Asymptotic Notation :

1. Big O Notation

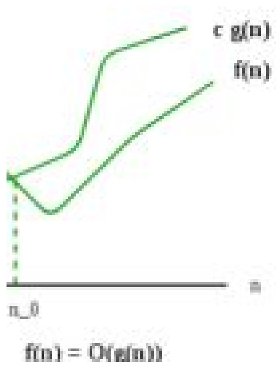
2. Omega Notation

3. Theta Notation

Big O Notation(very very important used everywhere) : upper bound of an algorithm

f(n) = O(g(n)), if f(n) <= cg(n) for all n , n >= n_o such that there exists two positive constants where c > 0 and n_o >= 1

So, if we say a = O(b) meaning is (b greater than a after taking c help)



Problem 1 :

f(n) = 5n

g(n) = n

f(n) = O(g(n)), it means f(n) <= c. (g(n))

5n <= c.n for all n, n >= 1

c = 5 $5n <= 5n$

f(n) = O(g(n))

now what should be the value of a constant "c"?

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Problem 2 :

$f(n) = n^2$

g(n) = n

f(n) = O(g(n)), it means f(n) <= cg(n) (Mathematical

definition) $n^2 <= c.n$ for all n, n >= 1 -------> equation 1

now what should be the value of constant "c"?

c = n // Is this a constant -> not at all because here c depends on the value of n.....bigger the value of n, bigger the value of c and smaller the value of n,smaller the value of c.

c = 3 4 6

$n^2 <= n^2$

f(n) is not equal to O(g(n))

Increasing order of complexities :

1. **Constant Complexity :** O(1)
2. **Logarithmic Complexity :** $O(\log_n)$
3. **Linear Complexity :** O(n)
4. **Quadratic Complexity :** $O(n^2)$
5. **Cubic Complexity :** $O(n^3)$

6. Polynomial Complexity : **$O(n^c)$; where c is constant**

7. Exponential Complexity : **$O(c^n)$; where c is constant**

8. $n! < n^n$

$2^n < n^n$

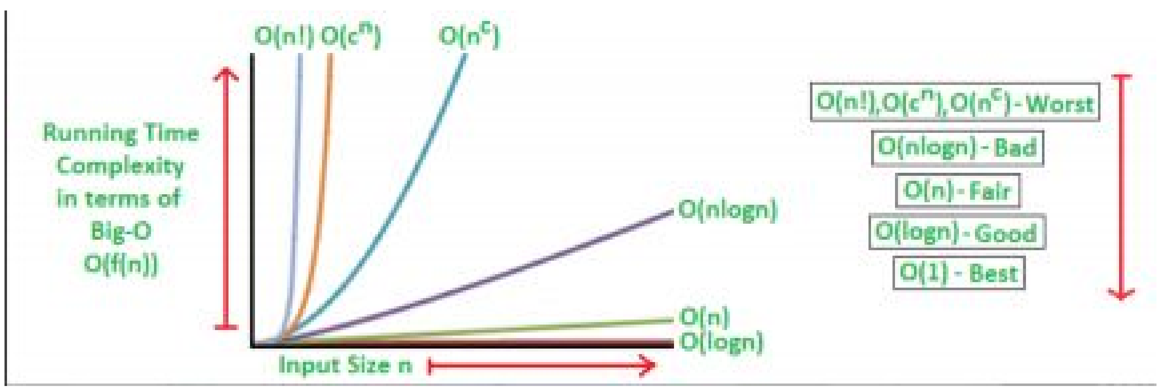
$n! > 2^n$

$2^n < n! < n^n$

OR

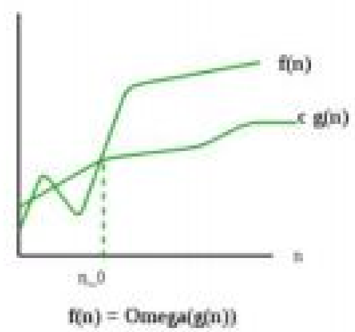
$2^n = O(n!)$; $n! = O(n^n)$

Complexity Graph(Focus very carefully)



Omega Notation :

Let f(n) = omega(g(n)) if and only if f(n) >= c(g(n)) for all n, $n>=n_0$ such that there exists two positive constants c > 0 and $n_o >= 1$.



Problem 1 :

f(n) = n g(n) = 5n

f(n) = omega(g(n))

What should be the value of c ?

f(n) >= c(g(n))

n >= c. 5n

c = 1/5 - constant

f(n) = omega(g(n))

Problem 2 :

f(n) = 5n g(n) = n

f(n) = omega(g(n))

What should be the value of c ?

f(n) >= c. omega(g(n))

5n >= c.n

c = 1 => 5n >= 1.n
f(n) = omega(g(n))

Problem 3 :

$f(n) = n^2$ $g(n) = n^2 + n + 10$

f(n) = omega(g(n))

What should be the value of c ?

f(n) >= c.g(n)

$n^2 >= c.(n^2+n+10)$

c = 1/2

$n^2 >= 1/2(n^2 + n + 10)$

f(n) = omega(g(n))

Problem 4 :

$f(n) = n$ $g(n) = n^2$

f(n) = omega(g(n))

What should be the value of c ?

f(n) >= g(n)

$n >= c.n^2$

c = 1/n -> means it is not a constant

That's why f(n) is not equal to omega(g(n)).

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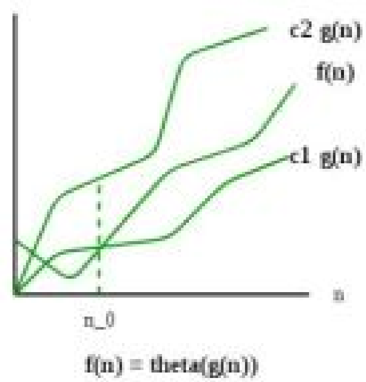
- Theta Notation : which satisfies both bigO and omega

Let f(n) = theta(g(n)) if and only if $f(n) >= c_1g(n)$(omega) and $f(n) <= c_2(g(n))$(Big O) for all n; $n >= n_0$ such that there exists three constants.

$c_1 > 0$

$c_2 > 0$

$n_0 >= 1$



Problem 1 :

f(n) = n g(n) = 5n

f(n) = theta(g(n))

1. Omega -> f(n) >= c.g(n)

n >= c. 5n

c = 1/5 - constant

f(n) = omega(g(n))

2. Big O -> f(n) <= c.g(n)

n <= c.5n

c = 1 - constant

f(n) = O(g(n))

Thus because f(n) holds true for both omega as well as O, thus f(n) = theta(g(n))

Problem 2 :

f(n) = n - 10 g(n) = n + 10

f(n) = theta(g(n))

1. Omega -> f(n) >= c.g(n)

n-10 >= c.(n+10)

c = 1/2 - constant

f(n) = omega(g(n))

2. Big O -> f(n) <= c.g(n)

n-10 <= c.(n+10)

c = 1

f(n) = O(g(n))

We can say that f(n) = theta(g(n)) as it holds true for both omega as well as big O.

Problem 3 :

f(n) = n g(n) = n

f(n) = theta(g(n))

1. Omega -> f(n) >= c.g(n)

n >= c.n

c = 1 - constant

Thus, f(n) = omega(g(n))

2. Big O -> f(n) <= c.g(n)

n <= c.n

c = 1 - constant

Thus, f(n) = O(g(n))

And thus f(n) = theta(g(n)) as it holds true for both omega and Big

O

Problem 4 :

f(n) = n g(n) = n^2

f(n) = theta(g(n))

1. Omega -> f(n) >= c.g(n)

n >= c.n^2

c = 1/n - Not a constant

f(n) is not theta(g(n))
