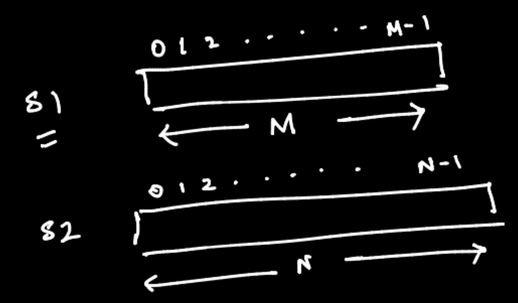
↳ S_1 and S_2 → length of LCS M ≠ N



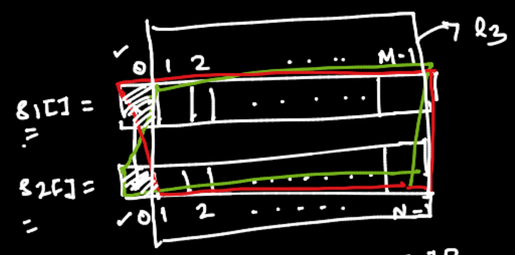
$\Rightarrow$ time $= 2^M \cdot M \sim O(2^M \cdot M)$

Let $f(i, j)$ denote the length of the LCS b/w $S_1[i \ldots M-1]$ and $S_2[j \ldots N-1]$ then

$$f(i,j) = \begin{cases} \max\left(f(i+1,j),\ f(i,j+1)\right) & ; S_1[i] \neq S_2[j] \\ 1 + f(i+1, j+1) & \text{otherwise} \end{cases}$$

$f(M, N)$ or $f(M, j)$ or $f(i, N)$ = 0

$i = M$ or $j = N$



$S_1[0] == S_2[0]$?

match → $1 + LCS(S_1[1 \ldots M-1], S_2[1 \ldots N-1])$

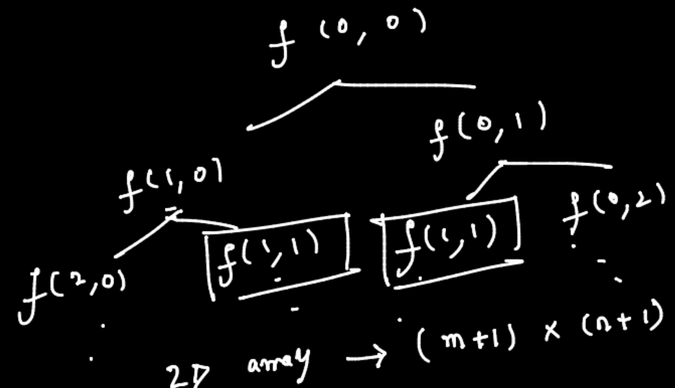
do not match → $LCS(S_1, S_2) = LCS(S_1, S_2[1 \ldots N-1])$ or $LCS(S_1[1 \ldots M-1], S_2)$

$= \max(l_1, l_2)$

DP?

S_1 = A B C D (0 1 2 3)

S_2 = W X Y Z (0 1 2 3)

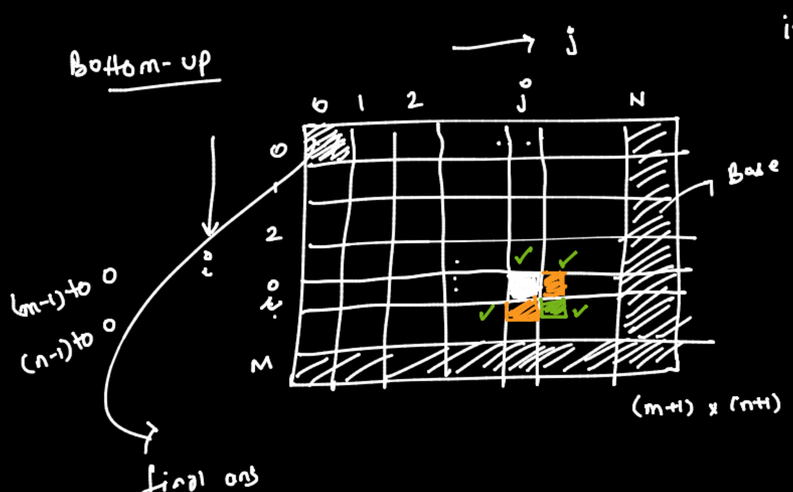


2D array → (m+1) × (n+1)

↳ i and j

$0 \leq i \leq m$ and $0 \leq j \leq n$

Bottom-up



$$f(i,j) = \max\begin{cases} f(i+1,j) \\ f(i,j+1) \\ \text{or} \\ f(i+1,j+1)+1 \end{cases}$$

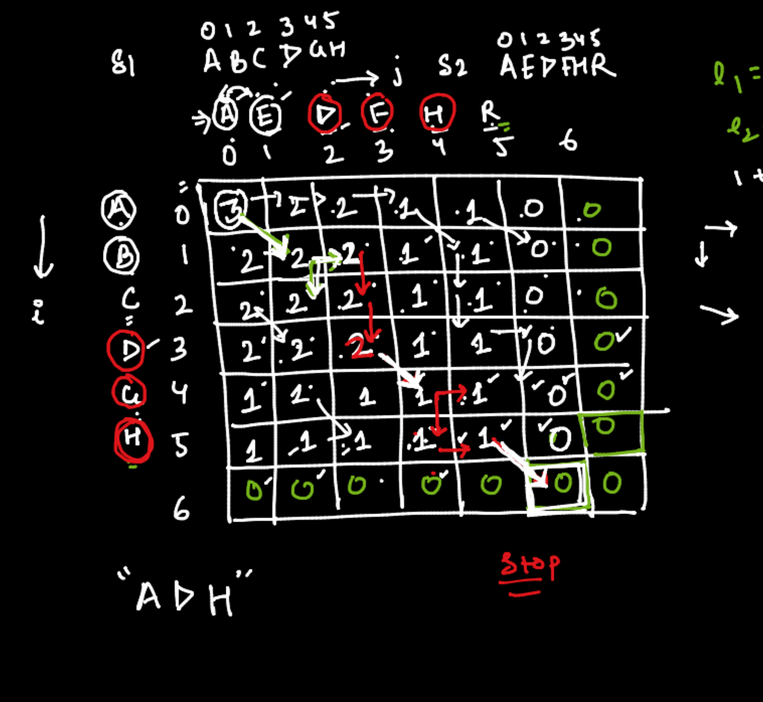
$(i,j) \rightarrow \begin{cases} i+1,j \\ i,j+1 \\ i+1,j+1 \end{cases}$

Evaluation order

row-wise bottom to top

and each row

right → left



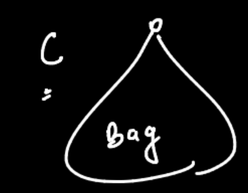
$l_1 = dp(i+1)(j)$

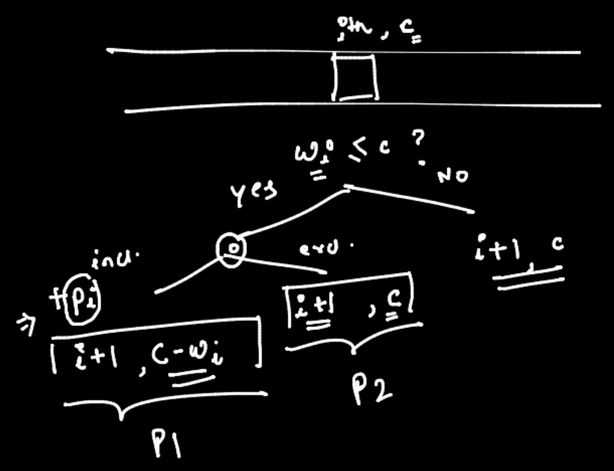
$l_2 = dp(i)(j+1)$

$1 + l_3 \Rightarrow dp(i+1)(j+1)$

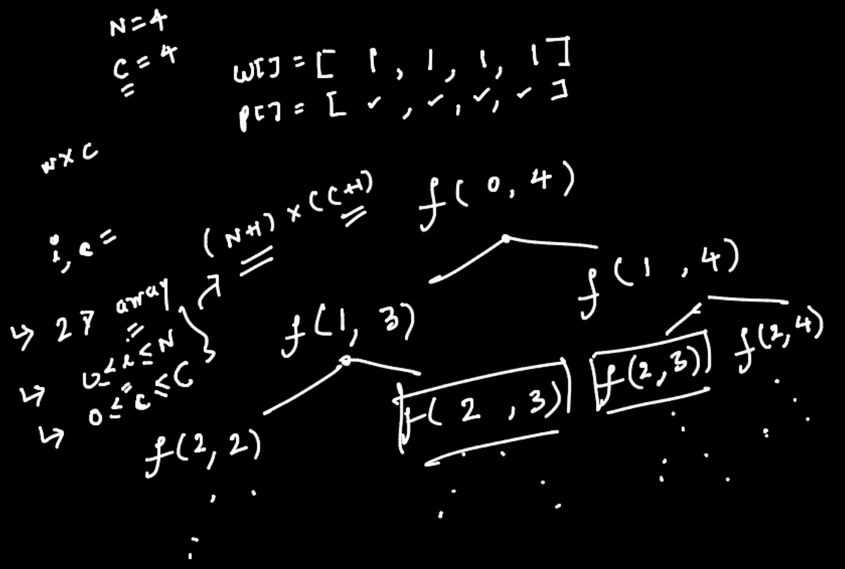
0/1 Knapsack

$N \rightarrow \begin{cases} p[\,] = \{ p_0, p_1, \ldots p_{N-1} \} \\ w[\,] = \{ w_0, w_1, \ldots w_{N-1} \} \end{cases}$





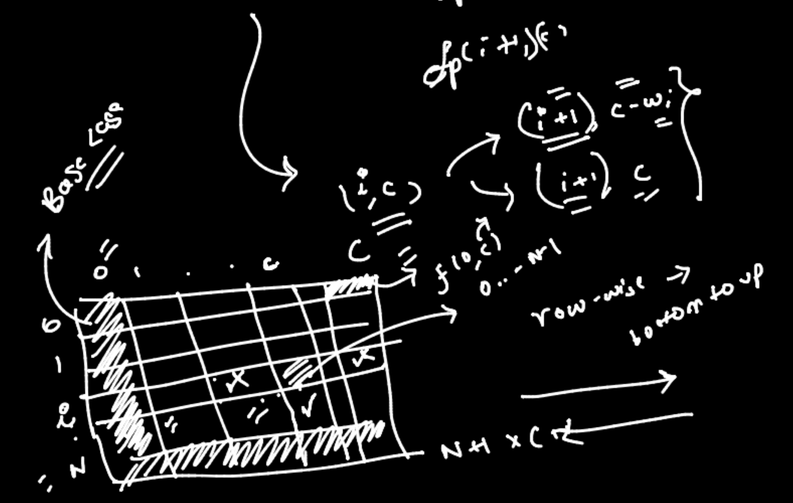
$\min(p_i + P_1, P_2)$

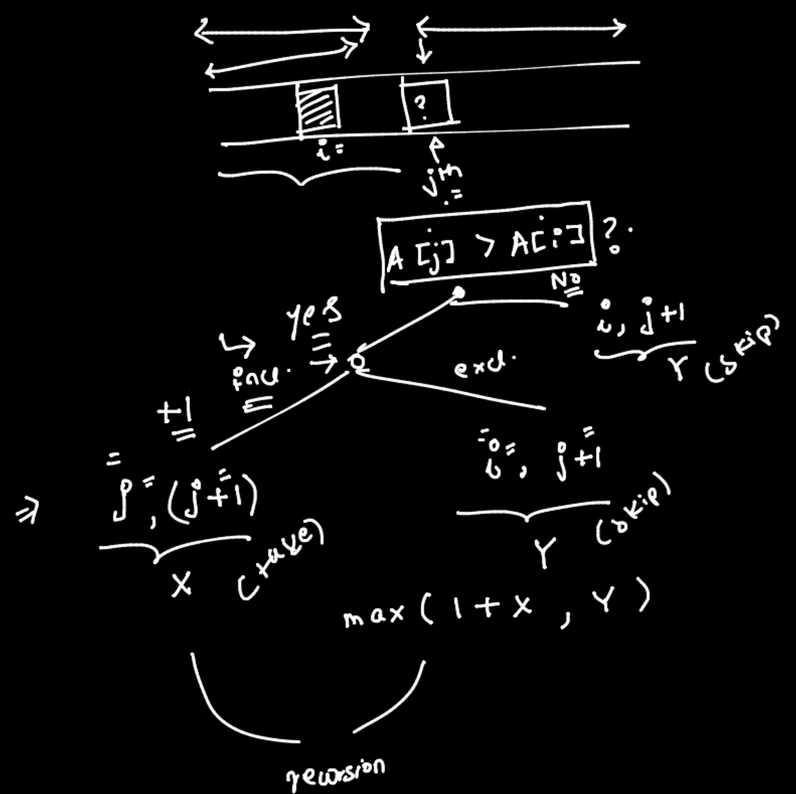


$f(i, c)$ = denotes the max profit you can make by choosing items $[i \ldots n-1]$ s.t. current KS capacity is c

$$f(i,c) = \max \begin{cases} p_i + f(i+1, c-w_i) & ; \ w_i \le c \\ f(i+1, c) & ; \ w_i > c \\ f(i+1, c) = 0 & ; \ i = N \text{ or } c = 0 \end{cases}$$

$$dp(i)(c) = \max \begin{cases} p_i + dp(i+1)(c-w_i) & w_i \le c \\ dp(i+1)(c) \end{cases}$$





$f(i,j)$ → denote the length of LIS in $A[j \ldots N]$ st. every element in LIS of $A[j \ldots N] > A[i]$

$i < j$

$A[i \ldots N][j]$

$$f(i,j) = \max \begin{cases} 1 + f(j, j+1) & : A[j] > A[i] \\ f(i, j+1) & : \text{otherwise} \end{cases}$$

$f(i, j+1)$: $j > N$

0

$f(0, 1)$

↳ find the length of LIS of $A[1 \ldots N]$ (st every element is $>$ $A[0]$)

$A[1 \ldots N]$

⇒

"INT_MIN" → sentinel