Assignment 2

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50) Let X and Y be i.i.d random variables uniformly distributed on (0,4). Then $P(X > Y|X < 2Y)$ is

Ans According to Bayes Theorem, for any 2 random variable A and B,

$$P(A|B) = \frac{P(A, B)}{P(B)} \tag{1}$$

Thus,

$$P(X > Y|X < 2Y) = \frac{P((X > Y), (X < 2Y))}{P(X < 2Y)} \tag{2}$$

Joint Probability Density Function

Two random variables A, B are jointly continuous if there exists a function $f_{XY}(x, y)$ such that

$$P((X, Y) \in A) = \iint_A f_{XY}(x, y)dxdy \tag{3}$$

where (X,Y) $\in$ A is the area of interest.

We will approach this question using the above two concepts. For that, we need to find the joint probability density function of X and Y

Since X,Y $\sim$ Uniform(0,4),

$$F_X(x) = \begin{cases} 0 & \text{for x<0} \\ x/4 & \text{for } 0 \leq \text{x} \leq 4 \\ 1 & \text{for x>4} \end{cases} \tag{4}$$

$$F_Y(y) = \begin{cases} 0 & \text{for y<0} \\ y/4 & \text{for } 0 \leq \text{y} \leq 4 \\ 1 & \text{for y>4} \end{cases} \tag{5}$$

Given that X and Y are independent random variables,
$F_{XY}(x, y) = F_X(x)F_Y(y)$

$$F_{XY}(x,y) = \begin{cases} 0 & \text{for y<0 or x<0} \\ xy/16 & \text{for } 0 \leq \text{x} \leq 4,\ 0 \leq \text{y} \leq 4 \\ x/4 & \text{for } 0 \leq \text{x} \leq 4 \text{ , y>4} \\ y/4 & \text{for } 0 \leq \text{y} \leq 4 \text{ , x>4} \\ 1 & \text{for x>4, y>4} \end{cases} \tag{6}$$

To get joint probability density from joint cumulative function,

$$f_{XY}(x,y) = \frac{\partial^2 F_{XY}(x,y)}{\partial x \partial y} \tag{7}$$

$$\Rightarrow f_{XY}(x,y) = \begin{cases} 1/16 & \text{for } 0 \leq \text{x} \leq 4,\ 0 \leq \text{y} \leq 4 \end{cases} \tag{8}$$

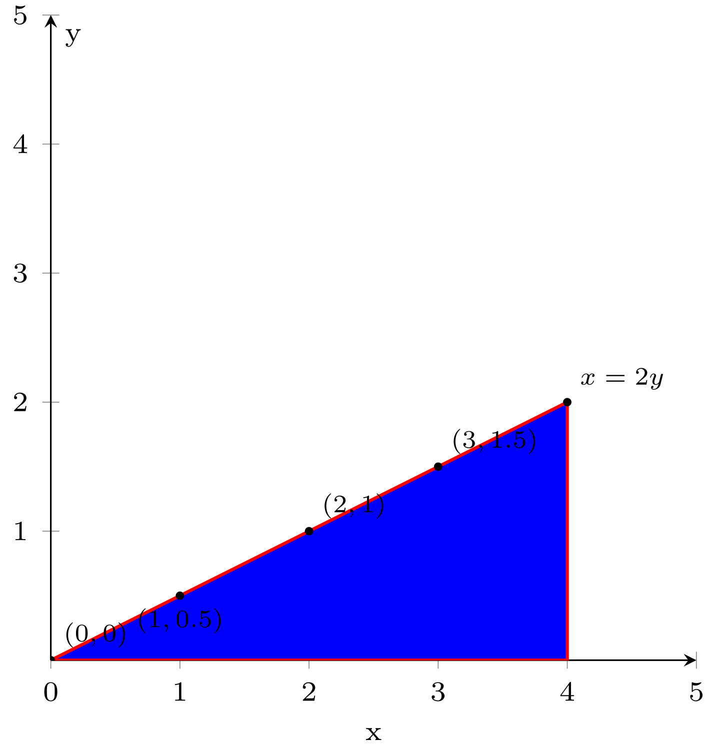
<u>Part a</u>: Finding P(X<2Y)

P(X<2Y) = 1 - P(X$\geq$ 2Y) = 1 - P(Y $\leq$ X/2)

$$\begin{aligned} P(Y \leq X/2) &= \iint_A f_{XY}(x,y)dxdy \\ &= \iint_A 1/16 dxdy \\ &= \int_0^4 \int_0^{x/2} \frac{1}{16} dydx \\ &= \int_0^4 \frac{x}{32} dx \\ &= \frac{4^2}{64} = \frac{1}{4} \end{aligned} \tag{9}$$

$$\Rightarrow P(X < 2Y) = 1 - \frac{1}{4} = \frac{3}{4} \tag{10}$$

Note: The graph below is line x=2y and the shaded area(2y<x) is taken for integration



<u>Part b</u>: Finding P(X>Y, X<2Y)

From the graph below, we need to integrate the joint probability density over the orange area. For that, we will integrate over the complete X > Y region and subtract the blue region (X ≥ 2Y) from it.

From equation 9, we already found the value under the blue region is $\frac{1}{4}$

$$
\begin{aligned}
P(X > Y, X < 2Y) &= \iint_A f_{XY}(x,y)dxdy \\
&= \iint_A 1/16dxdy \\
&= \int_0^4 \int_0^x \frac{1}{16}dydx - \int_0^4 \int_0^{x/2} \frac{1}{16}dydx \\
&= \int_0^4 \frac{x}{16}dx - \int_0^4 \frac{x}{32}dx \\
&= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}
\end{aligned}
\tag{11}
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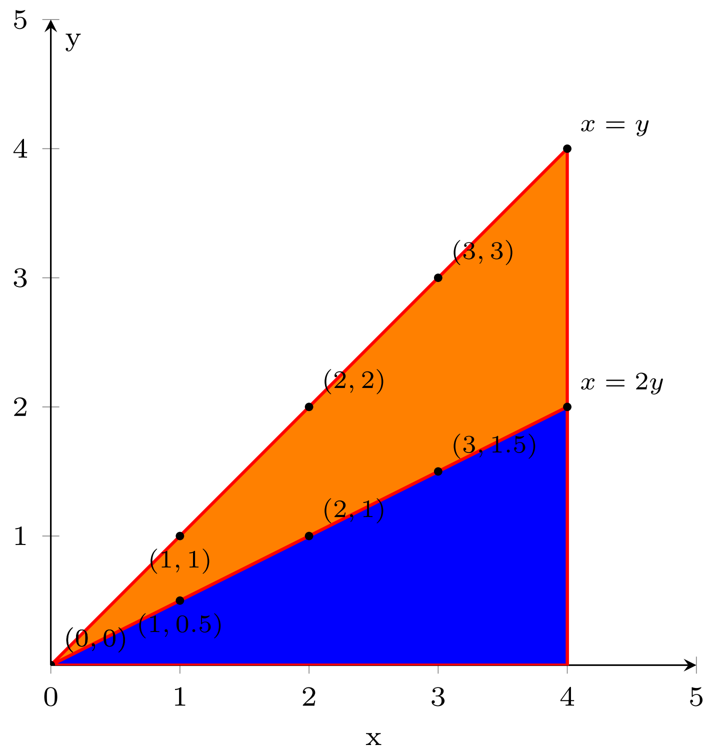

Substitute results from (10) and (11) in equation 2,

$$\Rightarrow P(X > Y|X < 2Y) = \frac{1/4}{3/4} = \frac{1}{3} \tag{12}$$