

## Assignment 6 Q56 (june 2018)

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56) To test the equality of effects of 10 schools against all alternatives, we take a random sample of 5 students from each school and note their marks in a common examination. "Between sum of squares" and "total sum of squares" are found to be 180 and 500 respectively. What is the p-value for the standard F-test?

**Ans**

Let  $X$  be  $n \times m$  matrix where  $n$  is number of students from each school and  $m$  is total number of schools.

Let  $\bar{X}$  be  $m \times 1$  column vector of mean marks of students of a particular school.

Let  $\bar{\bar{X}}$  be mean marks of all students.

$$\Rightarrow \bar{X} = (X_{m \times n}^T 1_{n \times 1})/n \quad (1)$$

$$\begin{aligned} \Rightarrow \bar{\bar{X}} &= (1_{1 \times m}^T \bar{X}_{m \times 1})/m \\ &= (1_{1 \times m}^T X_{m \times n}^T 1_{n \times 1})/mn \end{aligned} \quad (2)$$

The **Total sum of squares (SST)** is defined as -

$$\begin{aligned} SST &= Tr(X^T X) + mn(\bar{\bar{X}})^2 - 2\bar{\bar{X}}[1_{1 \times n}^T X_{n \times m} 1_{m \times 1}] \\ &= Tr(X^T X) - \frac{(1^T X^T 1)(1^T X^T 1)}{mn} \end{aligned} \quad (3)$$

The **Within sum of squares (SSW)** is defined as -

$$\begin{aligned} SSW &= Tr(X^T X) + n\bar{X}^T \bar{X} - 2[1_{1 \times n}^T X_{n \times m} \bar{X}_{m \times 1}] \\ &= Tr(X^T X) - \frac{1^T X X^T 1}{n} \end{aligned} \quad (4)$$

The **Between sum of squares (SSB)** is defined as -

$$\begin{aligned}
 SSB &= n[\bar{X}^T \bar{X} + m(\bar{\bar{X}})^2 - 2\bar{\bar{X}}(1_{1 \times m}^T \bar{X}_{m \times 1})] \\
 &= \frac{1^T X X^T 1}{n} - \frac{(1^T X^T 1)(1^T X^T 1)}{mn}
 \end{aligned} \tag{5}$$

Add equations 4 and 5-

$$\Rightarrow SSB + SSW = SST \tag{6}$$

Now, Let the null Hypothesis be  $H_0$  = All the schools are same (true mean of marks are same). We conduct F-test to verify this.

$$\begin{aligned}
 F &= \frac{Var_B}{Var_W} \\
 &= \frac{SSB/dof_b}{SSW/dof_w} \\
 &= \frac{SSB/(m-1)}{SSW/(m(n-1))}
 \end{aligned} \tag{7}$$

Given  $SSB = 180$  and  $SST = 500$ . From equation (6),  $SSB = 320$ . From question,  $m=10$  and  $n=5$

$$\begin{aligned}
 \Rightarrow F &= \frac{SSB/9}{SSW/40} \\
 &= \frac{\chi_9^2}{\chi_{40}^2} = \frac{180/9}{320/40} \\
 &= 2.5
 \end{aligned} \tag{8}$$

Therefore, p-value is -

$$P[F_{9,40} \geq 2.5] \tag{9}$$