* Linear Algebra - 3 *

Recap → Linear Algebra → Vectors
→ matrix

Matrices → $\boxed{\text{intuition}}$

$\boxed{\text{3blue1brown}}$
Essence of linear algebra
See

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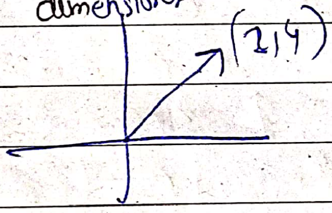
* Basis Vector -

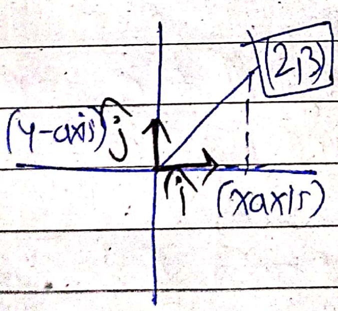
- vectors are set of numbers basically.

eg. $[\underbrace{2 \quad 3 \quad 4}_{\text{dimensions}}]$ $[2, 4]$



(2,4) → 2d vector
2d point.

- In 2D,



- $\hat{j}$ & $\hat{i}$ vectors called as unit vectors.
- Their distance from origin is 1.

→ $2\hat{i} + 3\hat{j}$.

- for, $[3, 4]$

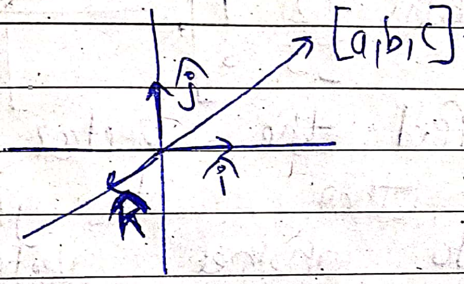
$\boxed{3\hat{i} + 4\hat{j}}$ → called as linear combination

So basically, any points we called them as vector but actually they are set of scalars. $[6, 7]$

$\uparrow$ Set of Scalars

When you multiply that set of scalars by basis vectors then you got 2 new vectors. And then you do vector addition in them. eg $\rightarrow$ $6i + 7j$

- In 2D, there are $\hat{i}, \hat{j}$.
- In 3D, $\hat{i}, \hat{j}, \hat{k}$.



$\boxed{a\hat{i} + b\hat{j} + c\hat{k}}$

* Linear Transformation -

matrices $\longrightarrow$ linear transformation

$\uparrow$

$$\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \longrightarrow \text{linear transformation.}$$

Any matrix can be called as linear transformation.

- What's Transformation?

- Imagine it as a fn.

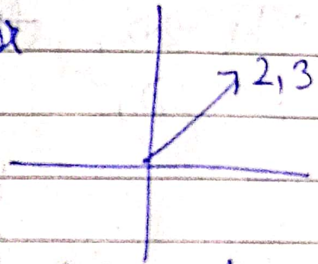
given input $\longrightarrow$ $f(x)$ $\longrightarrow$ o/p.

- Means transformation is like fn.
- When you give vector as a input to transformer / matrix then it gives you second vector.

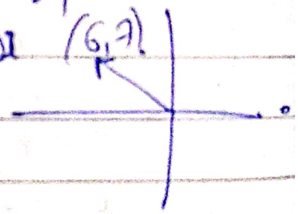
- Vector as input → transform / matrix → gives another vector

- Let's you have vector

and have matrix $\begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$.



- When you feed the vector to the matrix then it gives / transforms into another vector (6,7)



- Means, $(2,3) \longrightarrow \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} \longrightarrow (6,7)$

- Above no. doesn't have meaning, its just an example.

- This is what happens when you do a multiplication between a vector and a matrix.

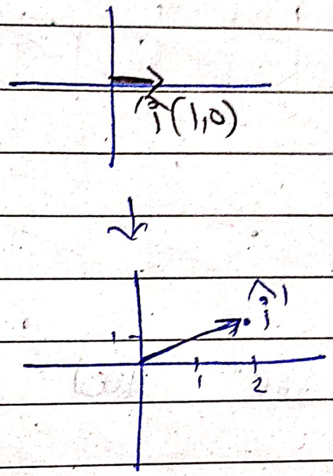
- Matrix can be considered as Transformations.
- Matrices acts as transformation on vectors.

- Suppose you have these matrix $\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$, then what's the meaning of the inside no.s.

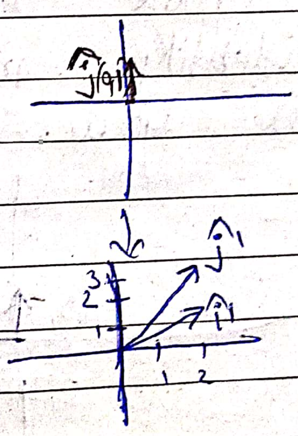
X $\begin{bmatrix}\boxed{\begin{matrix}2\\1\end{matrix}} & \begin{matrix}3\\2\end{matrix}\end{bmatrix}$ Y

Suppose, x is the i/p vector — Matrix — o/p

- This tells, when transformation applies then, in your 2D space your $\hat{i}$ vector whose current coordinate is (1,0) ~~that~~ will reached to (2,1).

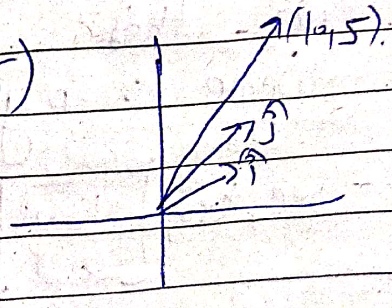


- for (3,2), $\hat{j}$ will go from current (0,1) to (3,2).



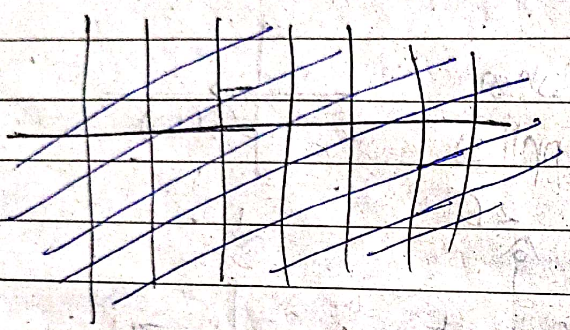
- Now, suppose you have $\begin{bmatrix}2\\2\end{bmatrix}$ for this, where it then in this 2D space will go. We'll multiply it with matrix

$$\begin{bmatrix}\boxed{2 \quad 3}\\ \boxed{1 \quad 2}\end{bmatrix}\begin{bmatrix}\boxed{\begin{matrix}2\\2\end{matrix}}\end{bmatrix} = (10,5)$$



- The whole point of this discussion is when you apply a matrix on a co-ordinate system it will basically transform the entire co-ordinate system.

- for visualization, key drawing

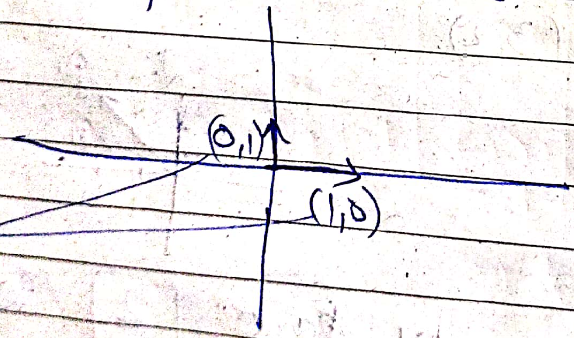


$$\begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

<u>Transformation.</u>

- See tool, for more understanding

- Suppose your matrix is identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ & this is your co-ordinate system



<u>Basis vector</u>

- When we are applying this matrix on this co-ordinate space, then where my basis vector goes.

- See, for $\hat{i}$, $\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{2\times2} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{2\times1} = [1, 0]$

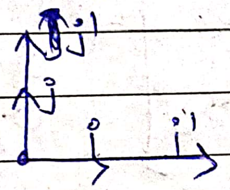
so $\hat{i}$ not change

- See, for $\hat{j}$, $\underset{2\times2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} [0,1]$

$\hat{j}$ will not change.

- Your x-axis & y-axis doesn't change after transformation means that is Identity matrix.
- No transformation happens by identity matrix.

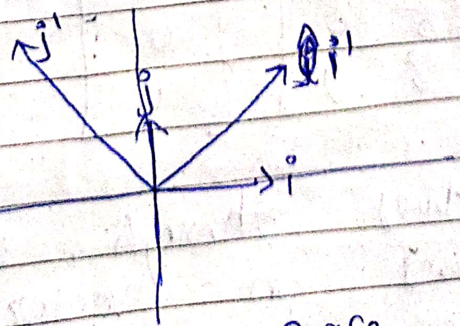
- Now, for matrix $\begin{bmatrix} \boxed{\begin{matrix} 2 \\ 0 \end{matrix}} & \begin{matrix} 0 \\ 2 \end{matrix} \end{bmatrix}$, your $\hat{i}$ & $\hat{j}$ vector will expand by 2 times in graph.



- Now, for $\begin{bmatrix} 2 & -2 \\ 2 & 4 \end{bmatrix}$ it will

- for $i = \begin{bmatrix} 2 & -2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = [2,2]$

for $j = \begin{bmatrix} 2 & -2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [-2,4]$

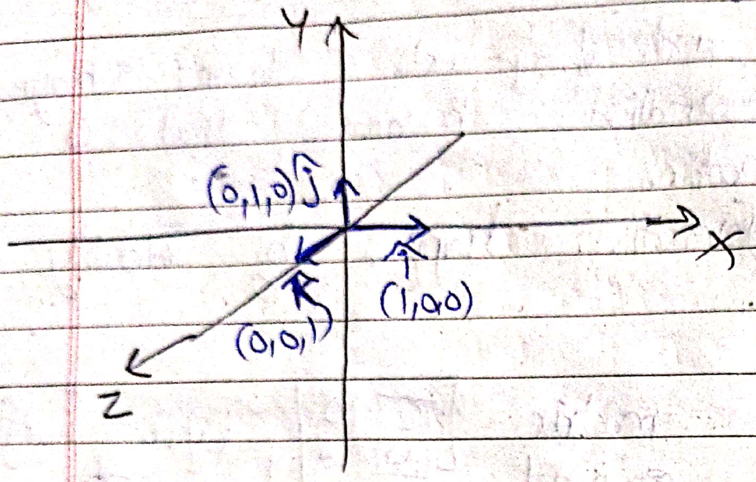


- new vector space

* Linear Transformation in 3D *

In 3D space,

(imagine there are grid lines in 3D)



for i,
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$= [1, 4, 7]$$

for j,
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = [2, 5, 8]$$

for k,
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = [3, 6, 9]$$

- By matrices through, we're applying transformations on vector spaces / co-ordinates by which transformations happen.
- There are diff types of transformation.

① Why linear transformation says?

∘ **Remember two things** –

1) While transformation, origin doesn't change.

2) After transformation, grid lines will parallel to each other not intersecting each other.

These two only true | happen when we're using linear transformation not applying like fn like square, log etc

Till now we understand matrices in visualization.

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* Matrix Multiplication as Composition *

$$\overset{A}{\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}} \cdot \overset{B}{\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}}$$

$2\times\boxed{2}$ c   r $\boxed{2}\times 2$

$$\begin{bmatrix}- & -\\- & -\end{bmatrix}$$

2×2

– As we know, matrix is transformation.

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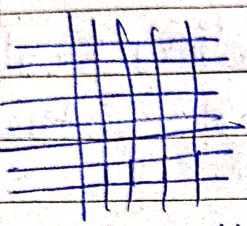
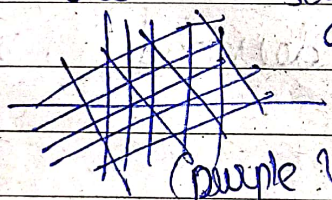
- By multiplying these two matrices we are getting matrix ~~and~~ but those two matrices are also transformations.

- It means, if you had vector space



- When you're doing $\boxed{A.B}$ on vector space then some transformation will happen then that transformation's geometric meaning is like that.

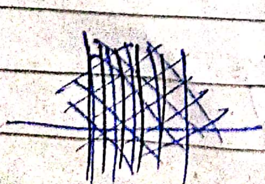
- first B transformation is applying on vector space and after that let's say it became like this



(purple wala co-ordinate system).

Then you're applying A transformation on that.

- In original red (color) → B
↓
purple (color) → A → Green.



- In short, $\boxed{A.B}$ - red → A.B → green.

* Test of Commutative law *

1. ◦ $A.B \neq B.A$, matrix multiplication is not commutative. **Order Matters.**

- Suppose, you have one co-ordinate space  let's called it C.

- When we say $A.B$, then we're doing

$C \rightarrow B \rightarrow C' \rightarrow A \rightarrow C''$

~~if we~~

- even if we do, $C \rightarrow A \rightarrow C' \rightarrow B \rightarrow C''$

After transformations, these two C'' are not same, they'll be different.

Note → Sometimes it could be same but can't guarantee that.

2. **Small Assignment** → Why associative is true? $A(BC) = (AB)C$ for matrix multiplication

* Determinants *

Recap → ◦ determinant is a Scalar value computed from a Square matrix that carries important information about the matrix.

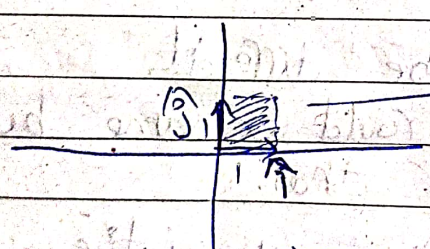
◦ Has several uses but one of them is calculating the volume scaling factor for linear transformations.

◦ 1 X 1   2 X 2   3 X 3

$$\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10 - 12 = \boxed{-2}$$

← Scalar no.

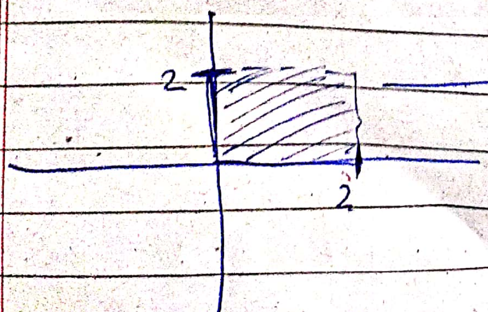
◦ Suppose you have below co-ordinate space (c) system & have $\hat{i}$ & $\hat{j}$ unit vector



→ area of this is 1.

$\boxed{C}$

– applying $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ transformation,



→ Total area of this → 4. (2x2)

$\boxed{C'}$

• $\det(A) = \frac{\text{area}'}{\text{area}} = \left|\frac{4}{1}\right| = \boxed{4}$

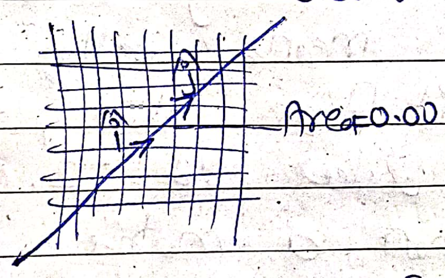
This is scalar no.

- determinants tells essentially that in 2D space when you apply matrix transformation then area's how much bigger it would be.

- In our case, 4 times bigger. It tells the ratio of streching / squeezing.

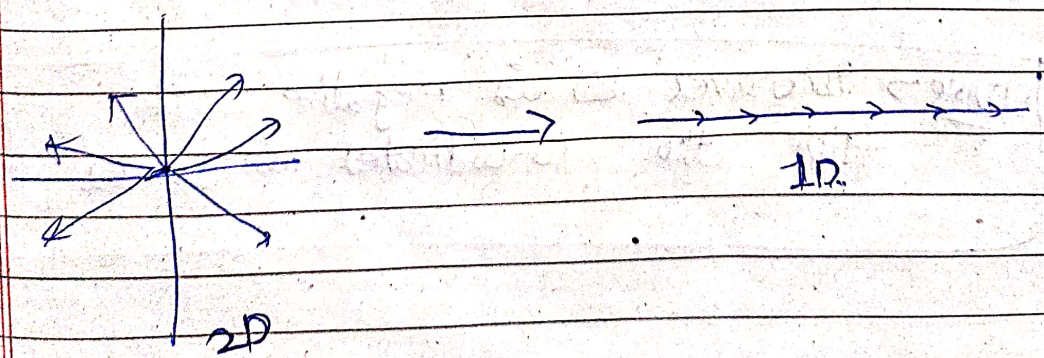
• $\begin{vmatrix} 2 & 4 \\ 2 & 4 \end{vmatrix} = 8 - 8 = 0.$

• What zero looks like?

→ 

• Means, in this transformation, you're all points in 2D space comes into 1 line.

• 2D space to number line (1D space).



Recap → if Δ is zero then you can't calculate inverse.

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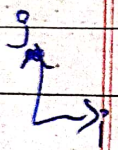
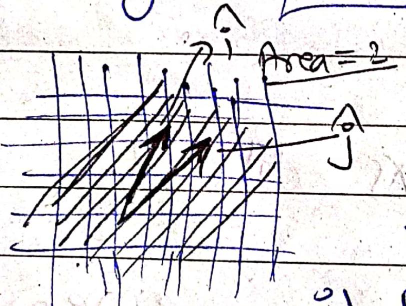
Q.1] Why determinant is possible only for Square matrix?

→ The interpretation of the determinant as a scaling factor is only meaningful for square matrices because the input & output spaces must have the same dimensions for this concept to be applicable.

Q2] What does it mean to have a negative determinant?

→ eg → $\boxed{-2}$ for $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = 10-12 = \boxed{-2}$



→ 

if we see, î & ĵ position changes after transformation means scaling happens but it flips also, that why scalar is -2. (negative)

Q3] What happens when a matrix is singular?

⇒ $\Delta = 0$, means 2D → 1D.

Note → whenever Δ is negative, it flips the co-ordinates in spaces

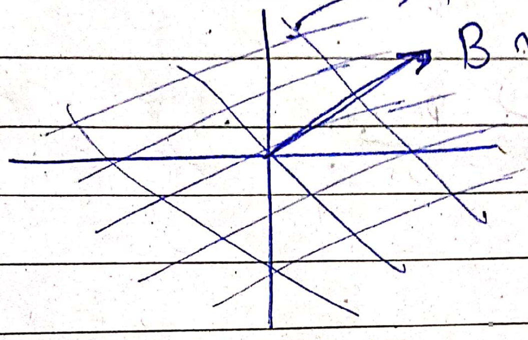
* Inverse *

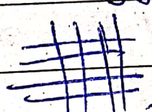
- $\boxed{A \rightarrow A^{-1}}$

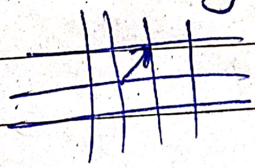
- Inverse is reverting the transformation.

- eg → We know, $\boxed{AX = B}$

→ A transformation. (Red)
→ B vector (5,6). (purple).



We have (A) Red transformation & vector (5,6). now we have to revert the transformation so that we become normal/ blue one  . Want to know that where B will go after reverting. And where B will go that will be our X.

 .

eg →

$$\begin{array}{ccc} A & X & & B \\ \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} & = & \begin{bmatrix} 5 \\ 6 \end{bmatrix} \end{array}$$

Q.1) Why $A \cdot A^{-1} = I$?

Q.2) Why inverse is possible for square matrix only?

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* Transformation for non-square matrix *

* ~~Why~~ Why only square matrix has inverse.

* Why inverse is possible for non-singular matrices only.

Just see pdf for this.

peace