Assignment 3 - Q54 (dec 2018)

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Q54 (dec2018) To test the hypothesis H_0 against H_1 using the test statistic T, the proposed test procedure is not to support H_0 if T is large. Based on a given sample, the p value of the test statistic is computed to be 0.05 assuming that the distribution of T is N(0,1) under H_0. If the distribution of T under H_0 is the t-distribution with 10 degrees of freedom instead, the p-value will be

Ans Let 'n' observations of a sample come from Normal distribution $N(\mu, \sigma^2)$. Then their sampling distribution is

$$\bar{X} = \frac{1}{n}(\Sigma_1^n X_i) \tag{1}$$

The sample mean and variance will be

$$\mu_{\bar{X}} = \frac{1}{n}(\Sigma E(X)) = \mu \tag{2}$$

$$\sigma_{\bar{X}}^2 = \frac{1}{n^2}(\Sigma Var(X)) = \frac{\sigma^2}{n} \tag{3}$$

Given initially, Z test is performed with p-value 0.05 Thus,

$$P(Z > z_\alpha) = 0.05 \tag{4}$$

where,

$$z_\alpha = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \tag{5}$$

We need to find the value of - $P(T > t_\alpha)$
where,

$$t_\alpha = \frac{\bar{X} - \mu}{s/\sqrt{n}} \tag{6}$$

s is the standard deviation of given observations.

A few properties listed below -

$$T = Z\frac{\sqrt{\nu}}{\sqrt{V}} \tag{7}$$

$$\frac{s}{\sigma} = \frac{\sqrt{V}}{\sqrt{\nu}} \tag{8}$$

where $\nu(degree of freedom) = n - 1$ and V is chi-squared distribution with ν degrees of freedom.

Substitute eq 7 and 8 in $P(T > t_\alpha)$

$$\begin{aligned} P(T > t_\alpha) &= P(\frac{Z\sqrt{\nu}}{\sqrt{V}} > t_\alpha) \\ &= P(\frac{Z\sqrt{\nu}}{\sqrt{V}} > \frac{z_\alpha \sigma}{s}) \\ &= P(\frac{Z\sqrt{\nu}}{\sqrt{V}} > \frac{z_\alpha \sqrt{\nu}}{\sqrt{V}}) \\ &= P(Z > z_\alpha) = 0.05 \end{aligned} \tag{9}$$