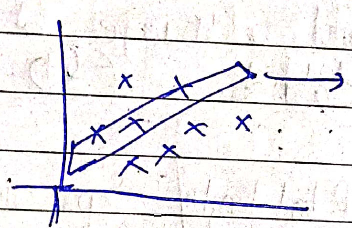
* Session - Linear Algebra - 1 *

- Linear Algebra has Scalars, Vectors, Matrices and tensors.

- ML? → ② reasons

i) Generalizing concepts or data in higher dimensions.

Not only 1D, 2D, 3D, can reach to n-Dimensions.



→ hyperplane

ii) Data Representation -

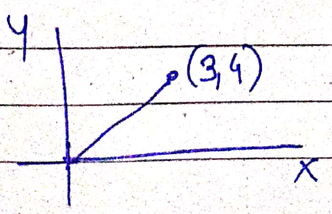
- → tabular
- → text
- → image
- → video

this type of data are represent by linear algebra and one can build ml model after that.

- What are Vectors?

- 2D Co-ordinate   a = [3,4]

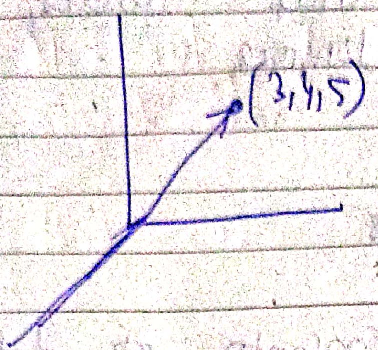
x component   y component



What is Vector?

→ It is a point in a particular co-ordinate system.

- a (3,4,5) in 3D space.



- n-dimensions

$$a = [x_1, x_2, x_3, \ldots, x_n]$$

• lets understand usage of vector by iris dataset. ~~(4D)~~ ~~dat~~

Sepal length (SL)	Petal length (PL)	Sepal width (SW)	Petal width (PW)	Species
5.2	2.1	7.2	9.1	Setosa
3.1	4.2	7	6.5	Versicolor
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

→ each datapoint convert into feature vector like for this datapoint

[5.2, 2.1, 7.2, 9.1] → send to ml model & predict species

↓

4 Dimensional vector.

~~But~~ ~~for~~ ∘ you write vector into numbers only.

Let's understand by titanic dataset

age	fare	sex	embarked
17	62	$\boxed{M}$	1

↓ For this you should use encode, for M → 0
For F → 1

Then featured vector be like
[17, 62, 0, 1]

* <u>Vector</u> <u>example</u> <u>in</u> <u>ML</u> -

∘ You have to make movie recommender system.

eg: movie name (Matrix) $\xrightarrow{\text{gives 5 movies related}}$ 1, 2, 3, 4, 5

name	Summary
m1	S1
m2	S2
m3	S3

→

	Summary's
m1	Hi How are you
m2	My name is Camp
m3	this is 2023

↓

So, how gonna you convert it into featured vectors.
↓
Do it using BOW technique

BOW → Bag of Words

① find Unique words –

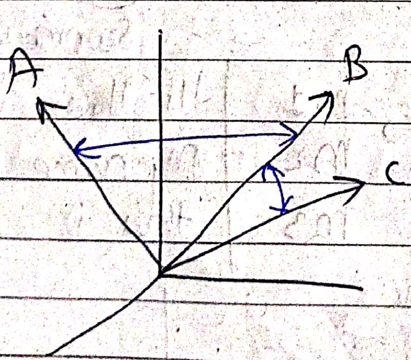
Hi | how | are | you | my | name | is | Camp |
~~this | 2023~~

this | 2023

- Now you'll go to every movie and ask how many times "Hi" are there, "How" are there.

Hi	how	are	you	my	name	is	campusx	this	2023
1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	0	0
0	0	0	0	0	0	1	0	1	0

<u>Note</u> → This 3 feature vector is in 10 dimensional data.



if Someone ask to recommend a movie like B then you'll tell movie C not A due to smaller distance

* Row vector and Column vector

- $a = [x_1\ x_2\ x_3\ \dots\ x_n] \rightarrow$ row Vector

$1 \times n$: 1 → no. of rows, n → no. of columns.

- $b = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \rightarrow$ Column Vector

$n \times 1$: n → no. of rows, 1 → no. of columns.

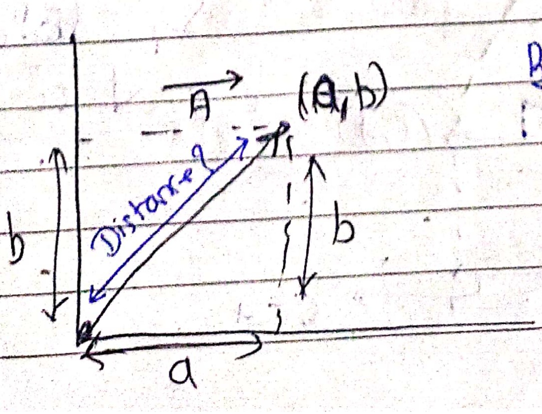
Eg →

SL	SW	PL	PW	Species
—	—	—	—	—
—	—	—	—	—
	—	—	—	—
⋮	⋮	⋮	⋮	⋮
—	—	—	—	—

row vector

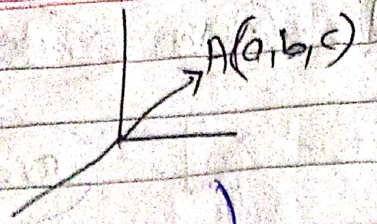
Column vector

* Distance from Origin ~~(...)~~ ||A||

- 

By using pythagoreus,

$$\boxed{d = \sqrt{a^2 + b^2}}$$



$$\boxed{d = \sqrt{a^2+b^2+c^2}}$$

This is also true for n-dimensionals

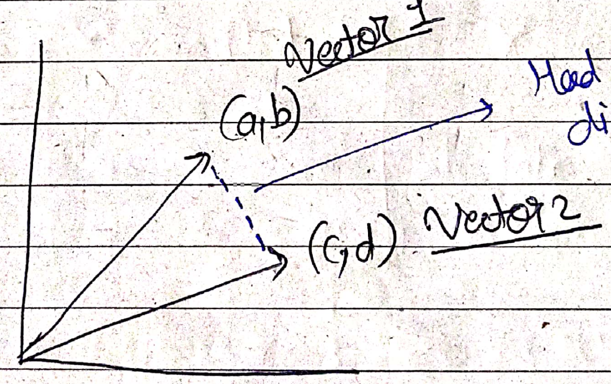
Suppose → $A = [x_1 \; x_2 \; --- \; x_n]$

$$d = \sqrt{x_1^2 + x_2^2 + -- x_n^2}$$

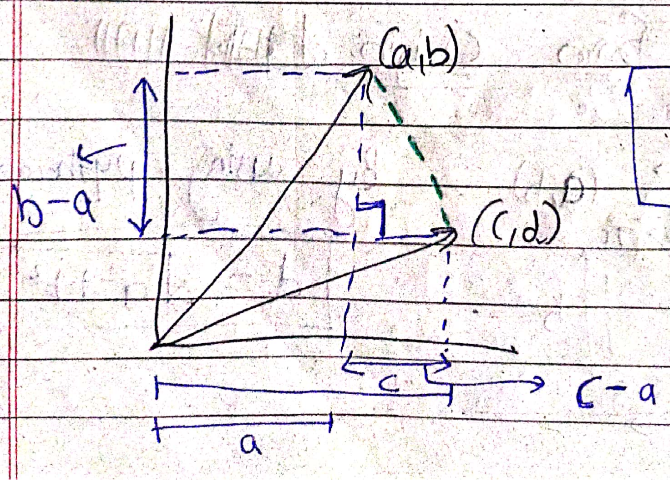
- We also called this ~~many~~ magnitude of vector. Notation → $||A||$

* Euclidean Distance -

(1)



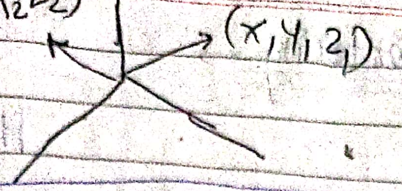
Need to find this distance, which is euclidean distance



$$\boxed{\text{So, distance} = \sqrt{(c-a)^2 + (d-b)^2}}$$

2-D

② for (x_2, y_2, z_2)



$$\boxed{\text{So, } dist = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

↓

for n-dimensions.

- Note → K-nearest neighbour uses euclidean distance to classify the data points.

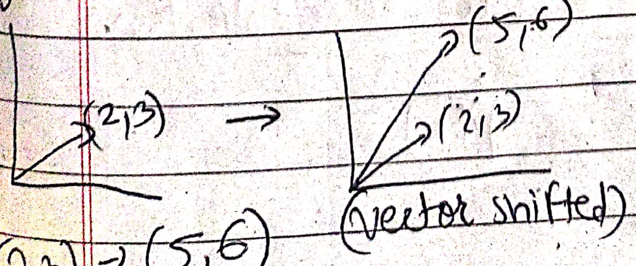
* Scalar Addition / Subtraction (Shifting)

[Scalar addition / Subtraction] [Scalar multiplication / division]

$v = [v_1, v_2, \ldots v_n] \xleftarrow{\text{add}} \boxed{n}$

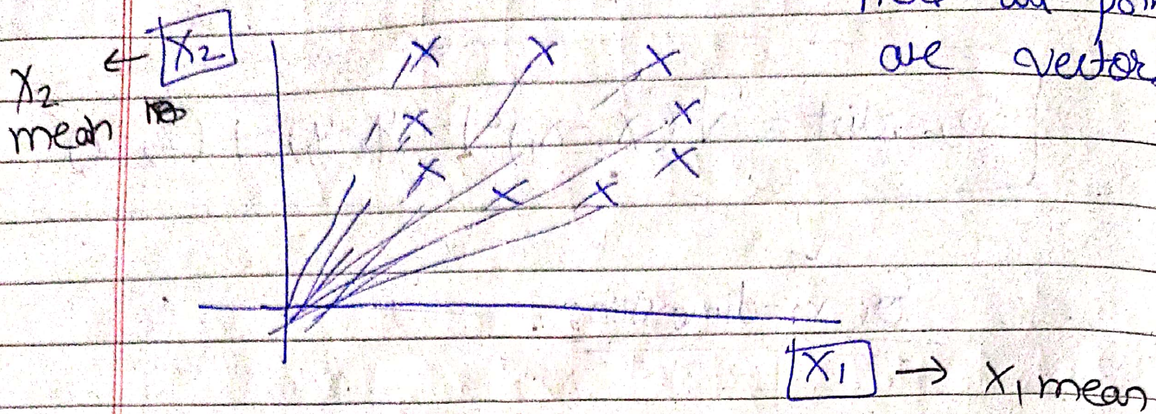
So, $v = [v_1+n, v_2+n \ldots v_n+n]$

eg → Suppose, Scalar = 3



$(2,3) \rightarrow (5,6)$

Mean centering →



Here all points are vectors

- Calculate X_1 mean & X_2 mean.

		X_1	X_2
Vector	v_1	3	4
	v_2	5	6
	v_3	7	8

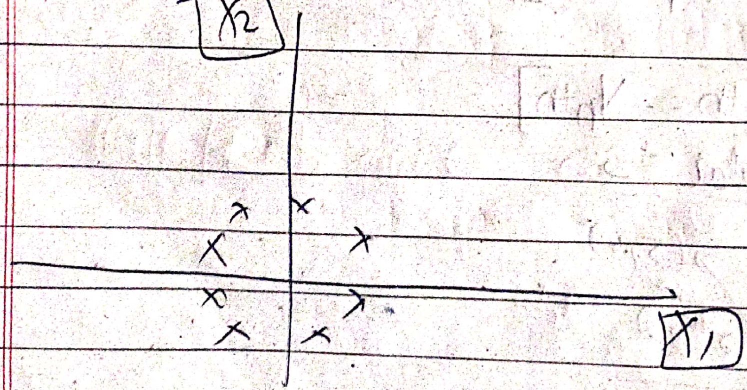
→ X_2 mean $= 6$

X_1 mean $= \frac{15}{3} = 5$

vectors

$X_1 - X_1$mean
$X_2 - X_2$mean
Scalars.

X_1 shifted	X_2 shifted
−2	−2
0	0
2	2



- Mean centering is a useful pre-processing technique in various ml appⁿ.

- It improves performance, convergence & interpretability of the model.

* **Dot Product** - (Multiplication between two vectors)

Vector A = $[a_1, a_2, a_3 -- a_n]$
Vector B = $[b_1, b_2, b_3 -- b_n]$

Dot product — result in Scalar

Cross product — result in vector.

- $\boxed{a_1b_1 + a_2b_2 + a_3b_3 + \cdots a_nb_n}$

It is a dot product of 2 vectors.

Matrix Multiplication -

$[a_1, a_2, --- a_n]_{1\times n}$ row

$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{n\times 1}$ Column

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad a^T B$$

So, $\boxed{A \cdot B = A^T B}$

- **Rules while forming Dot product →**

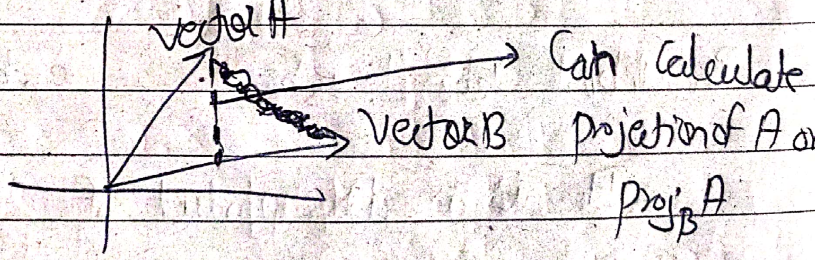
1) Commutative law
$$A \cdot B = B \cdot A$$

2) Distributive law
$$\boxed{A \cdot (B + C) = A \cdot B + A \cdot C}$$

- **Where to use dot product →**

→ Compute similarity between 2 vectors.

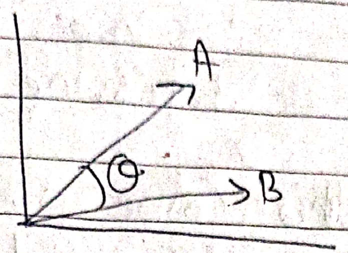
→ Can ~~calcul~~ Calculate projections.



→ Can perform matrix multiplication.

- Geometric intuition -

* Angle between 2 vectors



$$\boxed{A \cdot B = \|A\| \, \|B\| \cos\theta}$$

$\|A\|$ → dist of A from origin

$\|B\|$ → dist of B from origin

from this, we can conclude
A, B both vectors are Non-Zero

1) $A \cdot B = 0$

from this → $\|A\| \, \|B\| \cos\theta = 0$

we conclude that $\boxed{\theta = 90^\circ}$

So, we ~~can~~ conclude $A \perp B$ → perpendicular or orthogonal.

means A & B are dissimilar.

Suppose you have A B → $A = [a_1 \; a_2 \; \text{--} \; a_n]$

$B = [b_1 \; b_2 \; \text{--} \; b_n]$

from this value you can find angle.

How? →

$$\cos\theta = \frac{A \cdot B}{\|A\| \, \|B\|}$$

$$\Rightarrow \theta = \cos^{-1}\left\{ \frac{A \cdot B}{\|A\| \, \|B\|} \right\}$$

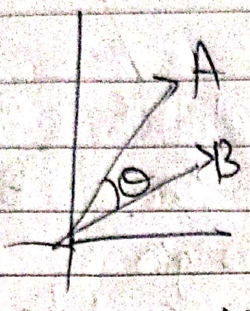
By this formula, we can find angle between two vectors.

◦ Cosine Similarity -

$$\boxed{\cos\theta = \frac{A \cdot B}{\|A\| \; \|B\|}}$$

Cos of angle betⁿ A and B are called Cosine similarity

Here cosine similarity value is value okay.



By this Cosine similarity we can find similarity of vectors

value in $\boxed{-1 \text{ to } 1}$

θ=0

means Same direction

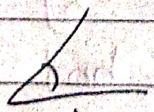
↓

Similar

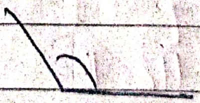
Value -1 — θ=180° → means polar opposite

Value 0 — θ=90° → means orthogonal means don't have any similarity

value $\boxed{0 - 1}$ value $\boxed{-1 \quad 0}$



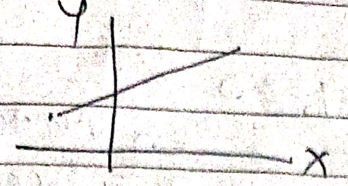
actue angle



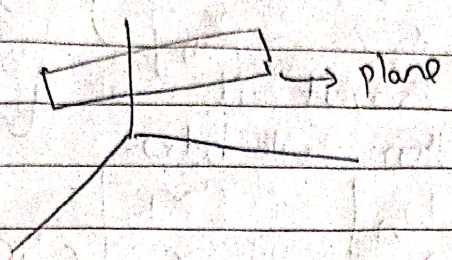
obtuse angle

* Eqn. of a Hyperplane -

2D →

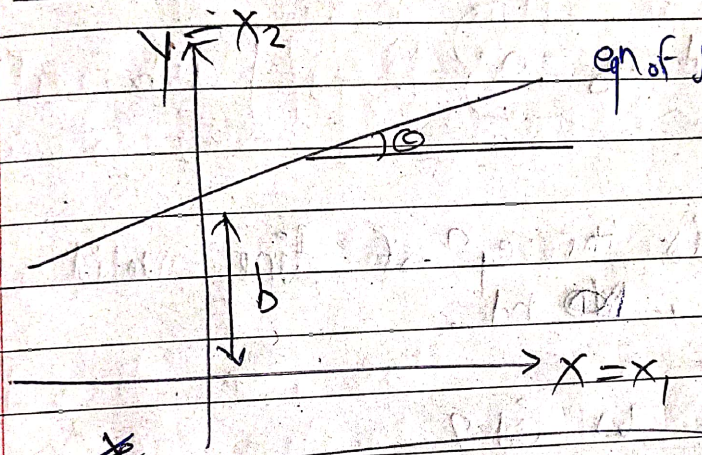


3D →



4D, 5D, nD → Hyperplane.

(for higher dimensions, we need hyperplane).



eqn. of line $\boxed{y = mx + b}$ → $\boxed{\text{general form}}$

$$\boxed{ax + by + c = 0}$$

$$ax_1 + bx_2 + c = 0$$

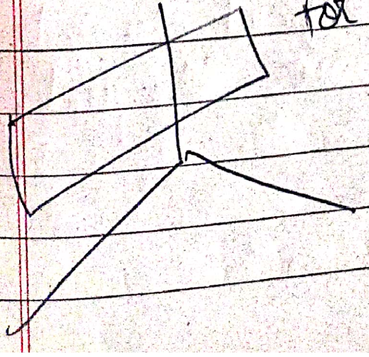
$$\boxed{w_1x_1 + w_2x_2 + w_0 = 0}$$

~~Just note~~

$$ax + by + c = 0$$

$$y = \frac{-a}{b}x - \frac{c}{b}$$

$$m = \frac{-a}{b}, \quad b = \frac{-c}{b}$$



for 3D → $\boxed{w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0}$

Hyperplane notation in high dimensions is π (pie)

$$\boxed{w_1 x_1 + w_2 x_2 + \dots + w_n x_n + w_0 = 0}$$

$$\Downarrow$$

$$w \cdot x + w_0 = 0$$

$w = [w_1, w_2, \dots w_n]$
$x = [x_1, x_2 \dots x_n]$

Generally, vectors look like this $\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$ $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

If you want to convert it into matrix form then write below-

$[w_1, w_2 \dots w_n]$ (row vector) $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ (col^n vector)

$$w^T x + w_0 = 0 \quad \dots \quad \boxed{a \cdot b = a^T b}$$

This is the eq^n of line valid in any 2D, 3D, ~~4D~~ nd.

Note- What w is?

We know, $\boxed{w^T x = 0}$

$$w^T x = w \cdot x = \|w\| \|x\| \cos\theta = 0$$

(w is always perpendicular)