# Recurrence Relation Solving : Substitution Method

**Substitution Method :** **Substitute the given function repeatedly until the given function is removed.**

**Problem 1 :**

**$T(n) = 1$ if $n=1$**

**$T(n-1)+n$ if $n>1$**

$T(n) = T(n-1)+n$ $1^{st}$ time

$=T(n-2)+n-1+n$ $2^{nd}$ time

$=T(n-3)+n-2+n-1+n$ $3^{rd}$time

K times=n-1

$n-k=1$

$n-1=k$

$=T(n-k)+(n-k+1)+(n-k+2)+....+n-2+n-1+n$ k times

$=T(n-(n-1))+(n-(n-1)+1)+(n-(n-1)+2)+....+n-2+n-1+n$

$=T(n-n+1)+(n-n+1+1)+(n-n+1+2)+....+n-2+n-1+n$

$=T(1)+(2)+(3)+....+n-2+n-1+n$ => sum of n natural numbers

$=n(n+1)/2$

$=(n^2+n)/2$

$=O(n^2)$

**Problem 2 :**

**$T(n) = 1$ if $n=1$**

**$T(n-1).n$ if $n>1$**

$T(n) = T(n-1).n$ 1st time

$=T(n-2)(n-1).n$ 2nd time

$=T(n-3)(n-2)(n-1).n$ 3rd time

K times=n-1

$n-k=1$

$n-1=k$

$=T(n-k)\ (n-k+1)\ (n-k+2) \ldots\ (n-2)\ (n-1).n$ k times

$=T(n-(n-1))\ (n-(n-1)+1)\ (n-(n-1)+2) \ldots\ (n-2)(n-1).n$

$=T(n-n+1)\ (n-n+1+1)\ (n-n+1+2) \ldots\ (n-2)(n-1).n$

$=T(1)\ (2)\ (3) \ldots\ (n-2)(n-1)n$ $\Rightarrow n!=O(n!)$

$5!=5.4.3.2.1.=120$

$n^n>n!$

$=O(n^n)$

**Problem 3 :**

**$T(n) = 1$ if $n=0$**

**$T(n-2)+n^2$ if $n>0$**

$T(n) = T(n-2)+n^2$ 1st time

$=T(n-4)+(n-2)^2+n^2$ 2nd time

$=T(n-6)+(n-4)^2+(n-2)^2+n^2$ 3rdtime

K times=$n-2K$

$n-2k=0$

$n/2=k$

$=T(n-2k)+(n-2k+2)^2+ \dots. (n-2)^2+n^2$ k times

$=T(n-2(n/2))+(n-2(n/2)+2)^2+ \dots. (n-2)^2+n^2$

$=T(n-2(n/2))+(n-2(n/2)+2)^2+ \dots. (n-2)^2+n^2$

$=T(0)+2^2+4^2+6^2 \dots. (n-2)^2+n^2$

$=1+2^2(1^2+2^2+\dots.+(n/2)^2)$

$=O(n^3)$