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NPTEL ONLINE  
CERTIFICATION COURSE

# VLSI Physical Design with Timing Analysis

## Lecture – 5: Graph Searching Algorithms

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# Contents

- Graph Search Algorithms
  - Depth First Search(DFS)
  - Breadth First Search(BFS)



# Graph Search Algorithms

- Systematically follow the edges of the graph to visit the vertices of the graph.
- Used to discover the structure of the graph.
- There are two popular algorithms for graph search:
  - Breadth First Search(BFS)
  - Depth First Search(DFS)



# Depth First Search

1. Create a stack to keep track of vertices to visit.
2. Choose a starting vertex and push it onto the stack.
3. While the stack is not empty, do the following:
  - a. Check if the vertex on the top of the stack has any unvisited adjacent vertices.
  - b. If yes, push an unvisited adjacent vertex onto the stack.
  - c. If no unvisited adjacent vertices are left for the current vertex, pop it from the stack.
4. Repeat steps 2 and 3 until the stack is empty, which means all vertices have been visited.

# Depth First Search

- Algorithm:

- DFS(G)

- for each vertex  $u \in G.V$

- $u.color = WHITE$  ✓

- $u.\pi = NIL$  ✓

- time = 0

- for each vertex  $u \in G.V$

- if  $u.color == WHITE$

- » DFS-VISIT(G,u)

$G(V, E)$   
↑ ↑

Paint all vertices white and initialize their attributes to NIL.

Check each vertex in  $V$ , and when a white vertex is found, visit it by calling DFS-VISIT.

# Depth First Search

- DFS-VISIT( $G, u$ )
  - $\text{time} = \text{time} + 1$
  - $u.d = \text{time}$
  - $u.\text{color} = \text{GRAY}$
- **for** each vertex  $v$  in  $G.\text{Adj}[u]$ 
  - **if**  $v.\text{color} == \text{WHITE}$
  - $v.\pi = u$
  - DFS-VISIT( $G, v$ )
- $\text{time} = \text{time} + 1$
- $u.f = \text{time}$
- $u.\text{color} = \text{BLACK}$

Increment the global variable time, record the new value of time as the discovery time  $u.d$ , and paint  $u$  **GRAY**.

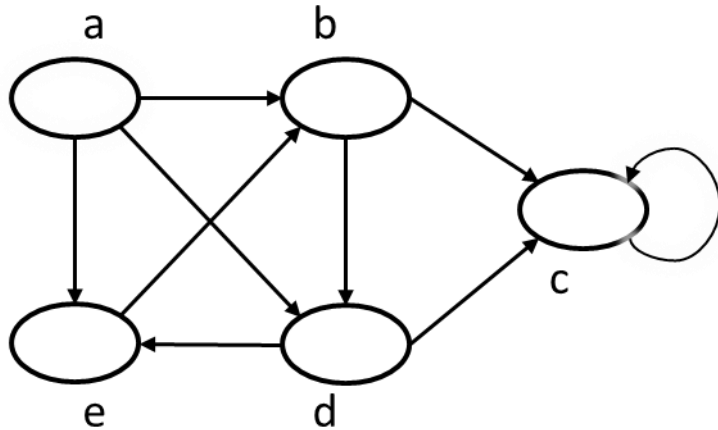
examine each vertex  $v$  adjacent to  $u$  and recursively visit  $v$  if it is white.

Finally, after every edge leaving  $u$  has been explored, increment time, record the finish time in  $u.f$ , and paint  $u$  **BLACK**.

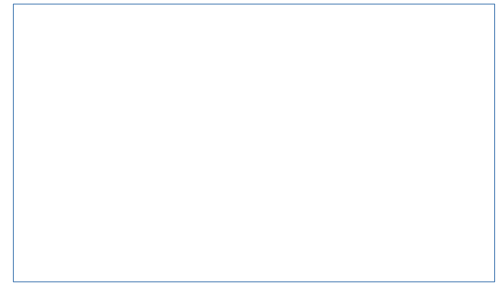
# DFS: Example - 1

$$G = (V, E)$$

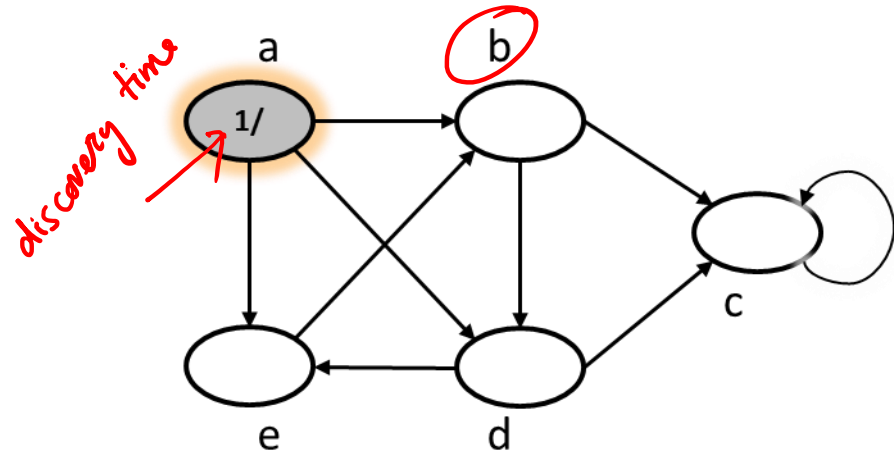
$$V := \{a, b, c, d, e\}$$



Input Graph

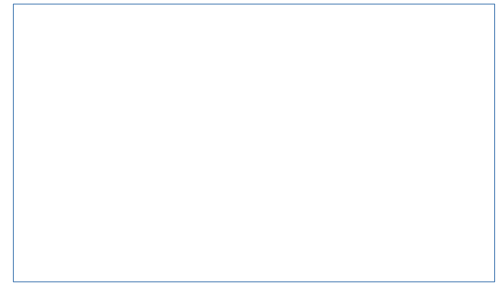


# DFS: Example - 1



a

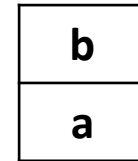
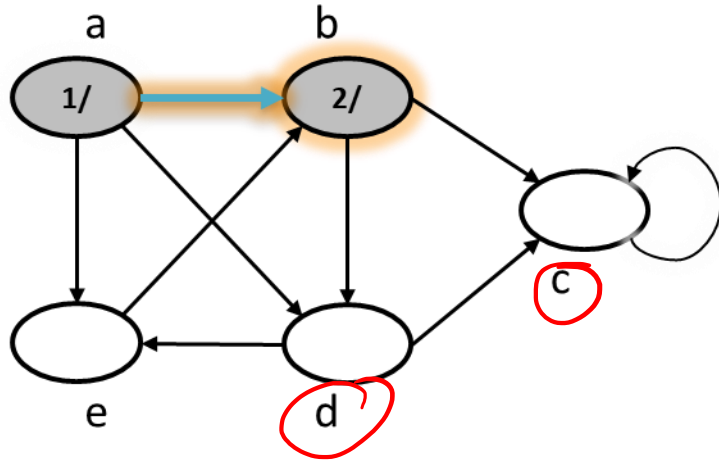
Stack



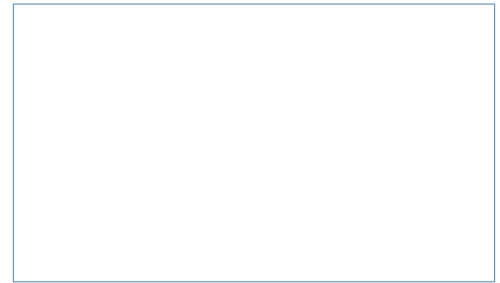


# DFS: Example - 1

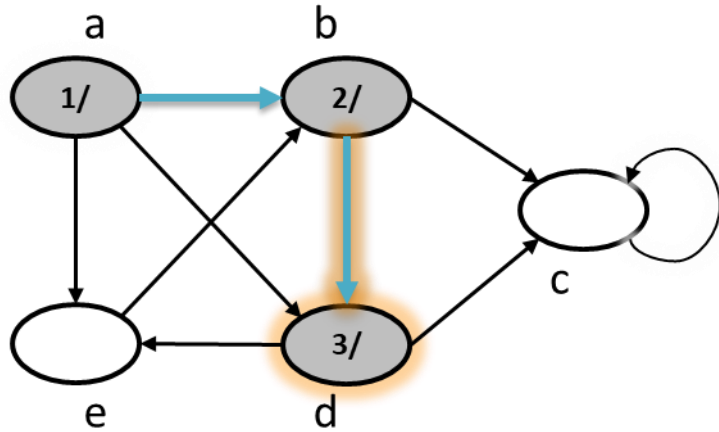
LIFO Data structure



Stack

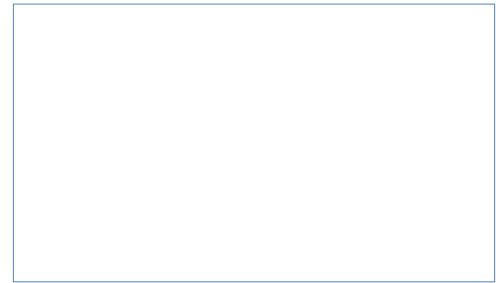


# DFS: Example - 1

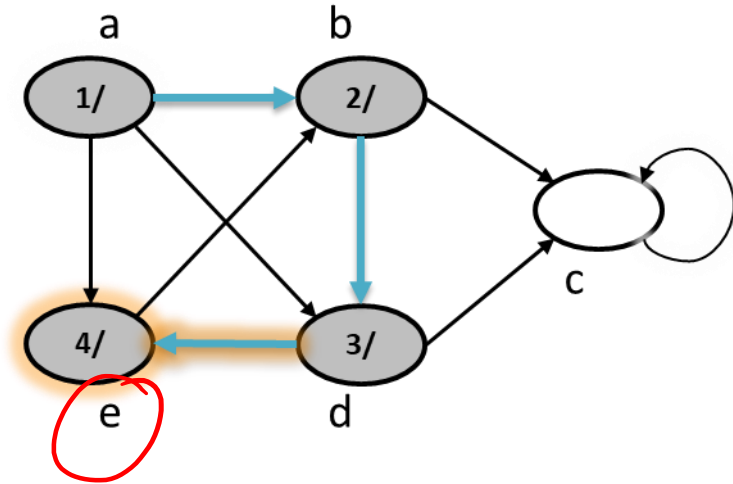


d
b
a

Stack

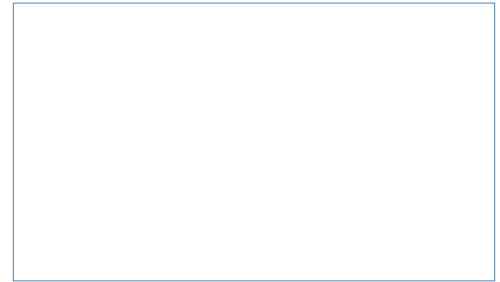


# Example - 1

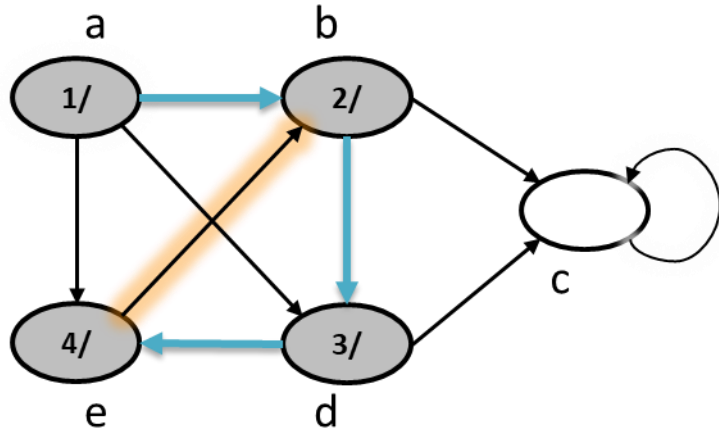


e
d
b
a

Stack

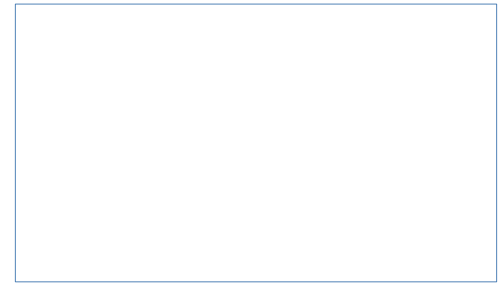


# DFS: Example - 1

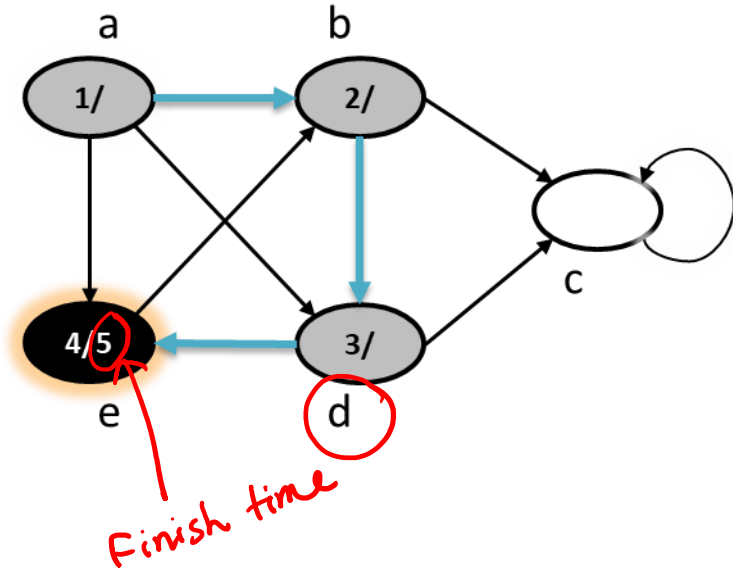


e
d
b
a

Stack

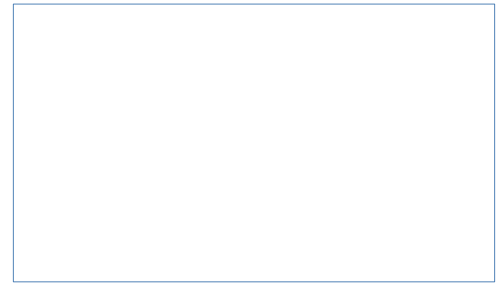


# DFS: Example - 1

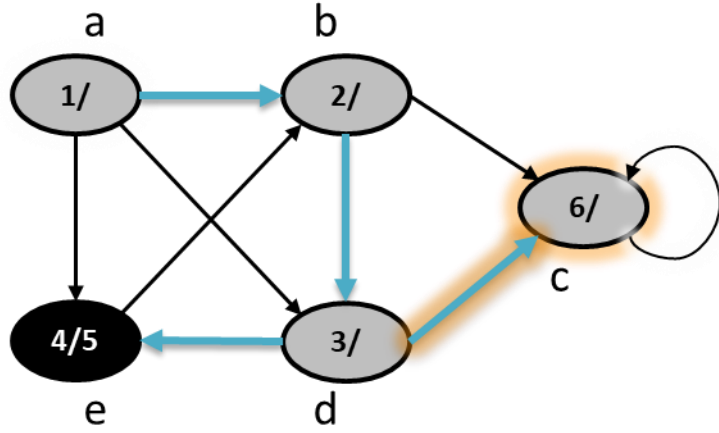


d
b
a

Stack

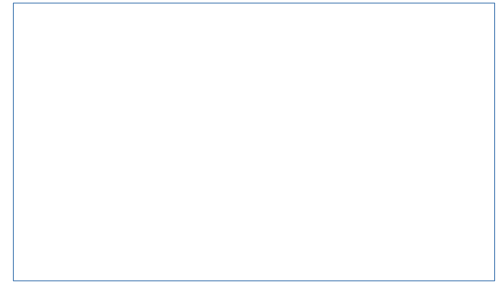


# DFS: Example - 1

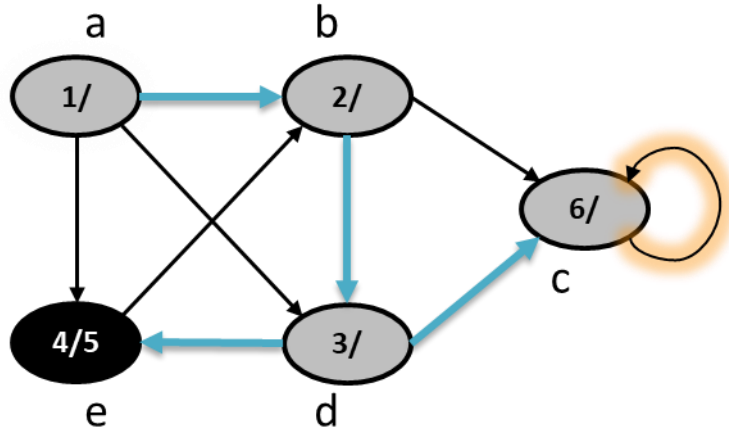


c
d
b
a

Stack

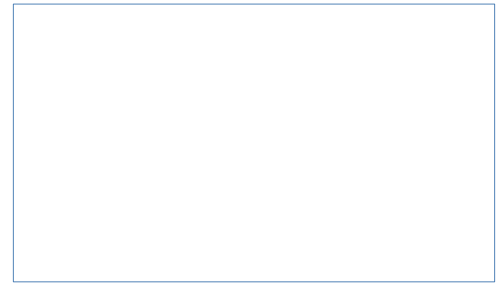


# DFS: Example - 1

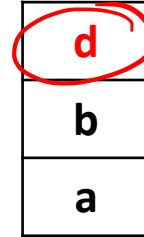
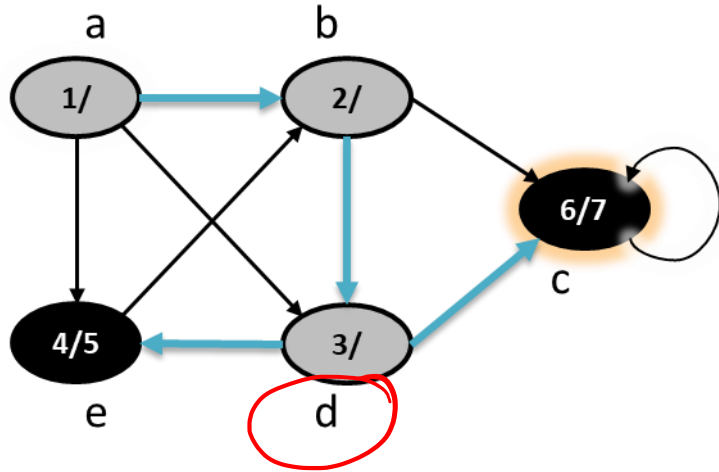


<b>c</b>
d
b
a

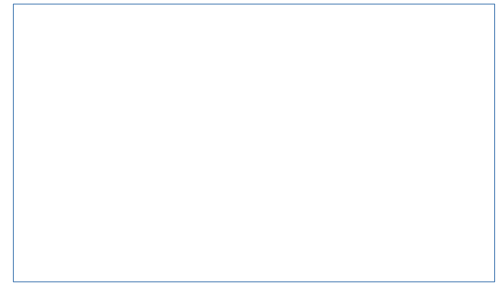
**Stack**



# DFS: Example - 1

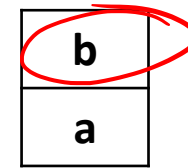
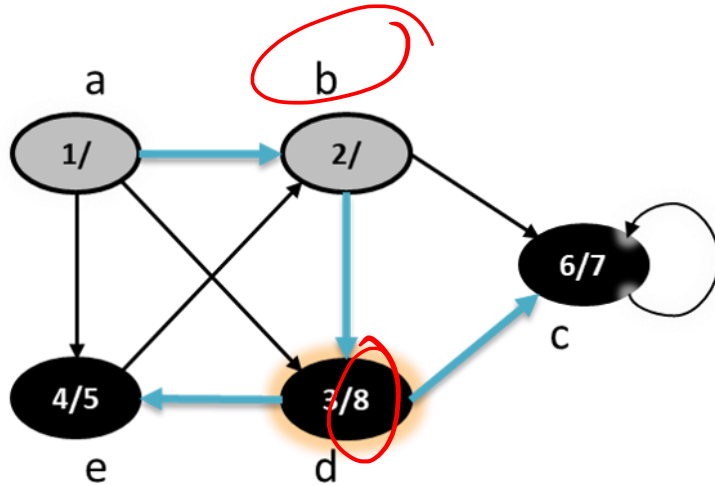


Stack

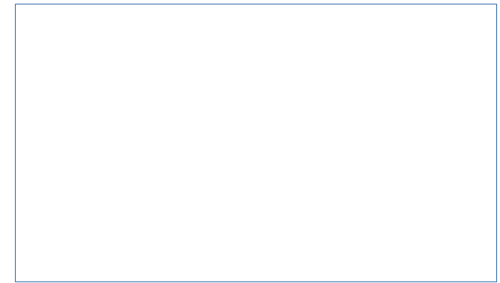




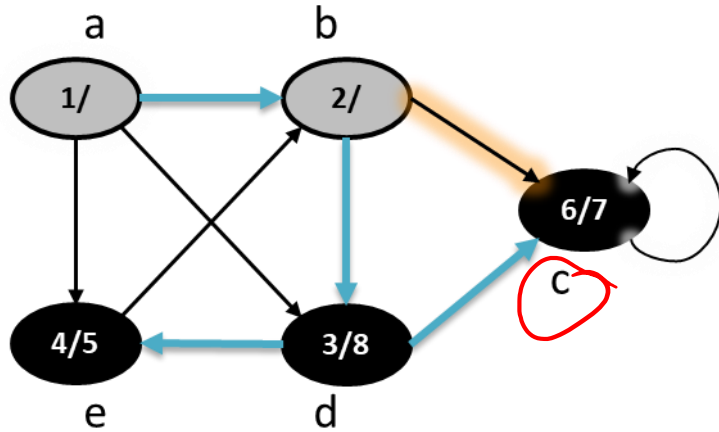
# DFS: Example - 1



Stack

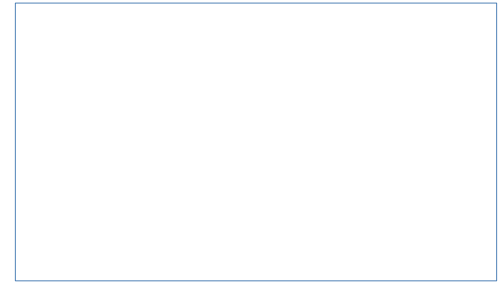


# DFS: Example - 1

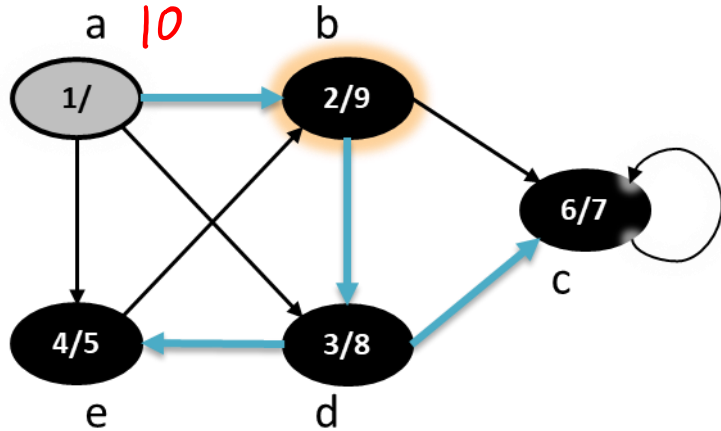


<b>b</b>
<b>a</b>

Stack

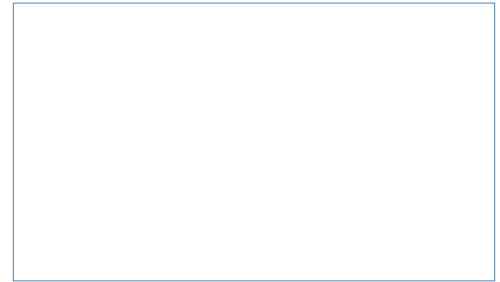


# DFS: Example - 1

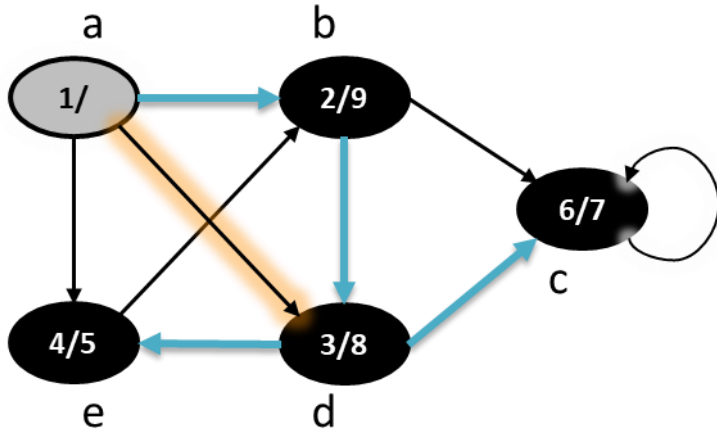


a

Stack

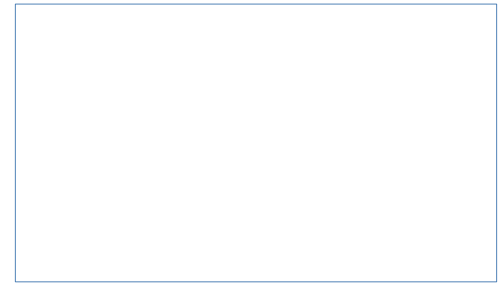


# DFS: Example - 1

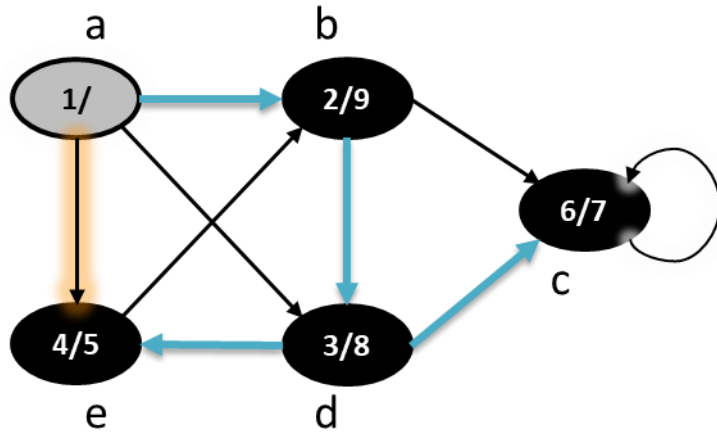


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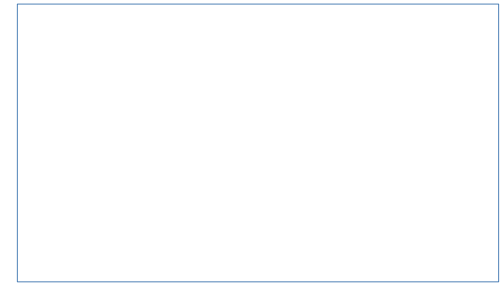


# DFS: Example - 1

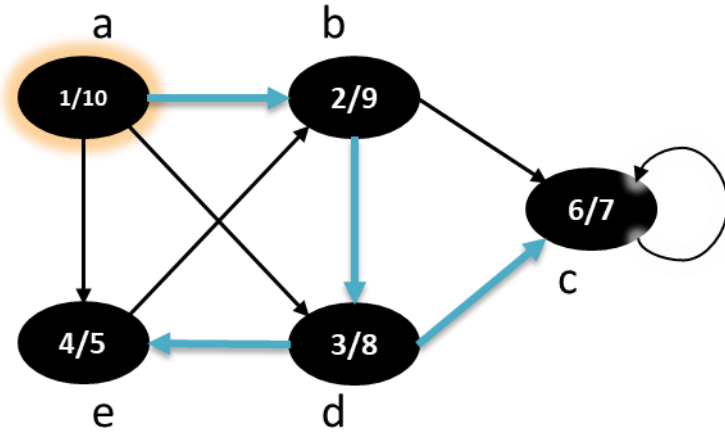


a

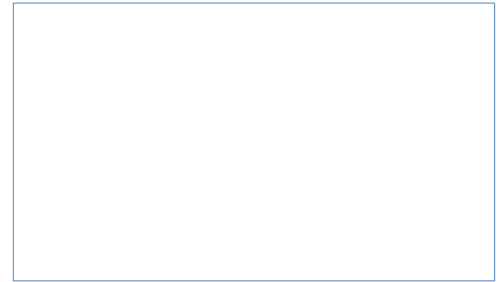
Stack



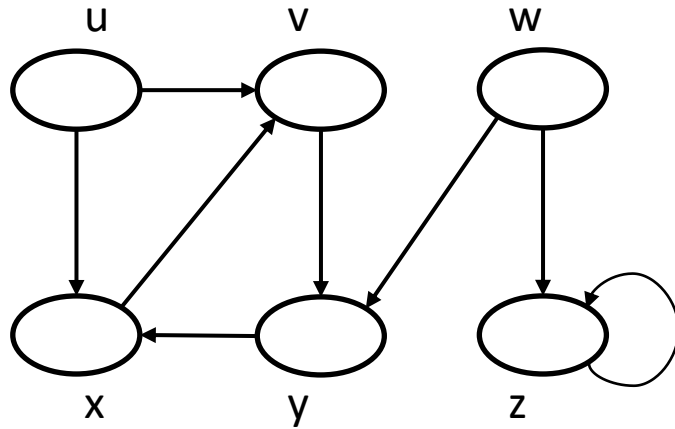
# DFS: Example - 1



Stack

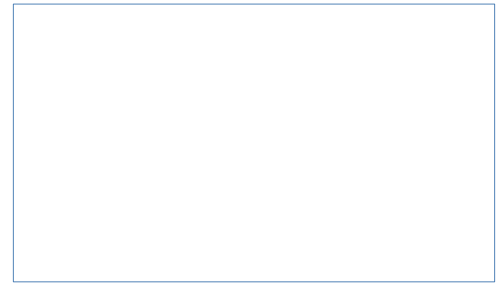


## DFS: Example - 2

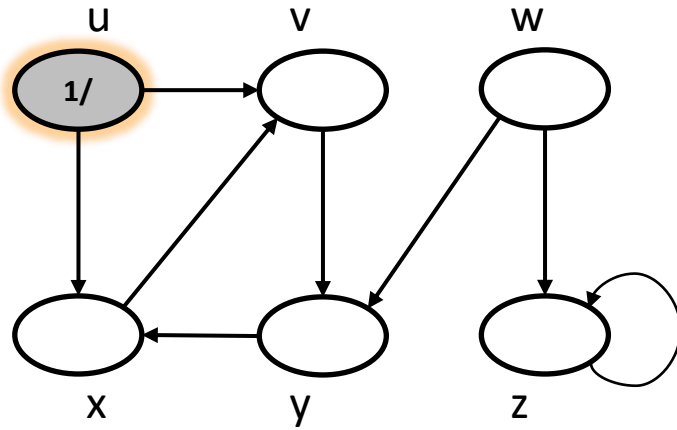


**Input Graph**

Source :Cormen, Leiserson, Rivest, Stein “Introduction to Algorithms”

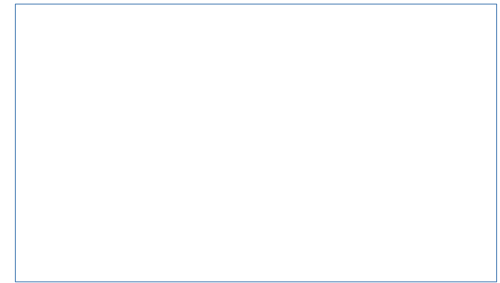


## DFS: Example - 2



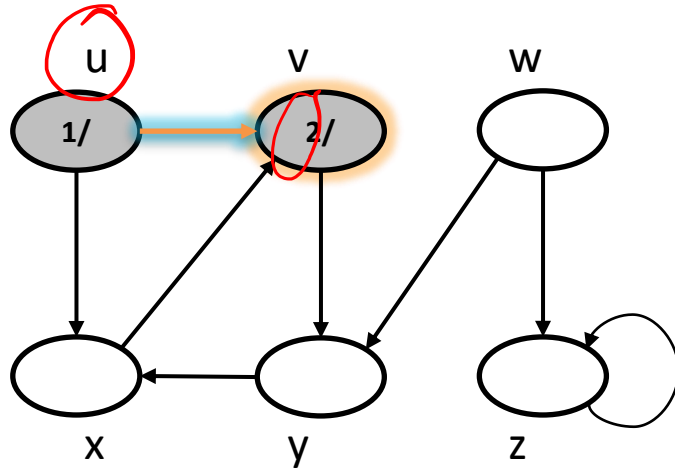
u

**Stack**



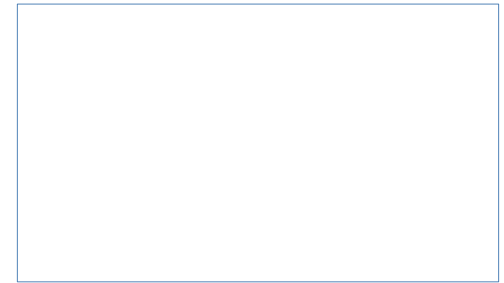


## DFS: Example - 2

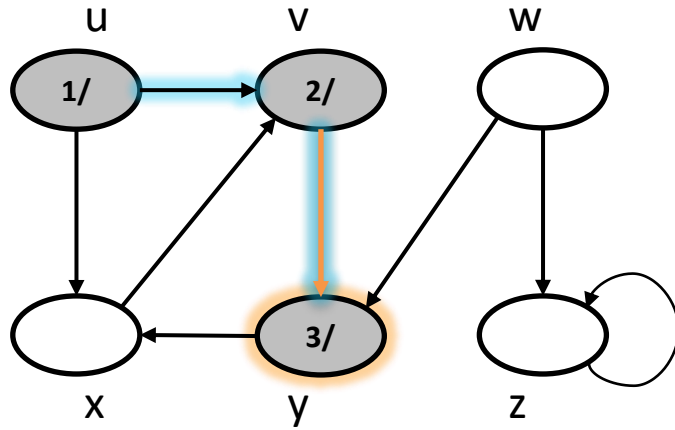


$v$
$u$

**Stack**

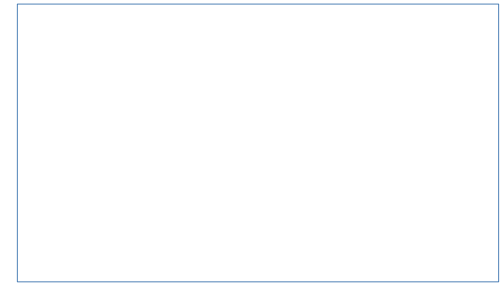


## DFS: Example - 2

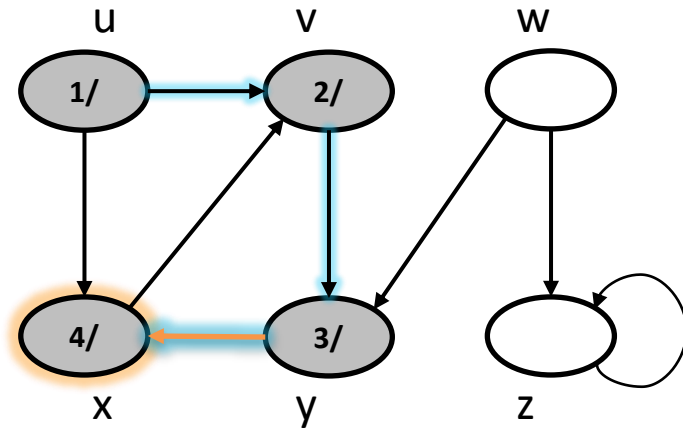


y
v
u

**Stack**

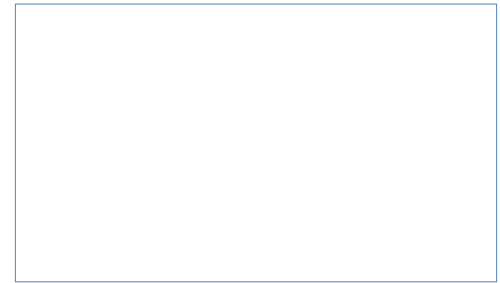


## DFS: Example - 2

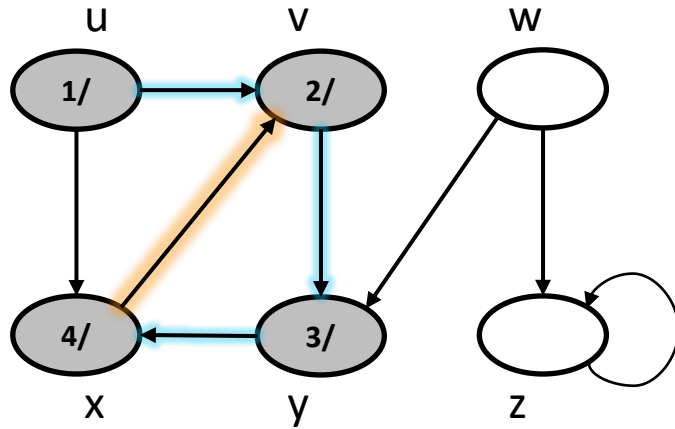


x
y
v
u

**Stack**

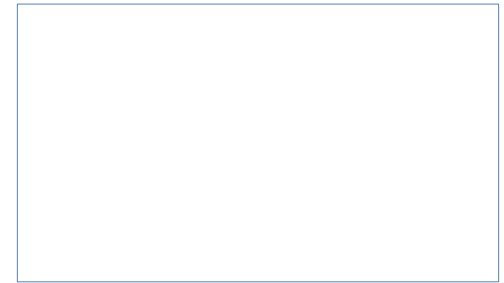


## DFS: Example - 2

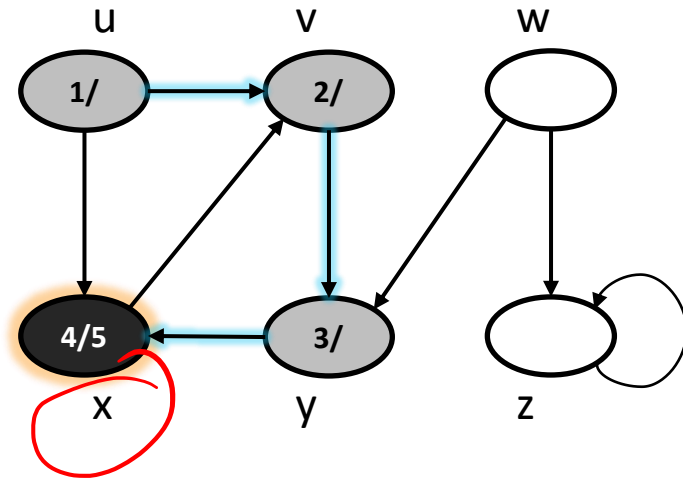


x
y
v
u

Stack

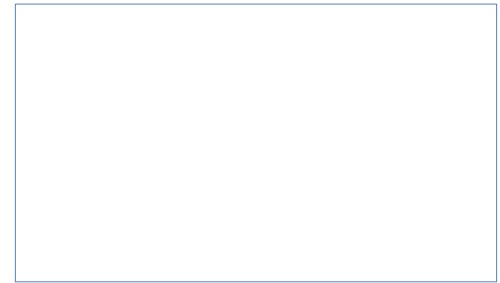


## DFS: Example - 2

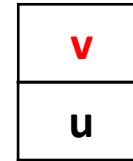
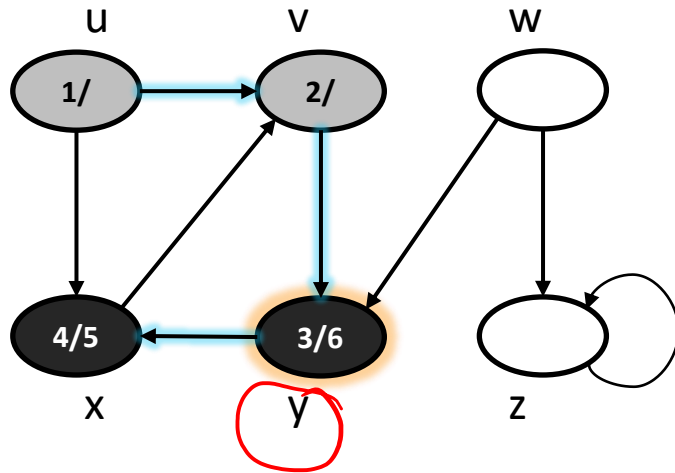


y
v
u

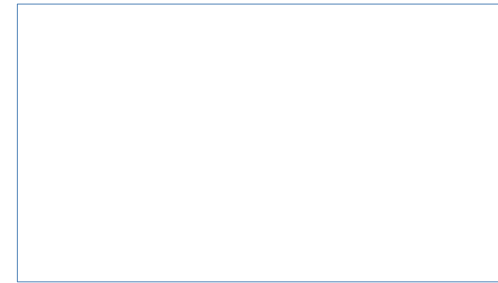
Stack



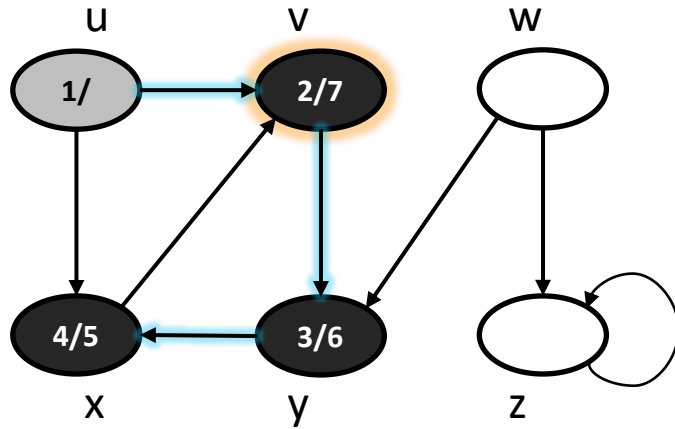
## DFS: Example - 2



**Stack**

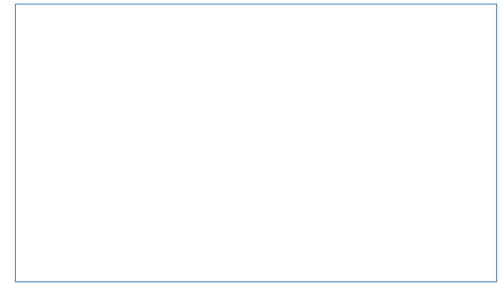


## DFS: Example - 2

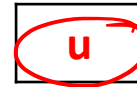
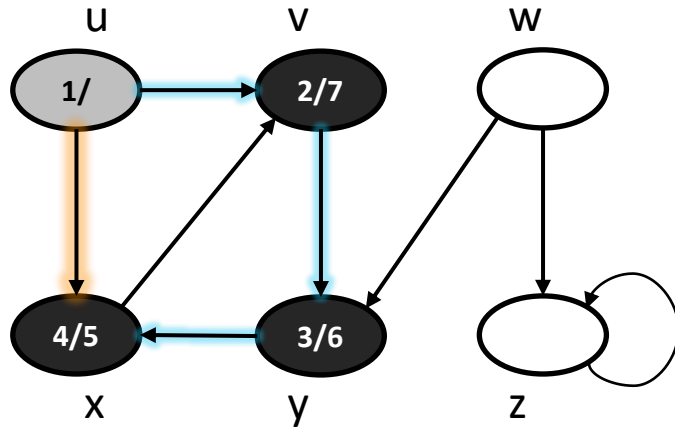


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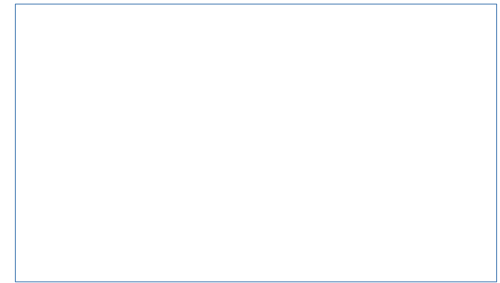
Stack



## DFS: Example - 2

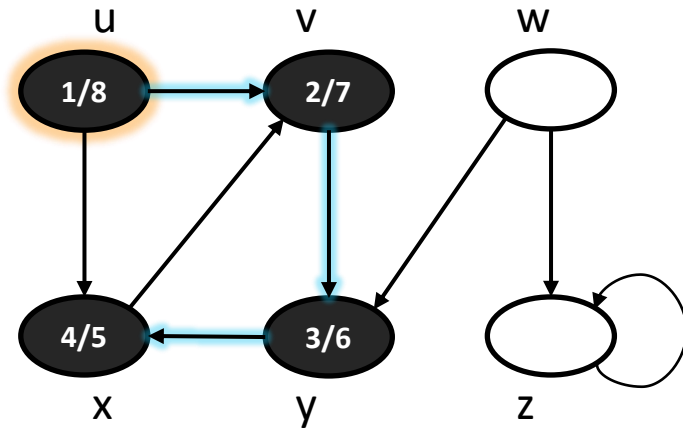


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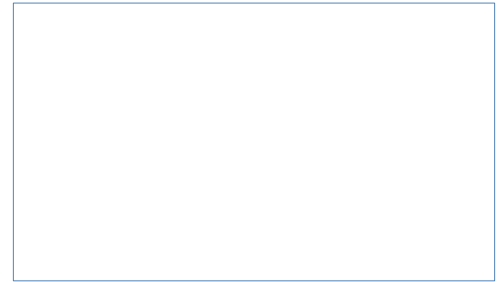




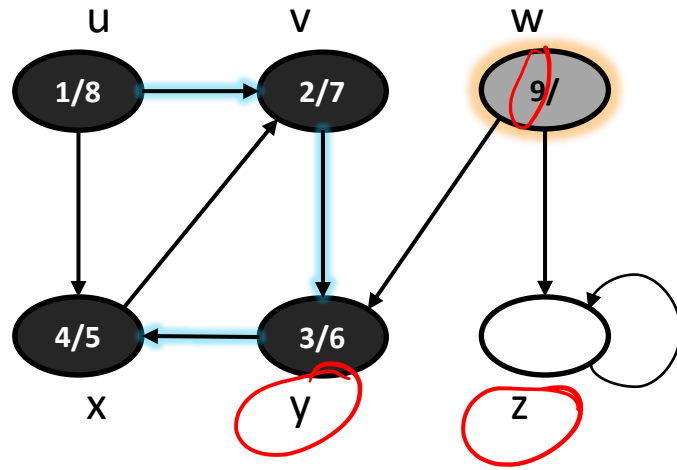
## DFS: Example - 2



**Stack**

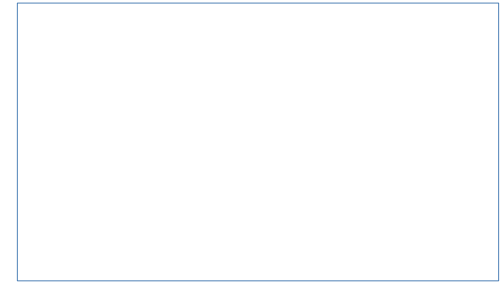


## DFS: Example - 2

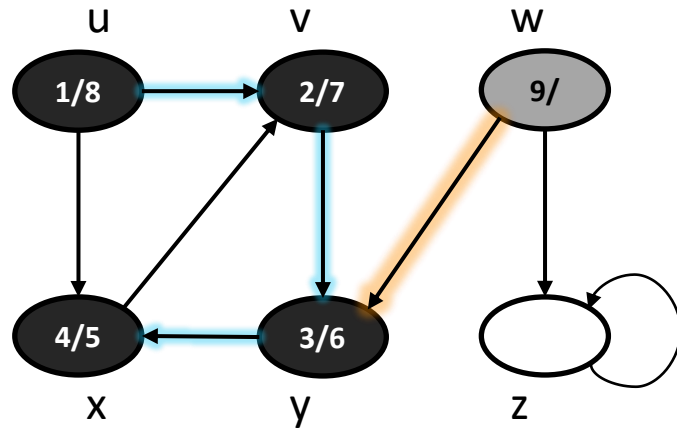


w

Stack

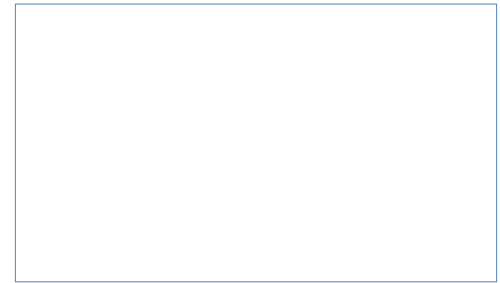


## DFS: Example - 2

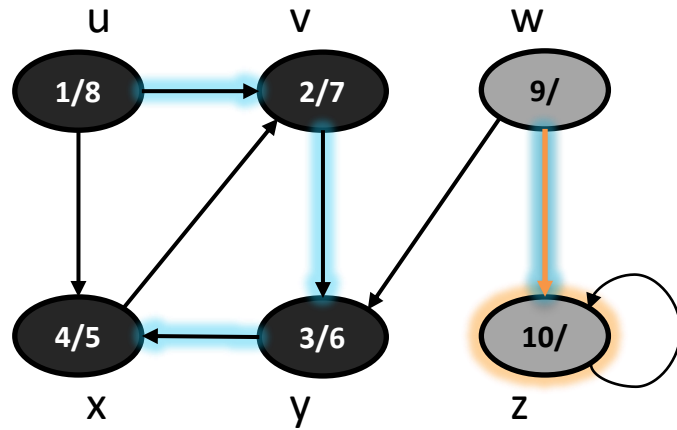


w

Stack

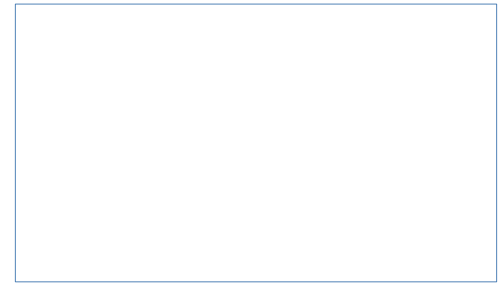


## DFS: Example - 2

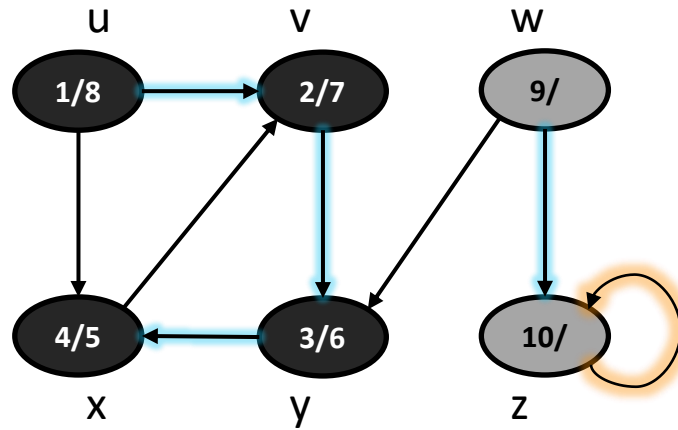


<b>z</b>
<b>w</b>

**Stack**

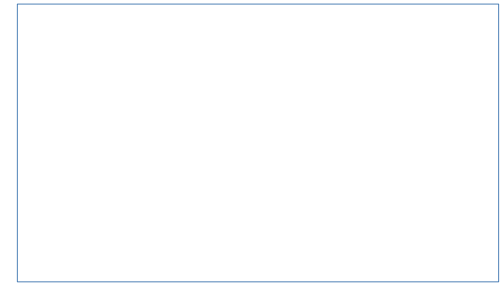


## DFS: Example - 2

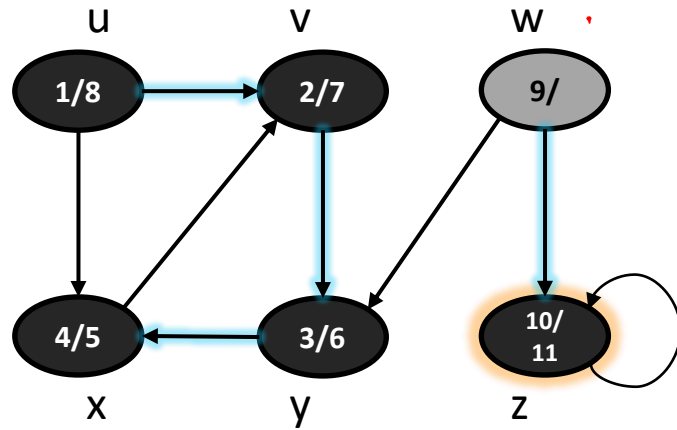


<b>z</b>
<b>w</b>

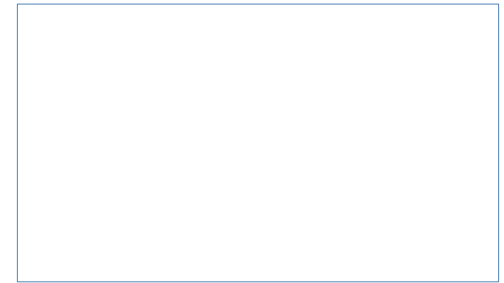
**Stack**



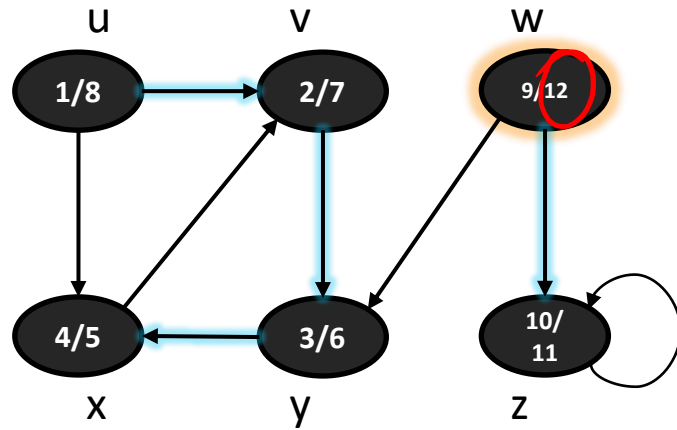
## DFS: Example - 2



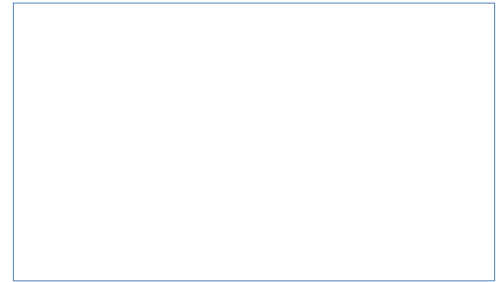
**Stack**



## DFS: Example - 2



Stack



# Breadth First Search(BFS)

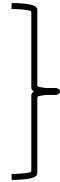
- BFS( $G, s$ )

- for each vertex  $u \in G.V - \{s\}$

- $u.color = WHITE$

- $u.d = \infty$

- $u.\pi = NIL$



With the exception of the source vertex  $s$ , paint every vertex white, set  $u.d = 1$  for each vertex  $u$ , and set the parent of every vertex to be NIL.

- $s.color = GRAY$

- $s.d = 0$

- $s.\pi = NIL$



Because the source vertex  $s$  is always the first vertex discovered, paint  $s$  gray, set  $s.d$  to 0, and set the predecessor of  $s$  to NIL.

- $Q = \emptyset$

- ENQUEUE( $Q, s$ )



Create the queue  $Q$ , initially containing just the source vertex.



# Breadth First Search(BFS)

- while  $Q \neq \emptyset$ 
  - $U = \text{DEQUEUE}(Q)$
  - for each vertex  $v$  in  $G.\text{Adj}[u]$ 
    - if  $v.\text{color} = \text{WHITE}$ 
      - »  $v.\text{color} = \text{GRAY}$
      - »  $v.d = u.d + 1$
      - »  $v.\pi = u$
      - »  $\text{ENQUEUE}(Q, v)$
  - $u.\text{color} = \text{BLACK}$

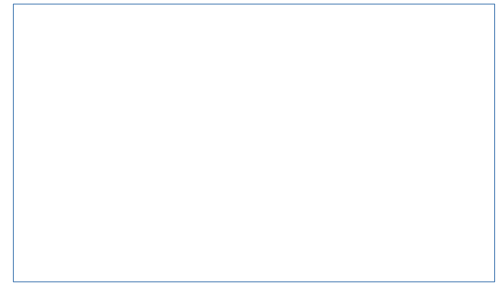
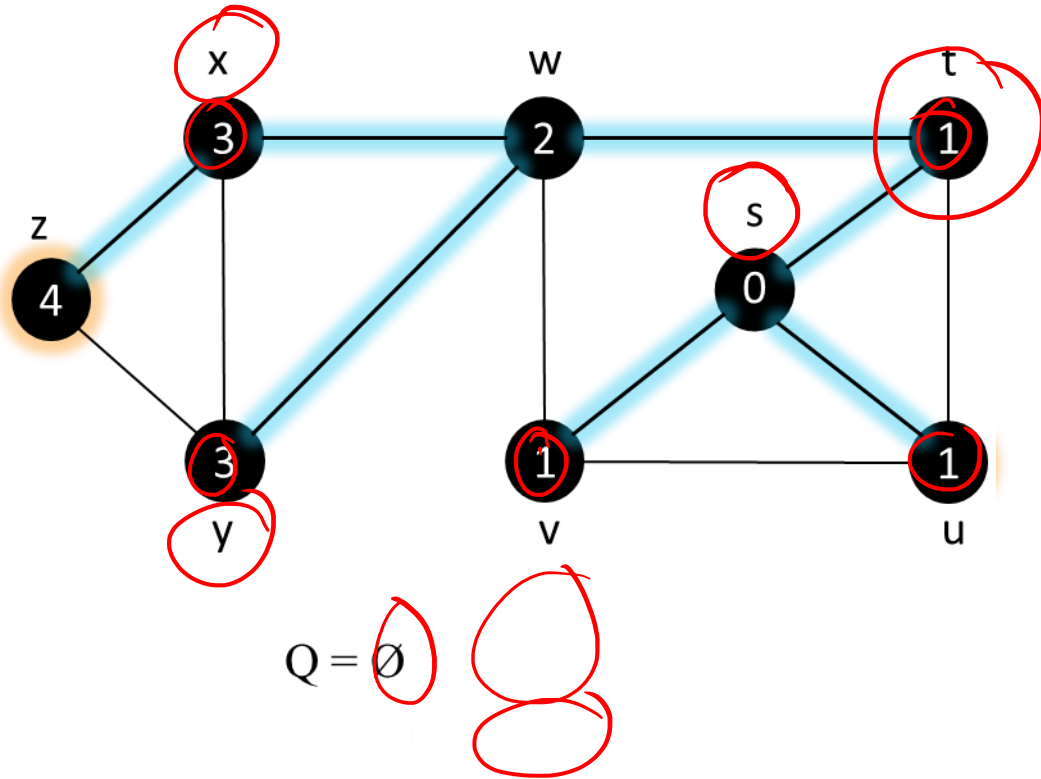
Determines the grey vertex  $u$  at the head of the queue  $Q$  and removes it from  $Q$ .

If  $v$  is white, then it has not yet been discovered. Paint vertex  $v$  gray, set  $v$ 's distance  $v.d$  to  $u.d + 1$ , record  $u$  as  $v$ 's parent  $v.\pi$  and place  $v$  at the tail of the queue  $Q$ .

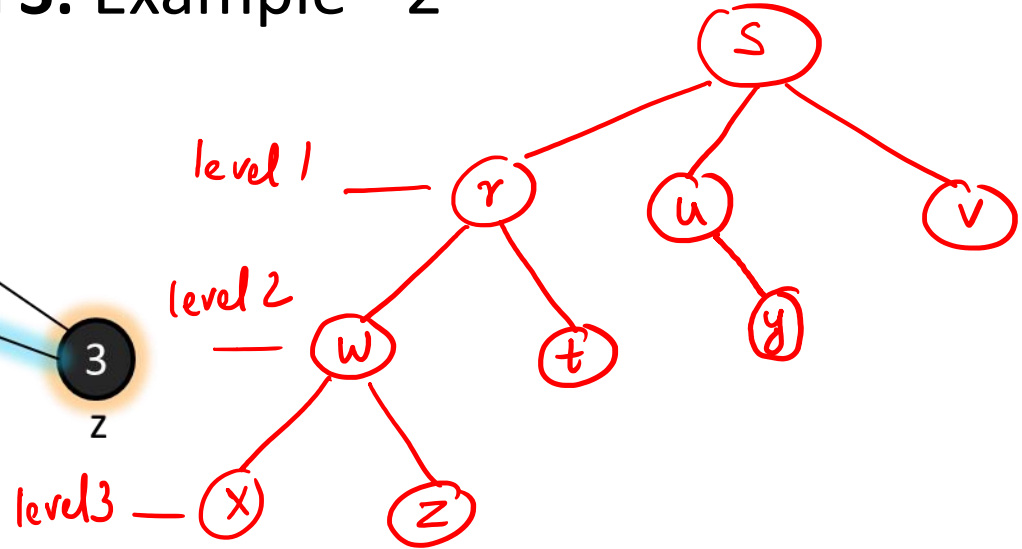
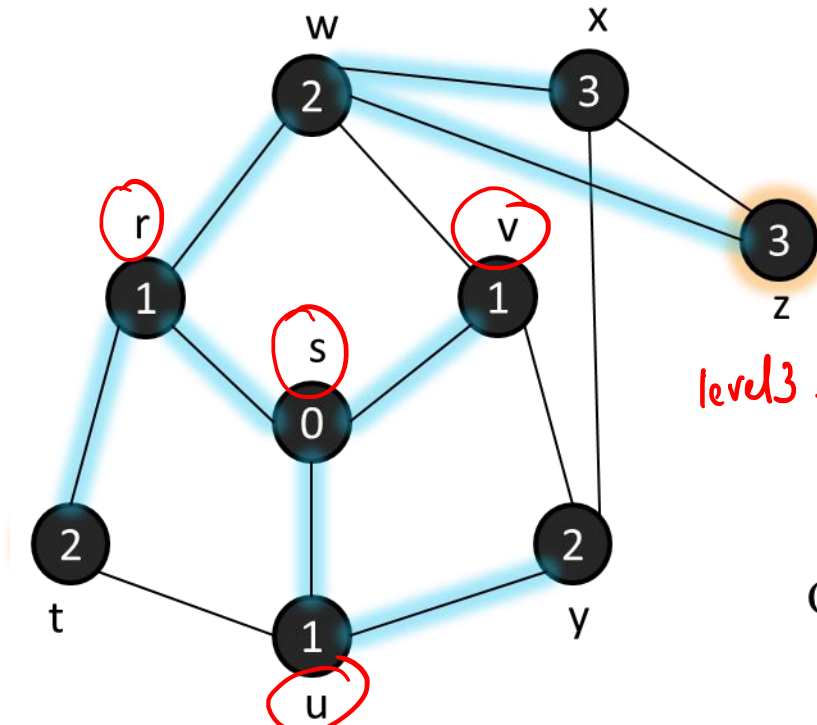
Once the procedure has examined all the vertices on  $u$ 's adjacency list, blacken  $u$ , indicating that  $u$  is now behind the frontier. .

## BFS: Example - 1

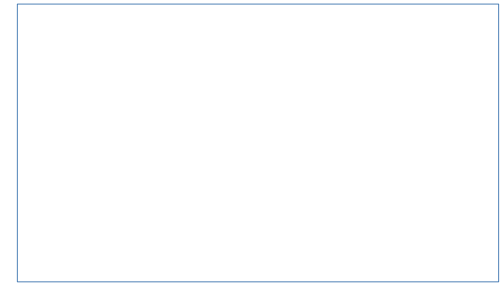
FIFO Data Structure



## BFS: Example - 2



$Q = \emptyset$



Source :Cormen, Leiserson, Rivest, Stein "Introduction to Algorithms"

# Summary

- Discussed Graph search algorithms: DFS and BFS



# Thank You

