Number System

\# In general a Number with decimal points is represented by a series of coefficent as

$a_n a_{n-1} \cdots a_2 a_1 a_0 \cdot a_{-1} a_{-2} \cdots a_{-m}$

This can be represented in base r system as follows-

$a_n r^n + a_{n-1} r^{n-1} + \cdots + a_2 r^2 + a_1 r^1 + a_0 r^0 + a_{-1} r^{-1} + \cdots + a_{-m} r^{-m}$

$\rightarrow (11010.11)_2 \rightarrow (26.75)_{10}$

$\rightarrow (4021.2)_5 \rightarrow (511.4)_{10}$

$\Rightarrow (B65F)_{16} \Rightarrow (46687)_{10}$

$\Rightarrow (521)_8 \Rightarrow (344)_{10}$

\# Decimal Number System Base → r → 10 → radix
⇒ coefficent 0, 1, 2 -- 8, 9.

⇒ Binary " " " r → 2
" 0, 1

⇒ octal " " Base → 8
" 0, 1, 2, — 7

⇒ Hexadecimal " " Base → 16
coeffcient 0 --- 9, A - F

A → 10
B → 11
C → 12
D → 13
E → 14
F → 15

Note - Generally we write subscript but somes not Than Highest NO. value will give you the radix + one $(325)_r \rightarrow$ radix $= 5+1 = 6$

$\Rightarrow (34)_r = (28)_{10} \rightarrow 3r + 4 = 28 \Rightarrow r = 8$

$\Rightarrow (xy)_8 = (24)_{10} \rightarrow x, y = ? \Rightarrow 8x + y = 24$

$x = 3, y = 0.$

P.T.O.

decimal to binary

(1) $(41)_{10}$

2	41	1
2	20	0
2	10	0
2	5	1
2	2	0
	1	1

$\Rightarrow (101001)_2$

(ii) $(.835)_2$

	Integer	Fraction	Coefficient
			1 (MSB).
$.835 \times 2$	$= 1.670$		1
$.670 \times 2$	$= 1.340$		0
$.340 \times 2$	$= .680$		1
$.680 \times 2$	$= 1.360$		0
$.360 \times 2$	$= .720$		

$(.11010)_2$

⇒ Decimal to octal

$8\,\overline{)26}$ 2 (LSB)
3 3 (MSB)

$(26)_{10} = (32)_8$

$8\,\overline{)4096}$ 0
$8\,\overline{)512}$ 0
$8\,\overline{)64}$ 0
$8\,\overline{)8}$ ↑ 0
1 1

$(4098)_{10} = (10000)_8$

⇒ $(\cdot 53125)_{10}$

Integer ↓ Coefficient

$0\cdot 53125 \times 8 = 4\cdot 250$ 4 (MSB)

$\cdot 250 \times 8 = 2\cdot 00$ 2

$(\cdot 53125)_{10} = (\cdot 42)_8$

⇒ Decimal to Hexadecimal

$16\,\overline{)26}$ ↑ A
1 1

$(26)_{10} = (1A)_{16}$

$16\,\overline{)5000}$ 8
$16\,\overline{)312}$ 8
$16\,\overline{)19}$ 3 ↑
1 1

$(1388)_{16}$

⇒ $(\cdot 53125)_{10}$ to Hexadecimal

Coeff

$(\cdot 53125) \times 16 = 8\cdot 5$ 8 (MSB)

$\cdot 5 \times 16 = 8\cdot 00$ 8 (LSB

$(\cdot 53125)_{10} = (\cdot 88)_{16}$

⇒ Decimal / Binary / octal / Hexadecimal

Decimal	Binary	octal	Hexadecimal
26	11010	011 010 = $(32)_8$	0001 1010 = $(1A)_{16}$
34	100010	= $(42)_8$	$(22)_{16}$
	pair of three		
·53125	·10001000	·100 010 00	= $(\cdot 88)_{16}$
	pair of four	·420	

p.T.O

$\Rightarrow$ In a Number system of radix r, determine x, y and r, given x and y are successive numbers

$$(xy)_r = (25)_{10} \text{ and } (yx)_r = (31)_{10}$$

$$y = x+1$$

$xr + y = 25$ put $y = x+1$

$xr + x + 1 = 25 \rightarrow xr + x = 24$

$yr + x = 31 \rightarrow xr + x + r = 31$

$r = 31 - 24 = 7$

$\Leftarrow$ and $x = 3,\ y = 4,\ r = 7$

$\Rightarrow (212)_x = \cancel{28}_{10} \Rightarrow 2x^2 + x + 2 = \cancel{28}$

$x = 3$

$\Rightarrow (1000)_x = [112]^3 \Rightarrow x^3 = 3^3 \Rightarrow x = 3$

$\Rightarrow (\sqrt{41})_r = 5_{\cancel{10}} \Rightarrow 4r + 1 = 25 = r = 6$ (squaring)

$\Rightarrow 23_r + 12_r = 101_r \Rightarrow r = 4, -1$

$2r + 3 + r + 2 = r^2 + 1$

Number Base Conversion →.

(1) Decimal to Binary —.

		Remainder
2	26	0 (LSB)
2	13	1
2	6	0
2	3	1 ↑
	1	1 (MSB)

$\Rightarrow$

2^5	2^4	2^3	2^2	2^1	2^0
32	16	8	4	2	1

$\Rightarrow (39)_{10}$

$(100111)_2$

$(26)_{10} = (11010)_2$

$\Rightarrow (.53125)_{10}$

	Integer part	Fractional part	Co-efficient
.53125 × 2	1	.06250	1 (MSB)
.06250 × 2	0	.12500	0
.12500 × 2	0	.2500	0
.2500 × 2	0	.5000	0
.5000 × 2	1	.0000	1 (LSB)

$\Rightarrow (.53125)_{10} = (.10001)_2$

Addition in Base r system.

If Addition equal to the value of Base r then Addition is zero with one carry and if it > r then Addition is equal to that (NO. − r) with one carry

NO digit →	Addition	Carry
$x+y = r$	0	1
$x+y > r$	$(x+y)-r$	1
$x+y < r$	$x+y$	0

```
  101101 → 45
  100110 → 38
  101110 → 46
10000001 → 129
```

	S	C
1+0 →	1	0
0+1 →	1	0
0+0 →	0	0
1+1 →	0	1

⇒ $(253)_8$
$(126)_8$
$(401)_8$

⇒ $(19)_{16} \to (25)_{10}$
$(1A)_{16} \to (26)_{10}$
$(33)_{16} \to (51)_{10}$

⇒ Multiplication.. in Binary.., Substraction.

```
  1011 × 101
    1011
   0000x
  1011xx
  110111
```

⇒
```
  101101
− 100111
  000110
```
1−0 ⇒ 1
0−1 ⇒ 1,1
0−0 = 0
1−1 = 0

⇒ $(437)_8$ → $(3+8) = 11$
− $(355)_8$ -5
  062 6 With one Borrow..

⇒
```
  DAD
− BED
  1C0
```
A+16 = 26
−14
C ← 12

⇒
```
  638A
− 549B
  12EF
```

if	S	B
$x=y$	0	0
$x>y$	$x-y$	0
$x<y$	$(x+r)-y$	1

Complements — Complements are used in digital computers

* To Simplify substraction operation

* for logical manipulation.

There are two types of complements in a Number system of Base r

(1) r's complement → consider a positive No. N as Base r with the Integer part having n digit.

r's Complement of $N = r^n - N$ for $N \neq 0$

$= 0$ for $N = 0$

⇒ 10's complement of $32890_{10} = 10^5 - 32890$ ⇒ $n = 5$

$= 67110$

" " " $.2897_{10} = 10^0 - .2897$

$= .7103$, $n = 0$

" " " $24.528_{10} = 10^2 - 24.528$

$= 75.472$

⇒ 8's comp of $(2350)_8 = (8^4)_{10} - 2350_8$

$(4096)_{10}$

$= 10000_8 - 2350_8$

$= 5430_8$

$$\begin{array}{r} 10000 \\ 2350 \\ \hline 05430 \end{array}$$

⇒ 2's " " $101100_2 = (2^6)_{10} - 101100$

$= 1000000 - 101100$

$= 010100$

Note: (i) 10's Complement of a decimal Number is obtained by retaining all least significant zeros and substracting the first non zero least significant digit from 10 and then substracting all other higher significant digit from 9.

⇒ (2) (i) If LSB = 1 (for Binary 2's comp.) Replacing each 0, by 1 and 1 by 0 except LSB

(ii) If LSB = 0, 2's complement can be obtained by scanning the No. from LSB to MSB, retaining the bit as it is up to first occurrence of 1's and then complement all other bit

Complement	Retain
0 1 1 0 0	1 0 0
↓ ↓ ↓ ↓ ↓	↓ ↓ ↓
1 0 0 1 1	1 0 0

c ← Retain

0 1 1 0 0 1 1 1

↓

1 0 0 1 1 0 0 1

$(r-1)$'s complement → N – Positive No. n – No of integer digit, No of fraction digit = m

$(r-1)$'s complement $= r^n - r^{-m} - N$

⇒ 9's complement of $42530_{10} = 10^5 - 10^0 - 42530$

$= (57469)_{10}$

⇒ " " of $.2645_{10} = 1 - 10^{-4} - .2645$

$= .7354_{10}$

⇒ 1's " " $101101_2 = 2^6 - 2^0 - 101101_2$

$= 1000000 - 0000001 - 101101$

$= 010010_2$

⇒ 1's " " $.0110_2 = 0.1001_2$

Note: (i) $(r-1)$'s complement can be achieved by substracting each bit, 1 ~~or~~ Digit by. $(r-1)$. i.e. from 9 for Decimal, or from 1 for binary.

in binary complement the each bit for 1's complement

⇒ (ii) r's Complement may be obtained by adding r^{-m} to the least significant deget of $(r-1)$'s complement

(iii) r/$r-1$'s complement of [r's / $(r-1)$'s] complement is the Number itself

i.e. r's complement of $N = r^n - N$

and r's " of $r^n - N = r^n - (r^n - N)$
$= N$

Substraction with r's complement —

The Digital hardware perform substraction by using complement and addition.

Substraction of two +ve No. of Base r $(M-N)$.

* Add minuend M to r's complement of substraind N
* observe end carry:
  → If it results, ignore it, the result is positive
  → If it does not result, the result is Negative take r's compliment and place Negative sign for correct result.

Example: 1) Substract 73.28 from 89.11

$M = 89.11$ ← Minuend

10's comp of $N = 26.72$ ← Substrahend
+
1 15.83

Ignore EAC   Difference $= +15.83$

(ii) If $M = 73.28$ $N = 89.11$

$M = 73.28$

10's comp $N = 10.89$
+
84.17

NO EAC

result $= -15.83$ ← 10's comp of 84.17

⇒ M-N for M = 1010100 , N = 1000100

$$\begin{array}{rr} M = & 1010100 \\ \text{2's Comp of } N = + & 0111100 \\ \hline & 10010000 \end{array}$$ result

ignore EAC = + 0010000

⇒ M-N if M = 1000100 , N = 1010100

$$\begin{array}{rr} M = & 1000100 \\ \text{2's comp of } N = + & 0101100 \\ \hline & 1110000 \end{array}$$

No EAC

Result = − 0010000 ← 2's comp of 1110000

Substraction using (r-1)'s complements.

* perform (M-N) ← Both +ve No and base r
* Add M to (r-1)'s complement of Substrahend N
* observe end carry
  → If it results, add one to LSD, result is positive ⇒ M>N
  → If it does not result, take (r-1)'s complement and place a negative sign.

i) M = 89.11, N = 73.28

$$\begin{array}{rr} M = & 89.11 \\ \text{9's comp of } N = + & 26.71 \\ \hline & 1\,15.82 \\ \text{EAC} & \to 1 \\ \hline & 15.83 \end{array}$$

Result = + 15.83

ii) M = 73.28

$$\begin{array}{rr} \text{9's comp of } N = & 10.88 \\ \hline & 84.16 \end{array}$$

No EAC

Result = − 15.83 ← 9's Comp. of 84.16

⇒ (iii) DO. 1010100 − 1000100

$$
\begin{array}{rr}
M = & 1010100 \\
\text{1's comp of } N = & +\,0111011 \\
\hline
\text{EAC} \rightarrow & 1\,0001111 \\
 & 1 \\
\hline
 & 0010000
\end{array}
$$

Result = +10000

(iv) DO. 1000100 − 1010100

$$
\begin{array}{rr}
M = & 1000100 \\
\text{1's comp of } N = & +\,0101011 \\
\hline
\text{NO EAC} \rightarrow & 1101111
\end{array}
$$

Result = −10000 ← 1's comp. of 1101111

Comparing 1's complement with 2's complements.

* finding 1's complement is easier as 2's complements. 1's comp. can be achieved by digital means 0→1, or 1→0
* During substraction 2's complement method is easier 2's complements requires only one addition., while 1's complements " two arthematic additions when EAC occurs
* 1's compliments having additional disadvantage of having two arthematic zeros. one with all 0's and one with all 1's, while 2's complements has only one arthematic zero. 1's compliments zero may be +ve or −ve
* How ever too 1's complement is useful in logical manipulation. change of 1 by 0 or vice versa is equivalent to logical inversion operation. 2's complements are useful for only arthematic operations