# Assignment 2

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**50) Let X and Y be i.i.d random variables uniformly distributed on (0,4). Then $P(X > Y|X < 2Y)$ is**

**Ans** According to Bayes Theorem, for any 2 random variable A and B,

$$P(A|B) = \frac{P(A,B)}{P(B)} \tag{1}$$

Thus,

$$P(X > Y|X < 2Y) = \frac{P((X > Y),(X < 2Y))}{P(X < 2Y)} \tag{2}$$

Probability Density Function

For a random variable X, the pdf of Uniform distribution (0,4) is given by

$$f_X(x) = \begin{cases} 1/4 & \text{for } 0 \leq \text{x} \leq 4 \\ 0 & \text{for x<0 or x>4} \end{cases} \tag{3}$$

Cumulative Density Function

For a random variable X, the cdf of Uniform distribution (0,4) is given by

$$F_X(x) = \begin{cases} 0 & \text{for x<0} \\ x/4 & \text{for } 0 \leq \text{x} \leq 4 \\ 1 & \text{for x>4} \end{cases} \tag{4}$$

Replacing the variable x with y and 2y:

$$F_X(y) = P(X \leq y) = \begin{cases} 0 & \text{for y<0} \\ y/4 & \text{for } 0 \leq \text{y} \leq 4 \\ 1 & \text{for y>4} \end{cases} \tag{5}$$

$$F_X(2y) = P(X \leq 2y) = \begin{cases} 0 & \text{for y<0} \\ y/2 & \text{for } 0 \leq \text{y} \leq 2 \\ 1 & \text{for y>2} \end{cases} \tag{6}$$

Since y is a variable belonging to random variable Y, we take expectation of the cdf with respect to Y as follows:

$$\begin{aligned} P(X < Y) &= E_Y[F_X(y)] \\ &= \int F_X(y) f_Y(y) dy \\ &= \int_0^4 \frac{y}{4} \times \frac{1}{4} dy \\ &= \frac{1}{2} \end{aligned} \tag{7}$$

$$\begin{aligned} P(X < 2Y) &= E_Y[F_X(2y)] \\ &= \int F_X(2y) f_Y(y) dy \\ &= \int_0^2 \frac{y}{2} \times \frac{1}{4} dy + \int_2^4 \frac{1}{4} dy \\ &= \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \end{aligned} \tag{8}$$

The numerator of equation 2 is:

$$\begin{aligned} P((X > Y), (X < 2Y)) &= P(Y < X < 2Y) \\ &= E_Y[F_X(2y)] - E_Y[F_X(y)] \\ &= P(X < 2Y) - P(X < Y) \\ &= \frac{3}{4} - \frac{1}{2} \\ &= \frac{1}{4} \end{aligned} \tag{9}$$

$$\begin{aligned} \Rightarrow P(X > Y | X < 2Y) &= \frac{P((X > Y), (X < 2Y))}{P(X < 2Y)} \\ &= \frac{1/4}{3/4} \\ &= \frac{1}{3} \end{aligned} \tag{10}$$

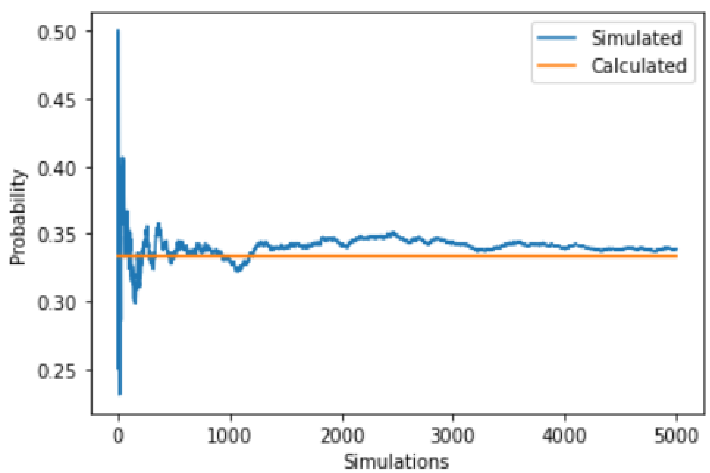


Figure 1: Final Probability vs number of simulations