# Boolean function Simplification Technique.

* Boolean Algebra
* K-Map
* Quine McCluskey method

⇒ Boolean Algebra → By applying theorems & reduction Technique.

# K-Map → Graphical method of simplifying logical expression. in Gray Code Representation.

⇒ Two variable K-map.

| $x$ | $y$ | min term |  |
|---|---|---|---|
| 0 | 0 | $x'y'$ | $m_0$ |
| 0 | 1 | $x'y$ | $m_1$ |
| 1 | 0 | $xy'$ | $m_2$ |
| 1 | 1 | $xy$ | $m_3$ |

| $x \backslash y$ | $\bar{y}$ | $y$ |
|---|---|---|
| $\bar{x}$ | $m_0$ | $m_1$ |
| $x$ | $m_2$ | $m_3$ |

$F = xy$

| | | |
|---|---|---|
| | | |
| | | 1 |

$F = \Sigma(1,2,3)$

| $x \backslash y$ | | |
|---|---|---|
| | | 1 |
| | 1 | 1 |

$F = x + y$

| $x \backslash y$ | | |
|---|---|---|
| | 1 | 1 |
| | 1 | 1 |

$F = 1$

| $x \backslash y$ | | |
|---|---|---|
| | | |
| | | |

$F = \{\phi\}$.

⇒ Three variable K-map

| $x \backslash yz$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | $yz$ | $y\bar{z}$ |
|---|---|---|---|---|
| $\bar{x}$ | $m_0$ | $m_1$ | $m_3$ | $m_2$ |
| $x$ | $m_4$ | $m_5$ | $m_7$ | $m_6$ |

$F = x'yz + x'yz' + xy'z' + xy'z$

| $x \backslash yz$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | $yz$ | $y\bar{z}$ |
|---|---|---|---|---|
| $\bar{x}$ | | | 1 | 1 |
| $x$ | 1 | 1 | | |

$F = x'y + xy'$

⇒ $F = A'C + A'B + ABC' + BC$

| $A \backslash BC$ | $\bar{A}\bar{B}$ | $\bar{A}B$ | $AB$ | $A\bar{B}$ |
|---|---|---|---|---|
| $\bar{A}$ | | 1 | 1 | 1 |
| $A$ | | 1 | 1 | |

$F = C + A'B$

⇒ $F = \Sigma(0, 2, 4, 5, 6)$

| $A \backslash BC$ | | | | |
|---|---|---|---|---|
| | 1 | | | 1 |
| | 1 | 1 | | 1 |

$F = C' + AB'$

⇒

| $A \backslash BC$ | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
|---|---|---|---|---|
| $\bar{A}$ | | 1 | 1 | |
| $A$ | | | 1 | 1 |

$F = \bar{A}C + AB$

# Four variable map. →

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 3 | 2 |
| 01 | 4 | 5 | 7 | 6 |
| 11 | 12 | 13 | 15 | 14 |
| 10 | 8 | 9 | 11 | 10 |

C (columns 11, 10); B (rows 01, 11); A (rows 11, 10); D (columns 01, 11)

5.2 (1)

⇒ A ckt is a even parity generator 3 I/P.

| x | y | z | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

$F = \Sigma(1, 2, 4, 7)$

$F = \Pi(0, 3, 5, 6)$

| x \ yz | | | | |
|---|---|---|---|---|
| | | 1 | | 1 |
| | 1 | | 1 | |

$F = x \oplus y \oplus z$

① $f(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

| wx \ yz | | | | |
|---|---|---|---|---|
| | 1 | 1 | | 1 |
| | 1 | 1 | | 1 |
| | 1 | 1 | | 1 |
| | 1 | 1 | | |

$f = y' + w'z' + xz'$

② $F = A'B'C' + B'CD' + A'BCD' + AB'C'$

| | | | |
|---|---|---|---|
| 1 | 1 | | 1 |
| | | | 1 |
| | | | |
| 1 | 1 | | 1 |

$f = B'D' + B'C' + A'CD'$

# product of Sums Simplification →.

$F(A.B.C.D) = (0, 1, 2, 5, 8, 9, 10)$

| AB \ CD | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | 1 | 1 | 0 | 1 |
| $\bar{A}B$ | 0 | 1 | 0 | 0 |
| $AB$ | 0 | 0 | 0 | 0 |
| $A\bar{B}$ | 1 | 1 | 0 | 1 |

$f = B'C' + B'D' + A'C'D$

Combine 0's will give you.

$F' = \cancel{AB} + CD + BD'$

apply demorgan's Theorems.

$F = \cancel{(A'+B')}(C'+D')(B'+D)$

#

| x | y | z | F | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | |
| 0 | 0 | 1 | 1 | — Represent the min term |
| 0 | 1 | 0 | 0 | |
| 0 | 1 | 1 | 1 | |
| 1 | 0 | 0 | 1 | |
| 1 | 0 | 1 | 0 | — Represent the max term. |
| 1 | 1 | 0 | 1 | |
| 1 | 1 | 1 | 0 | |

$F(x,y,z) = \Sigma(1,3,4,6)$

$F(x,y,z) = \Pi(0,2,5,7)$

| x \ yz | $\bar{y}\bar{z}$ | $\bar{y}z$ | $yz$ | $y\bar{z}$ |
|---|---|---|---|---|
| $\bar{x}$ | 0 | 1 | 1 | 0 |
| $x$ | 1 | 0 | 0 | 1 |

in SOP → $F = xz' + x'z$

in POS → $F' = xz + x'z'$

$F = (x'+z')(x+z)$

Note → If some function given in the form of POS take the complement and then mark 0's in the in the squares.

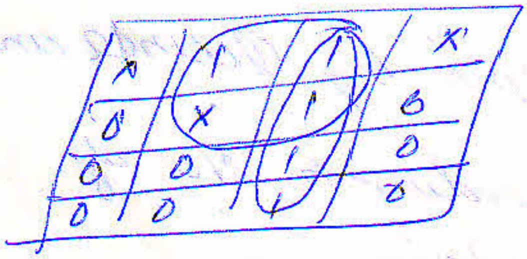
$F = (A'+B'+C)(B+D)$

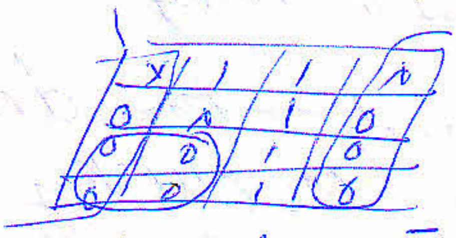
$F' = ABC' + B'D'$

and Remaining by one (1's).

⇒ Don't Care Condition → in certain situations certain combination of input variables never occurs. e.g. in BCD code has six combinations which are not used. these refer to Don't care conditions.. These condition may provide further simplification of Boolean function. Don't care may be assumed to be 0/1 → X 5.4

$$f(w,x,y,z) = \Sigma(1,3,7,11,15) + d\Sigma(w,x,y,z)\,(0,2,5).$$



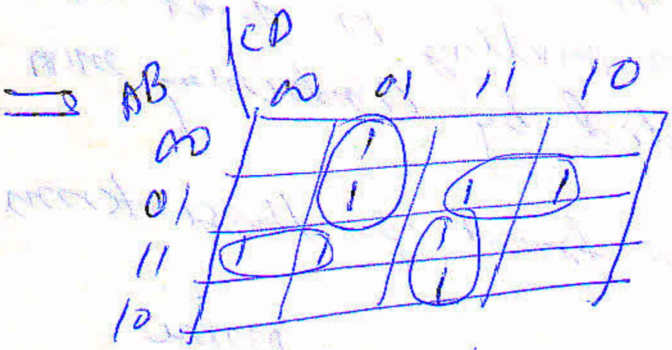


$$f = \bar{w}z + yz$$

$$f' = z' + w\bar{y}$$

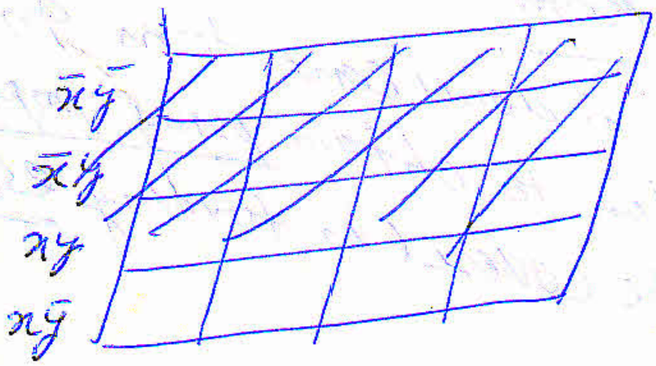
$$f = z(\bar{w} + y)$$

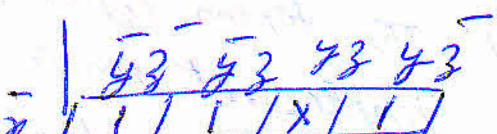
⇒



$$\Sigma = ABC' + A'B'C + BCD + A'BC$$

⇒ $f = y' + x'z'$, $d = yz + xy$





$$f = 1$$

$\Rightarrow$ $F = \prod(4,5,6,7,12)\ ;\ d(1,2,3,9,11,14)$

| $AB \backslash CD$ | $\bar{C}\bar{D}$ | $\bar{C}D$ | $CD$ | $C\bar{D}$ |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ | 1 | X | X | X |
| $\bar{A}B$ | 0 | 0 | 0 | 0 |
| $AB$ | 0 | 1 | 1 | X |
| $A\bar{B}$ | | X | X | 1 |

$SOP = \bar{A}B + AD + AC$

$POS\ F' = \bar{A}B + B\bar{D}$

$F = (A+\bar{B})(\bar{B}+D)$

$\Rightarrow$ 5 variable K-map $\Rightarrow$ Implement Using gates $\Rightarrow$.

① $F(x,y,z) = \Sigma(1,3,5,7) + \Sigma d(4,6)$

| $x \backslash yz$ | $\bar{y}\bar{z}$ | $\bar{y}z$ | $yz$ | $y\bar{z}$ |
|---|---|---|---|---|
| $\bar{x}$ | | 1 | 1 | X |
| $x$ | X | 1 | 1 | |

$F = \bar{x}z$ Ans.

② $F(A,B,C,D) = \Sigma(5,\cancel{7},13,15) + \Sigma d(12,14)$

| $AB \backslash CD$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | 1 | 1 | |
| 11 | d | 1 | 1 | d |
| 10 | | | | |

$F = \bar{B} + \bar{D}$