rIndex - sin // l subtree sin → rIndex-1  
    rSize = ein - rIndex // r subtree rIndex+1 → ein  
}
```

```
root.left = tree(in, post, sin, rIndex-1, sIndex, sIndex+lSize-1)  
root.right = tree(in, post, rIndex+1, ein, sIndex+lSize, eIndex-1)
```

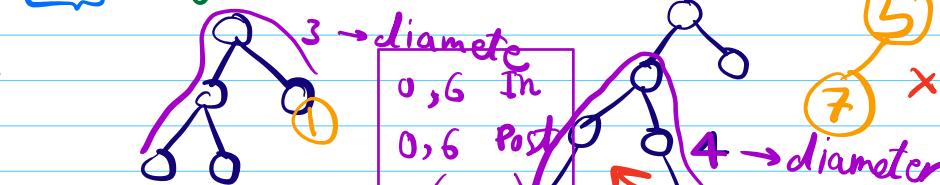
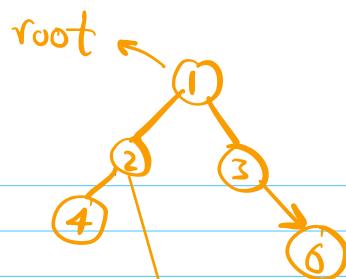
ret root

Optional
LVR8

diameter of a binary tree:

longest path of any two nodes
LRV8: 4, 7, 5, 2, 6, 3, 10

and $\Delta = 3$



Hint: use height

