

Assignment 2

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24 January 2022

50) Let X and Y be i.i.d random variables uniformly distributed on (0,4). Then $P(X > Y|X < 2Y)$ is

Ans According to Bayes Theorem, for any 2 random variable A and B,

$$P(A|B) = \frac{P(A, B)}{P(B)} \quad (1)$$

Thus,

$$P(X > Y|X < 2Y) = \frac{P((X > Y), (X < 2Y))}{P(X < 2Y)} \quad (2)$$

Joint Probability Density Function

Two random variables A, B are jointly continuous if there exists a function $f_{XY}(x, y)$ such that

$$P((X, Y) \in A) = \iint_A f_{XY}(x, y) dx dy \quad (3)$$

where $(X, Y) \in A$ is the area of interest.

We will approach this question using the above two concepts. For that, we need to find the joint probability density function of X and Y

Since $X, Y \sim \text{Uniform}(0, 4)$,

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ x/4 & \text{for } 0 \leq x \leq 4 \\ 1 & \text{for } x > 4 \end{cases} \quad (4)$$

$$F_Y(y) = \begin{cases} 0 & \text{for } y < 0 \\ y/4 & \text{for } 0 \leq y \leq 4 \\ 1 & \text{for } y > 4 \end{cases} \quad (5)$$

Given that X and Y are independent random variables,
 $F_{XY}(x, y) = F_X(x)F_Y(y)$

$$F_{XY}(x, y) = \begin{cases} 0 & \text{for } y < 0 \text{ or } x < 0 \\ xy/16 & \text{for } 0 \leq x \leq 4, 0 \leq y \leq 4 \\ x/4 & \text{for } 0 \leq x \leq 4, y > 4 \\ y/4 & \text{for } 0 \leq y \leq 4, x > 4 \\ 1 & \text{for } x > 4, y > 4 \end{cases} \quad (6)$$

To get joint probability density from joint cumulative function,

$$f_{XY}(x, y) = \frac{\partial^2 F_{XY}(x, y)}{\partial x \partial y} \quad (7)$$

$$\Rightarrow f_{XY}(x, y) = \begin{cases} 1/16 & \text{for } 0 \leq x \leq 4, 0 \leq y \leq 4 \end{cases} \quad (8)$$

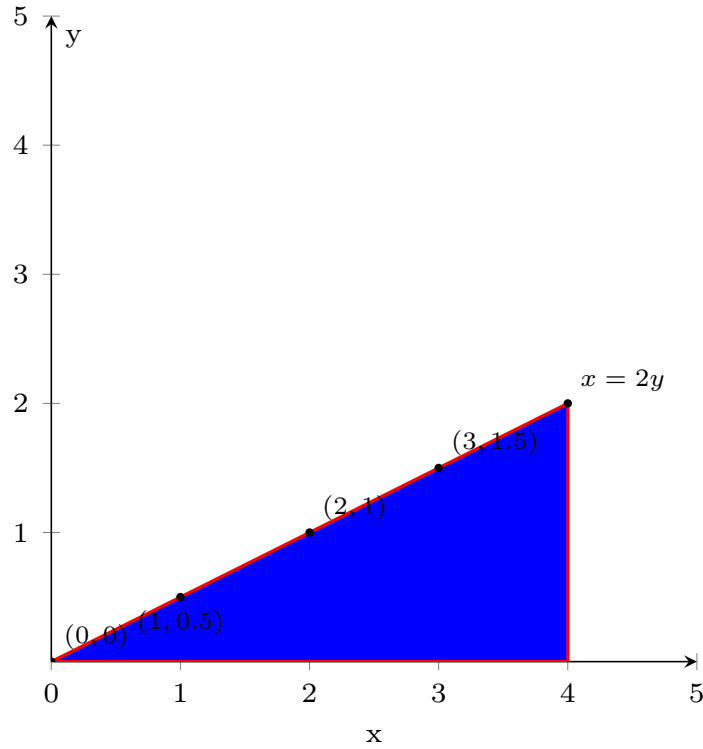
Part a: Finding $P(X < 2Y)$

$$P(X < 2Y) = 1 - P(X \geq 2Y) = 1 - P(Y \leq X/2)$$

$$\begin{aligned} P(Y \leq X/2) &= \iint_A f_{XY}(x, y) dx dy \\ &= \iint_A 1/16 dx dy \\ &= \int_0^4 \int_0^{x/2} \frac{1}{16} dy dx \\ &= \int_0^4 \frac{x}{32} dx \\ &= \frac{4^2}{64} = \frac{1}{4} \end{aligned} \quad (9)$$

$$\Rightarrow P(X < 2Y) = 1 - \frac{1}{4} = \frac{3}{4} \quad (10)$$

Note: The graph below is line $x=2y$ and the shaded area ($2y < x$) is taken for integration

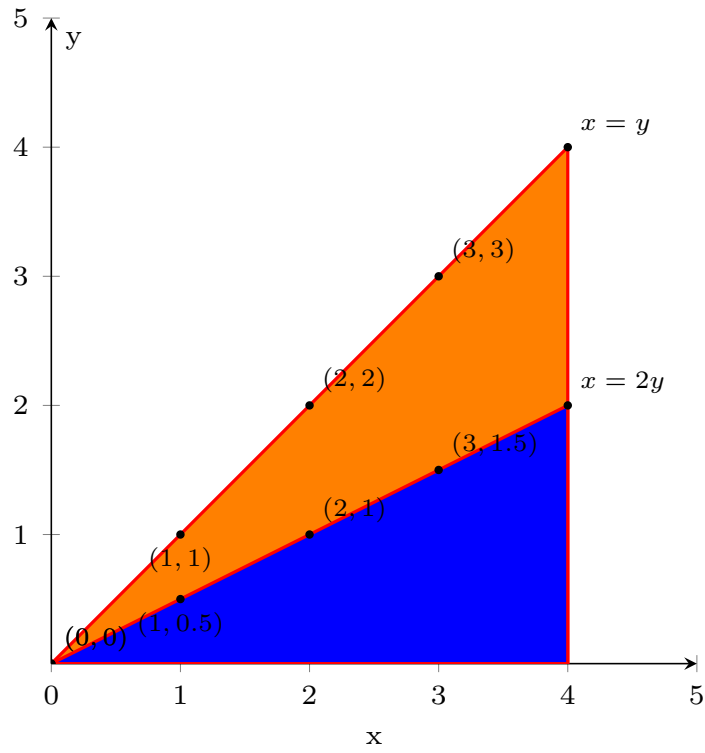


Part b: Finding $P(X > Y, X < 2Y)$

From the graph below, we need to integrate the joint probability density over the orange area. For that, we will integrate over the complete $X > Y$ region and subtract the blue region ($X \geq 2Y$) from it.

From equation 9, we already found the value under the blue region is $\frac{1}{4}$

$$\begin{aligned}
 P(X > Y, Y \leq X/2) &= \iint_A f_{XY}(x, y) dx dy \\
 &= \iint_A 1/16 dx dy \\
 &= \int_0^4 \int_0^x \frac{1}{16} dy dx - \int_0^4 \int_0^{x/2} \frac{1}{16} dy dx \quad (11) \\
 &= \int_0^4 \frac{x}{16} dx - \int_0^4 \frac{x}{32} dx \\
 &= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}
 \end{aligned}$$



Substitute results from (10) and (11) in equation 2,

$$\Rightarrow P(X > Y | X < 2Y) = \frac{1/4}{3/4} = \frac{1}{3} \quad (12)$$