

## # Combinational circuit

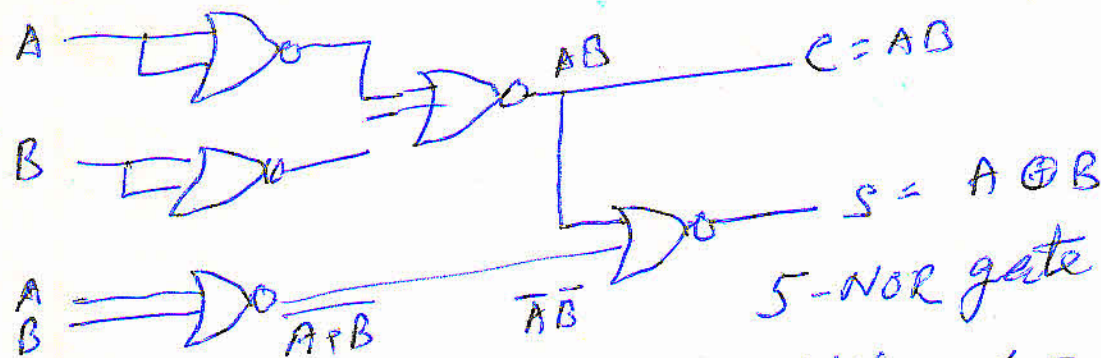
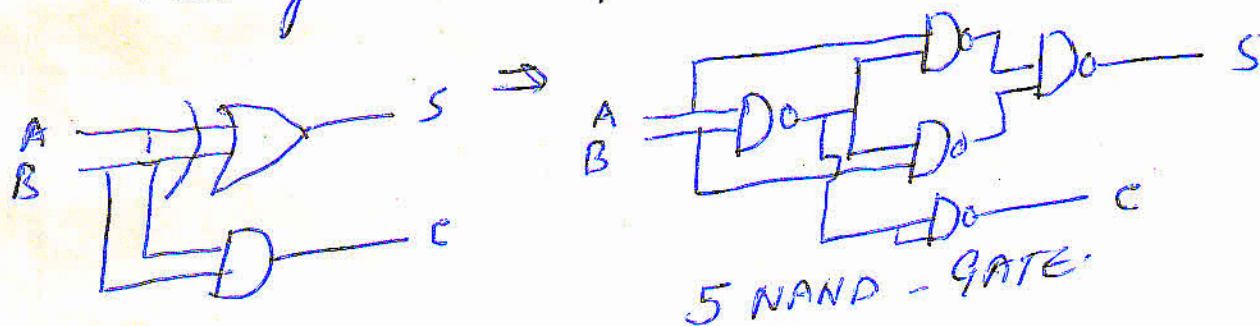
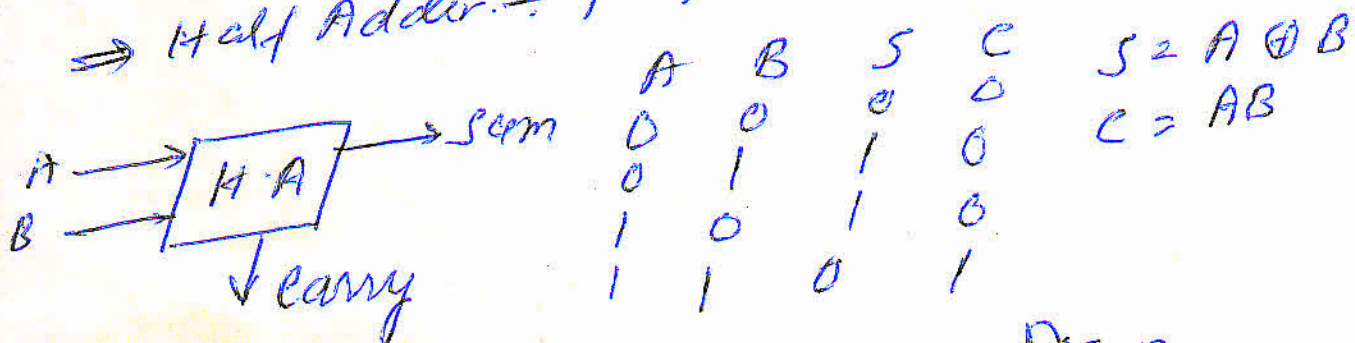
There is no feedback element. present o/p depend only up on present i/p. (NO. memory element)

### # Design of Combinational circuit

1. Identify i/p and o/p
2. Construct truth table
3. write logical operation
4. simplify logical "
5. Construct the logic ckt

### # Arithmetic circuit - HA, HS, FA, FS

⇒ Half Adder: perform addition of two bits



⇒ Full Adder → perform addition of 3 bits  
 Two input ~~X~~ and ~~Y~~ represent two significant bits to be added and third input Z represent carry from lower significant position.

⇒

| x | y | z | S | C |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$$S = \Sigma(1, 2, 4, 7)$$

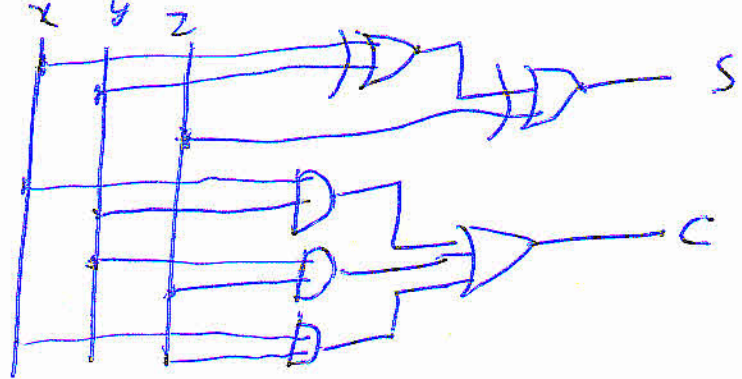
$$C = \Sigma(3, 5, 6, 7)$$

|           | $\bar{y}\bar{z}$ | $\bar{y}z$ | $y\bar{z}$ | $yz$ |
|-----------|------------------|------------|------------|------|
| $\bar{x}$ |                  | 1          |            | 1    |
| $x$       | 1                |            | 1          |      |

$$S = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$$

$$= \bar{x}(y \oplus z) + x(y \oplus z)$$

$$S = x \oplus y \oplus z$$



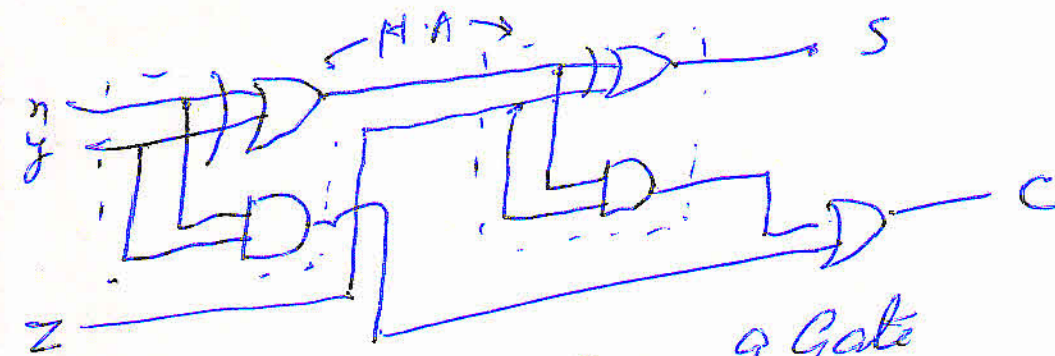
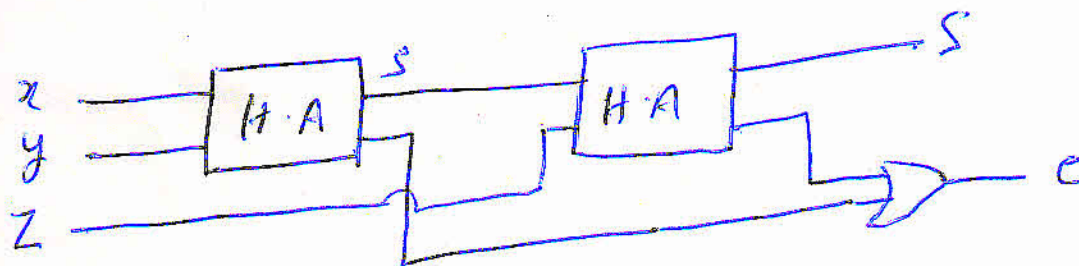
|           | $\bar{y}\bar{z}$ | $\bar{y}z$ | $y\bar{z}$ | $yz$ |
|-----------|------------------|------------|------------|------|
| $\bar{x}$ |                  |            | 1          |      |
| $x$       |                  | 1          |            | 1    |

for carry

$$C = xy + yz + zx$$

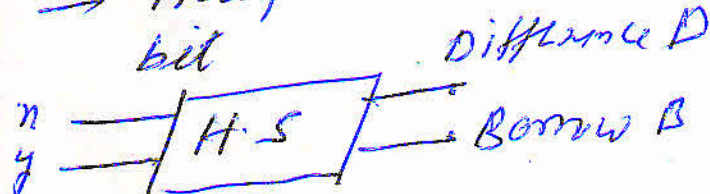
$$= xy + \bar{x}yz + x\bar{y}z$$

$$= xy + (x \oplus y)z$$



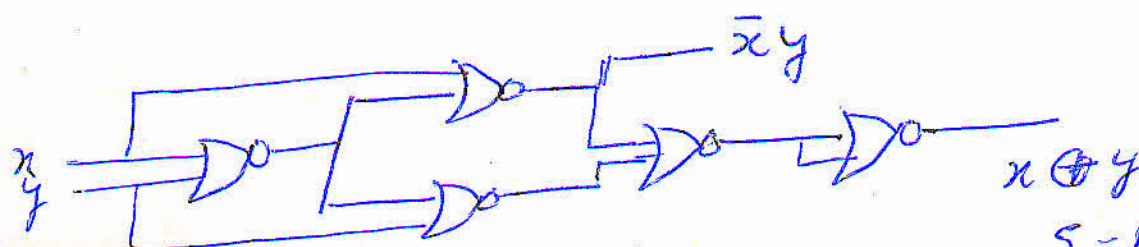
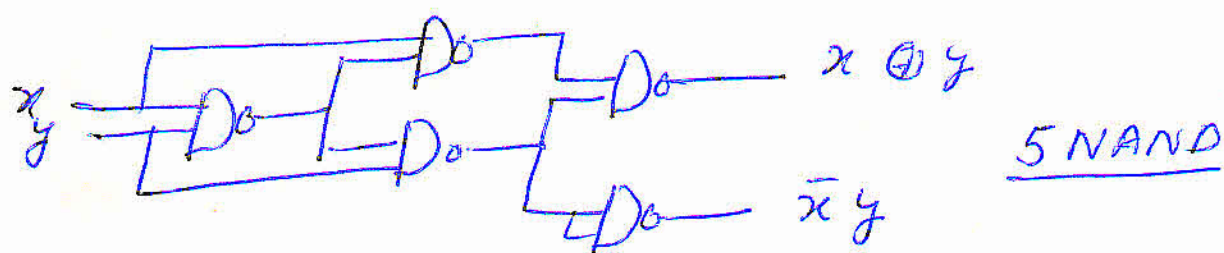
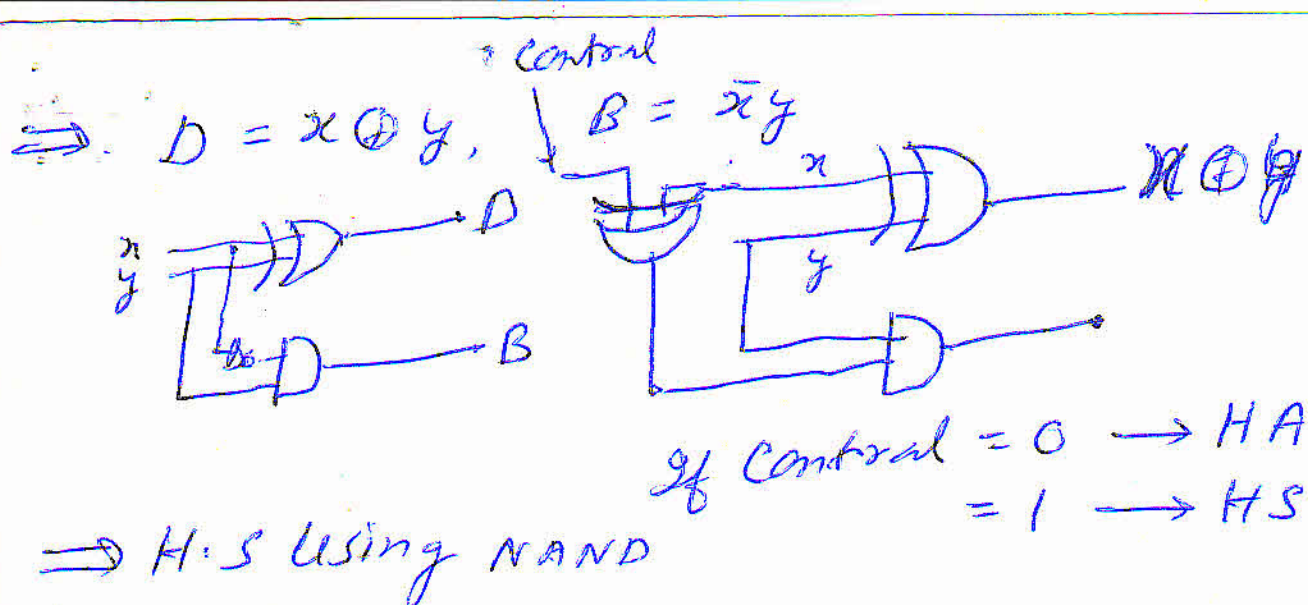
\* F.A by NAND — 9 Gate  
NOR — 9 Gate

⇒ Half Subtractor — perform subtraction of two bit



| x | y | D | B |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |





$\Rightarrow$  Design a HA/H.S controlled i.e. it behave as Half Subtractor for  $S=0$  and HA for  $S=1$

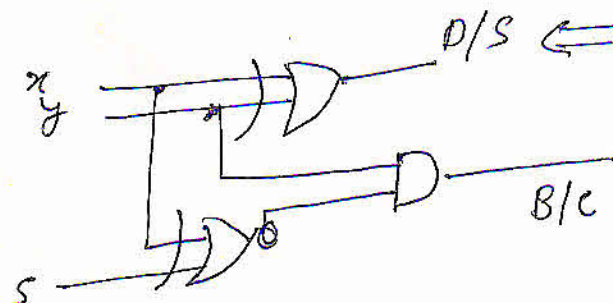
|    | S | x | y | S/D | B/C |
|----|---|---|---|-----|-----|
| HS | 0 | 0 | 0 | 0   | 0   |
|    | 0 | 0 | 1 | 1   | 0   |
|    | 0 | 1 | 0 | 0   | 0   |
|    | 0 | 1 | 1 | 0   | 0   |
| HA | 1 | 0 | 0 | 0   | 0   |
|    | 1 | 0 | 1 | 1   | 0   |
|    | 1 | 1 | 0 | 1   | 0   |
|    | 1 | 1 | 1 | 0   | 1   |

| S         | $\bar{x}\bar{y}$ | $\bar{x}y$ | $x\bar{y}$ | $xy$ |
|-----------|------------------|------------|------------|------|
| $\bar{S}$ |                  | 1          |            | 1    |
| S         |                  | 1          |            | 1    |

$$D/S = \bar{x}y + x\bar{y} = x \oplus y$$

| S         | $\bar{x}\bar{y}$ | $\bar{x}y$ | $x\bar{y}$ | $xy$ |
|-----------|------------------|------------|------------|------|
| $\bar{S}$ |                  | 1          |            |      |
| S         |                  |            | 1          |      |

$$B/C = \bar{S}\bar{x}y + Sx\bar{y} = y \cdot \overline{x \oplus y} = y \cdot x \odot y$$

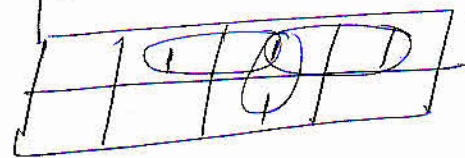


# Full Subtractor → perform subtraction between two bits taking into account that a 1 have been borrowed from a lower significant stage

| Stage | Min | Sub | Borrow | D | B |
|-------|-----|-----|--------|---|---|
| 0     | 0   | 0   | 0      | 0 | 0 |
| 0     | 0   | 0   | 1      | 1 | 1 |
| 0     | 0   | 1   | 0      | 1 | 1 |
| 0     | 0   | 1   | 1      | 0 | 1 |
| 1     | 0   | 0   | 0      | 1 | 0 |
| 1     | 0   | 0   | 1      | 0 | 0 |
| 1     | 1   | 0   | 0      | 0 | 0 |
| 1     | 1   | 1   | 1      | 1 | 1 |

|           | $\bar{y}z$ | $y\bar{z}$ | $yz$ | $\bar{y}\bar{z}$ |
|-----------|------------|------------|------|------------------|
| $\bar{x}$ | 0          | 1          | 0    | 1                |
| $x$       | 1          | 0          | 1    | 0                |

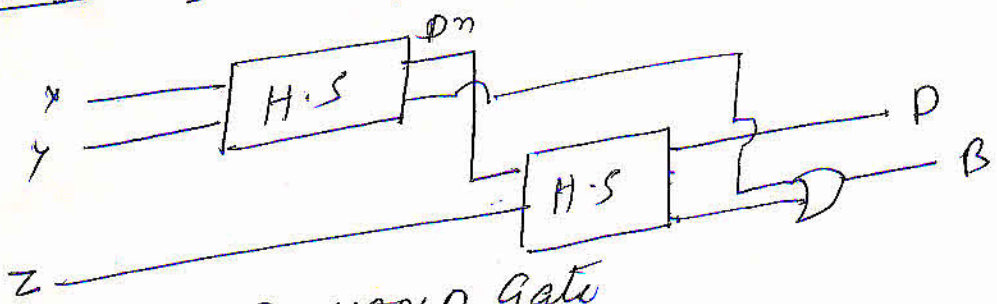
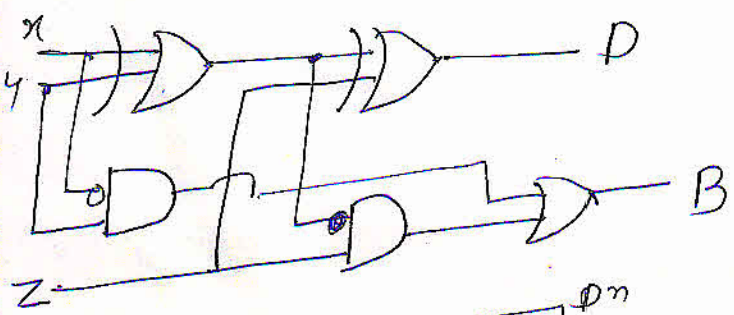
$D = x \oplus y \oplus z$



$B = \bar{x}z + \bar{x}y + yz$

$B = \bar{x}y + \bar{x}\bar{y}z + xyz$

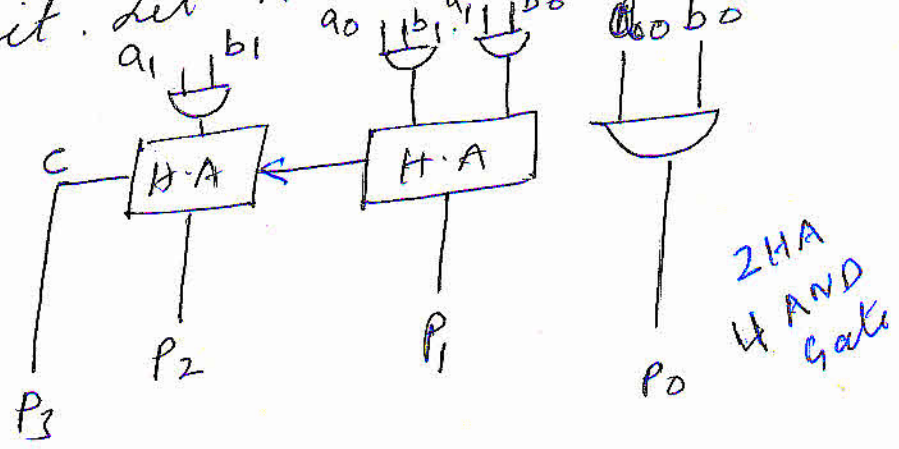
$B = \bar{x}y + [\bar{x} \oplus y]z$



⇒ F.S → 9 NAND Gate  
→ 9 NOR Gate.

# Two bit multiplier ⇒ In general A is of m bit (m ≠ 1) and B is of n bit (n ≠ 1) then product of AB will have (m+n) bit. let  $A = a_1 a_0$   $B = b_1 b_0$

$$\begin{array}{r}
 A \cdot B = \begin{array}{r} a_1 a_0 \\ b_1 b_0 \\ \hline a_1 b_0 \quad a_0 b_0 \\ + a_1 b_1 \quad b_1 a_0 \quad x \\ \hline P_3 \quad P_2 \quad P_1 \quad P_0 \end{array}
 \end{array}$$





# Three bit multiplier.

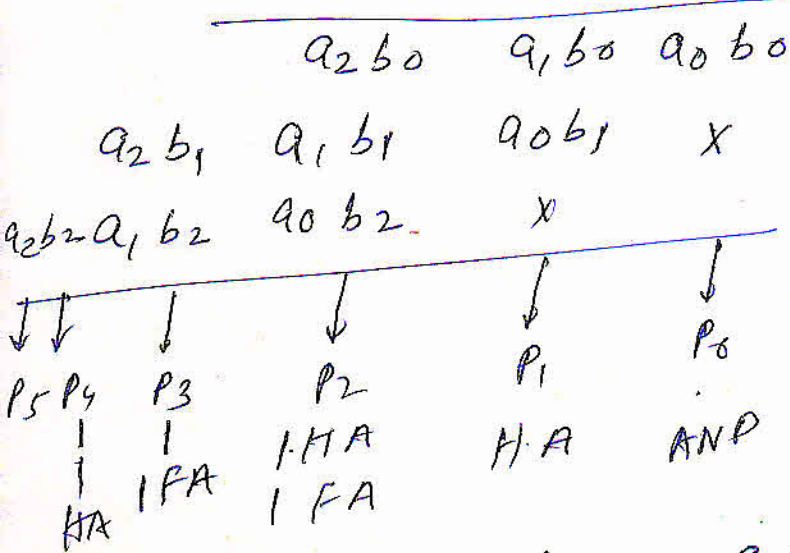
$$A = a_2 a_1 a_0$$

$$B = b_2 b_1 b_0$$

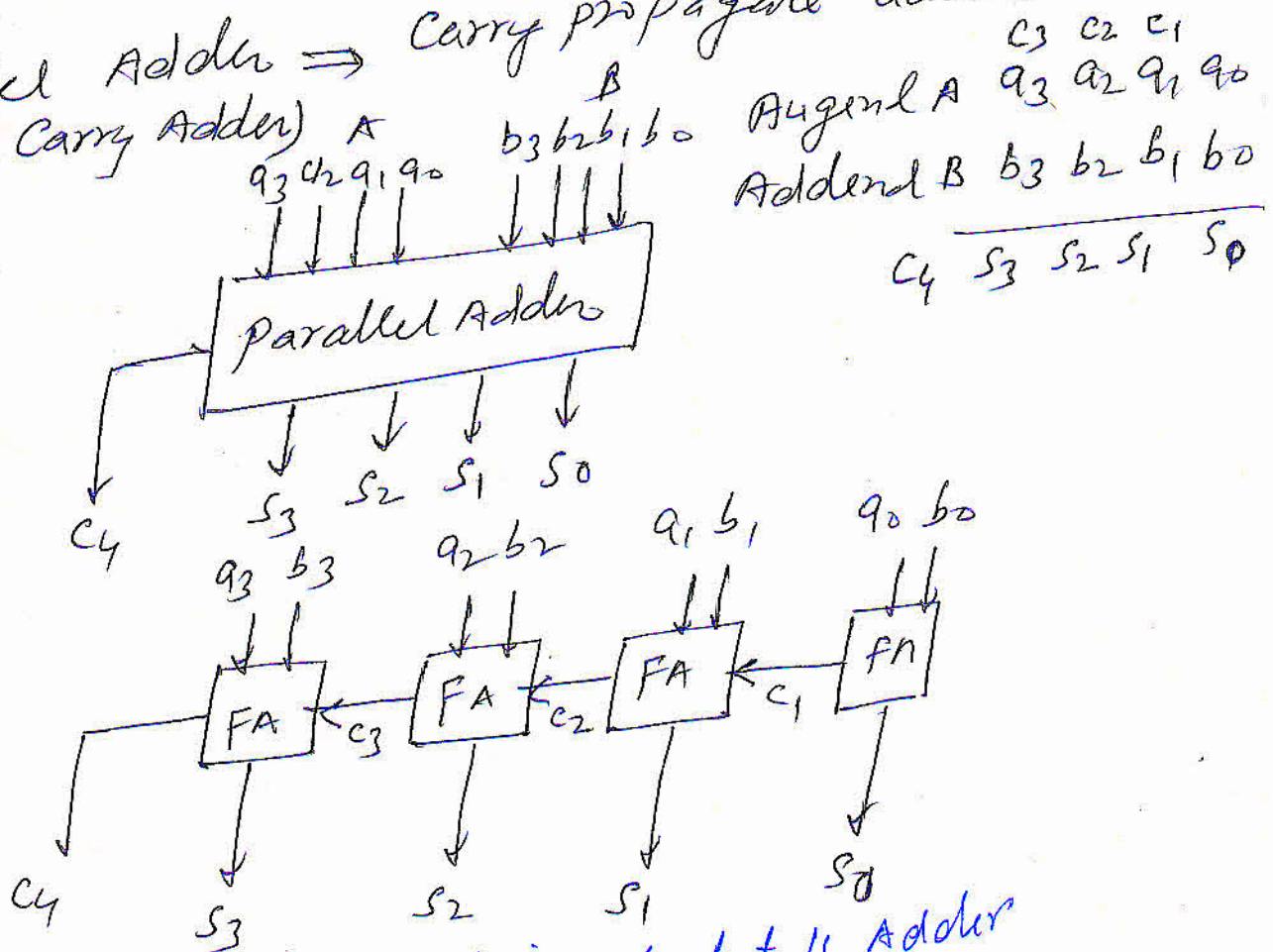
Two  $\rightarrow$  FA

Three  $\rightarrow$  HA

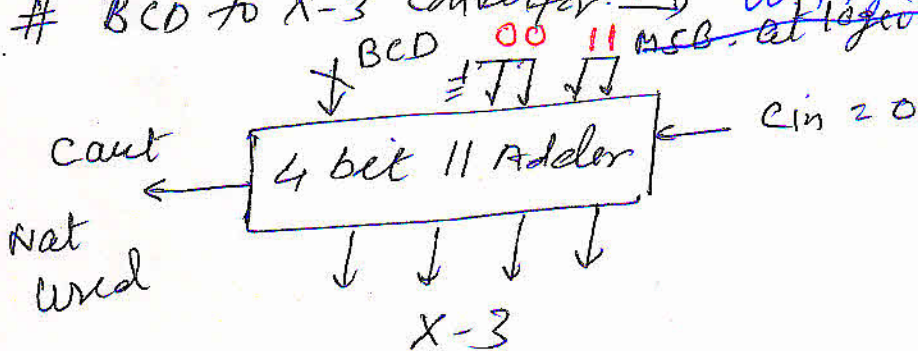
one  $\rightarrow$  AND GATE



# Parallel Adder  $\Rightarrow$  Carry propagate adder  
(ripple Carry Adder)



# BCD to X-3 Converter  $\Rightarrow$  Using 4 bit 11 Adder



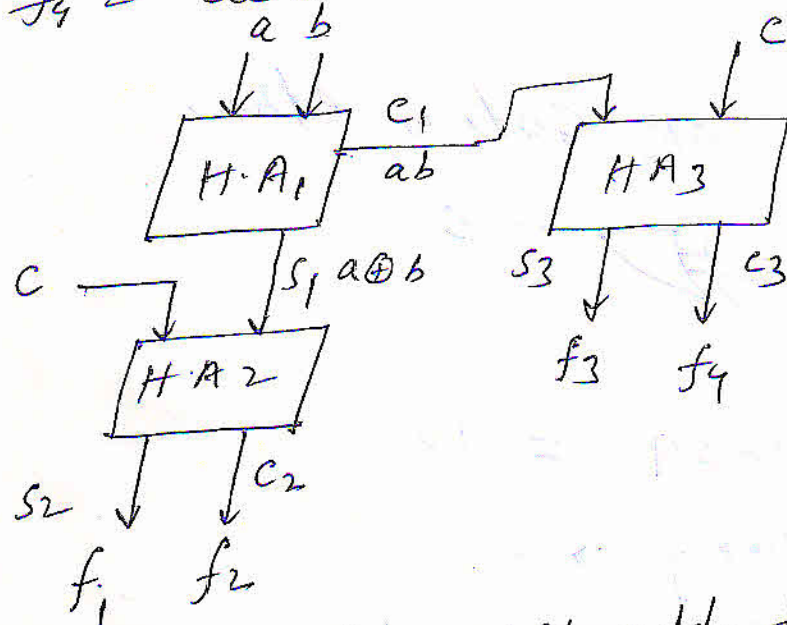
⇒ Implement following 4 functions using H.A.

$$f_1 = a \oplus b \oplus c$$

$$f_2 = \bar{a}bc + a\bar{b}c = [a \oplus b]c$$

$$f_3 = ab\bar{c} + (\bar{a} + \bar{b})c = ab\bar{c} + \bar{a}bc + \bar{a}b\bar{c} = ab \oplus c$$

$$f_4 = abc$$



# BCD Adder — It adds two BCD digits and provide the result in BCD (sum digit). It must include correction logic if  $\text{sum} > 9$  or carry is generated than 0110 is to be added to the result. The following K-map gives the design of a ckt that decide whether the sum is a valid BCD code or not.

| $s_3s_2$             | $s_1s_0$ | $\bar{s}_1s_0$ | $\bar{s}_1s_0$ | $s_1s_0$ | $s_1s_0$ |
|----------------------|----------|----------------|----------------|----------|----------|
| $\bar{s}_3\bar{s}_2$ | 0        | 0              | 0              | 0        | 0        |
| $\bar{s}_3s_2$       | 0        | 0              | 0              | 0        | 0        |
| $s_3\bar{s}_2$       | 1        | 1              | 1              | 1        | 1        |
| $s_3s_2$             | 0        | 0              | 1              | 1        | 1        |

$$f = s_3s_2 + s_3s_1$$

$$f = s_3(s_1 + s_2)$$

How Many don't care terms will be in BCD Adder.

Total term.  $2^9 = 512$  A B A B  
A and B can vary from 00 to 99  
= 100

100 with  $C_m = 0$   
100 "  $C_m = 1$ , will contribute  
200 valid terms.

$$\text{Don't care} = 512 - 200 = 312$$



If  $Sum > 9$

When ever the o/p of AND gate is 0 or vice versa the Carry from first adder is 1

Here  $0 \nrightarrow 1$

If  $Sum \leq 9$

then Input to En-OR Gate is either 0,0, or 1,1 Hence

$0,0$  or  $1,1 \rightarrow 0$

Example

$9 \Rightarrow 1001$   $Sum > 9 = 12$   
 $3 \Rightarrow 0011$

$1100$   
 $0110$

Carry  $\rightarrow 0$

$S_3(s_2 + s_1) \rightarrow 1$

Correction

$0001$   $0010$   
BCD BCD

$0 \nrightarrow 1$   
 $1 \nrightarrow 1$

$0110$   
 $0011$   
 $1001$

$Sum < 9$

Carry  $\rightarrow 0$

$S_3(s_2 + s_1) \rightarrow 0$

$0 \nrightarrow 0$   
 $0 \nrightarrow 0$

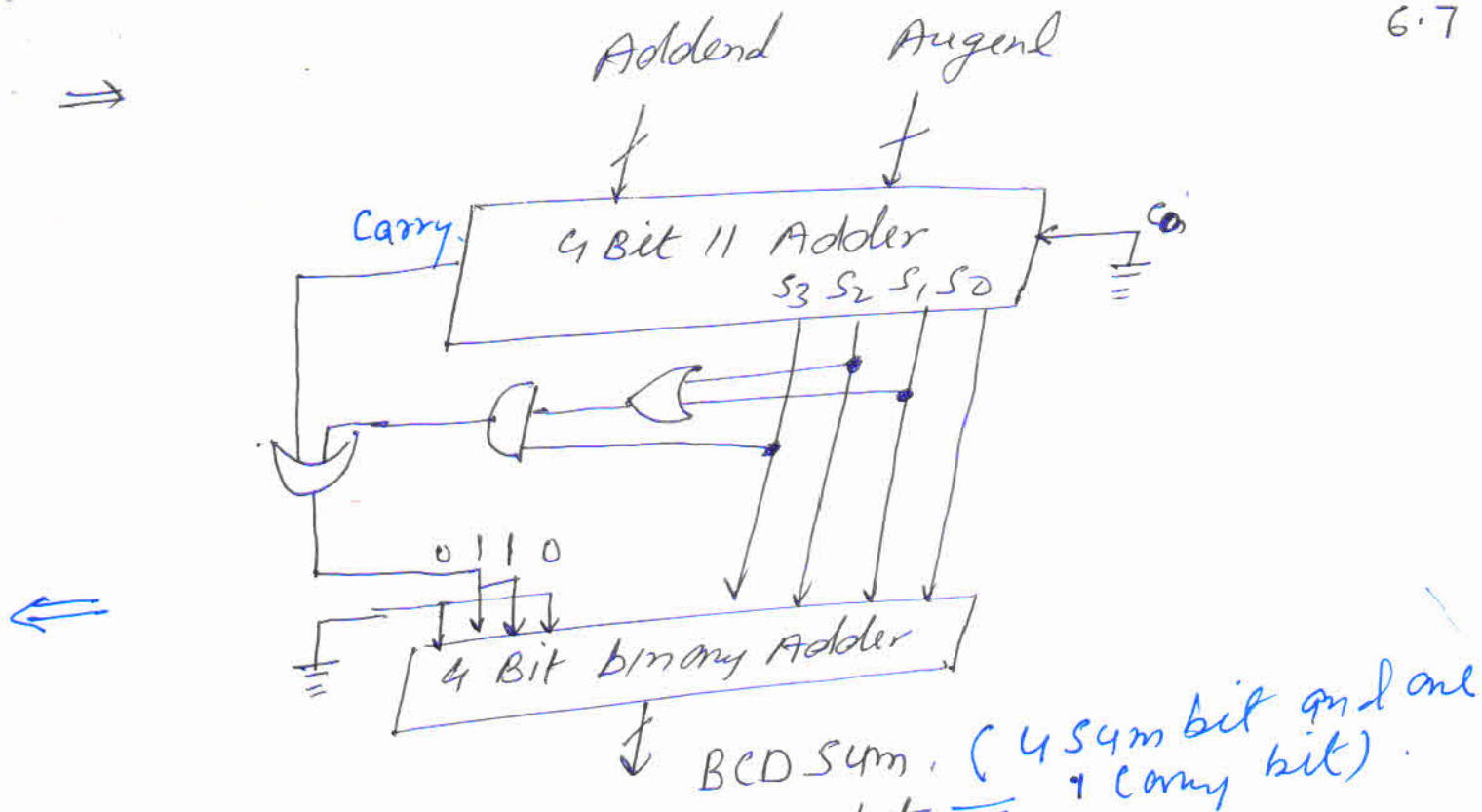
$9 \rightarrow BCD \rightarrow 1001$   
 $9 - " \rightarrow 1001$   
 $1 \leftarrow \text{Carry} \rightarrow 0010$

$S_3(s_2 + s_1) \rightarrow 0$  +  $0910 \rightarrow \text{correction}$

$0 \nrightarrow 1$

$0001$   $1000$   
BCD of BCD  $\rightarrow 8$

$\Rightarrow 18$

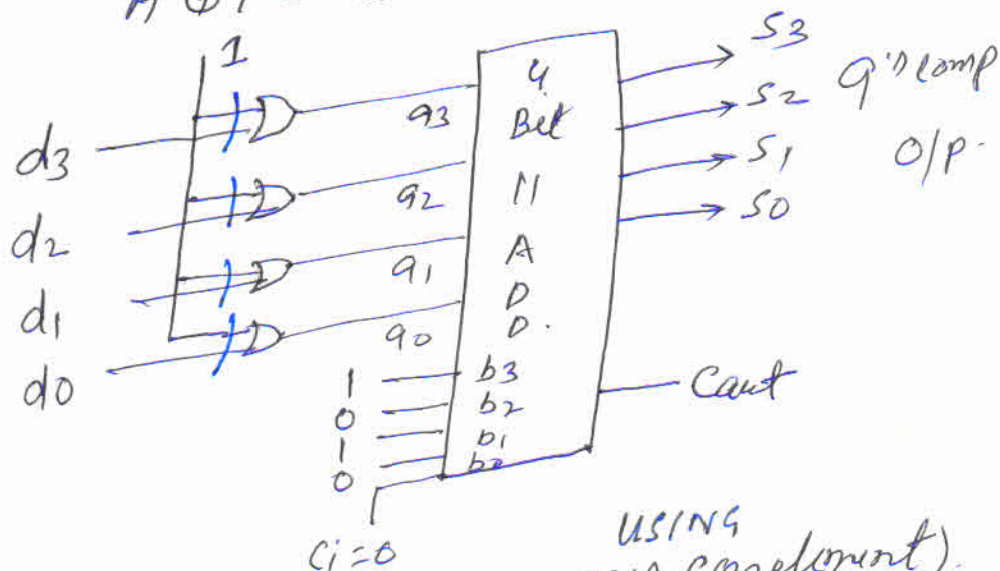


# 9's Complement of a BCD data =

9 - in decimal = 1's Complement + 1010

1's comp. of a binary data can be achieved by -

$$A \oplus 1 = A'$$



BCD of 4  
0100  
9's complement = 5

in binary  
1's comp. → 1011

$$\begin{array}{r} 1011 \\ + 1010 \\ \hline 0101 = 5 \text{ in BCD} \end{array}$$

Ci = 0

# Adder/subtractor (2's complement).

\* EX-OR acts as Inverter for Control = 1

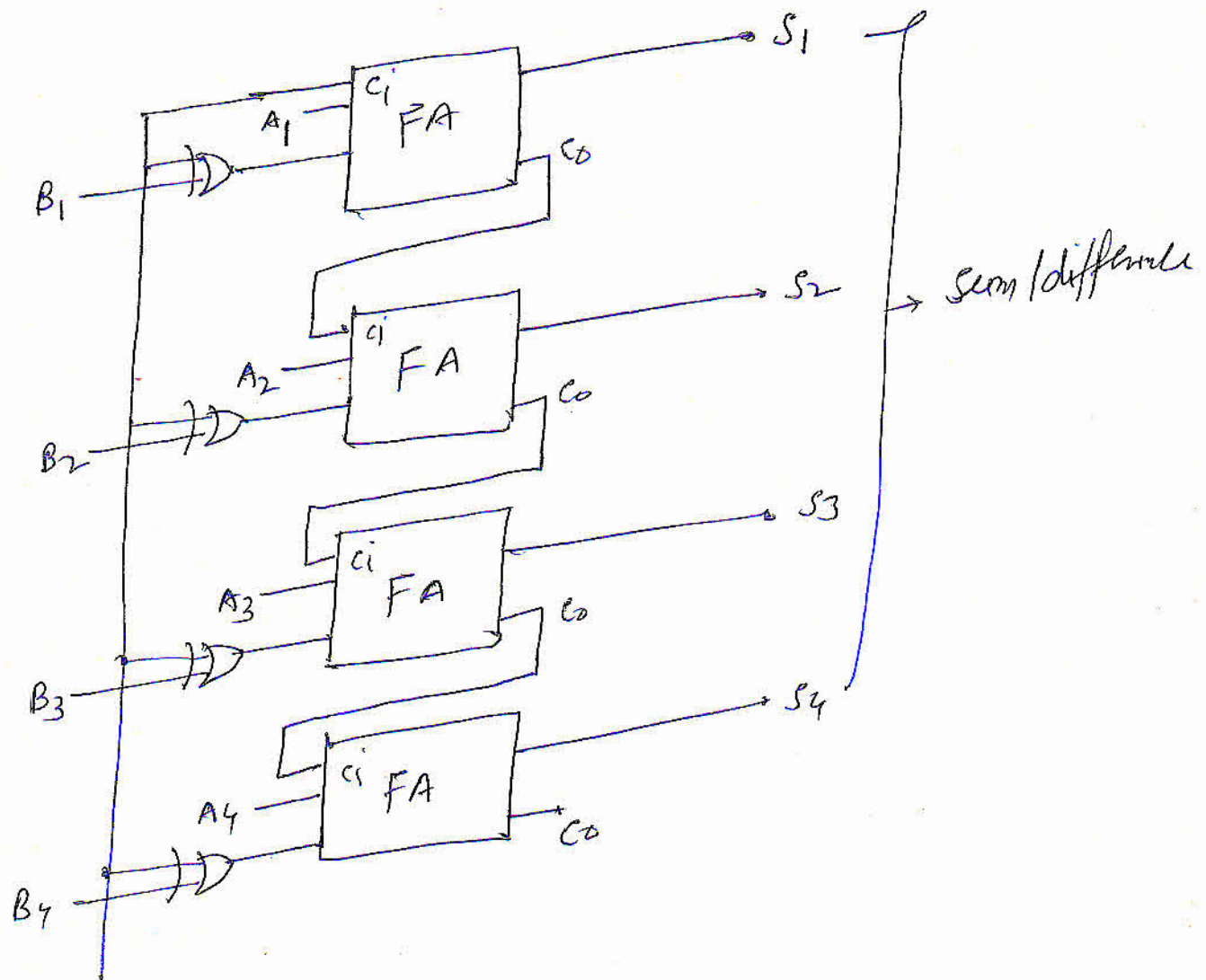
\* EX-OR acts as buffer " " = 0

A<sub>4</sub> A<sub>3</sub> A<sub>2</sub> A<sub>1</sub> A<sub>0</sub>

B<sub>4</sub> B<sub>3</sub> B<sub>2</sub> B<sub>1</sub> B<sub>0</sub>

+/-





# speed limitation of binary Parallel adder

1st level gate  
2nd level gate

$b_4 a_4$   $b_3 a_3$   $b_2 a_2$   $b_1 a_1$   $a_i$   $b_i$   $P_i$   $g_i$   $S_i$   $c_{i+1}$

$c_5$   $c_4$   $c_3$   $c_2$   $c_1$   $c_i$   $c_{i+1}$

$S_4$   $S_3$   $S_2$   $S_1$

\* The signal from input carry  $c_i$  to output  $c_{i+1}$  propagate through an AND gate and OR gate which constitutes two gate levels.

\* If there are 4 FAs in 11 adder o/p carry  $c_5$  would have  $2 \times 4 = 8$  gate levels from  $c_1$  to  $c_5$

\*  $a_i$  and  $b_i$  are simultaneously available at each FA for  $S_i$  to be in steady state. The previous

carry has to propagate through two gate levels.

\* The signals  $P_i$  and  $G_i$  settle to their steady state value after propagation through respective gates i.e. only one level (HA).  
 $P_i$  and  $G_i$  are common to all FAs and depend on only  $a_i$  and  $b_i$ ,  $FA_1, FA_2, FA_3, FA_4$  all provide  $P_i$  and  $G_i$  simultaneously after one gate delay.

$$\Rightarrow \text{Total propagation time} = \text{Delay of one (HA)} + 2 \times n \times \text{each gate Delay}$$

$$\text{in general total propagation delay} = (2n+1) \text{ times delay of each gate}$$

$$\text{for 4 bit 11 Adder} = (2 \times 4 + 1) 10ns = 90ns$$

each gate delay = 10ns

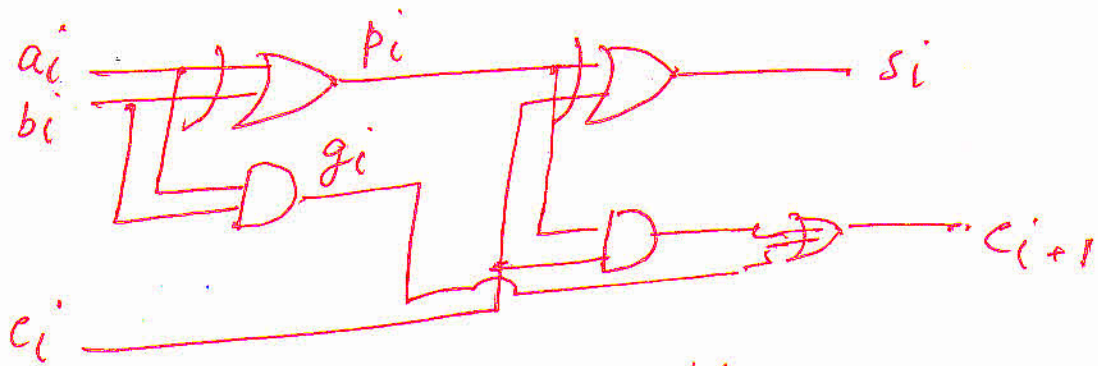
- $C_1 \text{ to } C_2 = 30ns$
- $C_2 \text{ to } C_3 = 20ns$
- $C_3 \text{ to } C_4 = 20ns$
- $C_4 \text{ to } C_5 = 20ns = 90ns$

$\Rightarrow$  The Carry propagation time can be reduced by using faster gates but there is always a physical limit.

$\Rightarrow$  Another way is to increase hardware complexity so as to reduce Carry propagation time.

# Carry look ahead Adder — in this type of adder all carries get generated simultaneously.





Let us define two Variable

$p_i = a_i \oplus b_i \rightarrow$  is called carry propagate

$g_i = a_i b_i \rightarrow$  is "generates"

$$s_i = p_i \oplus c_i$$

$$c_{i+1} = g_i + p_i c_i$$

$$c_2 = g_1 + p_1 c_1$$

$$c_3 = g_2 + p_2 c_2 = g_2 + p_2 (g_1 + p_1 c_1)$$

$$c_3 = g_2 + p_2 g_1 + p_2 p_1 c_1$$

$$c_4 = g_3 + p_3 c_3 = g_3 + p_3 (g_2 + p_2 g_1 + p_2 p_1 c_1)$$

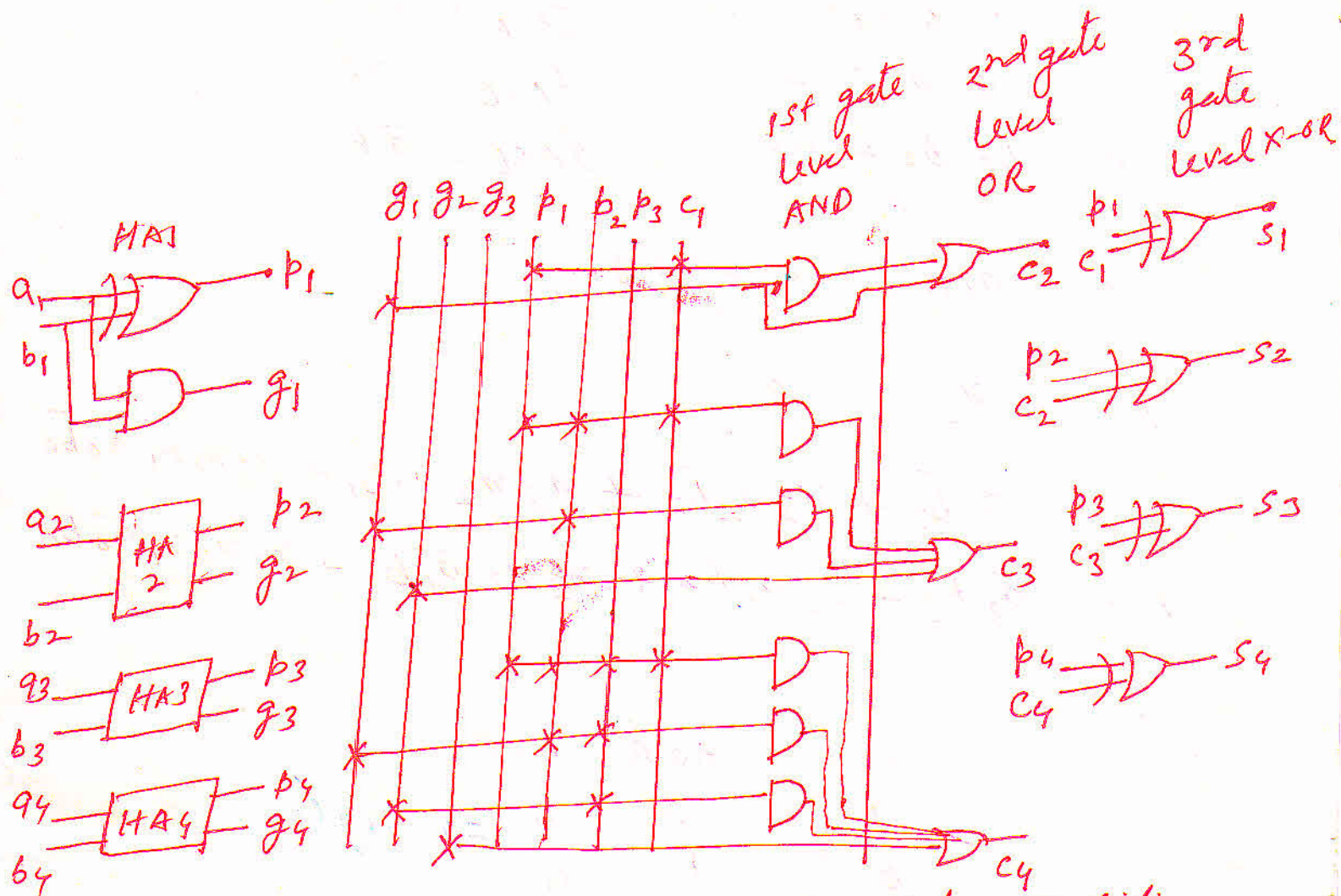
$$c_4 = g_3 + p_3 g_2 + p_3 p_2 g_1 + p_3 p_2 p_1 c_1$$

\* Note that  $c_2, c_3, c_4$  can be expressed in SOP and has two level AND-OR / Two level NAND-NAND implementation.

\* Also note that  $c_4$  does not have to wait for  $c_3$  and  $c_2$  to propagate. In fact  $c_4$  is propagates at the same time as  $c_2$  and  $c_3$

\*  $p_i$  is Responsible for propagating carry from  $c_i$  to  $c_{i+1}$

\*  $g_i$  is independent of  $c_i$



propagation delay of Carry look ahead Adder (4 bit)

= Delay of one HA + Delay of 3 gate levels.  
(AND - OR - EX-OR)

$P.D_{EX-OR} = 20 \text{ ns}$  and  $P.D_{AND/OR} = 10 \text{ ns}$

Then  $P.D_{\text{Carry look ahead}} = \frac{20}{\text{HA}} + 10 + 10 + 20 = 60 \text{ ns}$

Delay H.A = Delay X-OR

While  $P.D_{\text{Parallel adder}} = \text{Delay of one H.A} + 4(\text{AND} + \text{OR})$   
 $= 20 + 4(10 + 10)$   
 $= 100 \text{ ns}$



# # Magnitude Comparator:-

Consider a 4 bit no.

$$A = a_3 a_2 a_1 a_0$$

$$B = b_3 b_2 b_1 b_0$$

one bit comparator

$$f_{A=B} = \bar{a}b + a\bar{b} = a \odot b$$

$$f_{A>B} = a\bar{b}$$

$$f_{A<B} = \bar{a}b$$

$$\text{let } x_i = a_i \odot b_i \quad (i = 0, 1, 2, 3)$$

$$f_{A=B} = x_3 x_2 x_1 x_0$$

$$f_{A>B} = a_3 \bar{b}_3 + x_3 a_2 \bar{b}_2 + x_3 x_2 a_1 \bar{b}_1 + x_3 x_2 x_1 a_0 \bar{b}_0$$

$$f_{A<B} = \bar{a}_3 b_3 + x_3 \bar{a}_2 b_2 + x_3 x_2 \bar{a}_1 b_1 + x_3 x_2 x_1 \bar{a}_0 b_0$$

one bit

| A | B | A=B | A<B | A>B |
|---|---|-----|-----|-----|
| 0 | 0 | 1   | 0   | 0   |
| 0 | 1 | 0   | 1   | 0   |
| 1 | 0 | 0   | 0   | 1   |
| 1 | 1 | 1   | 0   | 0   |

⇒ If

$$A = B$$

$$A = 9 \quad 1001$$

$$B = 9 \quad 1001$$

$$A > B$$

$$A = 13 \quad 1101$$

$$B = 11 \quad 1011$$

$$x_3 = 0$$

$$x_2 = 0$$

$$x_1 = 0$$

$$x_0 = 0$$

$$a_i \odot b_i = x_i$$

⇒ If

$$A < B$$

$$1011$$

$$1101$$

$$x_3 = 1$$

$$x_2 = 0$$

$$x_1 = 0$$

$$x_0 = 0$$