Assignment 7 Q105 (dec 2017)

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53) Consider a markov chain with 5 states 1,2,3,4,5 and transition matrix P =

$$\begin{pmatrix} 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 1/7 & 0 & 0 & 6/7 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/3 & 0 & 0 & 2/3 & 0 \\ 0 & 5/8 & 0 & 0 & 3/8 \end{pmatrix} \tag{1}$$

Which of the following is true?
1. 3 and 1 communicating class
2. 1 and 4 communicating class
3. 4 and 2 communicating class
4. 2 and 5 communicating class

Ans

If i,j are states in Markov chain and i is accessible from j and j is accessible from i (written as $i \leftrightarrow j$), then we say i and j communicate. A communication class C $\subseteq$ S is a set of states whose members communicate,i.e. i $\leftrightarrow$ j for all i, j belongs to C, and no state in C communicates with any state not in C.

From the matrix we observe clearly that we can travel from 2 to 5 and vice versa. Its also true for 1 and 4. Thus 2,5 and 1,4 are communicating classes.

If the matrix is huge, we can solve this using Kosaraju's algorithm. It is implemented in Python.