$\rightarrow [1, 2, 5]$ amount = 11

11 coins of 1 unit $\rightarrow$ $1 + 1 + 1 + 1 \ldots\ldots + 1 \Rightarrow \underline{\underline{11}}$

$\rightarrow \underbrace{2 + 2 + 2 + 2 + 2 + 1}_{6 \text{ coins}} \rightarrow \underline{\underline{11}}$

Best way $\rightarrow \underbrace{5 + 5 + 1}_{3 \text{ coins}} \rightarrow \underline{\underline{11}}$

coins = [2] amount = 3 $\rightarrow$ -1

X

coins → [1, 2, 5] amount → 12

5 5 2

$12 \xrightarrow{-5} 7 \xrightarrow{-5} 2 \xrightarrow{-2} 0$

coins → [1, 6, 7] amount = 12

7 1 1

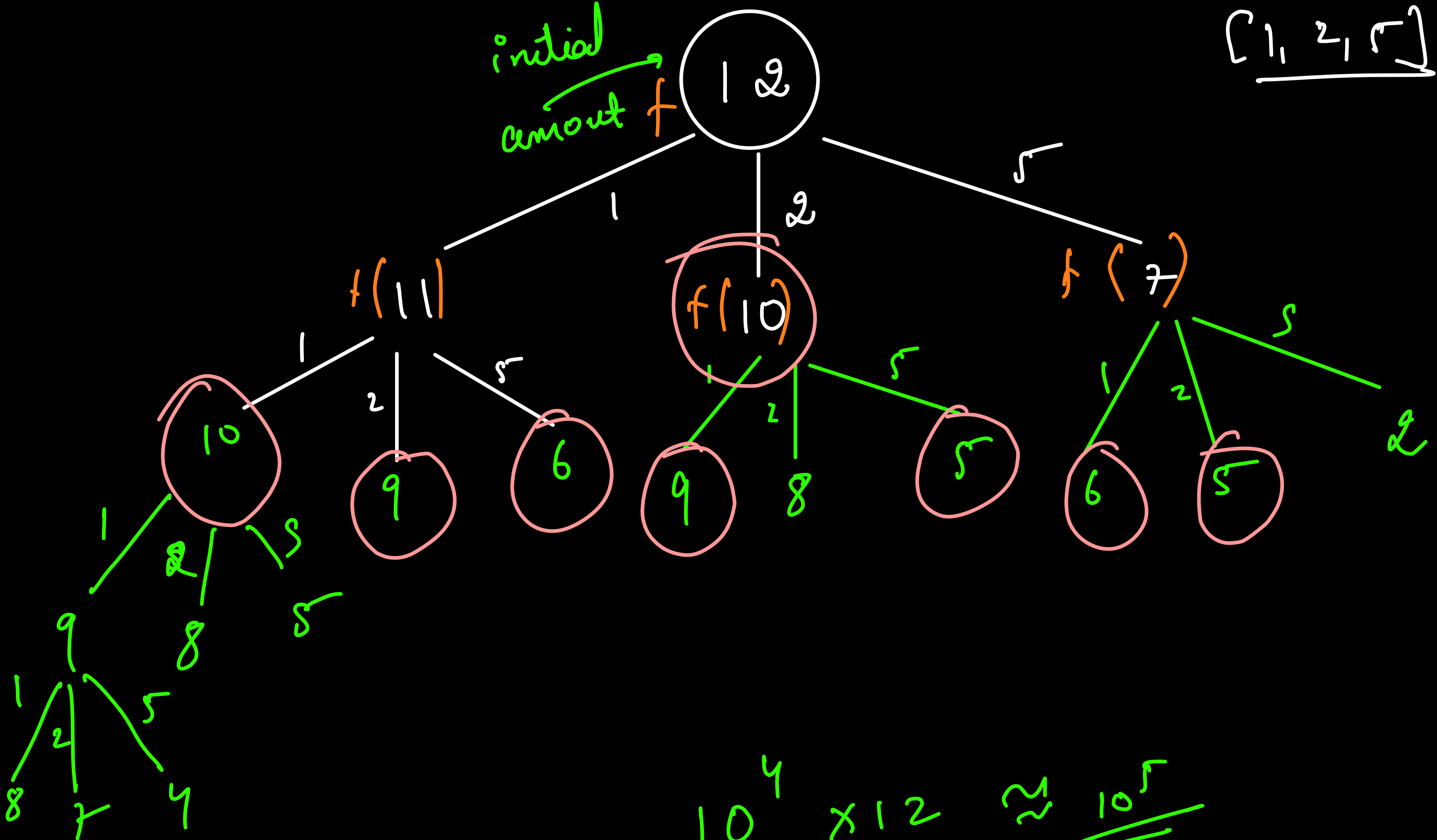
1 1 1

$12 \xrightarrow{-7} 5 \xrightarrow{-1} 4 \xrightarrow{-1} 3 \xrightarrow{-1} 2 \xrightarrow{-1} 1 \xrightarrow{-1} 0$

$12 \xrightarrow{-6} 6 \xrightarrow{-6} 0$

6 6 → 2 coins

Brute force



$\underline{10^4} \times 12 \approx \underline{10^5}$

Space → $O(\text{amount})$

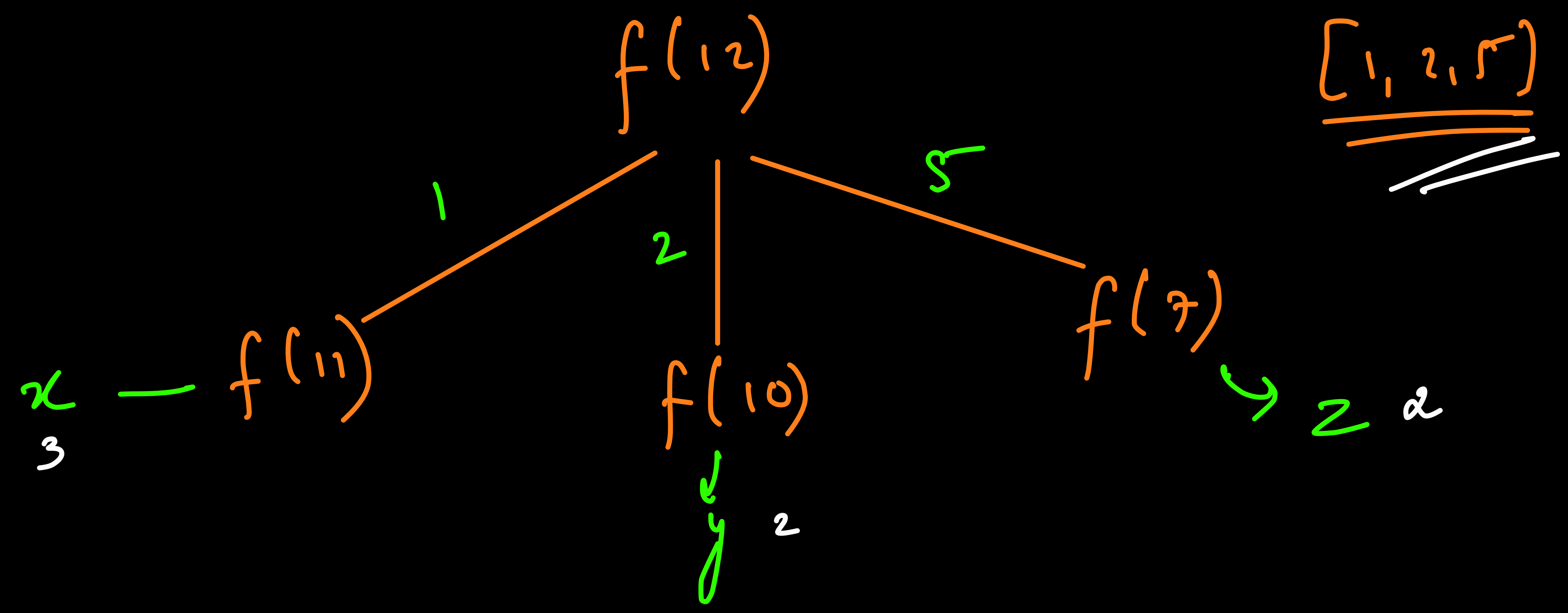
$O(n \times \text{amount})$

$$f(\text{amount}) = 1 + \min\left(f(\text{amount} - \text{coins}[i])\right)$$

returns min no. of coins req'd to give a change of given amount

$\forall\, i \in [0, n-1]$ and $\text{coins}[i] \le \text{amount}$

$$1 + \min\begin{pmatrix} f(\text{amount} - \text{coins}[0]) \\ f(\text{amount} - \text{coins}[1]) \\ \vdots \\ f(\text{amount} - \text{coins}[n-1]) \end{pmatrix}$$



$x \rightarrow$ min no. of coins reqd to make 11 $\rightarrow$ 3   5 + 5 + 1

$y \rightarrow$ min no. of coins reqd to make 10 $\rightarrow$ 2   5 + 5

$z \rightarrow$ min no. of coins reqd to make 7 $\rightarrow$ 2   5 + 2

$$f(12) = \min(x, y, z) + 1$$
$$= \min(3, 2, 2) + 1$$
$$= 2 + 1 \rightarrow 3$$

$\infty \to$ max-value   amount $\to 3$ ∞ [2]

$$f(3) \to \text{if } (\text{am} \to \infty) \to \infty$$

$f\ \infty$

$$f(3) \xrightarrow{-2} f(1)$$

$$f(3) = \underline{1 +} \underline{(-1)} \to 0 \ \times$$

Perfect Squares

n

min no. of perfect sq integer rep to sum upto n.

12

1+1+1+1 ...... 1 → 12

12 time

12

4+4+4 → 3

12

9+1+1+1 → 4

13

9+1+1+1+1 → 5

9+4 2

99% $\rightarrow$ coin change

$n \le 10^4$

coins $[1, 4, 9, 16, 25, 36, \ldots\ldots\ldots]$

pf_sq

$$f(n) = 1 + \min\left(f(n - pf\_sq[i])\right)$$

$\forall\, i \in [0, n-1]$ and

$n \geq pf\_sq[i]$

days → [1, 4, 6, 7, 8, 20]

[2, 7, 15] ← ticket cost

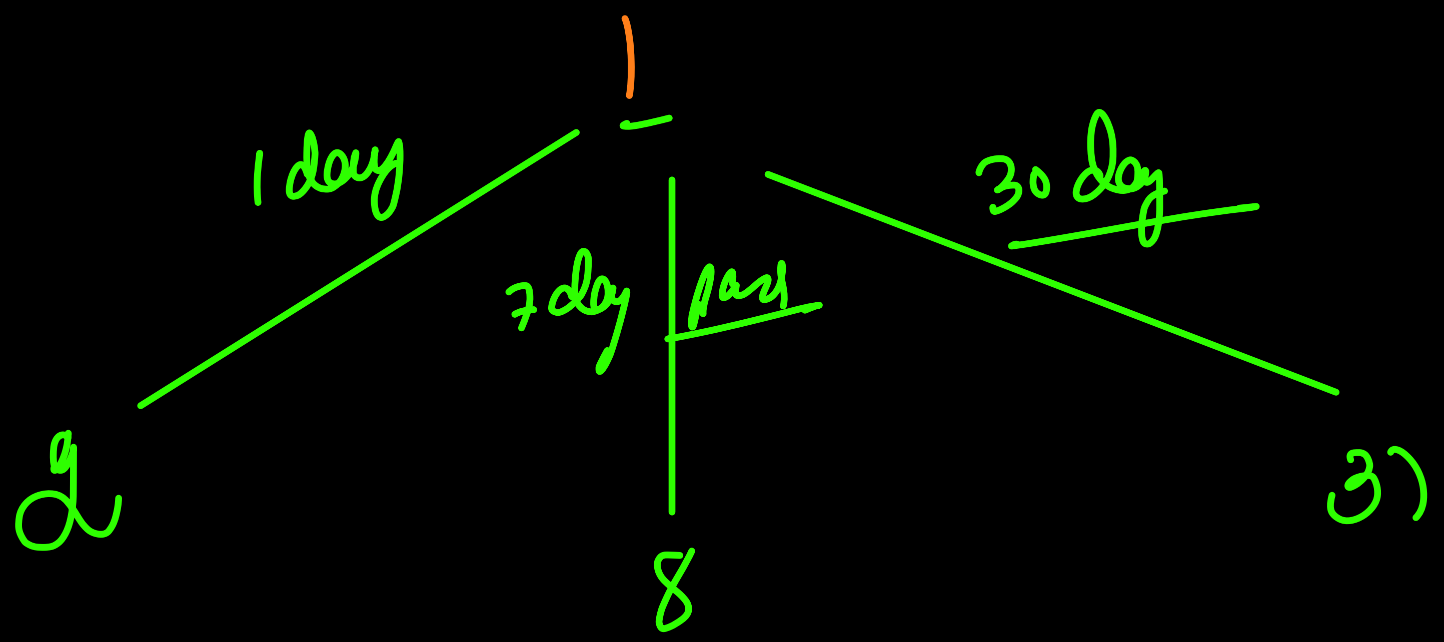
1 7 30

[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 30, 31]   [2, 7, 15]

15 + 2 → 17

2 + 2 + 2 + 2 + 2 + 2 → 12

7 + 2 + 2 → 11



if we buy a 7 day pass, the next pass we need will be on day 8 or more ≥ 8

$f(1) \rightarrow$ ans

1 day pass → cost[0]
7 day pass → cost[1]
30 day pass → cost[2]

$f(d)$ = min $\begin{cases} cost[0] + f(d+1) \\ cost[1] + f(d+7) \\ cost[2] + f(d+30) \end{cases}$

$\downarrow$

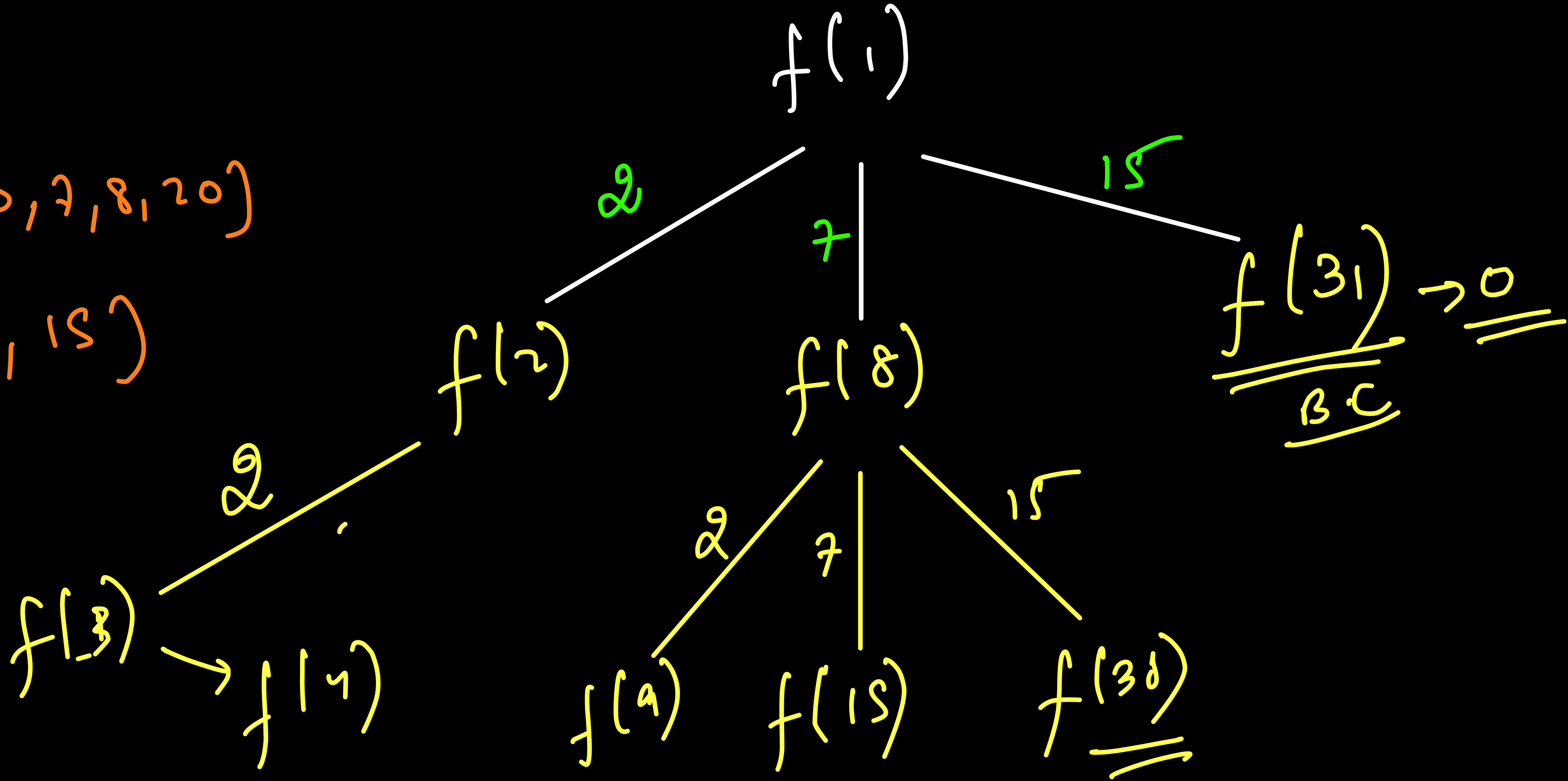
min dollars reqd to start from 'd' day & complete the trip

Base Case $\rightarrow$ $d > days[n-1] \longrightarrow 0$

if d is not part of travel itinerary $\longrightarrow f(d+1)$

[1, 4, 6, 7, 8, 20]

(2, 7, 15)



$$f(1) = \min \begin{cases} 2 + f(2) \rightarrow 2 + 9 \rightarrow 11 \\ 7 + f(8) \rightarrow 7 + 4 \rightarrow 11 \\ 15 + f(31) \rightarrow 15 + 0 \rightarrow 15 \end{cases}$$

11