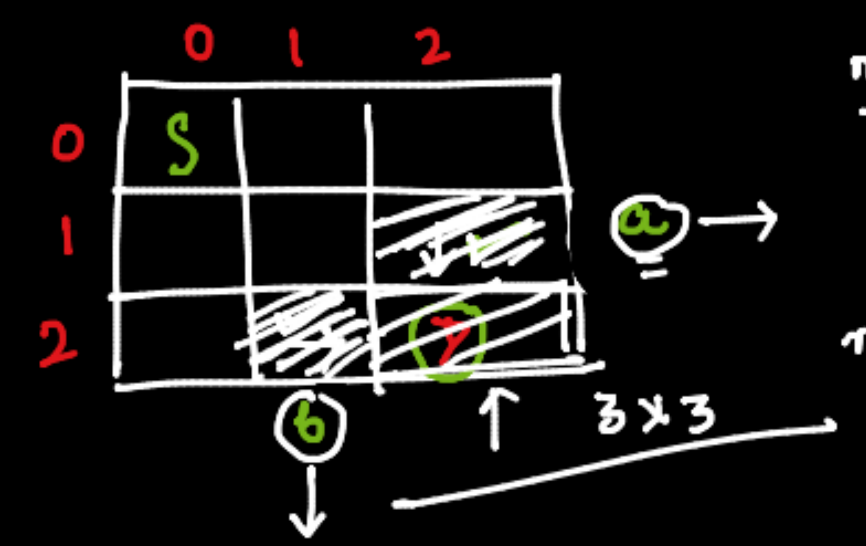


(x, y)

$\min(a, b) + $ value at D

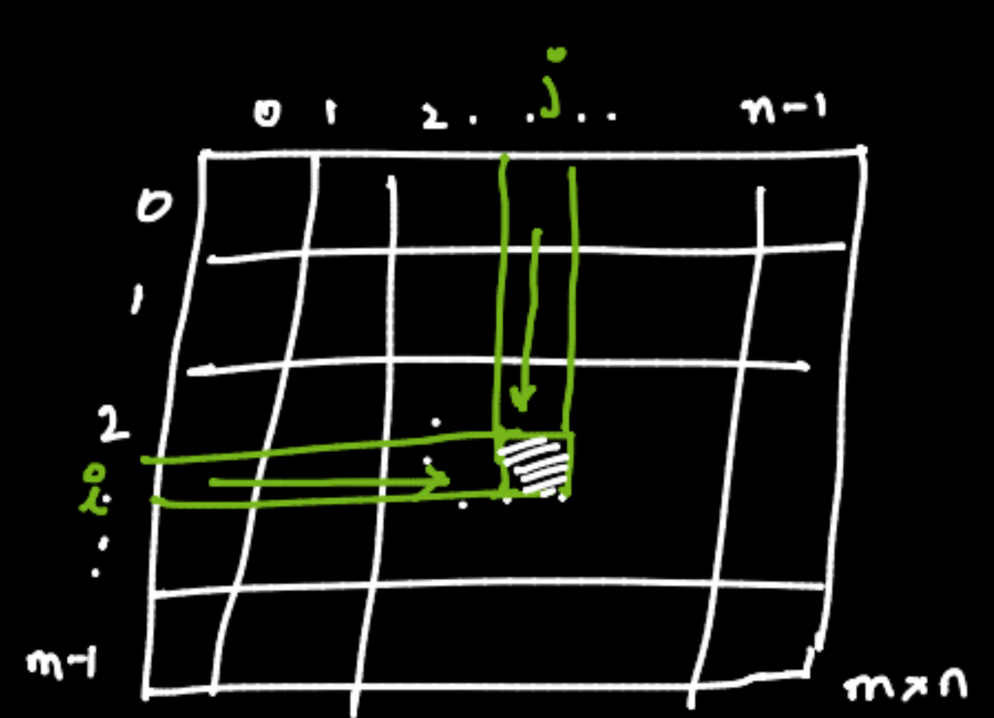
recursion

⇒ min sum path that takes you from S to D is same as the min. sum path through which you reach D from S

$f(i, j) \rightarrow$ denote the min sum path through which you can reach (i, j) from $(0, 0)$

$$f(i, j) = \underbrace{\text{grid}[i][j]}_{\text{value at the dest'n cell}} + \min(f(i-1, j), f(i, j-1))$$

$f(0, 0) = \text{grid}[0][0]$ $\quad i = 0$ and $j = 0$



DP ?

Recursion

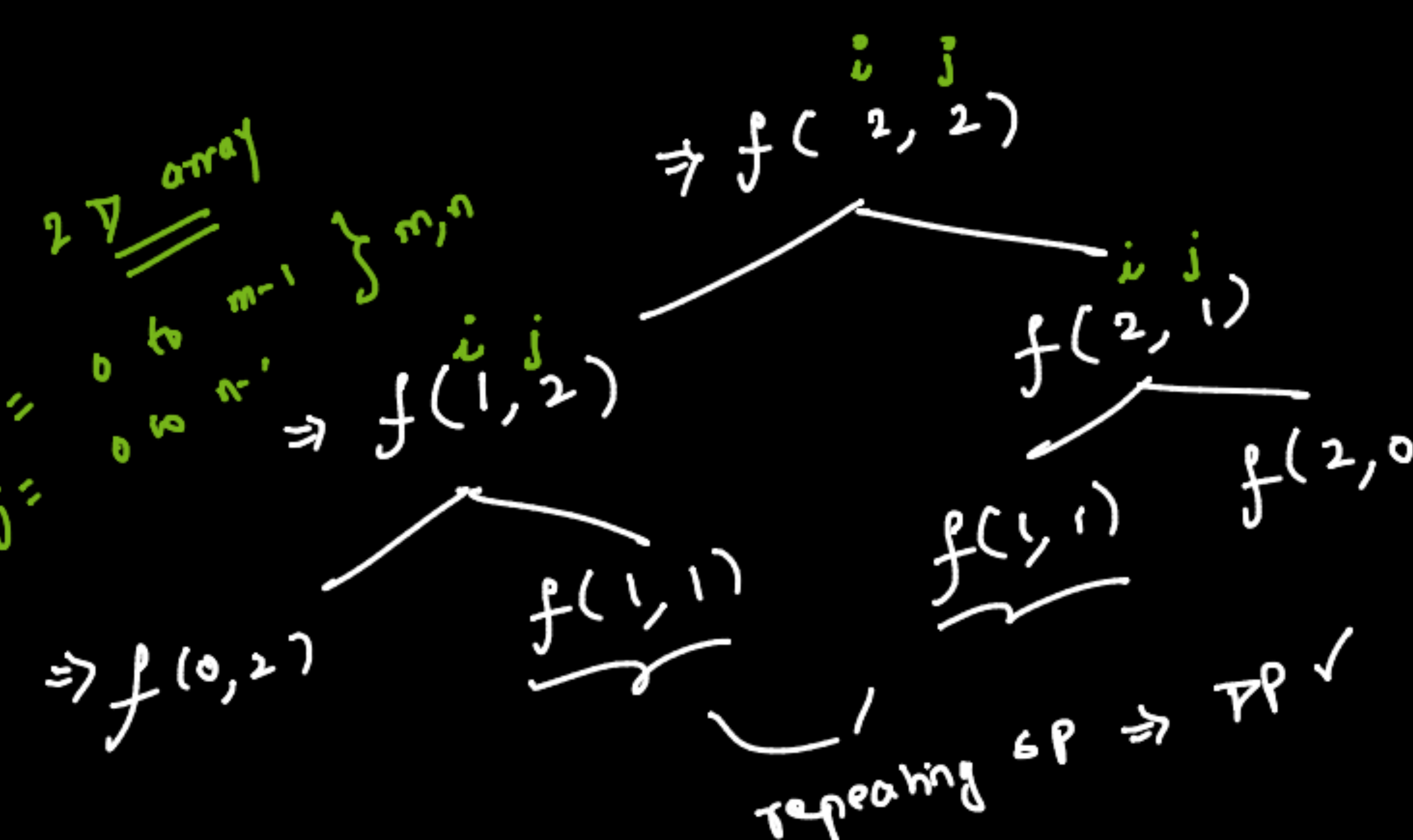
⇒ $m, n = 3 \times 3$

$x, y = (2, 2)$

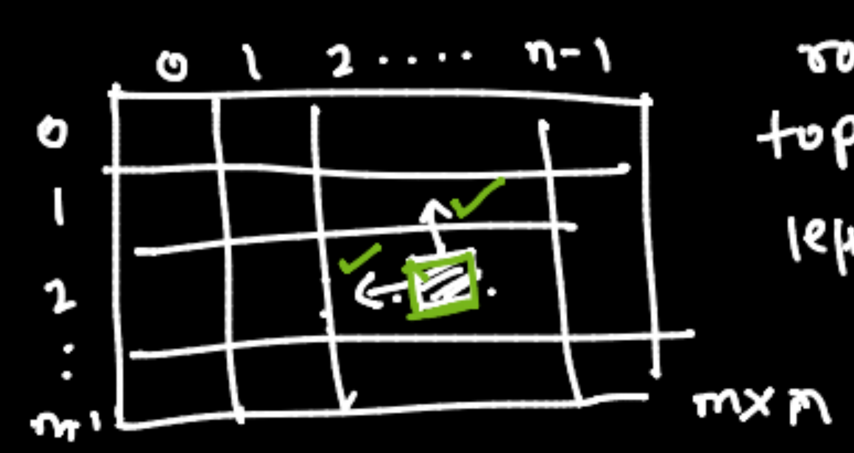
⇒ 2D array } m,n

⇒ $i =$ 0 to m-1

$j =$ 0 to n-1



Bottom-up



⇒ Evaluation Order

row-wise

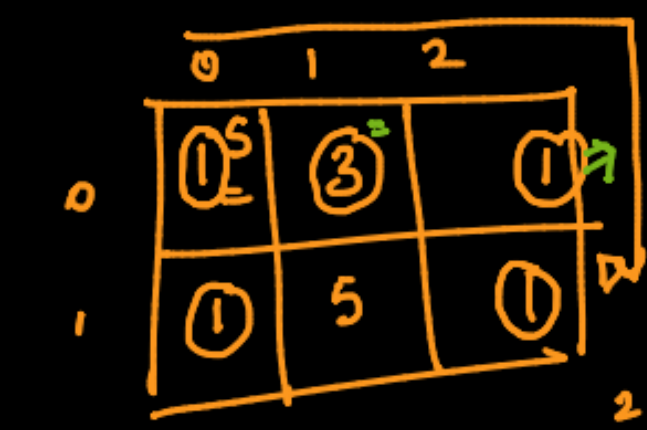
top-down

left-right each row

$$\Rightarrow f(i, j) = grid[i][j] + \min(f(i-1, j),\ f(i, j-1))$$

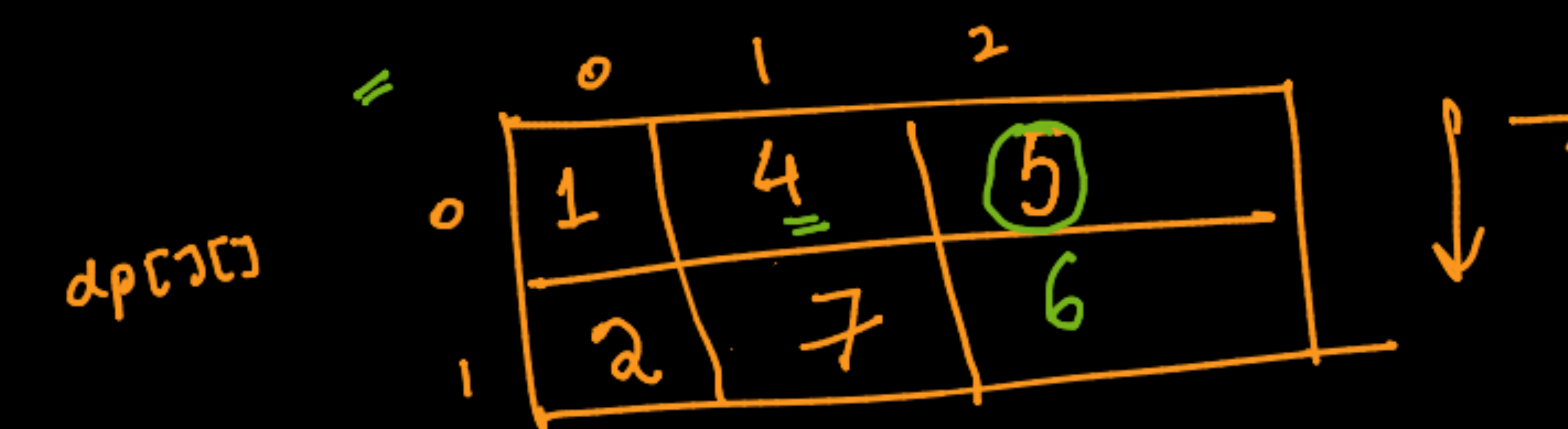
Dependencies

$$\Rightarrow dp[i][j] = grid[i][j] + \min(dp[i-1][j],\ dp[i][j-1])$$



$\Rightarrow i == 0 \Rightarrow grid[i][j] + dp[i][j-1]$

$= j == 0 \Rightarrow grid[i][j] + dp[i-1][j]$



$$dp[i][j] = g[i][j] + \min(5, 7)$$

2

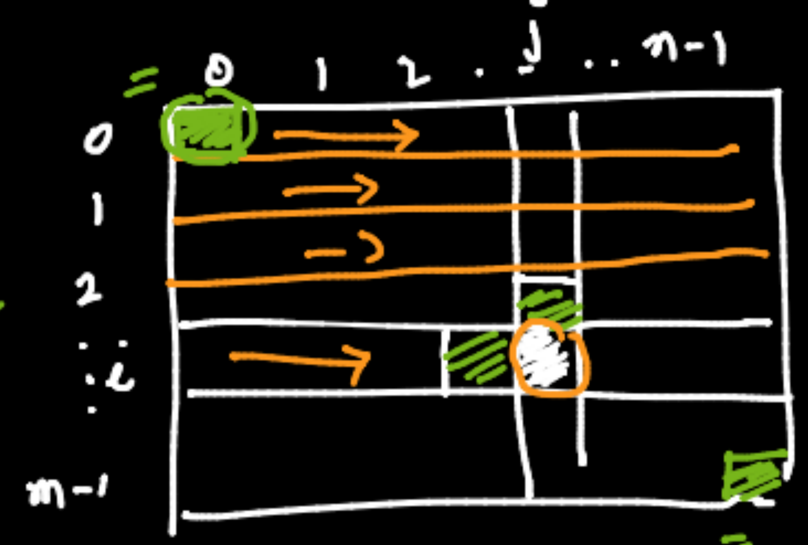
Dependency

$$f(i, j) = grid[i][j] + \min(f(i-1, j),\ f(i, j-1))$$

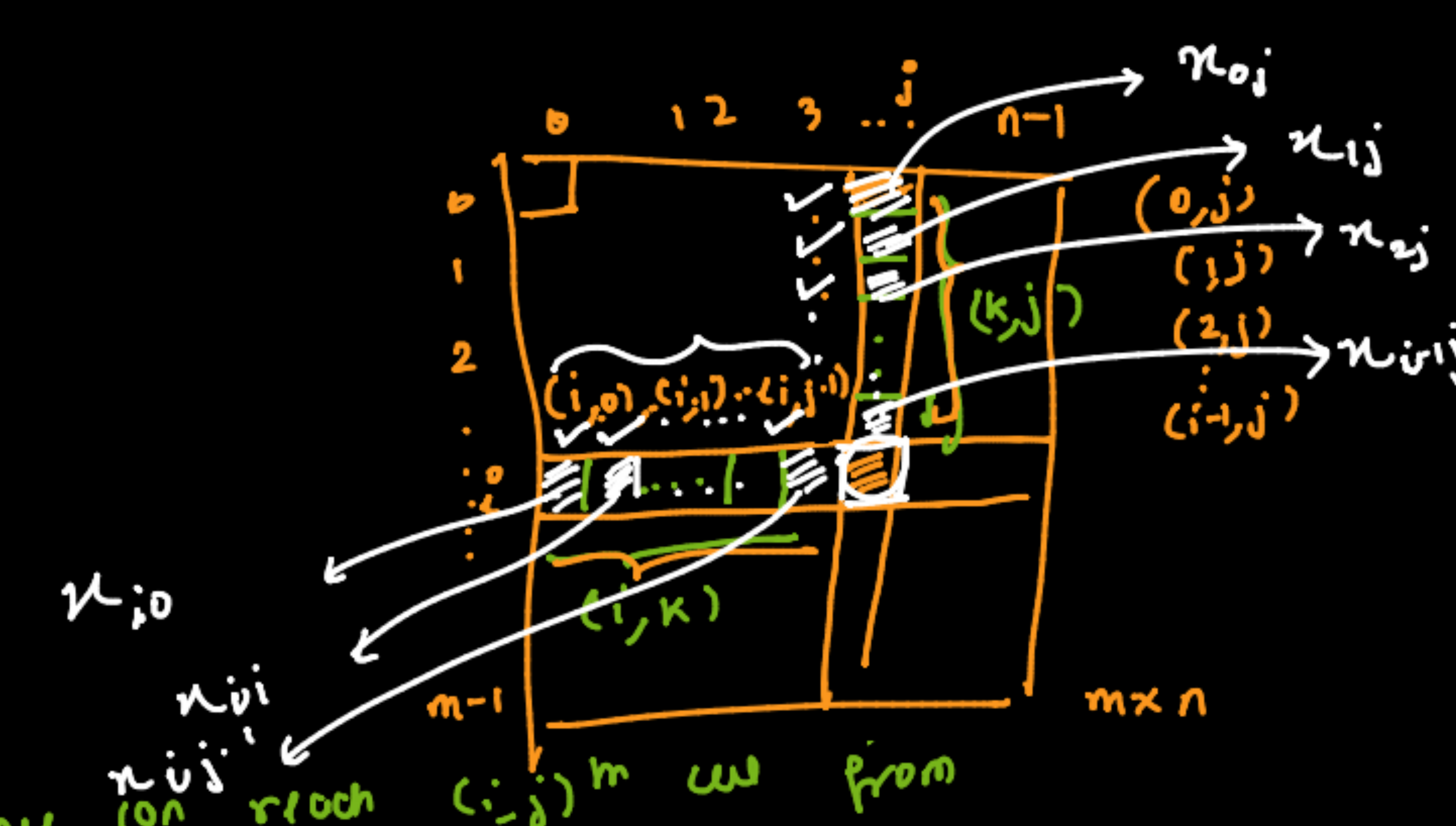
$$\Rightarrow dp[i][j] = grid[i][j] + \min(\boxed{dp[i-1][j]},\ dp[i][j-1])$$

Evaluation Order

(x, y)



row-wise top to down, and each row left-right



You can reach (i, j) from

$\Rightarrow (k, j)$ st $0 \le k < i$

$\Rightarrow (i, k)$ st $0 \le k < j$

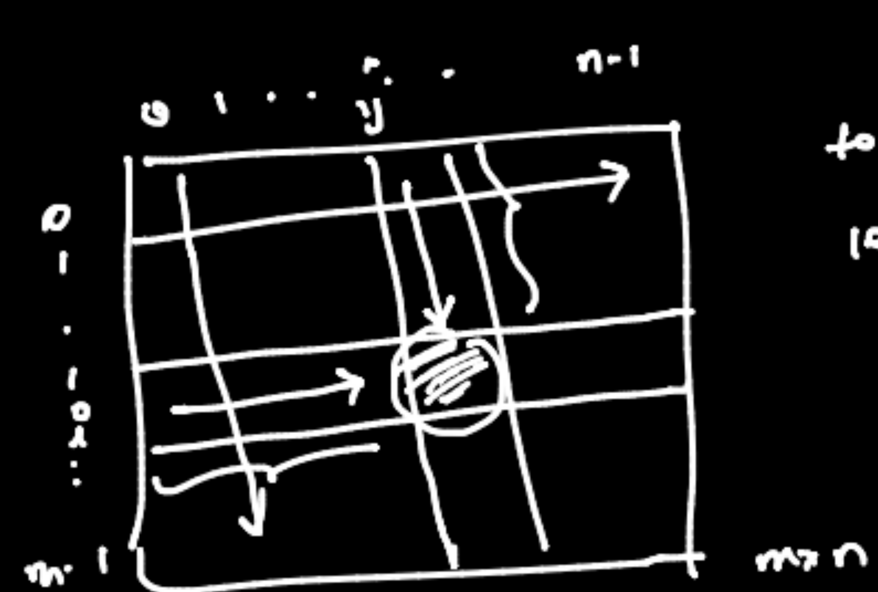
$$\left[\sum_{k=0}^{i-1} x_{kj} + \sum_{k=0}^{j-1} x_{ik}\right]$$

$f(i,j)$ denotes # ways in which you can reach the (i,j)th cell from $(0,0)$

$$f(i,j) = \sum_{k=0}^{j-1} f(i,k) + \sum_{k=0}^{i-1} f(k,j)$$

Base case

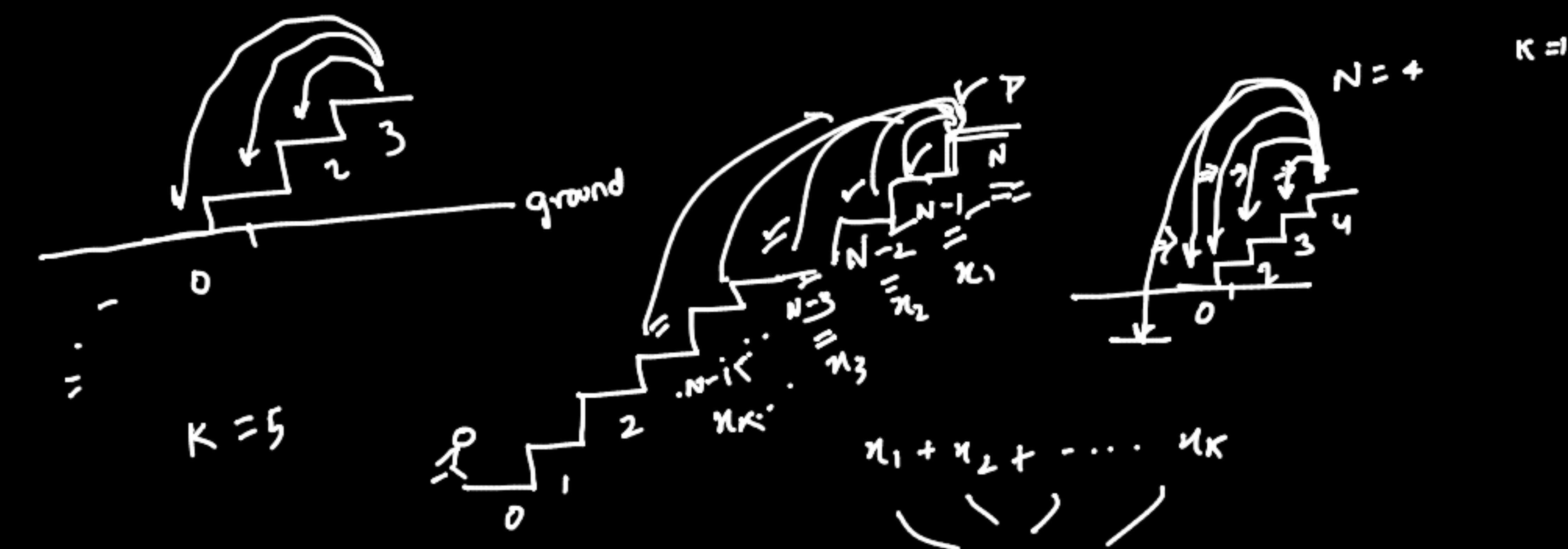
$f(0,0) = 1$ $i=0$ and $j=0$



Evaluation order

top-down

left-right



K=5

K=10

$x_1 + x_2 + \dots x_K$

recursion

$f(n)$ denotes the # ways I can reach the top of ladder

$$f(n) = \sum_{i=1}^{K} f(n-i) \mid (n-i) \geq 0$$

$f(0) = 1$ ← Base $n=0$

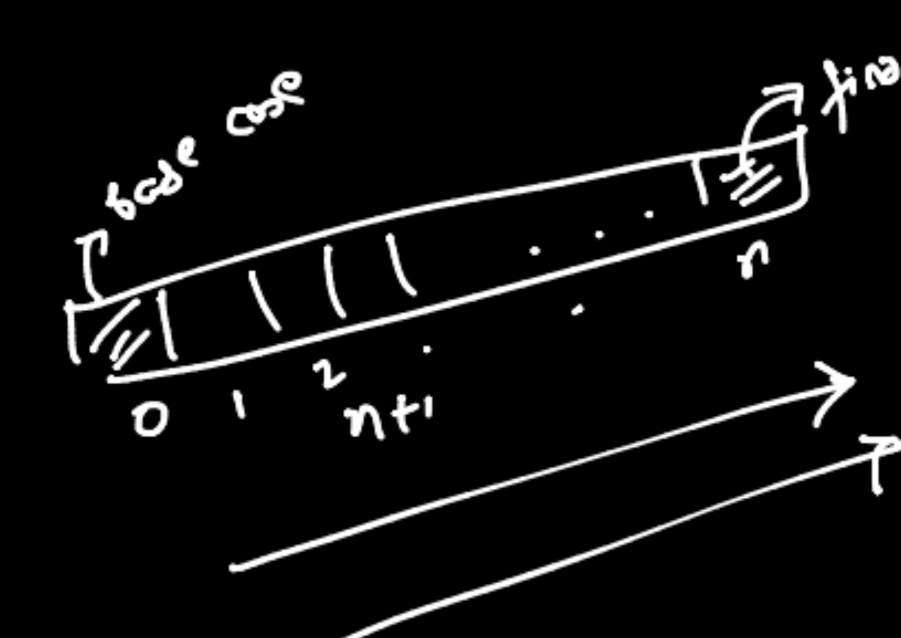
$f(1) = 1$

$f(1)=1$ K=3

$f(2)=2$ K=3

$$f(2) = \sum_{i=1}^{3} f(n-i) = f(1) + f(0) + \cancel{f(-1)} = 1 + 1 = 2$$

$f(n) = \sum f(n-i)$



$$f(n) = \sum_{j=1}^{k} f(n-j)$$

$$\Downarrow$$

$$\Rightarrow dp[i] = \sum_{j=1}^{k} dp[i-j]$$

$$\Rightarrow dp[i] = \underbrace{dp[i-1] + dp[i-2] + \dots + dp[i-k+1]} + dp[i-k]$$

$$dp(i) - dp(i-k)$$

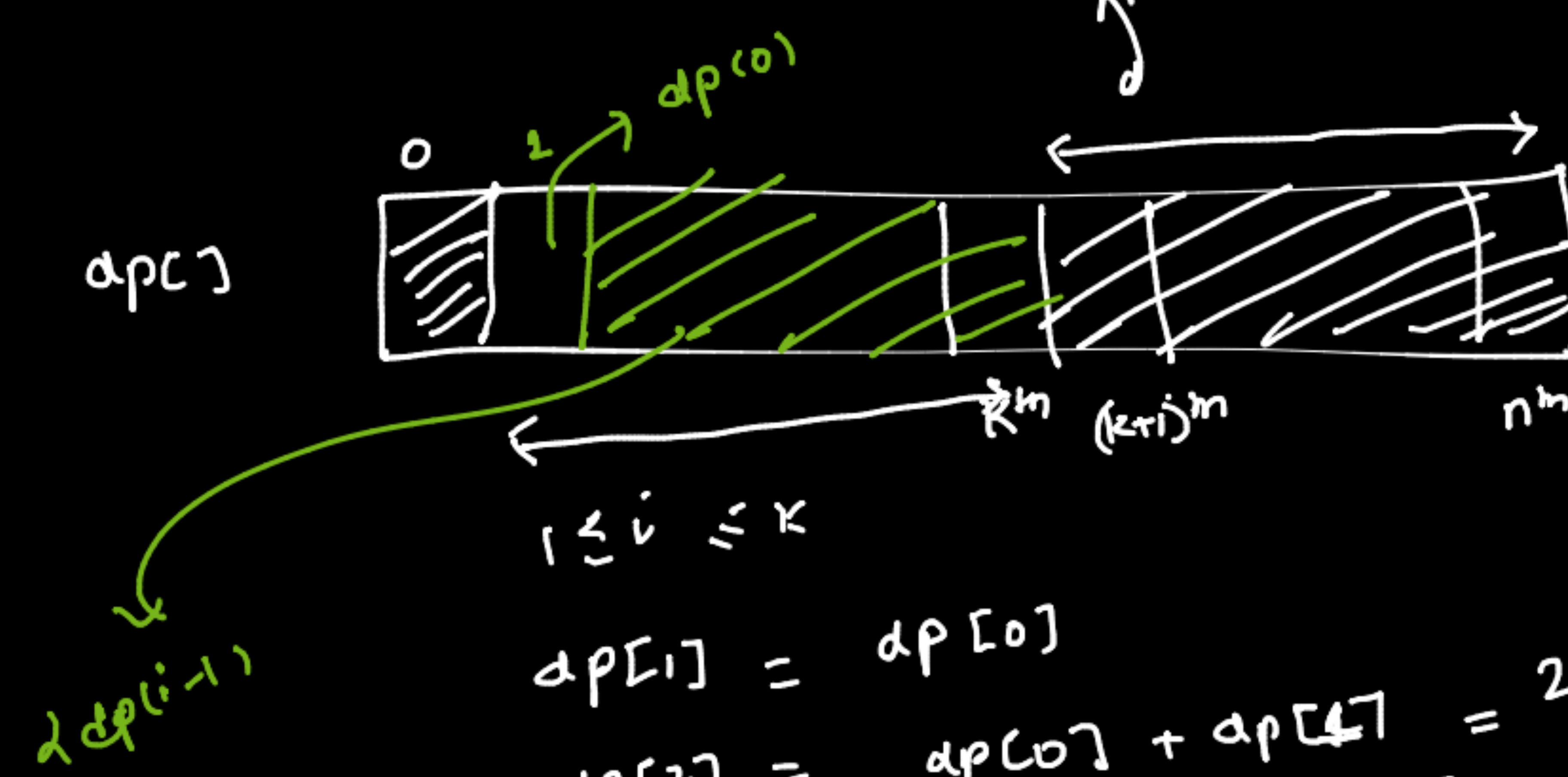
$$\Rightarrow dp[i+1] = dp[i] + \underbrace{dp[i-1] + \dots + dp[i-k+1]}_{dp[i] - dp[i-k]}$$

$$= dp[i] + dp[i] - dp[i-k]$$

$$\Rightarrow dp[i+1] = 2dp[i] - dp[i-k]$$

$$\Rightarrow dp[i] = 2dp[i-1] - dp[i-k-1]$$

$$\left[\, dp[i] = 2dp[i-1] - dp[i-k-1] \,\right]$$



$$dp[1] = dp[0]$$

$$dp[2] = dp[0] + dp[1] = 2dp[1]$$

$$dp[3] = dp(0) + dp(1) + dp(2) = 2dp(2)$$

$$dp(4) = dp(0) + dp(1) + dp(2) + dp(3) = 2*dp(3)$$

$$\vdots$$

$$dp(k) = dp(0) + \dots + dp(k-1) = 2*dp(k-1)$$

$$\Rightarrow dp[i] = 2dp[i-1] \; ; \; 2 \le i \le k$$