Boolean Algebra — George Boole — 1854

It deals with variable that take on two discrete values 0/1. A, B, C, x, y, z,

* Basic logical operation — OR, AND, NOT

OR (Parallel switching ckt)	AND (Series switching ckt)	NOT
$x+0=x$	$x\cdot 1=x$	$x\to\bar{x}$
$x+1=1$	$x\cdot 0=0$	$\bar{x}\to x$
$x+\bar{x}=1$	$x\cdot x=x$	$1\to 0$
$x+x=x$	$x\cdot\bar{x}=0$	$0\to 1$

→ Dual

$$+ \longrightarrow \cdot$$
$$\cdot \longrightarrow +$$

$$x+yz+xy\bar{z} \xrightarrow{\text{Dual}} x\cdot(y+z)\cdot(x+y+\bar{z})$$

⇒ Theorems —

* Commutative —

$$x+y=y+x \xrightarrow{\text{dual}} xy=yx$$

* Associative:

$$x+(y+z)=(x+y)+z \longrightarrow x(yz)=(xy)z$$

* Distributive

$$x(y+z)=xy+xz \longrightarrow x+yz=(x+y)(x+z)$$

* Absorption:

$$x+xy=x \longrightarrow x(x+y)\to x$$

* De Morgan: $\overline{(x+y+z)}=\bar{x}\cdot\bar{y}\cdot\bar{z} \longrightarrow \overline{xyz}=(\bar{x}+\bar{y}+\bar{z})$
* Transposition: $(A+B)(A+C)=A\cdot A+B\cdot C$
$=A+BC$

$\Rightarrow A + \bar{A}B \Rightarrow (A+\bar{A})(A+B) = A+B$

$A\cdot(\bar{A}+B) = A\bar{A} + AB = AB$

$AB + \bar{A}C + BC = (A+BC)(B+BC) + \bar{A}C$

$= (A+B)(A+C)B + \bar{A}C$

Redundant term

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$		1	1	
A			1	1

$\Rightarrow AB + \bar{A}C$

$AB + \bar{A}C + B(A+\bar{A})C$

$AB + \bar{A}C + ABC + \bar{A}BC$

$AB(1+C) + \bar{A}C(1+B) = AB + \bar{A}C$

$\Rightarrow$ Consensus Theorem :- 3 Variable
2 - terms
1 Variable complementary / uncompl.

comple.

$\rightarrow AC + A\bar{B} + BC$ $= A\bar{B} + BC$

A — two terms
B — " "
C — " "

$\rightarrow AB + BC + A\bar{C} = A\bar{C} + BC$

comple.

$\rightarrow (A+B)(B+C)(\bar{A}+C) = (\bar{A}+C)(A+B)$

comp.

$\rightarrow \bar{A}\bar{B} + \bar{A}C + \bar{B}\bar{C} \Rightarrow \bar{A}C + \bar{B}\bar{C}$

comp.

$\rightarrow AB + BC + \bar{A}B \rightarrow$ Not containing variable 2 times. Hence can not apply consensus theorem.

$B(A+\bar{A}) + BC$

$B + BC = B(1+C) = B$

Boolean function Repersention — If there are n varible than 2^n possible combination of these varible. Where as 2^{2^n} distinct Bolean function can be generated.

* Min term (product term) → A product term in which all varible appear either in complemented or uncomplemented form.
* Max term: (Sum term) → A sum term in which all the literals appear in complemented or uncomplemented form
* Boolean function expressed as sum of min terms or product of max term are said to be in Canonical form.

x	y	z	AND Term (MIN term) Term	Designation	OR Terms Max Term Term	Designation
0	0	0	$\bar{x}\bar{y}\bar{z}$	m_0	$x+y+z$	M_0
0	0	1	$\bar{x}\bar{y}z$	m_1	$x+y+\bar{z}$	M_1
0	1	0	$\bar{x}y\bar{z}$	m_2	.	.
0	1	1	$\bar{x}yz$	m_3	.	.
1	0	0	$x\bar{y}\bar{z}$	m_4	.	.
1	0	1	$x\bar{y}z$	m_5	.	.
1	1	0	$xy\bar{z}$	m_6	.	.
1	1	1	xyz	m_7	$\bar{x}+\bar{y}+\bar{z}$	M_7

$$\Rightarrow \quad (m_J)' = M_J$$

Note: If Boolean function expressed by terms containing one, two or any No. of literals then it is called Standard form.

⇒ Standard form

logic 1 ← SOP   POS ← logic 0

* $f(x,y,z) = 1$ has 8 min term, 0 max term
* $f(x,y,z) = 0$ has 0 " " 8 max "
* $f(a,b,c) = a + \bar{b}c$ is in SOP form but not in canonical form.

$$a(b+\bar{b}) = ab + a\bar{b} = ab(c+\bar{c}) + a\bar{b}(c+\bar{c})$$
$$= abc + ab\bar{c} + a\bar{b}c + a\bar{b}\bar{c}$$

$$(a+\bar{a})(\bar{b}\cdot c) = (a+\bar{a})\bar{b}c = a\bar{b}c + \bar{a}\bar{b}c$$

$$f(a,b,c) = abc + ab\bar{c} + a\bar{b}c + a\bar{b}\bar{c} + \bar{a}\bar{b}c$$
$$= \Sigma(1,4,5,6,7) = \Sigma = m_1 + m_4 + m_5 + m_6 + m_7$$

* $f(x,y,z) = xy + x'z$ ← SOP | or $(xy+x')(xy+z)$
$(\bar{x}+y)(x+z)(y+z)$

convert in POS

$$f_{dual} = (x+y)(x'+z) = xx' + xz + x'y + yz$$

$$(f_{dual})_{dual} = (x+z)(\bar{x}+y)(y+z)$$
$$= (x+z+yy')(\bar{x}+y+zz')(xx'+y+z)$$
$$= (x+y+z)(x+y'+z)(\bar{x}+y+z)(\bar{x}+y+z')$$
$$(x+y+z)(x'+y+z)\ \cancel{x}$$

$$= (x+y+z)(x+y'+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})$$
$$= M_0 \quad M_2 \quad M_4 \quad M_5$$

$f(x,y,z) = \Pi(0,2,4,5)$ → 4 Max Term.

* $f(A,B,C,D) = (AB+CD)(A'B'+C'D')$

Neither SOP or POS

⇒ * If there are two Boolean function F_1 and F_2 then $G = F_1 + F_2$ — contain all the min terms of F_1 and F_2

$E = F_1 \cdot F_2$ → " only common min terms of F_1 and F_2

⇒ dual → Dual are used to convert +ve logic to neg logic

$+ \longrightarrow \cdot$ Keep literal as it

$\cdot \longrightarrow +$

$1 \longleftrightarrow 0$

Self dual → If the dual of a function is function itself then it is called self dual. Max possible self dual function with n variable are $\underline{2^{2^{n-1}}}$.

Complements →

(I) AND ⟷ OR } → dual

(II) $1 \longleftrightarrow 0$

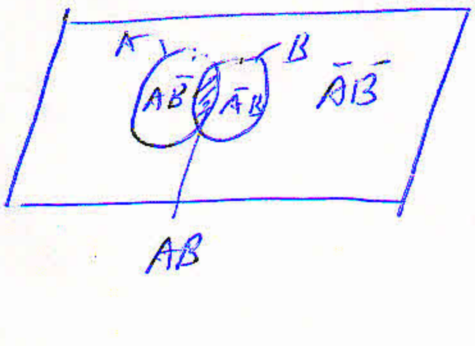
(III) Complements each literals

$f = \bar{x} \cdot y \cdot \bar{z} + x \bar{y} z$

$f_{dual} = (\bar{x} + y + \bar{z})(x + \bar{y} + z)$

$f' = (x + \bar{y} + z)(\bar{x} + y + \bar{z})$

Venn diagram :—



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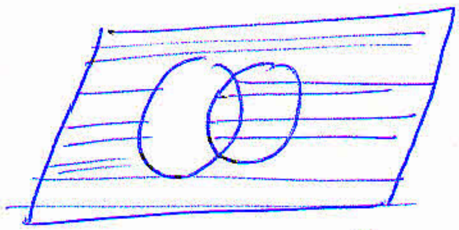
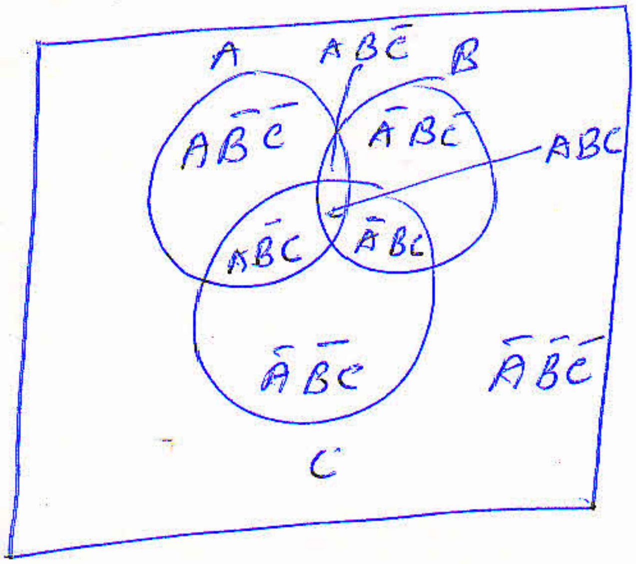
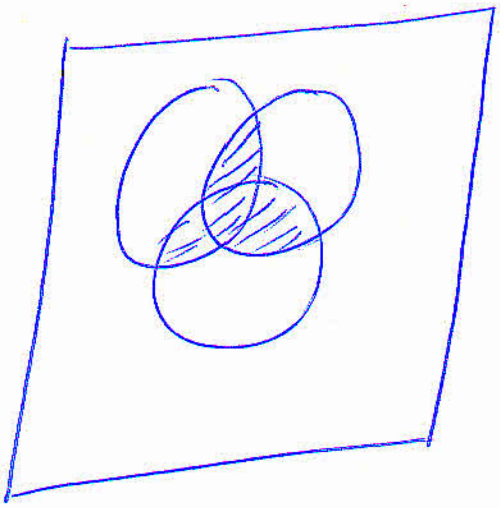


$AB + \bar{A}\bar{B} + \bar{A}B$

$= \bar{A} + B$

⇒





$ABC + \bar{A}BC + A\bar{B}C$
$+ AB\bar{C}$
$= AB + BC + AC$

⇒ TRUTH TAable ⇒

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

in SOP $F = m_1 + m_4 + m_7$

$= \bar{x}\bar{y}z + x\bar{y}\bar{z} + xyz$

$F = \Sigma[1, 4, 7]$

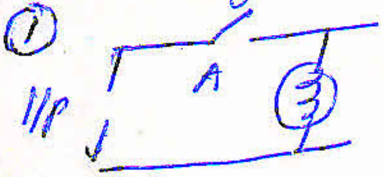
in ~~Sop~~ POS

$F = M_0 M_2 M_3 M_5 M_6$

$= \Pi(0, 2, 3, 5, 6)$

$F = (x+y+z)(x+\bar{y}+z)$
$(x+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})$
$(\bar{x}+\bar{y}+z)$

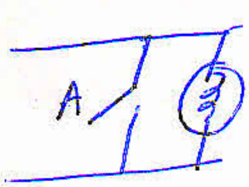
⇒ Switching Circuit Repersentation →

① 

$Y = A$ Buffer

A	Y
0	0
1	1

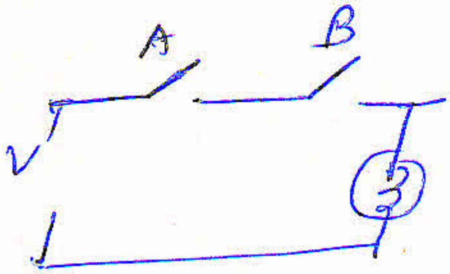
open → logic 0
close → logic 1
or vice versa.

⑪ 

$Y = \bar{A}$ Invertor

A	Y
0	1
1	0

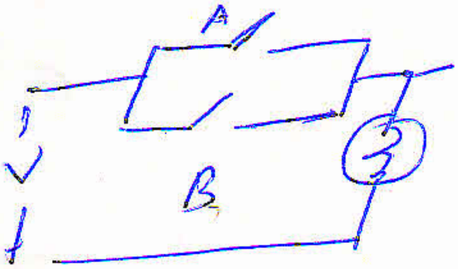
⇒ (3)



$Y = AB$

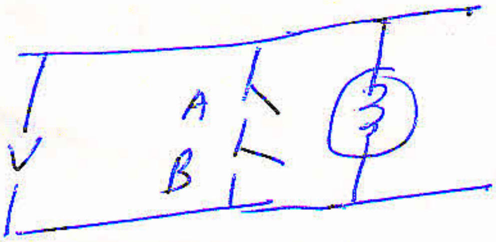
AND Gate.
(Switch Are in Series).

(4)



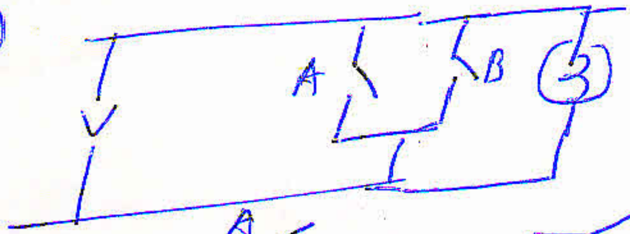
$Y = A + B$ OR Gate
(Swich are in parallel)

(5)



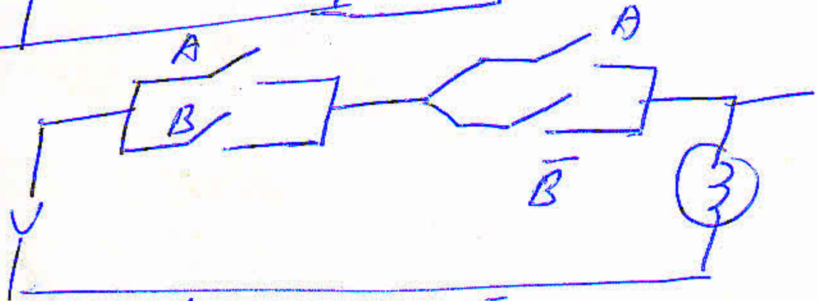
$Y = \overline{AB}$ NAND gate

(6)



$Y = \overline{A+B}$ NOR gate

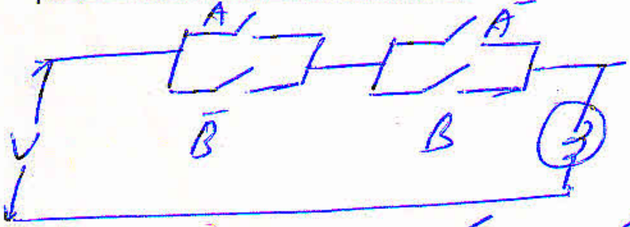
(7)



$Y = (A+B)(\bar{A}+\bar{B})$
$= A\bar{B} + \bar{A}B$
$= A \oplus B$

EX-OR

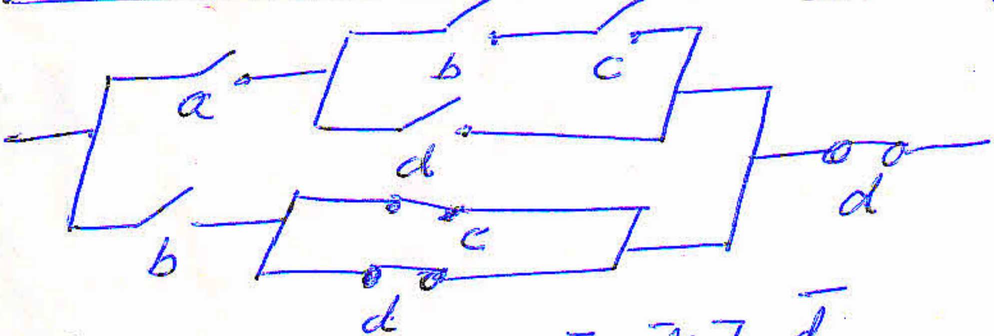
(8)



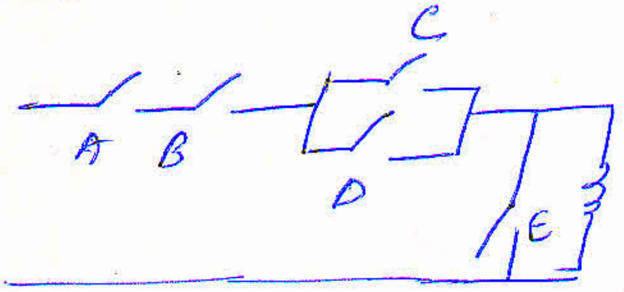
$Y = A \odot B$

EX-NOR

⇒



$[a\cdot(bc+d) + b\cdot(\bar{c}+\bar{d})]\cdot\bar{d}$
$= abc\bar{d} + b\bar{c}\bar{d} + b\bar{d}$

$\Rightarrow$ 

$\Rightarrow Y = AB(C+D)\bar{E}$

$\Rightarrow$ A logic ckt have three input A, B, C and O/P Y have logic 1 from the following condition.

1) A & B are false
II) A " C " True
III) A, B & C " false
IV) A, B & C " True

$$Y = \bar{A}\bar{B} + AC + \bar{A}\bar{B}\bar{C} + ABC$$

$$= \bar{A}\bar{B}(1+\bar{C}) + AC(1+B)$$

$$= \bar{A}\bar{B} + AC$$

$\Rightarrow$ Example $\Rightarrow$

* $AB + ABC + A\bar{B} = A$

$AB(1+C) + A\bar{B} = AB + A\bar{B} = A(B+\bar{B}) = A$

* $(a+b)(b+d)(a+c)(c+d) = bc + ad$

$\underset{x}{(b} + \underset{z}{ad)}(\underset{y}{c} + \underset{z}{ad)} = bc + ad$ $\quad (xy+z)$

* $\overline{(\bar{A}+B)}\,\overline{\bar{A}\bar{B}\bar{C}} = \overline{A+B+C} = \bar{A}\bar{B}\bar{C}$

$\underline{\bar{A}\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{B}\bar{C}} = \bar{A}\bar{B}\bar{C} = \overline{A+B+C}$

* $\overline{a + \overline{b + cd}} = \bar{a}b + \bar{a}\bar{c} + \bar{a}\bar{d}$

$$\overline{a + \overline{b + (\bar{c}+\bar{d})}} = \overline{a + \bar{b}\cdot cd} = \bar{a}\cdot(b + \bar{c} + \bar{d})$$

$$= \bar{a}b + \bar{a}\bar{c} + \bar{a}\bar{d}$$