Lecture-2

Revision of Lecture-1

1. Introduction to FoLT (Completed)
2. What are Logic- Propositional (Completed)
3. Types of operators for Logic (Completed)
4. Fuzzy Logic
5. Propositional Equivalences
6. Predicates and Quantifiers
7. Rules of Inference
8. Introduction to proofs
9. Normal forms

Example question based on lecture -1

An island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people A and B. What are A and B if A says "B is a knight" and B says "The two of us are opposite types?

Let: p: *A* is knight ; q: *B* is knight ; ⌐p: *A* is knave ; ⌐q: *B* is knave

CASE1: <u>A is knight </u>:

P is true so q is also true since A is knight(and knight always says truth)

However if B is a knight, then B’s statement that A and B are opposite is also true so it can be written as

$$(p \wedge \neg q) \vee (\neg p \wedge q) \text{ (and it should be true)}$$

(but the statement is false because both ‘A’ and ‘B’ are knight so we can conclude that A is not knight i.e p is false)

CASE2: <u>B is knight </u>:

CASE3: <u>A is knave </u>:

Fuzzy Logic

- **Fuzzy** refers to things which are not clear or are vague. In the real world many times we encounter a situation when we can’t determine whether the state is true or false, their fuzzy logic provides a very valuable flexibility for reasoning.
- In Fuzzy logic , a proposition has a truth value Between 0 and 1.

Application

- It is used for decision making support systems and personal evaluation in the large company business.
- Fuzzy logic are used in Natural language processing and various intensive applications in Artificial Intelligence.

Operations

- Truth values that are between 0 and 1 indicate varying degrees of truth.
- E.g: the truth value 0.8 can be assigned to the statement “Fred is happy” because Fred is happy most of the time
- The truth value 0.4 can be assigned to the statement “John is happy,” because John is happy slightly less than half the time.
- Truth value of the negation of a proposition in fuzzy logic is 1 minus the truth value of the proposition
- The truth value of the conjunction of two propositions in fuzzy logic is the minimum of the truth values of the two propositions.
- The truth value of the disjunction of two propositions in fuzzy logic is the maximum of the truth values of the two propositions.

Propositional Equivalence

Two *syntactically* (*i.e.,* textually) different compound propositions may be the *semantically* identical (*i.e.,* have the same meaning). We call them *equivalent*.

Topic to cover:

- Various *equivalence rules* or *laws*.
- How to *prove* equivalences using *symbolic derivations*.

Tautologies and Contradictions

A *tautology* is a compound proposition that is **true** *no matter what* the truth values of its atomic propositions are!

Ex. $p \vee \neg p$ [What is its truth table?]

A *contradiction* is a compound proposition that is **false** no matter what! *Ex.* $p \wedge \neg p$ [Truth table?]

Other compound props. are *contingencies*.

Logical Equivalence

Compound proposition *p* is *logically equivalent* to compound proposition *q*, written $p \Leftrightarrow q$, **IFF** the compound proposition $p \leftrightarrow q$ is a tautology.

Compound propositions *p* and *q* are logically equivalent to each other **IFF** *p* and *q* contain the same truth values as each other in <u>all</u> rows of their truth tables.

Proving Equivalence via Truth Tables

Ex. Prove that $p \vee q \Leftrightarrow \neg(\neg p \wedge \neg q)$.

p	q	$p \vee q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
F	F	F	T	T	T	F
F	T	T	T	F	F	T
T	F	T	F	T	F	T
T	T	T	F	F	F	T

- $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent. T/F

TABLE 4 **Truth Tables for $\neg p \vee q$ and $p \rightarrow q$.**

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

- Is $p \lor (q \land r)$ and $(p \lor q) \land (p \lor r)$ are logically equivalent.

- This is the *distributive law* of disjunction over conjunction

TABLE 5 A Demonstration That $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ Are Logically Equivalent.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Equivalence Laws

- They provide a pattern or template that can be used to match all or part of a much more complicated proposition and to find an equivalence for it.
- Just like algebra property equivalence property of Prepositions can also be defined.

Equivalence Laws - Examples

- *Identity*: $p \wedge \mathbf{T} \Leftrightarrow p$ $\quad p \vee \mathbf{F} \Leftrightarrow p$
- *Domination*: $p \vee \mathbf{T} \Leftrightarrow \mathbf{T}$ $\quad p \wedge \mathbf{F} \Leftrightarrow \mathbf{F}$
- *Idempotent*: $p \vee p \Leftrightarrow p$ $\quad p \wedge p \Leftrightarrow p$
- *Double negation:* $\neg\neg p \Leftrightarrow p$
- *Commutative:* $p \vee q \Leftrightarrow q \vee p$ $\quad p \wedge q \Leftrightarrow q \wedge p$
- *Associative:* $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

  $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

More Equivalence Laws

- *Distributive*: $p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)$
  $p \land (q \lor r) \Leftrightarrow (p \land q) \lor (p \land r)$
- *De Morgan's*:
  $\neg(p \land q) \Leftrightarrow \neg p \lor \neg q$
  $\neg(p \lor q) \Leftrightarrow \neg p \land \neg q$
- *Trivial tautology/contradiction*:
  $p \lor \neg p \Leftrightarrow \mathbf{T}$ $\quad$ $p \land \neg p \Leftrightarrow \mathbf{F}$

Augustus
De Morgan
(1806-1871)

e.g

Use De Morgan's laws to express the negations of "Mick has a cellphone and he has a laptop computer"

- Let p be “Mick has a cellphone” and q be “Mick has a laptop computer.”
- So “Mick has a cellphone and he has a laptop computer” can be represented by $p \land q$
- De Morgan’s laws, $\neg(p \land q)$ is equivalent to $\neg p \lor \neg q$
- the negation of our original statement as “Mick does not have a cellphone or he does not have a laptop computer

Defining Operators via Equivalences

Using equivalences, we can *define* operators in terms of other operators.

- Exclusive or: $p \oplus q \Leftrightarrow (p \vee q) \wedge \neg(p \wedge q)$
  $p \oplus q \Leftrightarrow (p \wedge \neg q) \vee (q \wedge \neg p)$
- Implies: $p \rightarrow q \Leftrightarrow \neg p \vee q$
- Biconditional: $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$
  $p \leftrightarrow q \Leftrightarrow \neg(p \oplus q)$

Show that $(p \land q) \rightarrow (p \lor q)$ is a tautology.

- To show that this statement is a tautology , we can use truth table

$$
\begin{aligned}
(p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) \\
&\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{by the first De Morgan law} \\
&\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{by the associative and commutative laws for disjunction} \\
&\equiv \mathbf{T} \vee \mathbf{T} && \text{the commutative law for disjunction} \\
&\equiv \mathbf{T} && \text{by the domination law}
\end{aligned}
$$

An Example Problem

- Check using a symbolic derivation whether $(p \wedge \neg q) \rightarrow (p \oplus r) \Leftrightarrow \neg p \vee q \vee \neg r$.

$(p \wedge \neg q) \rightarrow (p \oplus r)$ [Expand definition of $\rightarrow$]

$\Leftrightarrow \underline{\neg}(p \wedge \neg q) \underline{\vee} (p \oplus r)$ [Expand defn. of $\oplus$]

$\Leftrightarrow \neg(p \wedge \neg q) \vee (\underline{(p \vee r) \wedge \neg(p \wedge r)})$

[DeMorgan's Law]

$\Leftrightarrow \underline{(\neg p \vee q)} \vee ((p \vee r) \wedge \neg(p \wedge r))$

cont.

Example Continued...

$(\neg p \vee q) \vee ((p \vee r) \wedge \neg(p \wedge r)) \Leftrightarrow$ [$\vee$ commutes]

$\Leftrightarrow (\underline{q \vee \neg p}) \vee ((p \vee r) \wedge \neg(p \wedge r))$ [$\vee$ associative]

$\Leftrightarrow q \vee \underline{(}\neg p \vee ((p \vee r) \wedge \neg(p \wedge r)))$ [distrib. $\vee$ over $\wedge$]

$\Leftrightarrow q \vee (((\neg p \vee (p \vee r)) \wedge (\underline{\neg p \vee} \neg(p \wedge r)))$

[assoc.] $\Leftrightarrow q \vee (((\neg p \vee p) \vee r) \wedge (\neg p \vee \neg(p \wedge r)))$

[trivail taut.] $\Leftrightarrow q \vee ((\mathbf{T} \vee r) \wedge (\neg p \vee \neg(p \wedge r)))$

[domination] $\Leftrightarrow q \vee (\mathbf{T} \wedge (\neg p \vee \neg(p \wedge r)))$

[identity] $\Leftrightarrow q \vee (\neg p \vee \neg(p \wedge r)) \Leftrightarrow$ *cont.*

End of Long Example

$q \vee (\neg p \vee \neg(p \wedge r))$

[DeMorgan's] $\Leftrightarrow q \vee (\neg p \vee (\neg p \vee \neg r))$

[Assoc.] $\Leftrightarrow q \vee ((\neg p \vee \neg p) \vee \neg r)$

[Idempotent] $\Leftrightarrow q \vee (\neg p \vee \neg r)$

[Assoc.] $\Leftrightarrow (q \vee \neg p) \vee \neg r$

[Commut.] $\Leftrightarrow \neg p \vee q \vee \neg r$

(Which was to be shown.)

Review: Propositional Logic

- Atomic propositions: $p, q, r, \ldots$
- Boolean operators: $\neg \wedge \vee \oplus \rightarrow \leftrightarrow$
- Compound propositions: $s :\equiv (p \wedge \neg q) \vee r$
- Equivalences: $p \wedge \neg q \Leftrightarrow \neg(p \rightarrow q)$
- Proving equivalences using:
  - Truth tables.
  - Symbolic derivations. $p \Leftrightarrow q \Leftrightarrow r \ldots$