Assignment 5 Q53 (june 2018)

Abhishek Ajit Sabnis

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53) Suppose that the lifetime of an electric bulb follows an exponential distribution with mean θ hours. In order to estimate θ, n bulbs are switched on at the same time. After t hours, n-m(>0) bulbs are found in functioning state. If the lifetimes of other m(>0) bulbs are noted as $x_1, x_2, ..., x_m$ respectively, then the maximum likelihood estimate of θ is given by

Ans
Probability distribution function of Exponential distribution is given by -

$$f_X(x) = \lambda e^{-\lambda x} \tag{1}$$

Given mean is θ. We know

$$E(X) = \frac{1}{\lambda} = \theta \tag{2}$$

Substitute in eq(1) we get,

$$f_X(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \tag{3}$$

Likelihood function -

$$L(x_1, ...x_n) = \prod f_X(x_i) = (\frac{1}{\theta})^n exp(\frac{-1}{\theta} \sum_{i=1}^{n} x_i) \tag{4}$$

Take logarithm of the likelihood function -

$$log(L) = l = nlog(\frac{1}{\theta}) - \frac{1}{\theta} \sum_{i=1}^{n} x_i \tag{5}$$

Maximizing the log likelihood function-

$$\frac{dl}{d\theta} = \frac{-n}{\theta} + \frac{1}{\theta^2} \sum x_i = 0 \tag{6}$$

$$\Rightarrow \theta = \frac{\sum_{i=1}^{n} x_i}{n} \tag{7}$$

Given (n-m) bulbs are running with life of t hours and m bulbs have life $x_1, ..., x_m$

$$\therefore \theta = \frac{(n-m)t + \sum_{i=1}^{m} x_i}{n} \tag{8}$$

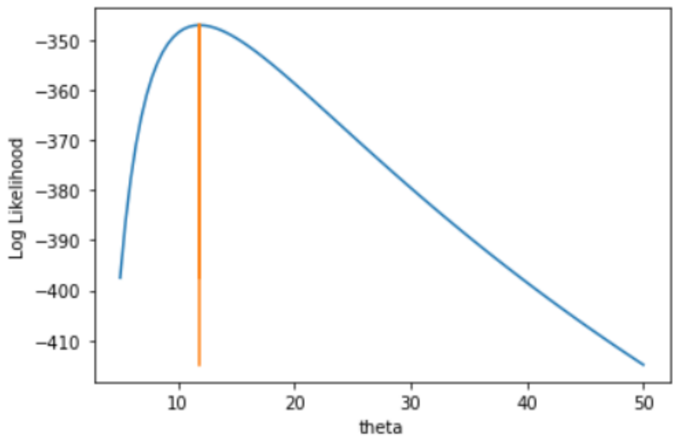

Figure 1: Theta vs Log Likelihood

The above graph is log likelihood function plotted for various theta. The following variables are chosen - n=100, m=45, t=15.

The graph has global maximum at theta calculated by equation(8)