

Teacher Energized Resource Manual

Class : 8th
Subject : Mathematics



CENTRAL BOARD OF SECONDARY EDUCATION

Preface

In consonance with the move towards outcome-based education where focus is on developing competencies in students, the Central Board of Secondary Education is delighted to share the *Teacher Energized Resource Manual* that will aid teachers in aligning their classroom transaction to a competency framework.

Each chapter of the Resource Manual corresponds to the respective chapters in the NCERT textbooks. The chapters have been chunked by concept; these concepts have been linked to the NCERT Learning Outcomes; and an attempt has been made to delineate Learning Objectives for each concept. Every chapter has a set of assessment items, where two items have been provided as examples for each Learning Objective. Teachers can use these to assess if the learner has acquired the related concept. Needless to say, the items are illustrative examples to demonstrate how competency-based items can be prepared to measure Learning Objectives and Outcomes. The variety in item forms is suggestive of the ways in which a particular concept can be assessed to identify if the learner has attained different competencies. We trust and hope that teachers would be able to generate many more similar test items for use in practice.

Your observations, insights and comments as you use this Resource Manual are welcome. Please encourage your students to voice their suggestions as well. These inputs would be helpful to improve this Manual as these are incorporated in the subsequent editions. All possible efforts have been made to remove technical errors and present the Manual in a form that the teachers would find it easy and comfortable to use.

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HOW TO USE THIS MANUAL

The goal of the Teacher Energized Resource Manual (TERM) is to provide teachers with competency-based education resources aligned to NCERT textbooks that would support them in the attainment of desired Learning Outcomes and development of requisite competencies of the learner. The TERM has equal number of corresponding chapters as NCERT Textbooks with listing of Concepts, Learning Outcomes developed by NCERT and Learning Objectives. Competency based test items for each corresponding Learning Objective and sample activities for enrichment have been provided.

Learning Objectives:

Each chapter has a *Learning Objectives* table. The table also lists the Concepts covered in the chapter. Learning Objectives are broken down competencies that a learner would have acquired by the end of the chapter. They are a combination of skills and what the learner would use this skill for. For example, the first Learning Objective in the table below relates to the skill of *application* and the students will use this competency to obtain the highest common factor of 2 positive integers. Teachers can use these specific Learning Objectives to identify if a student has acquired the associated skill and understands how that skill can be used.

Concepts	Learning Objectives	Learning Outcomes
Euclid's Division	Apply Euclid Division Algorithm in order to obtain HCF of 2 positive integers in the context of the given problem	Generalises properties of numbers and relations among them studied earlier, to evolve results, such as, Euclid's division algorithm, fundamental theorem of arithmetic in order to apply them to solve problems related to real life contexts
	Apply Euclid Division Algorithm in order to prove results of positive integers in the form of $ax+b$, where a and b are integers	
Fundamental Theorem of Arithmetic	Use the Fundamental Theorem of Arithmetic in order to calculate HCF and LCM of the given numbers in the context of the given problem	
	Recall the properties of irrational number in order to prove that whether the sum/difference/product/quotient of 2 numbers is irrational or not	
Irrational Numbers	Apply theorems of irrational number in order to prove whether a given number is irrational or not	
	Apply theorems of rational numbers in order to find out about the nature of their decimal representation and their factors	
Decimal Representation of Irrational Numbers		

Concepts:

The important concepts in a particular chapter are listed in the first section. Most often, they follow a logical order and present a sequence in which these are likely to be covered while teaching. In case, your teaching strategy is different and presents them in a different order, you need not worry. Teach the way, you consider the best. You only need to ensure their understanding and the attainment of desired learning objectives.

Learning Outcomes (NCERT):

A mapping of Learning Outcomes developed by the NCERT and Learning Objectives is provided in last column of the table. The Learning Outcomes have been developed by the NCERT. Each Learning Objective is mapped to NCERT Learning Outcomes and helps teachers to easily identify the larger outcome that a learner must be able to demonstrate at the end of the class/ chapter.

Test items:

For each Learning Objective, at least two competency-based test items have been provided. Although, the items in this resource manual are multiple choice questions, which assess developed competencies of a student rather than only knowledge, it must be kept in mind that there can be different kinds of assessment that can easily align with competency-based education. Teachers can use these items to assess if a learner has achieved a particular Learning Objective and can take necessary supportive actions. Teachers are also encouraged to form similar questions which assess skills of students.

LOB: Apply Euclid Division Algorithm in order to obtain HCF of 2 given numbers in the context of the given problem

1. A worker needs to pack 350 kg of rice and 150 kg of wheat in bags such that each bag weighs the same. Each bag should either contain rice or wheat. Which option shows the correct steps to find the greatest amount of rice/wheat the worker can pack in each bag?

Option 1: Step 1: $350 = 2(150) + 50$
Step 2: $150 = 3(50) + 0$
Step 3: Greatest amount: 50 kg

Option 2: Step 1: $350 = 2(150) + 50$
Step 2: $150 = 2(50) + 0$
Step 3: Greatest amount: 50 kg

Option 3: Step 1: $350 = 2(150) + 50$
Step 2: $150 = 3(50) + 0$
Step 3: Greatest amount: 150 kg

Option 4: Step 1: $350 = 2(150) + 50$
Step 2: $150 = 2(50) + 0$
Step 3: Greatest amount: 150 kg

Correct Answer: Option 1

Fig: 3

Suggested Teacher Resources

At the end of each chapter, certain activities have been suggested which can be carried out by the teachers with learners to explain a concept. These are only samples and teachers can use, adapt, as well as, create activities that align to a given concept.

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I. RATIONAL NUMBERS

QR Code:



Learning outcome and Learning Objectives:

Content area / Content area / Concepts	Learning Objectives	Learning Outcome
Introduction to Rational Numbers	Define rational number in order to identify whether the given number is a rational number or not	Explores patterns in arithmetic operations in order to generalise properties of addition, subtraction, multiplication and division for rational numbers
	Apply the properties of natural numbers, whole numbers and integers with respect to all the arithmetic operations in order to extend them for rational numbers	
	Use prior knowledge in order to define the additive and multiplicative identity of rational numbers.	
	Use prior knowledge of integers and fractions in order to define the additive and multiplicative inverse of rational numbers.	
	Apply Distributivity of multiplication over addition for rational numbers in order to simplify a given expression	
Representation of Rational Numbers on the Number Line	Extend the concepts of number line in order to represent rational number on the number line	
Rational Numbers between Two Rational Numbers	Calculate and find rational numbers between any two rational numbers in order to prove that there are infinite rational numbers between any two given rational numbers	Calculate rational numbers between any two given rational numbers in order to prove that there are infinite rational numbers between two rational numbers

Test items



LOB: Define rational number in order to identify whether the given number is a rational number or not.

1. Is -0.4 a rational number?

Option 1: No, because it cannot be written in the form of $\frac{p}{q}$.

Option 2: Yes, because it can be written in the form of $\frac{p}{q}$.

Option 3: Yes, because it is a negative number.

Option 4: No, because it is a decimal number.

Correct Answer: Option 2

2. Consider the expression below:

$$\frac{1}{\sqrt{k+7}}$$

Given that the expression represents a rational number, which of these is the least possible positive integer value of k ?

Option 1: 0

Option 2: 2

Option 3: 8

Option 4: 10

Correct Answer: Option 2

LOB: Apply the properties of natural numbers, whole numbers and integers with respect to all the arithmetic operations in order to extend them for rational numbers.

1. Which of these options completes the sentence below?

The equation $\frac{-25}{9} - \frac{7}{3} = \frac{-46}{9}$ shows that ____.

Option 1: rational numbers are not closed under subtraction

Option 2: rational numbers are not closed under division

Option 3: rational numbers are closed under subtraction

Option 4: rational numbers are closed under division

Correct Answer: Option 3

2. Consider the equation below.

$$\frac{5}{6} + \left(\frac{1}{8} + \frac{16}{5}\right) = A + \frac{1}{8}$$

Which of these is equivalent to A ?

Option 1: $\frac{5}{48} + \frac{16}{5}$

Option 2: $\frac{5}{6} + \frac{1}{8}$

Option 3: $\frac{1}{8} + \frac{16}{5}$

Option 4: $\frac{5}{6} + \frac{16}{5}$

Correct Answer: Option 4

LOB: Use prior knowledge in order to define the additive and multiplicative identity of rational numbers.

1. If the multiplicative identity of rational number $\frac{p}{q}$ is x , what could be the value of x ?

Option 1: $\frac{q}{p}$

Option 2: $\frac{p}{q}$

Option 3: 0

Option 4: 1

Correct Answer: Option 4

2. Which of these statements is true about the rational number $\frac{a}{b}$?

Option 1: The additive identity of rational number $\frac{a}{b}$ is $\frac{a}{b}$.

Option 2: The additive identity of rational number $\frac{a}{b}$ is $-\frac{a}{b}$.

Option 3: The additive identity of rational number $\frac{a}{b}$ is 0.

Option 4: The additive identity of rational number $\frac{a}{b}$ is 1.

Correct Answer: Option 3

LOB: Use prior knowledge of integers and fractions in order to define the additive and multiplicative inverse of rational numbers.

1. Consider the equation $\frac{m}{5} + \left(\frac{-m}{5}\right) = 0$.

Which of these is true?

Option 1: $\frac{-m}{5}$ is the additive inverse of $\frac{m}{5}$.

Option 2: $\frac{-m}{5}$ is the additive identity.

Option 3: $\frac{-m}{5}$ is the multiplicative inverse of $\frac{m}{5}$.

Option 4: $\frac{-m}{5}$ is the multiplicative identity.

Correct Answer: Option 1

2. Consider the equation $\frac{2}{3} \times (p - q) = 1$.

Which of these can be the values of p and q ?

Option 1: $p = \frac{3}{2}$ and $q = 1$

Option 2: $p = \frac{5}{2}$ and $q = 1$

Option 3: $p = \frac{3}{2}$ and $q = \frac{5}{2}$

Option 4: $p = \frac{5}{2}$ and $q = \frac{3}{2}$

Correct Answer: Option 2

LOB: Apply Distributivity of multiplication over addition for rational numbers in order to simplify a given expression

1. Which of these is equivalent for $\frac{16}{9} \times \left(\frac{5}{7} + \frac{13}{4}\right)$?

Option 1: $\frac{16}{9} \times \frac{5}{7} \times \frac{13}{4}$

Option 2: $\left(\frac{16}{9} \times \frac{5}{7}\right) + \frac{13}{4}$

Option 3: $\left(\frac{16}{9} \times \frac{5}{7}\right) + \left(\frac{16}{9} \times \frac{13}{4}\right)$

Option 4: $\left(\frac{16}{9} \times \frac{5}{7}\right) \times \left(\frac{16}{9} \times \frac{13}{4}\right)$

Correct Answer: Option 3

2. Consider the expression below.

$$\frac{-9}{4} \times \left(\frac{2}{7} + p\right)$$

If the expression is equal to $\frac{72}{35}$, what is the value of p ?

Option 1: $\frac{6}{5}$

Option 2: $\frac{2}{35}$

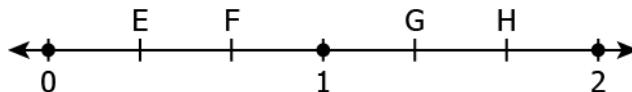
Option 3: $-\frac{6}{5}$

Option 4: $-\frac{2}{35}$

Correct Answer: Option 3

LOB: Extend the concepts of number line in order to represent rational number on the number line

- I. **Question Text:** Which of these letters on the given number line represents $\frac{5}{3}$?



Option 1: E

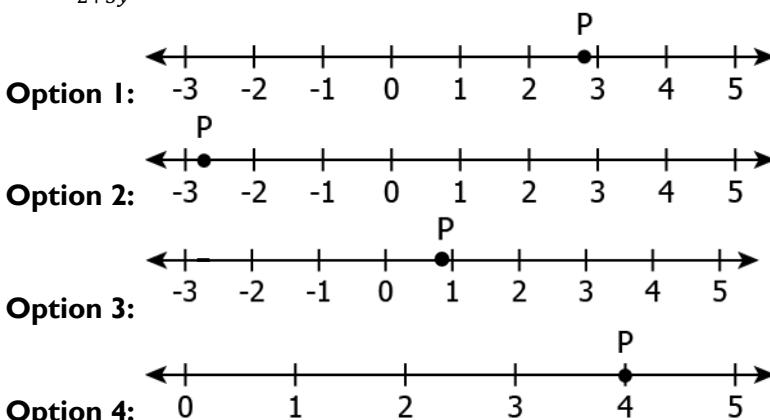
Option 2: F

Option 3: G

Option 4: H

Correct Answer: Option 4

2. If x and y are two natural numbers such that $x > 19$ and $y < 5$, which number line represents the location of a point $P = \frac{x}{2+5y}$ such that P has the least possible value?



Correct Answer: Option 3

LOB: Calculate and find rational numbers between any two rational numbers in order to prove that there are infinite rational numbers between any two given rational numbers.

- I. Which of the following rational numbers lies between $\frac{1}{10}$ and $\frac{9}{4}$?

Option 1: $\frac{21}{20}, \frac{1}{4}$ and $\frac{3}{80}$

Option 2: $\frac{1}{2}, \frac{15}{19}$ and $\frac{11}{15}$

Option 3: $\frac{17}{6}, \frac{19}{20}$ and $\frac{53}{4}$

Option 4: $\frac{49}{25}, \frac{17}{4}$ and $\frac{2}{3}$

Correct Answer: Option 2

2. If x and y are two non-zero positive rational numbers, which of these statements is always true?

Option 1: $\frac{x-y}{2}$ is a rational number between x and y .

Option 2: $\frac{xy}{2}$ is a rational number between x and y .

Option 3: $\frac{x+y}{2}$ is a rational number between x and y .

Option 4: $\frac{x+y}{2}$ is a rational number between x and y .

Correct Answer: Option 3

Suggested Teacher Resources



I

Lesson Plan



Objective	The students will be able to verify the closure and the commutative properties of rational numbers
Prerequisite	Properties of integers, rational numbers, arithmetic operations on rational numbers.
Vocabulary words	Rational number, closure, commutativity.
Materials required	Chalks, blackboard, duster, notebooks, pencil, chits, chart papers.
Procedure	<p>Preparation required before the class:</p> <ol style="list-style-type: none">1. Prepare around 12-20 chits mentioning a rational number on each of them. A sample has been given at the end of the chapter.2. Equally distribute the chits into two bowls.3. Arrange a dice wherein faces should show an arithmetic operator (+, -, ×, ÷), one face should mention- 'try again' and, the 6th face should mention-'skip'.4. The teacher will prepare chart papers mentioning the facts of closure property and the commutative property for rational numbers (a sample is given at the end). <p>Instructions:</p> <ol style="list-style-type: none">1. The class will be divided into two teams.2. The teacher will place the two bowls containing the chits on her table and will name it as Team 1 and Team 2.3. Both the teams will be given a working space on the board.4. Nothing will be erased.<ol style="list-style-type: none">a. This will be a timed activity.b. The teacher will paste the chart on the board and draw a scoreboard for both the teams.c. In each round, one representative from each team will come and pick out a chit from their respective bowl, for instance, Team 1 got the rational number $\frac{4}{5}$ and Team 2 got $1\frac{1}{3}$. They will write it on the board (in their allotted space).d. The teacher will then throw the dice.e. If the upward face shows an arithmetic operator, for instance, '+' comes up, the two students will need to add the numbers that they both got within 90 seconds, i.e., they will add $\frac{4}{5}$ and $1\frac{1}{3}$ in both the orders, i.e., $\frac{4}{5} + 1\frac{1}{3}$ and $1\frac{1}{3} + \frac{4}{5}$. It will be done similarly for all arithmetic operations.f. If the face shows- 'try again', then the two students will need to return their chits to the teacher and pick a new chit again. The teacher will then roll the dice again.g. If the face shows- 'skip', then the two students will need to return their chits and go back to their seats. The next student from both the teams will then come and take their turn.5. All the while, students sitting at their desks will also be doing the operations in their notebooks and will be the ones to decide if the students at the blackboard have done the operation correctly or not. The teacher will intervene if required.

6. In each round, if both the teams get the correct answer while doing the operation in both the orders, they will get 3 points each. Else, the Team getting the correct answer will get 5 points and the other will get 0.
7. The chits once used would not go back into the bowls.
8. After all the chits have been exhausted, the teacher will announce the winner as per the scoreboard.

The teacher will now engage the students in a discussion and bring their attention to the charts prepared for the closure and the commutative properties.

The teacher will first discuss the closure property of rational numbers under the four arithmetic operations.

Closure Property of Rational numbers		
S. No.	Statement	Yes/No
1	On adding two rational numbers, we always get a rational number	
2	On subtracting two rational numbers, we always get a rational number	
3	On multiplying two rational numbers, we always get a rational number	
4	On dividing two rational numbers, we always get a rational number	

The teacher will take up statements in the given order, and will discuss if the property holds true for each operation or not. The teacher would use the examples taken from the activity done earlier and highlight it from the board directly. For instance, for statement 1, the teacher could refer to the addition sums done on the board.

Similarly, the teacher could discuss the commutative property. During the discussion, the teacher and the students will simultaneously complete both the tables and provide non-examples where the property is not satisfied. The teacher could give non-examples herself if there isn't any on the board during the discussion, if required.

The completed tables would appear something like below:

Closure Property of Rational numbers		
S. No.	Statement	Yes/No
1	On adding two rational numbers, we always get a rational number	Yes
2	On subtracting two rational numbers, we always get a rational number	Yes
3	On multiplying two rational numbers, we always get a rational number	Yes
4	On dividing two rational numbers, we always get a rational number	No $5 \div 0$ is not defined.

Commutative Property of Rational numbers		
S. No.	Statement	Yes/No
1	Addition is commutative for rational numbers, i.e., For any two rational numbers a and b , $a + b = b + a$	Yes
2	Subtraction is commutative for rational numbers, i.e., For any two rational numbers a and b , $a - b = b - a$	No $5 - \frac{3}{2} = \frac{7}{2}$ $\frac{3}{2} - 5 = -\frac{7}{2}$
3	Multiplication is commutative for rational numbers, i.e., For any two rational numbers a and b , $a \times b = b \times a$	Yes
4	Division is commutative for rational numbers, i.e., For any two rational numbers a and b , $a \div b = b \div a$	No $5 \div 6 = \frac{5}{6}$ $6 \div 5 = \frac{6}{5}$

Suggestion: The teacher could do a similar exercise for the other properties.

2

Activity



Objective: To represent rational numbers on a number line.

Materials required: Chits prepared by the teacher, notebooks, scale, pencils.

Preparation required by the teacher before the class:

- The teacher will need to prepare chits (in abundance) mentioning a rational number on each of them.

Instructions:

- This activity will be done in pairs. The students can do it with their partners.
- The teacher will place all the chits in a bowl on her table.
- One volunteer from each pair will come forward and randomly choose 10 chits each.
- The students will have to draw number line in their notebooks and represent those 10 rational numbers on it.
- The teacher will be there to handle queries of the students.

Further tasks after the students have represented the 10 rational numbers on the number line:

- Arrange the 10 rational numbers in ascending order.
- Did any two numbers represent the same number?
- The teacher could ask the teams to interchange their chits and repeat the whole process once again. This way each team will get a variety of examples.
- After the class is over, the students will return the chits to the teacher.

Sample chits are given below:

$-\frac{20}{35}$	$\frac{9}{3}$	$\frac{8}{5}$	$\frac{12}{36}$	0	$-\frac{3}{9}$
$-\frac{2}{3}$	$\frac{2}{10}$	7	-1.5	$\frac{2}{3}$	$\frac{3}{9}$
$-\frac{9}{3}$	$-\frac{2}{3}$	$-\frac{12}{36}$	$-\frac{10}{15}$	$\frac{11}{7}$	$-\frac{5}{8}$

2. LINEAR EQUATIONS IN ONE VARIABLE

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Meaning of Linear Equation in one variable and its solution	<p>List the properties of algebraic expression and algebraic equation in order to differentiate between the two</p> <p>Identify the variable(s) and the highest power of the variable in a given algebraic equation in order to distinguish whether it is a linear equation in one variable or not</p> <p>Substitute the given values of variable in order to verify whether it is the solution of the equation or not</p>	Use variables in order to solve puzzles and daily life problems
Solving Equations which have Linear Expressions on one Side and Numbers on the other Side	Transpose terms to the other side in order to solve linear equations which have linear expression on one side and numbers on the other side	
Applications of Linear Equations with one variable	Write simple contextual problems as linear equations in one variable in order to find its solution	
Solving Equations having the Variable on both Sides	Transpose terms to the other side in order to solve linear equations in one variable which have	
Reducing Equations to Simpler Form	Simplify the given linear equation in one variable in order to solve them	
Equations Reducible to the Linear Form	Use cross multiplication in order to reduce certain equations into their linear form	

Test items



LOB: List the properties of algebraic expression and algebraic equation in order to differentiate between the two

- I. A set of equations and expressions is given below:

$$2x, x + 5, 3x - 2 = 5, xyz + 5x + 3y, 2y + 5 = 0$$

Which of the following options correctly represents the number of algebraic expressions and equations given in the set?

Option 1: Examples of algebraic expressions: 5

Examples of algebraic equations: 0

Option 2: Examples of algebraic expressions: 0

Examples of algebraic equations: 5

Option 3: Examples of algebraic expressions: 2

Examples of algebraic equations: 3

Option 4: Examples of algebraic expressions: 3

Examples of algebraic equations: 2

Correct Answer: Option 4

2. Consider the statements.

Statement A: $3x + 5$ is an example of algebraic expression because equality (=) sign is missing in it and it consists of variable, numbers, and operations.

Statement B: $2x - 6 = 11$ is an example of algebraic equation because it is an equality involving variables and has an equality sign.

Which of these statement(s) is/are correct?

Option 1: Only Statement A

Option 2: Only Statement B

Option 3: Both Statement A and Statement B

Option 4: Neither Statement A nor Statement B

Correct Answer: Option 3

LOB: Identify the variable(s) and the highest power of the variable in a given algebraic equation in order to distinguish whether it is a linear equation in one variable or not

- I. Which of these equations is a linear equation in one variable?

Option 1: $2x + 5 = 14x$

Option 2: $2x + 5y = 14$

Option 3: $2x^2 + 5 = 4$

Option 4: $\frac{2}{x} + 3x = 0$

Correct Answer: Option 1

2. Consider the equation as shown.

$$x^{m-3} + 15y^{2n-2} = 2$$

What should be the value of m and n such that the above equation is a linear equation in one variable?

Option 1: $m = 3, n = 3$

Option 2: $m = 3, n = 1.5$

Option 3: $m = 4, n = 2$

Option 4: $m = 4, n = 1.5$

Correct Answer: Option 2

LOB: Substitute the given values of variable in order to verify whether it is the solution of the equation or not

- I. Which of these is a solution of the equation $2(x - 9) = -14$?

Option 1: $x = 0$

Option 2: $x = 1$

Option 3: $x = 2$

Option 4: $x = 3$

Correct Answer: Option 3

2. Consider the equation shown below:

$5x - a = 14$; where a is any number

Given that $x = 4$ is a solution of the equation, what is the value of a ?

Option 1: 3

Option 2: 6

Option 3: 9

Option 4: 18

Correct Answer: Option 2

LOB: Transpose terms to the other side in order to solve linear equations which have linear expression on one side and numbers on the other side

- I. Which of these is the solution for the equation $3x + 13 = -8$?

Option 1: $x = -7$

Option 2: $x = 7$

Option 3: $x = \frac{5}{3}$

Option 4: $x = -\frac{5}{3}$

Correct Answer: Option 1

2. What is the solution of the equation $\frac{7t-3}{5} + \frac{t+11}{10} = \frac{1}{10} - \frac{3}{5}$?

Option 1: $t = -\frac{3}{2}$

Option 2: $t = -\frac{2}{3}$

Option 3: $t = \frac{2}{3}$

Option 4: $t = \frac{3}{2}$

Correct Answer: Option 2

LOB: Write simple contextual problems as linear equations in one variable in order to find its solution

- I. Anita has ₹1800 and saves ₹600 each week to buy a new guitar that costs ₹12000. If n represents the number of additional weeks, she needs to save to buy the guitar, which equation represents the situation and what will be the value of n ?

Option 1: $1800 + 600n = 12000$; $n = 23$

Option 2: $1800 + 600n = 12000$; $n = 17$

Option 3: $1800n + 600 = 12000$; $n = 6$

Option 4: $1800n + 600 = 12000$; $n = 7$

Correct Answer: Option 2

2. Raman sells printed t-shirts and plane t-shirts at his shop. He sells each printed t-shirt for ₹350 and each plane t-shirt for ₹230. On a day, he sells a total of 50 t-shirts for ₹14260. How many of each kind of t-shirt has he sold on that day?

Option 1: Printed t-shirts = 35 and Plane t-shirts = 15

Option 2: Printed t-shirts = 15 and Plane t-shirts = 35

Option 3: Printed t-shirts = 27 and Plane t-shirts = 23

Option 4: Printed t-shirts = 23 and Plane t-shirts = 27

Correct Answer: Option 4

LOB: Transpose terms to the other side in order to solve linear equations in one variable which have variable on both the sides

- I. What is the solution of the linear equation $(4x - 5) - (2 + 3x) = (7x - 6)$?

Option 1: $x = -\frac{13}{8}$

Option 2: $x = -\frac{1}{8}$

Option 3: $x = -\frac{13}{6}$

Option 4: $x = -\frac{1}{6}$

Correct Answer: Option 4

2. Which of these is the solution of the equation $\frac{5u-3}{6} - \frac{3u+2}{3} = \frac{u}{6} - 3$?

Option 1: $u = -12.5$

Option 2: $u = -5.5$

Option 3: $u = 5.5$

Option 4: $u = 12.5$

Correct Answer: Option 3

E8M0207: Simplify the given linear equation in one variable in order to solve them

- I. What is the solution of the linear equation $9k - 2(3 + 4k) = 3(2k - 1)$?

Option 1: $k = -\frac{9}{7}$

Option 2: $k = -\frac{3}{5}$

Option 3: $k = \frac{3}{5}$

Option 4: $k = \frac{9}{7}$

Correct Answer: Option 2

2. Which of these is the solution of the equation $3\left(\frac{x-5}{7}\right) - 2\left(\frac{2x-11}{3}\right) = \frac{1}{21}[2(x+3) - 5(x+1)]$?

Option 1: $x = \frac{55}{8}$

Option 2: $x = \frac{54}{11}$

Option 3: $x = \frac{27}{4}$

Option 4: $x = 5$

Correct Answer: Option 3

LOB: Use cross multiplication in order to reduce certain equations into their linear form

- I. What is the solution of the equation $\frac{2x+1}{2x-1} = 2$?

Option 1: $-\frac{3}{2}$

Option 2: $-\frac{2}{3}$

Option 3: $\frac{2}{3}$

Option 4: $\frac{3}{2}$

Correct Answer: Option 4

2. At a retail store, the cost of a plastic pot is ₹50 more than the cost of a broomstick. If the ratio of the cost of two pots to the cost of three broomsticks is 5:6, what is the cost of the pot?

Option 1: ₹150

Option 2: ₹200

Option 3: ₹250

Option 4: ₹300

Correct Answer: Option 3

Suggested Teacher Resources



I

Lesson Plan



Objective	Write simple contextual problems as linear equations in one variable in order to find its solution.
Prerequisite	Linear equation in one variable and the methods to find its solution
Vocabulary words	Equation, solution, coefficients, variables.
Materials required	Chalk and blackboard, Pen/pencil and notebook.
Procedure	<p>The teacher will begin by presenting a problem-</p> <p>Mrs. Ratna, the owner of a saree shop knows that there are 700 sarees in her shop but wants to reorganize how they are displayed on the counters. She needs to know how many cotton sarees; designer sarees and silk sarees are in the shop. She knows that the shop has four times as many cotton sarees as silk sarees and half as many designer sarees as cotton sarees. If these are the only types of sarees in the shop, how many of each type of saree are in the shop?</p> <p>The teacher will give 2-3 minutes to the students to work individually on the problem before initiating a discussion. The students will have to formulate an equation and then find its solution.</p> <p>In the given problem, there are basically 4 elements, i.e., cotton sarees, designer sarees, silk sarees and the total number of sarees.</p> <p>In the problem statement, we have been given the value of the total number of sarees. So, let's represent the total number of sarees by a long rectangle.</p> <p style="text-align: center;">700 sarees</p> <p>We have been given that there are four times as many cotton sarees as silk sarees and number of designer sarees is half the number of cotton sarees. So, let us represent silk sarees, cotton sarees and designer sarees in terms of rectangles.</p> <p>Silk sarees: <input type="text"/></p> <p>Cotton sarees: <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/></p> <p>Designer sarees: <input type="text"/> <input type="text"/></p> <p>This helps us in deciding that we should choose silk sarees as variable x because silk sarees have the fewest number in this case. Choosing a different type of saree would create fractions throughout the problem.</p> <p>So, if the number of silk sarees= x, Then, number of cotton sarees= $4x$ And, number of designer sarees= $2x$ If we again represent the situation in terms of rectangles, we get this</p>

700 sarees		
x	$4x$	$2x$

Now, we can easily write an equation which represents the above table/diagram. Taking x as the variable here, we can create a linear equation in one variable as

$$x + 4x + 2x = 700$$

Now to find the value of x , we need to solve this equation.

$$7x = 700$$

(dividing both sides by 7),

$$\frac{7x}{7} = \frac{700}{7}$$

$$x = 100$$

The value $x = 100$ here means that there are 100 silk sarees available in the shop.

Hence, the number of cotton sarees = $4x = 4 \times 100 = 400$

And the number of designer sarees = $2x = 2 \times 100 = 200$

This way Mrs. Ratna's problem could be solved.

The teacher could give different types of problems to the students wherein they need to first write an equation and then solve it. A suggestion is given below:

Nisha has 270 channels on her cable TV, Harsh has half the number of channels as Atul. If Nisha has three times the number of channels as Atul, how many channels does Harsh have?

This problem could also be approached in a manner similar to Mrs. Ratna's problem above. Here we can denote the number of channels with Nisha and Harsh both in terms of Atul's channels.

So, let us assume that the number of channels Atul has = a

Number of channels Harsh has = $a/2$

Number of channels Nisha has = $3a$

From the problem statement, we know that there are 270 channels in Nisha's cable TV which could also be represented by the equation given below:

$$3a = 270$$

$$\text{(Dividing both sides by 3)} \frac{3a}{3} = \frac{270}{3}$$

$$a = 90$$

Therefore, number of channels Atul has = 90

Number of channels Harsh has = $\frac{a}{2} = \frac{90}{2} = 45$

Thus, there are 45 channels on Harsh's cable TV.

Reference

<https://www.engageny.org/>

2

Activity



Objective: To solve a variety of linear equations in one variable.

Materials required: Blackboard, chalk, duster, 2 set of cards with linear equations in one variable written on them.

Preparation required:

- Select around 20 linear equations in one variable which you want the students to practice
- Prepare two cards for each equation.
- For example, for linear equation

$$4x + 10 = 3(x + 10)$$

Mention the expression given in LHS in the 1st card and the RHS in 2nd card. Both the cards should be numbered the same. That means, there will be two different cards with the same number but different linear expressions.

Like, there will be two cards numbered 5 but one of them will have $4x + 10$ and the other $3(x + 10)$ written over it.

- After creating both sets of cards, shuffle them together.

Instructions:

- Put the shuffled cards in a bowl or a box and let each student randomly pick one card.
- Ensure that there are even number of students so that no student gets left out from the activity.
- After all the students have each got a card, tell the students to pair up with the person who has got the other card for the same number.
- The students along with their respective partners should solve the equation by equating the expressions given on their cards in their notebooks.
- The teacher should ideally not give more than 5 minutes for the students to solve and then take the cards back in the bowl or the box.
- Now the teacher should randomly pick out any card, check its number so that the team who solved it initially will now have to come to the blackboard and discuss their solution with the others.
- This exercise would then continue for other equations as well.

Note: the current content material is for 30 students but the teachers can always create more equations if there are more than 40 students.

Number on card	Card 1	Card 2
1	$\frac{a+1}{4}$	$\frac{a-2}{3}$
2	$0.50(8p - 10)$	$1.50 p + 16$
3	$\frac{9q+6}{6q-9}$	$-\frac{9}{12}$
4	$\frac{15y}{2y-1}$	6
5	$4x + 10$	$3(x + 10)$
6	$3x - 4$	$1 - 2x$
7	$14b + 7$	-6
8	$6h + 16$	$2h - 12$
9	$30t - 10$	$20t + 20$
10	$21 + x$	$5x - 3(2x + 1)$
11	$j - \frac{j-1}{2}$	$1 - \frac{j-2}{3}$
12	$\frac{12x - 10}{4}$	$\frac{6x + 12}{4}$
13	$\frac{3t-2}{3} + \frac{2t+3}{2}$	$t + \frac{7}{6}$
14	$4t - 3 - (3t + 1)$	$5t - 4$
15	$\frac{2q-3}{4} - \frac{3q-5}{2}$	$q + \frac{3}{4}$

Reference: https://www.educationworld.com/a_tsl/archives/07-1/lesson016.shtml

3. UNDERSTANDING QUADRILATERALS

QR Code:



Learning outcome and Learning Objectives:

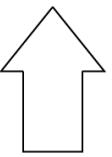
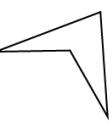
Content area / Concepts	Learning Objectives	Learning Outcome
Classification of Polygons	List the properties of a polygon in order to classify the given figures as a polygon	
	List the properties of different types of polygons in order to classify them as regular or irregular, concave or convex	
Angle sum property of polygons	Recall the angle sum property of triangle in order to extend it for quadrilaterals	Use angle sum property in order to solve problems related to angles of quadrilateral
	Relate the angle sum property of triangle and quadrilateral in order to extend it for an n-sided polygon	
	Apply angle sum property of a quadrilateral in order to find the measure of the unknown angle in a given quadrilateral	
Sum of the Measures of the Exterior Angles of a Polygon	Apply exterior angle property of a polygon in order to find the measure of the unknown angle in a given figure	
Kind of Quadrilaterals	List the properties of quadrilaterals in order to classify them as trapezium, kite and parallelogram	
Some special Parallelograms	Discuss the properties of a parallelogram in order describe the relation between its opposite sides, angles and diagonals.	Apply reasoning through activities such as constructing parallelograms, drawing their diagonals and measuring their sides and angles in order to verify properties of parallelograms
	Discuss the properties of a rhombus in order to classify it as special case of kite and parallelogram	
	Discuss the properties of a rectangle in order to show that it is a special case of parallelogram	
	Discuss the properties of a square in order to show it as special case of parallelogram, rhombus and rectangle	

Test items



LOB: List the properties of a polygon in order to classify the given figures as a polygon

- I. Consider the table below.

Figure A	Figure B	Figure C	Figure D
			

Which of these figures are polygons?

- Option 1:** Figure A and C
- Option 2:** Figure B and D
- Option 3:** Figure A, C and D
- Option 4:** Figure B, C and D

Correct Answer: Option I

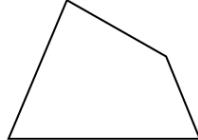
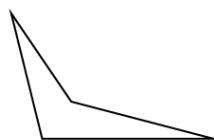
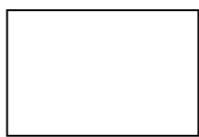
2. From each vertex of a polygon, all diagonals are drawn. If exactly nine diagonals are drawn, which polygon is drawn?

- Option 1:** Heptagon
- Option 2:** Pentagon
- Option 3:** Hexagon
- Option 4:** Octagon

Correct Answer: Option 3

LOB: List the properties of different types of polygons in order to classify them as regular or irregular, concave or convex

- I. Consider the polygons shown.



Which of these is a concave polygon?

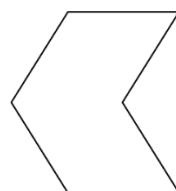
- Option 1:** Polygons W and Z
- Option 2:** Polygons W and Y
- Option 3:** Polygons X and Z
- Option 4:** Polygons W, Y and Z

Correct Answer: Option 2

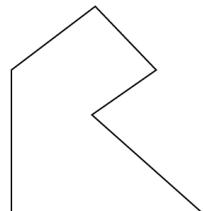
2. A student draws a polygon. The properties of the polygon are as shown.

- No diagonal lies in the exterior of the polygon.
- The length of one of the sides is twice the length of another side.

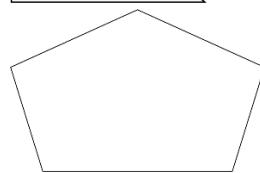
Which of these polygons could the student have drawn?



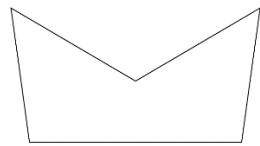
Option 1:



Option 2:



Option 3:

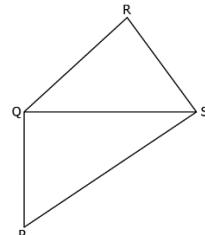


Option 4:

Correct Answer: Option 3

LOB: Recall the angle sum property of triangle in order to extend it for quadrilaterals

- I. Consider a quadrilateral PQRS shown.



Which expression represents the sum of the measures of all the angles of the quadrilateral PQRS?

Option 1: $2 \times 90^\circ$

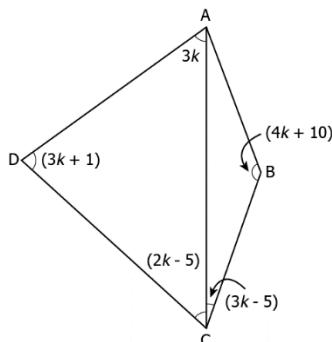
Option 2: $2 + 180^\circ$

Option 3: $2 \times 180^\circ$

Option 4: $2 + 90^\circ$

Correct Answer: Option 3

2. Consider the quadrilateral ABCD below.



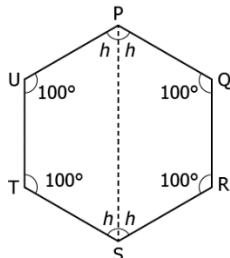
What is the measure of $\angle DAB$ of the quadrilateral ABCD?

- Option 1:** 14°
Option 2: 69°
Option 3: 83°
Option 4: 113°

Correct Answer: Option 3

LOB: Relate the angle sum property of triangle and quadrilateral in order to extend it for an n-sided polygon

- I. Consider a hexagon shown.

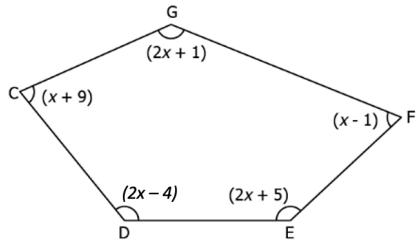


What is the measure of $\angle TSR$?

- Option 1:** 50°
Option 2: 80°
Option 3: 100°
Option 4: 160°

Correct Answer: Option 4

2. Consider a pentagon shown.



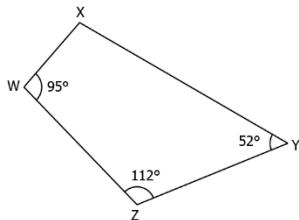
What is the measure of $\angle CGF$?

- Option 1:** 29°
Option 2: 33°
Option 3: 35°
Option 4: 37°

Correct Answer: Option 3

LOB: Apply angle sum property of a quadrilateral in order to find the measure of the unknown angle in a given quadrilateral

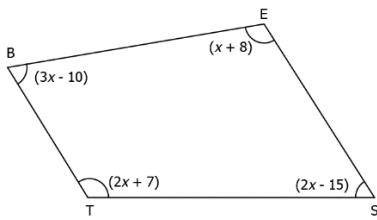
- I. What is the measure of $\angle X$ in the quadrilateral WXYZ?



- Option 1:** 101°
Option 2: 248°
Option 3: 259°
Option 4: 281°

Correct Answer: Option 1

2. Observe the quadrilateral below.



What is the measure of the smallest angle of the quadrilateral BEST?

Option 1: 54.25°

Option 2: 76.75°

Option 3: 99.5°

Option 4: 128.75°

Correct Answer: Option 1

LOB: Apply exterior angle property of a polygon in order to find the measure of the unknown angle in a given figure

- What is the measure of each exterior angle of a regular octagon?

Option 1: 22.5°

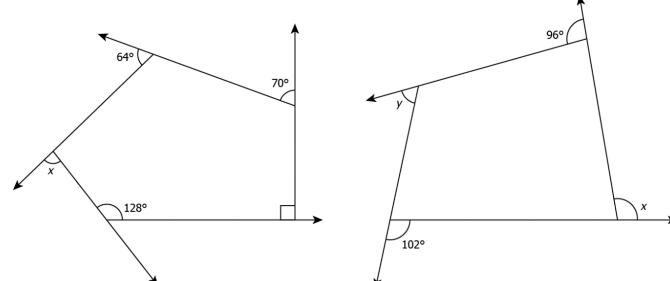
Option 2: 45°

Option 3: 67.5°

Option 4: 135°

Correct Answer: Option 2

- Consider two polygons shown.



What is the measure of y ?

Option 1: 66°

Option 2: 78°

Option 3: 84°

Option 4: 96°

Correct Answer: Option 2

LOB: List the properties of quadrilaterals in order to classify them as trapezium, kite and parallelogram

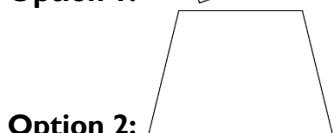
- Ankita draws a quadrilateral with properties given below.

- Two pairs of parallel sides.
- Opposite sides and opposite angles are equal.
- Diagonals of quadrilateral bisect each other.

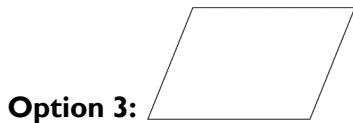
Which of these quadrilaterals she could have drawn?



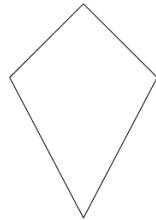
Option 1:



Option 2:



Option 3:



Option 4:

Correct Answer: Option 3

2. Two students are determining in which quadrilateral the measure of all interior angles can be calculated, given the measure of one interior angle.

Their response is as shown:

Student A: The quadrilateral will be kite.

Student B: The quadrilateral will be an isosceles trapezium.

Whose response is correct?

Option 1: Only Student A

Option 2: Only Student B

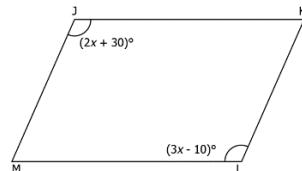
Option 3: Both the students

Option 4: Neither student A nor B

Correct Answer: Option 2

LOB: Discuss the properties of a parallelogram in order describe the relation between its opposite sides, angles and diagonals.

- I. Consider a parallelogram shown.



What is the measure of angle JML?

Option 1: 70°

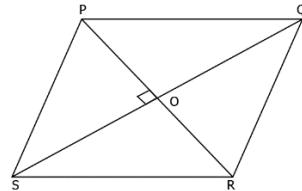
Option 2: 110°

Option 3: 125°

Option 4: 140°

Correct Answer: Option 1

2. Consider a parallelogram shown.



Given that the length of PR = 10 cm and SQ = 24 cm, what is the perimeter of the parallelogram PQRS?

Option 1: 26 cm

Option 2: 34 cm

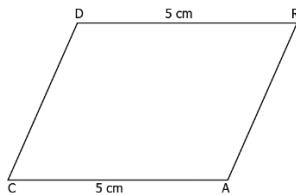
Option 3: 47 cm

Option 4: 52 cm

Correct Answer: Option 4

LOB: Discuss the properties of a rhombus in order to classify it as special case of kite and parallelogram

- I. Consider a parallelogram CARD with one pair of known sides.



For the parallelogram to be a rhombus, what should be the length of CD and AR?

Option 1: 2.5 cm

Option 2: 5 cm

Option 3: 10 cm

Option 4: 15 cm

Correct Answer: Option 2

2. Gagan draws a quadrilateral with diagonals perpendicular to each other at a point O.

What additional information is required to conclude that the quadrilateral drawn is a rhombus?

Option 1: Point O must be the midpoint of both the diagonals.

Option 2: Point O must be the midpoint of the longer diagonal.

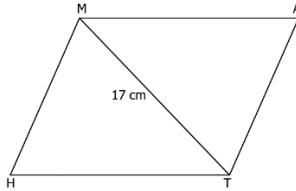
Option 3: Point O divides both the diagonals in the ratio 1:2.

Option 4: Point O must be the midpoint of the shorter diagonal.

Correct Answer: Option 1

LOB: Discuss the properties of a rectangle in order to show that it is a special case of parallelogram

- I. Consider a parallelogram MATH shown.



For the parallelogram to be a rectangle, what should be length of AH?

Option 1: equal to 17 cm

Option 2: less than 17 cm

Option 3: greater than 17 cm

Option 4: cannot be determined

Correct Answer: Option 1

2. Rajat constructs a parallelogram ABCD. What additional information does he need to construct a rectangle?

Option 1: $\angle D = 90^\circ$ and $AB = CD$

Option 2: $AC = BD$ and $AB = CD$

Option 3: $\angle D = 90^\circ$ and $AC = BD$

Option 4: $AB = BC$ and $CD = DA$

Correct Answer: Option 3

LOB: Discuss the properties of a square in order to show it as special case of parallelogram, rhombus and rectangle

- I. Which of these statements is true for a square?

Option 1: A rectangle with equal adjacent sides.

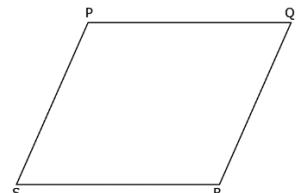
Option 2: A rectangle with 2 acute and 2 obtuse angles.

Option 3: A rectangle with diagonals of different length.

Option 4: A rectangle with diagonals bisecting each other.

Correct Answer: Option 1

2. Devika draws a parallelogram PQRS as shown below.



What additional information is required to make it a square?

Option 1:

- The measure of $\angle P = 90^\circ$.
- One pair of equal adjacent sides.
- Diagonals of equal length and are perpendicular to each other.

Option 2:

- The measure of $\angle P = 90^\circ$.
- Diagonals of equal length.

Option 3:

- The measure of $\angle P = 60^\circ$.
- One pair of equal adjacent sides.
- Diagonals of equal length and are perpendicular to each other.

Option 4:

- One pair of equal adjacent sides.
- Diagonals are perpendicular to each other.

Correct Answer: Option 1

Suggested Teacher Resources



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Lesson Plan

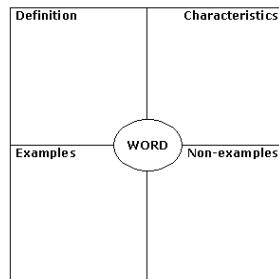


Objective	List the properties of a polygon in order to classify the given figures as a polygon								
Prerequisite	Line segments, curves, triangles								
Vocabulary words	Polygons, Quadrilaterals, simple closed curves								
Materials required	Blackboard, chalks, notebooks								
Procedure	<ul style="list-style-type: none">• The teacher will begin by drawing a set of examples and non-examples of polygons on the blackboard. A sample for the same has been given below. <table border="1"><thead><tr><th>Example of Polygons</th><th>Non-Examples</th></tr></thead><tbody><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></tbody></table> <ul style="list-style-type: none">• The teacher will tell that the figures in the first set are polygons whereas the figures in set 2 are not polygons.• The students should be given time to observe and analyse the two sets and note down any similarities within the polygons set and any differences between the two sets and come up with their own definition of a polygon.	Example of Polygons	Non-Examples						
Example of Polygons	Non-Examples								

The students will work in pairs and note down their observations. Some samples responses are given below:

- A few students might write about polygons using prior knowledge.
- A Few of them might write the differences between both of them by analysing their sides, nature of curves etc.
- In order to define the term ‘Polygon’, the teacher will draw the template (of the Frayer’s model) on the blackboard on the clean side without rubbing the examples and non- examples of polygons.

Frayer Model



- The students will make a similar table in their notebooks keeping the word ‘Polygon’ at the centre.
- The teacher will then engage the students in a discussion.

If the first set of figures (Polygons) is carefully observed, it could be seen that all the figures are closed figures whereas in set 2 not all figures are closed, some are open curves as well.

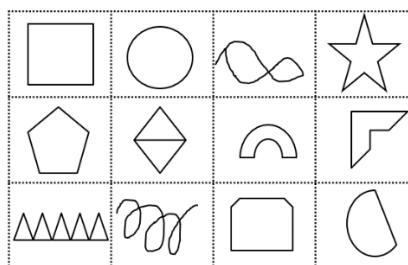
Another thing noticeable is that in set 1 (polygons) the curves do not cross each other even once hence they are simple closed curves whereas in set 2, we have such examples.

One major observation is that the figures in set 1 (polygons) are made up of only line segments only whereas the figures in set 2 show curved edges as well.

So, it could be said that any simple closed curve which is entirely made up of line segments (joined end to end), is a polygon.

- The students will have to fill in the table on their own based on the discussion
 - In the first part, the student will write the definition of Polygon in their own words.
 - In the second part, the students will write characteristics of polygons.
 - In the third part, students will give some examples of polygons.
 - In the fourth part, students will give some non- examples of polygons.

- The teacher will then summarize, and also fill in her version of the definition for the term ‘polygon’ on the board so that the students can cross check if they have noted down all the features of polygons.
- The teacher will then give examples of polygons with which the students are already familiar with like triangles, square, rectangle, hexagon, etc.
- The teacher will then show some more figures like below and ask the students to classify the given figures into polygons and non- polygons.



- Students should keep the definition of Polygons in front of them. The teacher should give a few minutes to the students for this task. Post which the teacher should ask volunteers about each figure one by one wherein volunteers have to give reasons of why the specific figure is a polygon or a non-polygon.

Suggestions

The teacher could do a similar exercise while discussing concave and convex polygons. And also, for regular and irregular polygons.

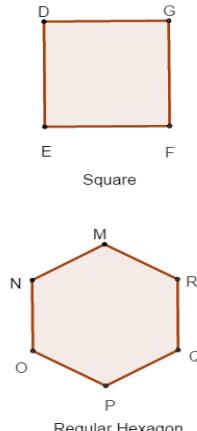
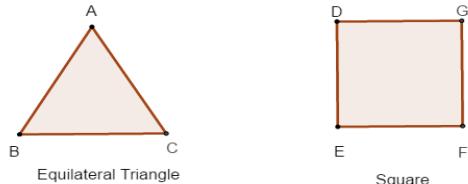
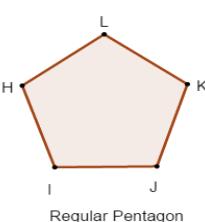
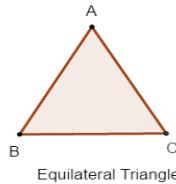
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Activity

Objective: To visualize the relation between the interior and the exterior angles in n-sided regular polygons (as the number of sides increases).

The teacher will need to arrange large cut-outs for regular polygons.

The teacher will also need sheets with different regular polygons pasted or printed on them to distribute to each student.



The teacher will begin by pasting a big chart paper on the board on which the cut-out of an enlarged equilateral triangle is pasted. She will engage the students in a discussion by asking a few questions as below. The students will be working on the sheets provided to them.

- Is this a regular polygon or an irregular polygon?
 - This is an Equilateral triangle. Hence it is a regular polygon.
- What can we say about the angles and sides of each of this polygon?
 - The measure of each angle is equal and the length of each side is equal in a regular polygon.
- What is the angle sum of a triangle?
 - The sum of the interior angles of a triangle is 180° .
- What is the measure of each interior angle of an equilateral triangle?
 - The teacher will encourage the students to measure each interior angle of the equilateral triangle on the sheets provided to them. It will come out to be 60° each.
- What is the sum of exterior angles of a polygon?
 - It is 360° .
- What is the measure of each exterior angle of an equilateral triangle?
 - The measure of each exterior angle of an equilateral triangle is 120° . The process done for interior angles will be repeated to measure the exterior angles.
- Now the teacher will take up a square, a regular pentagon and other regular polygons one by one and will repeat the same process as done for an equilateral triangle and record the measure of their interior and exterior angles.

Number of sides of the regular polygon	Measure of each interior angle	Measure of each exterior angle
3	60°	120°
4	90°	90°
5	108°	72°
6	120°	60°
:	:	:

n	$\frac{(n - 2) \times 180^\circ}{n}$	$\frac{360^\circ}{n}$
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- It will be noted that as the number of sides increases,
 - the angle sum increases as per the formula $n-2 \times 180^\circ$, where n is the number of sides.
 - the measure of each interior angle increases
 - the measure of each exterior angle decreases
- It will be observed that a regular polygon with the
 - Maximum exterior angle possible is in an equilateral triangle where each exterior angle measures 120° .
 - Minimum interior angle possible is in an equilateral triangle where each interior angle measures 60° .
- It can be an extension to encourage the students to think that if we keep on increasing the number of sides for a regular polygon, we will get a circle at some point.

4. PRACTICAL GEOMETRY

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Constructing a Quadrilateral	Discuss and list the minimum number of elements required in order to construct a unique quadrilateral	Use compasses and straight edge in order to construct a given quadrilateral
	List and execute steps of construction in order to construct a quadrilateral given the length of its four sides and a diagonal	
	List and execute steps of construction in order to construct a quadrilateral given the length of its three sides and two diagonals	
	List and execute steps of construction in order to construct a quadrilateral given the length of two adjacent sides and measures of three angles are known	
	List and execute steps of construction in order to construct a quadrilateral given the length of three sides and measures of two included angles are known	
Some Special Cases	Identify the minimum number of elements required in order to construct special cases of quadrilaterals	

Test items



LOB: Discuss and list the minimum number of elements required in order to construct a unique quadrilateral

- I. Which of the following options shows the minimum measurements required to construct a unique quadrilateral?

Option 1: Two adjacent sides and the included angle

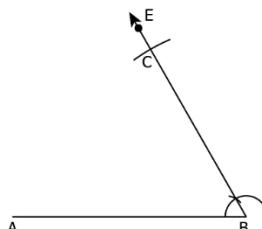
Option 2: Two adjacent sides and three angles

Option 3: Four sides and two diagonals

Option 4: Three sides and a diagonal

Correct Answer: Option 2

2. A part of construction of a quadrilateral ABCD is shown.



Which of the following options shows the minimum additional measurements required to construct the quadrilateral?

Option 1: AD and AC

Option 2: AD and CD

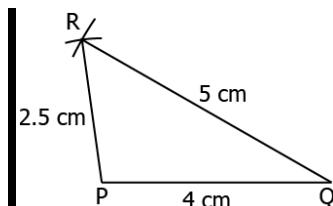
Option 3: AD, CD and $\angle A$

Option 4: AD, CD and $\angle C$

Correct Answer: Option 2

LOB: List and execute steps of construction in order to construct a quadrilateral given the length of its four sides and a diagonal

- I. A part of construction of a quadrilateral PQRS where $PQ = 4\text{ cm}$, $QR = 5\text{ cm}$, $RS = 7\text{ cm}$, $PS = 4.8\text{ cm}$ and $PR = 2.5\text{ cm}$ is shown below.



Which of the following options shows the next two steps required to construct the quadrilateral PQRS?

Option 1: Step 1: Taking R as the centre and radius equal to 7 cm, draw an arc. **Step 2:** Taking P as the centre and radius equal to 4.8 cm, draw an arc intersecting the arc drawn in previous step at S. Join PR and RS to get quadrilateral PQRS.

Option 2: Step 1: Taking P as the centre and radius equal to 7 cm, draw an arc. **Step 2:** Taking R as the centre and radius equal to 4.8 cm, draw an arc intersecting the arc drawn in previous step at S. Join PR and RS to get quadrilateral PQRS.

Option 3: Step 1: Taking Q as the centre and radius equal to 7 cm, draw an arc.

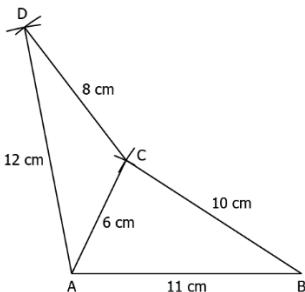
Step 2: Taking R as the centre and radius equal to 4.8 cm, draw an arc intersecting the arc drawn in previous step at S. Join PR and RS to get quadrilateral PQRS.

Option 4: Step 1: Taking R as the centre and radius equal to 7 cm, draw an arc.

Step 2: Taking Q as the centre and radius equal to 4.8 cm, draw an arc intersecting the arc drawn in previous step at S. Join PR and RS to get quadrilateral PQRS.

Correct Answer: Option 1

2. Mrinal constructs a quadrilateral ABCD where $AB = 11$ cm, $BC = 10$ cm, $CD = 8$ cm, $AD = 12$ cm and diagonal $AC = 6$ cm.



Is his construction correct?

Option 1: No, because in triangle ABC; $AB + BC > AC$, $AB + AC > BC$, $BC + AC > AB$ and in triangle ACD; $CD + AD > AC$, $CD + AC > AD$, $AC + AD > CD$.

Option 2: Yes, because in triangle ABC; $AB + BC > AC$, $AB + AC > BC$, $BC + AC > AB$ and in triangle ACD; $CD + AD > AC$, $CD + AC > AD$, $AC + AD > CD$.

Option 3: No, because $AB + BC + AC < AD + CD + AC$.

Option 4: Yes, because $AB + BC + AC < AD + CD + AC$.

Correct Answer: Option 2

LOB: List and execute steps of construction in order to construct a quadrilateral given the length of its three sides and two diagonals

- I. The first three steps to construct a quadrilateral WXYZ where $XY = 9$ cm, $WZ = 11$ cm, $YZ = 10$ cm, diagonal $WY = 11$ cm and diagonal $XZ = 14$ cm are listed below.

Step 1: Draw $WY = 11$ cm.

Step 2: Taking Y as the centre and radius equal to 10 cm, draw an arc.

Step 3: Taking W as the centre and radius equal to 11 cm, draw an arc intersecting the arc drawn in step 2 at Z. Join WZ and YZ.

Which option correctly shows the next two steps required to complete the construction?

Option 1: Step 4: Taking Y as the centre and radius equal to 14 cm, draw an arc opposite to Z.

Step 5: Taking Z as the centre and radius equal to 9 cm, draw an arc intersecting the arc drawn in step 4 at X. Join WX, XY and XZ to get the required quadrilateral WXYZ.

Option 2: Step 4: Taking W as the centre and radius equal to 9 cm, draw an arc opposite to Z.

Step 5: Taking Z as the centre and radius equal to 14 cm, draw an arc intersecting the arc drawn in step 4 at X. Join WX, XY and XZ to get the required quadrilateral WXYZ.

Option 3: Step 4: Taking Y as the centre and radius equal to 9 cm, draw an arc opposite to Z.

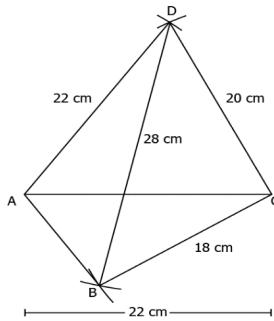
Step 5: Taking Z as the centre and radius equal to 14 cm, draw an arc intersecting the arc drawn in step 4 at X. Join WX, XY and XZ to get the required quadrilateral WXYZ.

Option 4: Step 4: Taking W as the centre and radius equal to 14 cm, draw an arc opposite to Z.

Step 5: Taking Z as the centre and radius equal to 9 cm, draw an arc intersecting the arc drawn in step 4 at X. Join WX, XY and XZ to get the required quadrilateral WXYZ.

Correct Answer: Option 3

2. Anamika constructs a quadrilateral ABCD where $BC = 18$ cm, $AD = 22$ cm, $CD = 20$ cm, diagonal $AC = 22$ cm and diagonal $BD = 28$ cm.



Which of the following options justifies that her construction is correct?

Option 1: $AC = AD$

Option 2: $BC + AD + CD > AC + BD$

Option 3: In triangle BCD; $BC + CD \neq BD$, $CD + BD \neq BC$, $BD + BC \neq CD$ and in triangle ACD; $AC + AD \neq CD$, $AC + CD \neq AD$, $CD + AD \neq AC$

Option 4: In triangle BCD; $BC + CD > BD$, $CD + BD > BC$, $BD + BC > CD$ and in triangle ACD; $AC + AD > CD$, $AC + CD > AD$, $CD + AD > AC$

Correct Answer: Option 4

LOB: List and execute steps of construction in order to construct a quadrilateral given the length of two adjacent sides and measures of three angles are known

1. **Question Text:** A teacher asked her students to construct a quadrilateral ABCD with $AB = 12 \text{ cm}$, $BC = 9 \text{ cm}$, $\angle A = 60^\circ$, $\angle B = 105^\circ$ and $\angle C = 105^\circ$. A student performed the following steps of construction.

Step 1: Draw $AB = 12 \text{ cm}$.

Step 2: Taking B as the centre, draw $\angle ABE = 105^\circ$.

Step 3: Taking A as the centre and radius equal to 9 cm, draw an arc intersecting BE at C.

Step 4: Taking A as the centre, draw $\angle BAF = 105^\circ$.

Step 5: Taking C as the centre, draw $\angle BCG = 60^\circ$.

The student made an error while constructing the quadrilateral. In which step did the student make the first error?

Option 1: Step 2

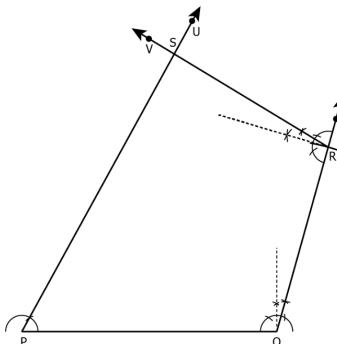
Option 2: Step 3

Option 3: Step 4

Option 4: Step 5

Correct Answer: Option 2

2. Richa constructed a quadrilateral PQRS based on the information given to her.



What information was provided to her?

Option 1: Measures of PQ , QR , $\angle P$, $\angle Q$ and $\angle R$

Option 2: Measures of PQ , $\angle P$, $\angle Q$, $\angle R$ and $\angle S$

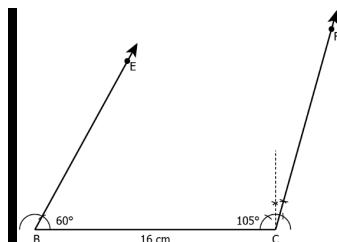
Option 3: Measures of PQ , QR , RS , PS and PR

Option 4: Measures of PQ , QR , RS , PS and QS

Correct Answer: Option 1

LOB: List and execute steps of construction in order to construct a quadrilateral given the length of three sides and measures of two included angles are

- I. A part of construction of a quadrilateral ABCD where $AB = 11 \text{ cm}$, $BC = 16 \text{ cm}$, $CD = 9 \text{ cm}$, $\angle B = 60^\circ$ and $\angle C = 105^\circ$ is shown below.



Which of these options shows the next three steps to complete the construction?

Option 1: Step 4: Taking B as the centre and radius equal to 11 cm, draw an arc intersecting BE at A.

Step 5: Taking C as the centre and radius equal to 9 cm, draw an arc intersecting CF at D.

Step 6: Join AD to get the required quadrilateral ABCD.

Option 2: Step 4: Taking B as the centre and radius equal to 9 cm, draw an arc intersecting BE at A.

Step 5: Taking C as the centre and radius equal to 11 cm, draw an arc intersecting CF at D.

Step 6: Join AD to get the required quadrilateral ABCD.

Option 3: Step 4: Taking F as the centre and radius equal to 11 cm, draw an arc intersecting BE at A.

Step 5: Taking E as the centre and radius equal to 9 cm, draw an arc intersecting CF at D.

Step 6: Join AD to get the required quadrilateral ABCD.

Option 4: Step 4: Taking F as the centre and radius equal to 9 cm, draw an arc intersecting BE at A.

Step 5: Taking E as the centre and radius equal to 11 cm, draw an arc intersecting CF at D.

Step 6: Join AD to get the required quadrilateral ABCD.

Correct Answer: Option 1

2. The steps to construct a quadrilateral WXYZ where $WX = 7 \text{ cm}$, $XY = 9 \text{ cm}$, $YZ = 12 \text{ cm}$, $\angle X = 105^\circ$ and $\angle Y = 75^\circ$ in a random order are as follows.

Step 1: Draw $XY = 9 \text{ cm}$.

Step 2: Taking Y as the centre, draw $\angle XYS = 75^\circ$.

Step 3: Taking X as the centre and radius equal to 7 cm, draw an arc intersecting XT at W.

Step 4: Taking X as the centre, draw $\angle YXT = 105^\circ$.

Step 5: Taking Y as the centre and radius equal to 12 cm, draw an arc intersecting YS at Z.

Step 6: Join WZ to get the required quadrilateral WXYZ.

Which two steps should be interchanged so that the construction becomes possible?

Vinay: Step 3 and Step 4

Chetan: Step 3 and Step 5

Who is/are correct?

Option 1: Only Vinay

Option 2: Only Chetan

Option 3: Both Vinay and Chetan

Option 4: Neither Vinay nor Chetan

Correct Answer: Option 3

LOB: Identify the minimum number of elements required in order to construct special cases of quadrilaterals

- I. What is the minimum number of elements required to construct a unique rectangle?

Option 1: One

Option 2: Two

Option 3: Three

Option 4: Four

Correct Answer: Option 2

2. Nikhil has to draw a quadrilateral in which each pair of the consecutive sides is congruent, but the opposite sides are not congruent. What is the minimum number of elements will he require to construct the quadrilateral?

Option 1: One

Option 2: Two

Option 3: Three

Option 4: Four

Correct Answer: Option 3

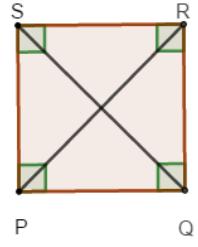
Suggested Teacher Resources



I

Lesson Plan

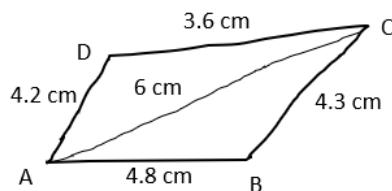


Objectives	List and execute steps of construction in order to construct a quadrilateral given the length of its four sides and a diagonal.
Prerequisite Knowledge	How to use a compass and a scale.
Material Required	<ul style="list-style-type: none">Chalk and Blackboard, A Big sized compass preferably from the Math lab for the teacher (to be able to demonstrate the construction on board)Pen/Pencil and notebook.Compass and scale for students.
Procedure	<p>Start the class by drawing a rough quadrilateral on the board. For instance, draw a square PQRS.</p>  <p>Discuss that a quadrilateral has ten parts in all: four sides, four angles and two diagonals. In square PQRS,</p> <ol style="list-style-type: none">Four sides: PQ, QR, RS and SPFour angles: $\angle P$, $\angle Q$, $\angle R$ and $\angle S$. In this case all angles are right angles.Two diagonals: PR and QS <p>Note: The teacher could take any general quadrilateral for the above discussion as well.</p> <p>To construct a quadrilateral, we need the measures of five specified. These are:</p> <ol style="list-style-type: none">When the length of four sides and a diagonal are givenWhen the length of two diagonals and three sides are givenWhen length of two adjacent sides and measures of three angles are given and,When the length of three sides and the measures of two angles between them are given. <p>The measures given should also satisfy the properties: Triangle inequality and the angle sum property of a quadrilateral. Only then the quadrilateral given could be drawn. In addition, to these four cases, special cases of parallelograms could be constructed with information of less than five measurements.</p> <p>Now the teacher could begin by discussing the first case and focus on the construction of a quadrilateral when we are given the length of four sides and one diagonal and demonstrate this construction using a compass, scale and chalk on the board.</p> <p>Sample question:</p>

Construct a quadrilateral ABCD in which $AB = 4.8 \text{ cm}$, $BC = 4.3 \text{ cm}$, $CD = 3.6 \text{ cm}$, $AD = 4.2 \text{ cm}$ and diagonal $AC = 6 \text{ cm}$.

It could be seen that here we have got five measures, i.e. Four sides and one diagonal. Before the actual construction, the rough figure is to be constructed.

Drawing a rough figure helps in avoiding mistakes during construction.



Now, we could see that the given quadrilateral ABCD satisfies the triangle inequality.

In $\triangle ABC$

$$AB + BC > AC$$

$$BC + AC > AB$$

$$AC + AB > BC$$

In $\triangle ADC$

$$AD + DC > AC$$

$$DC + AC > AD$$

$$AD + AC > DC$$

So now, the teacher would demonstrate the construction step by step on the board.

Steps of Construction:

Step 1: Since we have been given the length of all four sides, we can start the construction by taking any one of the four sides as the base.

For instance, we will start the construction by taking AB as the base, i.e., Draw $AB = 4.8 \text{ cm}$ by using a scale.

Step 2: Now, since we have started with AB, the $\triangle ABC$ should be constructed first. We will have to locate the point C.

We have been given that $AC = 6 \text{ cm}$, so using a compass and scale take a radius equal to 6cm and with A as center, draw an arc.

Step 3: With B as center and radius equal to 4.3 cm, draw another arc which intersects the previous arc drawn from point A. Name this point of intersection as C.

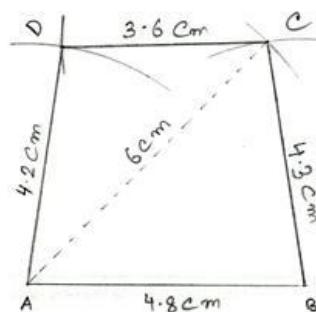
Step 4: Join AC and BC. We have now constructed the $\triangle ABC$.

Step 5: Now we have to construct $\triangle ADC$. We know that $AD = 4.2 \text{ cm}$ is given so, with A as center and radius equal to 4.2 cm, draw an arc.

Step 6: We know that $DC = 3.6 \text{ cm}$ is given so, with C as center and radius equal to 3.6 cm, draw another arc, which intersects the previous arc at point D.

Step 7: Join AD and CD.

Thus, ABCD is the required quadrilateral.



Note:

- i. The teacher would be required to note down the steps of construction on board.
- ii. The usage of the tools of construction at each step is to be carefully explained during the demonstration.

After the demonstration, the teacher would give some time to attend any queries.

The teacher would give another construction for the students to do in the class.
Construct a quadrilateral HEAR where HE=3.5cm, EA=4cm, AR=5cm, RH=4.5cm and ER=6.5cm.

2

Activity



Objective: List and execute steps of construction in order to construct a quadrilateral of the given measurements.

Materials required: Geometry box for students, chits, a bowl, drawing sheets, A4- sized white sheets.

Instructions:

1. The teacher will divide the class into six teams and will name them as Team A, Team B, Team C, Team D, Team E and Team F.
2. The teacher will prepare two chits each for the following four cases:
 - a. When the length of four sides and a diagonal are given
 - b. When the length of two diagonals and three sides are given
 - c. When length of two adjacent sides and measures of three angles are given and,
 - d. When the length of three sides and the measures of two angles between them are given.
3. One volunteer from each team will come to choose a chit in order to select the particular case of quadrilateral construction. The chits will be placed in a bowl.
4. Each team is required to give a name of the quadrilateral that they will construct. For instance, Team A declares that they will construct a quadrilateral named JUMP.
5. Each team will have to construct a quadrilateral as per the measures given by the other teams.
 - a. Suppose, Team A got the second case, i.e., when the length of two diagonals and three sides are given.
 - b. The other teams will one by one provide the measures required to construct the quadrilateral JUMP.
 - c. For instance, Team B could say take $JU=5\text{cm}$, Team C could say take $UM=4\text{cm}$, Team D could say take $MP=6\text{cm}$, Team E could say take $JM=6\text{cm}$ and Team F could say take $PU=5\text{cm}$.
 - d. Team A has to note down these measures.
 - e. Each team has to ensure that the measures/condition they give to others don't result in an invalid case of construction.
 - f. The teacher and the Team who has to do the construction, need to keep a check on the 5 measures being given.
 - g. The same process of getting the measurements will be repeated for all the teams.
6. After all the six teams have gotten the 5 measurements, only then all teams will start with the construction.
7. Each team will have to construct their quadrilateral on a drawing sheet or A4 sized white sheet. They also need to write down the steps of construction on another sheet.
8. The teams need to finish the task within 10 minutes. After that, two volunteers from each team will come forward and explain their steps of construction to the other teams.
9. All the while, the teacher will sort queries, if any.

5. DATA HANDLING

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Looking for Information	Recall the different types of graphical representation (namely pictograph, bar graph and double bar graph) of data in order to represent the given data in the most suitable representation and interpret them	
Organising raw data	Use tally marks in order to organise the given raw data in a frequency distribution table	
Grouping data	Use tally marks in order to prepare a grouped frequency distribution table for large ungrouped data	Draw and interpret bar graphs and pie charts in order to answer a variety of questions based on them
	Construct histogram in order to represent the given grouped data	
	Discuss the elements of the given histogram in order to interpret it	
Circle graph or Pie Chart	List and execute steps of construction in order to construct a circle graph	
	Read a given circle graph in order to infer a variety of information from it	
Chance and Probability	List all the possible outcomes of an experiment in order to define the equally likely outcomes	Conduct activities in order to makes hypotheses on chances of future events on the basis of its earlier occurrences or available data like, after repeated throws of dice and coins
	List all the possible outcomes of an event in order to calculate the probability of a given event	

Test items



LOB: Recall the different types of graphical representation (namely pictograph, bar graph and double bar graph) of data in order to represent the given data in the most suitable representation and interpret them

- I. Which of these can be represented by a double bar graph?

Option 1: The marks obtained by a student in different subjects of an exam.

Option 2: The number of visitors entering a museum every hour from 10 am to 5 pm.

Option 3: The percentage of budget allocated to different departments of an office.

Option 4: The number of girls and boys in different classes of a school.

Correct Answer: Option 4

2. Which of these graphs will be the best measure to represent the marks obtained by 5 students in two Mathematics tests and why?

Option 1: Bar graph; because it shows the relationship or comparison among discrete categories.

Option 2: Double bar graph; because it shows the relationship or comparison among two groups.

Option 3: Bar graph; because it shows the frequency of data divided into equal groups.

Option 4: Double bar graph; because it shows the frequency of data divided into equal groups.

Correct Answer: Option 2

LOB: Use tally marks in order to organize the given raw data in a frequency distribution table

- I. **Question Text:** Consider the data collected from students of a class to choose the language they prefer to study.

French, German, Spanish, Sanskrit, German, German, French, French, Spanish, Sanskrit, French, Sanskrit, Spanish, German, French, German, German, Sanskrit, German, Sanskrit, Sanskrit, Spanish, German, French, German, French, French, German, Sanskrit, Spanish

Which of these tables correctly represents the given data?

Option 1:

Languages	Number of Students
French	
German	
Spanish	
Sanskrit	

Option 2:

Languages	Number of Students
French	
German	
Spanish	
Sanskrit	

Option 3:

Languages	Number of Students
French	
German	
Spanish	
Sanskrit	

Option 4:

Languages	Number of Students
French	
German	
Spanish	
Sanskrit	

Correct Answer: Option 1

2. A study was conducted to determine the favourite sports of 20 students of a class. The results are:
- Cricket is the favourite sport of most students
 - Football is the least favourite sport among students
 - Badminton is more liked by the students than Basketball

Based on the given information, which of these can be the correct tally marks representing the given data?

Sports	Number of Students
Badminton	
Cricket	
Basketball	
Football	

Option 1:

Sports	Number of Students
Badminton	
Cricket	
Basketball	
Football	

Option 2:

Sports	Number of Students
Badminton	
Cricket	
Basketball	
Football	

Option 3:

Sports	Number of Students
Badminton	
Cricket	
Basketball	
Football	

Option 4:

Sports	Number of Students
Badminton	
Cricket	
Basketball	
Football	

Correct Answer: Option 4

LOB: Use tally marks in order to prepare a grouped frequency distribution table for large ungrouped data

- I. Consider the time taken (in seconds) by 30 students of a class to finish a 100 m race.

17.39, 16.50, 18.10, 17.20, 18.55, 19.45, 15.55, 15.20, 17.45, 19.20, 16.30, 18.58, 16.45, 18.30, 16.38, 16.45, 18.35, 16.52, 17.45, 17.20, 19.20, 16.40, 15.10, 15.40, 19.50, 18.20, 16.20, 16.58, 18.50, 19.58, 16.30

Which of these tables correctly represents the given data?

Time Taken (in seconds)	Number of Students
15 - 16	
16 - 17	
17 - 18	
18 - 19	
19 - 20	

Option 1:

Time Taken (in seconds)	Number of Students
15 - 16	
16 - 17	
17 - 18	
18 - 19	
19 - 20	

Option 2:

Time Taken (in seconds)	Number of Students
15 - 16	
16 - 17	
17 - 18	
18 - 19	
19 - 20	

Option 3:

Time Taken (in seconds)	Number of Students
15 - 16	
16 - 17	
17 - 18	
18 - 19	
19 - 20	

Option 4:

Correct Answer: Option 1

2. Sheena collected the data to find the average time spent by an employee to commute to the office with 25 employees. She finds that:

- highest number of employees take less than 60 minutes to commute
- fewest number of employees take more than 90 minutes to commute
- there are fewer number of employees who take less than 30 minutes to commute than the number of employees who take 60 minutes or more but less than 90 minutes.

Which of these tables correctly represents the data?

Time Taken (in seconds)	Number of Employees
0 - 30	
30 - 60	
60 - 90	
90 - 120	

Option 1:

Time Taken (in seconds)	Number of Employees
0 - 30	
30 - 60	
60 - 90	
90 - 120	

Option 2:

Time Taken (in seconds)	Number of Employees
0 - 30	
30 - 60	
60 - 90	
90 - 120	

Option 3:

Time Taken (in seconds)	Number of Employees
0 - 30	
30 - 60	
60 - 90	
90 - 120	

Option 4:

Correct Answer: Option 2

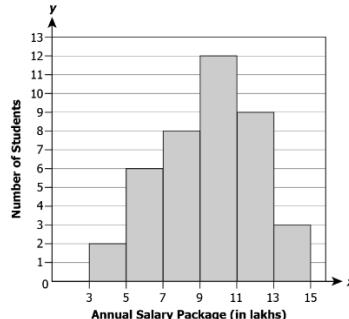
LOB: Construct histogram in order to represent the given grouped data

- I. The average salary package taken by students of a recent batch of 30 students in a college is as shown.

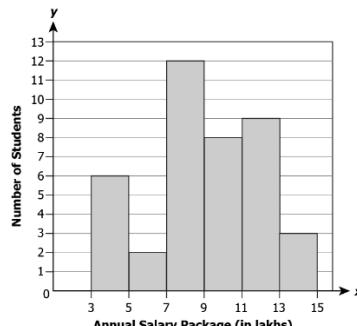
Annual salary package (in Lakhs)	Number of Students
3-5	2
5-7	6
7-9	12
9-11	8

11-13	9
13-15	3

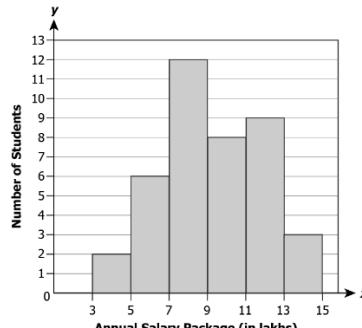
Which of these correctly represents the given data?



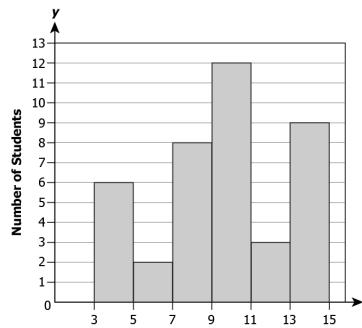
Option 1:



Option 2:



Option 3:



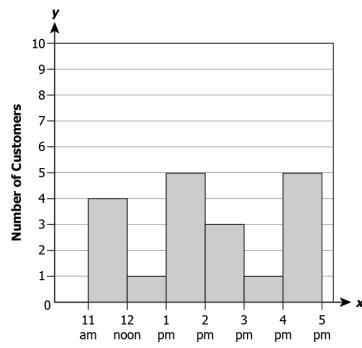
Option 4:

Correct Answer: Option 3

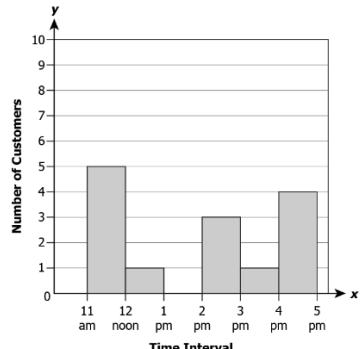
2. Consider the information for the number of calls received by a customer care executive between 11 am to 5 pm.

- most of the calls were received in the last hour of the day
- there are two instances in which the number of calls received per hour was the same
- No call was received in between 1 pm to 2 pm.

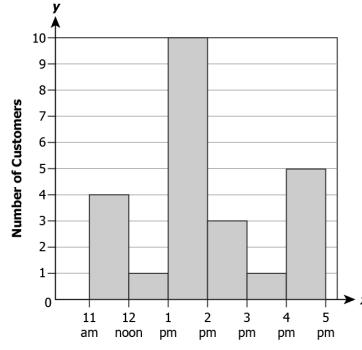
Based on the given information, which of these graphs correctly represents the data?



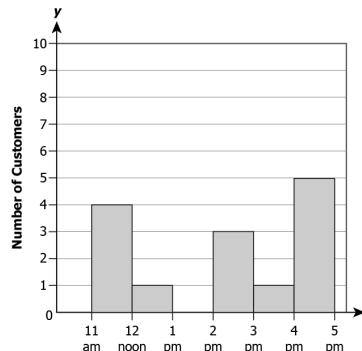
Option 1:



Option 2:



Option 3:

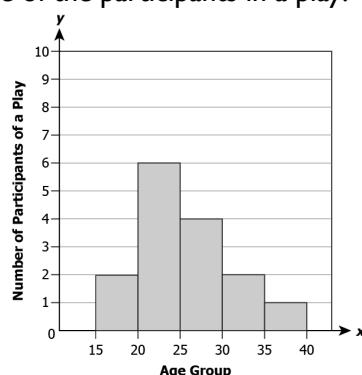


Option 4:

Correct Answer: Option 4

LOB: Discuss the elements of the given histogram in order to interpret it

- I. The histogram below shows the age of the participants in a play.



How many participants are below the age of 25 years?

Option 1: 10

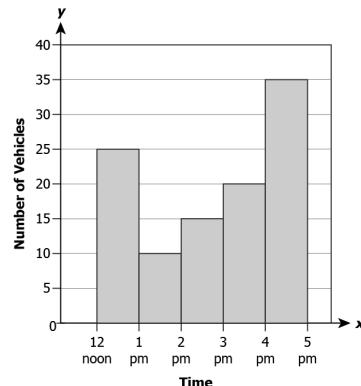
Option 2: 8

Option 3: 6

Option 4: 4

Correct Answer: Option 2

2. The histogram shown below represents the number of vehicles passing through a toll bridge each hour on a particular day.



Which of these is true about the given histogram?

Option 1: the number of vehicles passing through the toll before 3 pm is 50% of the number of vehicles passing through the toll after 3 pm

Option 2: the number of vehicles passing through the toll before 2 pm is 50% of the number of vehicles passing through the toll after 2 pm.

Option 3: the number of vehicles passing through the toll before 2 pm is equal to the number of vehicles passing through the toll after 2 pm

Option 4: the number of vehicles passing through the toll before 3 pm is equal to the number of vehicles passing through the toll after 3 pm

Correct Answer: Option 2

LOB: List and execute steps of construction in order to construct a circle graph

- I. The table below shows different pets and the number of students that own the pet.

Pets	Number of Students
Dog	12
Cat	8
Hamster	3
Rabbit	5

Akhil wants to represent the data in a circle graph, where each sector of the graph represents number of students that own a pet. Which of these is not a step to construct a circle graph?

Option 1: Find the total number of students.

Option 2: Find the fraction or percentage that each sector represents.

Option 3: Finding the degree the central angle of each sector.

Option 4: Finding the area of each sector.

Correct Answer: Option 4

2. The table below shows the number of different flowers sold by a florist on a day.

Flowers	Number of Flowers Sold
Rose	30
Tulip	10
Hibiscus	15
Lilly	25

Akriti represents the data on a circle graph in which each sector represents the number of sales of the flowers. Which of these represents the angle of each sector?

Option 1:

Sector	Angle
Rose	135°

Tulip	45°
Hibiscus	67.5°
Lilly	112.5°

Option 2:

Sector	Angle
Rose	135°
Tulip	45°
Hibiscus	112.5°
Lilly	67.5°

Option 3:

Sector	Angle
Rose	45°
Tulip	135°
Hibiscus	67.5°
Lilly	112.5°

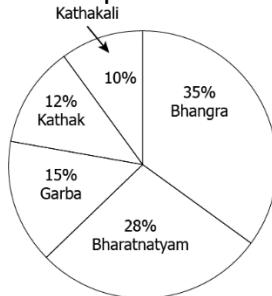
Option 4:

Sector	Angle
Rose	45°
Tulip	135°
Hibiscus	112.5°
Lilly	67.5°

Correct Answer: Option 1

LOB: Read a given circle graph in order to infer a variety of information from it

- I. The circle graph below shows the dance forms preferences of 300 students of a school.



How many students prefers to learn Bharatanatyam?

Option 1: 84

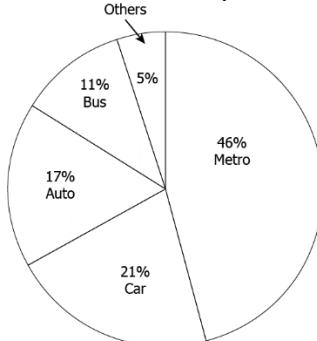
Option 2: 36

Option 3: 48

Option 4: 16

Correct Answer: Option 3

2. The data was collected to determine the mode of transport used by employees of an office as shown.



If 66 people commute by bus, how many more employees prefers to commute by metro than car?

Option 1: 600

Option 2: 276

Option 3: 165

Option 4: 150

Correct Answer: Option 4

LOB: List all the possible outcomes of an experiment in order to deduce that each of the outcomes has the same chance of occurring

1. A bag has 3 identical black balls marked with Alphabets A to C on it. If two balls are selected at random, which of these shows the possible outcomes?

Option 1: {AB, AC, BA, BC, CA, CB, AA, BB, CC}

Option 2: {AB, BC, CA}

Option 3: {AB, AC, BA, BC, CA, CB}

Option 4: {A, B, C}

Correct Answer: Option 3

2. Two spinners each divided into three sectors such that each sector is marked with number 1 to 3. The sample space is:

{11, 12, 13, 21, 22, 23, 31, 32, 33}

Consider the conditions below.

Condition 1: Each sector must be equal in area.

Condition 2: Spinners should be identical.

Which of these is required to prove that each outcome is equally likely?

Option 1: Only Condition 1

Option 2: Only Condition 2

Option 3: Both Condition 1 and 2

Option 4: Neither Condition 1 nor Condition 2

Correct Answer: Option 3

LOB: List all the possible outcomes of an event in order to calculate the probability of a given event

1. Jagat has 15 cards marked with 1 to 15 on it. Which of these is the list of all possible outcomes of getting a card with a number divisible by 3?

Option 1: {3, 6, 9, 12}

Option 2: {3}

Option 3: {3, 6, 9, 12, 15}

Option 4: {5, 10, 15}

Correct Answer: Option 3

2. A bag contains some identical balls. Out of those, 45% are blue, 30% are yellow and rest are red in colour. If a ball is picked at random, what is the probability that it is not a red ball?

Option 1: $\frac{1}{4}$

Option 2: $\frac{3}{4}$

Option 3: $\frac{1}{3}$

Option 4: $\frac{2}{3}$

Correct Answer: Option 2

Suggested Teacher Resources



I

Lesson Plan



Objectives	Construct histogram in order to represent the given data in a graph.																												
Prerequisite Knowledge	Knowledge of Tally marks, frequency, scale and intervals.																												
Material Required	Graph paper.																												
Time Required	45 minutes																												
Procedure	<p>Start the class with the following real-life problem to test the pre-requisite knowledge of the students-</p> <p>Frequency Table with Intervals</p> <p>The boys' and girls' basketball teams at your School wanted to raise money to help buy new uniforms. They decided to sell baseball caps with the school logo to family members and other interested fans. To obtain the correct cap size, students had to measure the head circumference (distance around the head) of the adults who wanted to order a cap. The following data set represents the head circumferences, in millimetres (mm), of the adults.</p> <p>513, 525, 531, 533, 535, 535, 542, 543, 546, 549, 551, 552, 552, 553, 554, 555, 560, 561, 563, 563, 563, 565, 565, 568, 568, 571, 571, 574, 577, 580, 583, 583, 584, 585, 591, 595, 598, 603, 612, 618</p> <p>The caps come in six sizes: XS, S, M, L, XL, and XXL. Each cap size covers an interval of head circumferences. The cap manufacturer gave students the table below that shows the interval of head circumferences for each cap size. The interval 510– 530 represents head circumferences from 510 mm to 530 mm, not including 530.</p> <table border="1"><thead><tr><th>Cap Sizes</th><th>Interval of Head Circumferences (millimetres)</th><th>Tally</th><th>Frequency</th></tr></thead><tbody><tr><td>XS</td><td>510 - 530</td><td></td><td></td></tr><tr><td>S</td><td>530 - 550</td><td></td><td></td></tr><tr><td>M</td><td>550 - 570</td><td></td><td></td></tr><tr><td>L</td><td>570 - 590</td><td></td><td></td></tr><tr><td>XL</td><td>590 - 610</td><td></td><td></td></tr><tr><td>XXL</td><td>610 - 630</td><td></td><td></td></tr></tbody></table> <p>Now ask the students to complete the following table. It will look something like this-</p>	Cap Sizes	Interval of Head Circumferences (millimetres)	Tally	Frequency	XS	510 - 530			S	530 - 550			M	550 - 570			L	570 - 590			XL	590 - 610			XXL	610 - 630		
Cap Sizes	Interval of Head Circumferences (millimetres)	Tally	Frequency																										
XS	510 - 530																												
S	530 - 550																												
M	550 - 570																												
L	570 - 590																												
XL	590 - 610																												
XXL	610 - 630																												

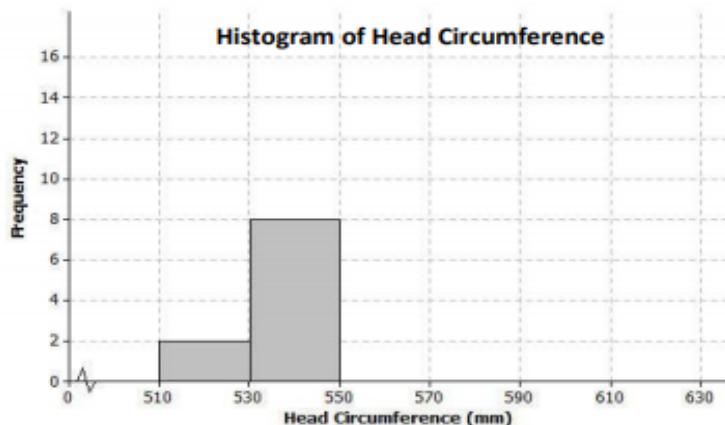
Cap Sizes	Interval of Head Circumferences (millimetres)	Tally	Frequency
XS	510-530		2
S	530-550	III	8
M	550-570	-	15
L	570-590	IIII	9
XL	590-610		4
XXL	610-630		2

Now help the students to construct a histogram using the following instructions-

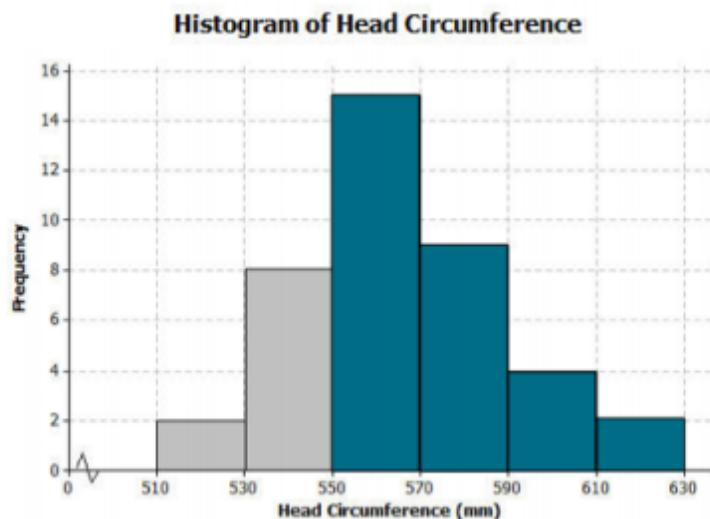
To make a histogram:

- Draw a horizontal line (x axis), and mark the intervals.
- Draw a vertical line (y axis), and label its Frequency.
- Mark the Frequency axis with a scale that starts at 0 and goes up to few levels up which are greater than the largest frequency in the frequency table.
- Draw a jagged line or broken line (it means, we are not showing the number between 0 and 510)
- on the x axis and start marking head circumference starting from 510 till the highest given head circumference with suitable given interval.
- For each interval, draw a bar over that interval that has a height equal to the frequency for that interval.

Demonstrate by drawing the first two bars of the histogram –



Now ask the students to complete the Histogram. It will look something like this



Based on Histogram, ask the following questions from students and drive the given answers through logical reasoning-

Which two intervals have the same frequency?

Which interval has the highest frequency?

What is the difference between the highest and the lowest head circumference interval?

What is the sum of frequency of head circumference 550mm and above?

What is the sum of frequency of head circumference 590mm and below?

How many head circumference intervals have frequency more than 4?

Source	https://www.engageny.org/file/45621/download/math-g6-m6-topic-a-lesson-4-teacher.pdf?token=eRh_Gq3k
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2

Activity



Objective: Students will be able to construct histograms.

Time Required: 60 minutes

Instructions:

- Teacher starts the activity by discussing the Corona pandemic, causes, symptoms, toll on the entire world etc.
- Teacher will then take the students to the computer lab and will ask them to surf the given information over web and complete the task:
Take the data of top 5 countries which are most affected by corona till 31st March 2020. Make 3 different histograms depicting the information mentioned below for each country:

- i) Number of People affected by the virus.
- ii) Number of people died.
- iii) Number of people recovered.

Choose an appropriate scale to construct the histograms.

Follow Up Question:

Combine all the above three graphs with suitable scale. And represent all the three lines with different colours.

6. SQUARES AND SQUARE ROOTS

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Properties of Square Numbers	<p>Define perfect squares in order to classify the given numbers as perfect squares or non-perfect squares</p> <p>Observe the number in order to find the unit place of its square</p> <p>Observe different number patterns in order to deduce square numbers</p> <p>Use the rule that there are exactly $2n$ non-perfect square numbers between the squares of the number n and $(n+1)$ in order to find how many numbers, lie between the squares of the given two consecutive numbers</p>	
Finding the Square of a Number	<p>Use the rule that a perfect square number (n^2) can be written as the sum of first n odd natural numbers in order to distinguish between square and non-square numbers</p> <p>Use Pythagoras theorem in order to find the Pythagorean triplet</p>	
Square Roots	<p>Apply inverse operations on a given perfect square in order to deduce square root of this number</p> <p>Use method of repeated subtraction in order to find the square root of the given square number</p> <p>Use prime factorization method in order to find the square root of the given perfect square</p> <p>Use prime factorization method in order to determine whether the given number is a perfect square or not</p> <p>Use prime factorization method in order to find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect square and then find the square root of the new number</p> <p>Use long division method in order to find the square root of the given perfect square number</p> <p>Use long division method in order to find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect square and then find the square root of the new number</p>	Apply different methods in order to find the squares, cubes, square roots and cube roots of a given number
Square Roots of Decimals	Use long division method in order to find the square root of the given decimal number	
Estimating Square Root	Use estimation in order to approximate the value of the square root of the given number to the nearest whole	

Test items



LOB: Define perfect squares in order to classify the given numbers as perfect squares or non-perfect squares

- Which of these is a non-perfect square number?

Option 1: 25

Option 2: 49

Option 3: 81

Option 4: 90

Correct Answer: Option 4

- Consider the given set of numbers.

{576, 615, 784, 961, 972, 1024, 1225, 1396}

Which of these options correctly classifies the numbers as perfect and non-perfect square numbers?

Option 1:

Perfect square numbers	Nonperfect square numbers
615, 972, 1396	576, 784, 961, 1024, 1225

Option 2:

Perfect square numbers	Nonperfect square numbers
576, 784, 1024, 1225	615, 961, 972, 1396

Option 3:

Perfect square numbers	Nonperfect square numbers
576, 784, 961, 1024, 1225	615, 972, 1396

Option 4:

Perfect square numbers	Nonperfect square numbers
615, 972, 1024, 1396	576, 784, 961, 1225

Correct Answer: Option 3

E0M0602: Observe the number in order to find the unit place of its square

- Which of these is the digit at unit's place in square of 1458?

Option 1: 1

Option 2: 4

Option 3: 5

Option 4: 6

Correct Answer: Option 2

- The digit at unit's place of a square of a number is 2 more than the digit at unit place of that number. Which of these can be the digit at the unit's place of the square of the number?

Option 1: 1

Option 2: 3

Option 3: 5

Option 4: 9

Correct Answer: Option 4

LOB: Observe different number patterns in order to deduce square numbers

- Which of these will give a square number?

Option 1: * + ***

*

**

Option 2: ** + ****

*

**

Option 3: *** + *****

*

**

Option 4: ** + *****

*

Correct Answer: Option 3

2. Consider a pattern shown.

$$4^2 = 16$$

$$34^2 = 1156$$

$$334^2 = 111556$$

$$3334^2 = 11115556$$

Which of these will never be a part of the pattern?

$$33334^2 = 1111155556$$

$$333334^2 = 111111555556$$

Option 1: $3333334^2 = 11111115555556$

$$333334^2 = 111111555556$$

$$3333334^2 = 11111115555556$$

Option 2: $33333334^2 = 1111111155555556$

$$3333334^2 = 11111115555556$$

$$33333334^2 = 1111111155555556$$

Option 3: $333333334^2 = 111111111555555556$

$$33333334^2 = 1111111155555556$$

$$333333334^2 = 111111111555555556$$

Option 4: $3333333334^2 = 11111111115555555556$

Correct Answer: Option 4

LOB: Use the rule that there are exactly $2n$ non perfect square numbers between the squares of the number n and $(n+1)$ in order to find how many numbers, lie between the squares of the given two consecutive numbers

- I. How many non-perfect square numbers lie between 8^2 and 9^2 ?

Option 1: 15

Option 2: 16

Option 3: 18

Option 4: 19

Correct Answer: Option 2

2. Consider the three consecutive numbers a , b and c . If there are 54 non-perfect square numbers between a and b , how many non-perfect square numbers lie between the numbers b and c ?

Option 1: 54

Option 2: 56

Option 3: 58

Option 4: 59

Correct Answer: Option 2

LOB: Use the rule that a perfect square number (n^2) can be written as the sum of first n odd natural numbers in order to distinguish between square and non-square numbers

- I. Which of these can be expressed as the sum of first 8 odd natural numbers?

Option 1: 4^2

Option 2: 8^2

Option 3: 9^2

Option 4: 16^2

Correct Answer: Option 2

2. Consider the expression below.

$1 + 3 + 5 + 7 \dots \dots + (2k + 1)$, where k is a natural number

Which of these options describes the given expression?

Option 1: The sum of the given expression is $(k + 1)^2$.

Option 2: The sum of the given expression is $(k - 1)^2$.

Option 3: The sum of the given expression is k^2 .

Option 4: The sum of the given expression is $(2k + 1)^2$.

Correct Answer: Option 1

LOB: Use Pythagoras theorem in order to find the Pythagorean triplet

- I. If one member of the Pythagorean triplet is 10, which of these are the remaining members of the triplet?

Option 1: 3 and 8

Option 2: 6 and 8

Option 3: 18 and 19

Option 4: 35 and 36

Correct Answer: Option 2

2. The product of the numbers 14 and 16 is determined by using $(p + 1)(p - 1) = p^2 - 1$, where p is one of the members of the Pythagorean triplet which is neither the largest nor the smallest. Which of the following represents all the members of the Pythagorean triplet?

Option 1: 4, 15 and 17

Option 2: 8, 15 and 17

Option 3: 14, 15 and 16

Option 4: 8, 63 and 64

Correct Answer: Option 2

LOB: Apply inverse operations on a given perfect square in order to deduce square root of this number

- I. What is the square root of 144?

Option 1: 6

Option 2: 12

Option 3: 24

Option 4: 72

Correct Answer: Option 2

2. An integer k is obtained by multiplying a number c by itself. Consider the two statements about the square root of k .

Statement I: The square root of k is c if $c > 0$.

Statement II: The square root of k is $-c$ if $c < 0$.

Which of these statements is correct?

Option 1: Only Statement I

Option 2: Only Statement II

Option 3: Both Statements I and II

Option 4: Neither Statement I nor II

Correct Answer: Option 3

LOB: Use method of repeated subtraction in order to find the square root of the given square number

- I. Some steps to find the square root of a number are shown below.

- $49 - 1 = 48$
- $48 - 3 = 45$
- $45 - 5 = 40$
- $40 - 7 = 33$
- $33 - 9 = 24$
- $24 - 11 = 13$
- $13 - 13 = 0$

What is the square root of the number?

Option 1: 7

Option 2: 13

Option 3: 25

Option 4: 49

Correct Answer: Option 1

2. The steps to determine the square root of a number m are shown below.

$$\begin{aligned}m - 1 &= m - 1 \\m - 1 - 3 &= m - 4 \\m - 4 - 5 &= m - 9 \\m - 9 - 7 &= m - 16 \\\cdot & \\\cdot & \\m - 81 - 19 &= 0\end{aligned}$$

Which of these options shows the value of m and its square root?

Option 1: $m = 25$ and $\sqrt{m} = 5$

Option 2: $m = 81$ and $\sqrt{m} = 9$

Option 3: $m = 100$ and $\sqrt{m} = 10$

Option 4: $m = 121$ and $\sqrt{m} = 11$

Correct Answer: Option 3

LOB: Use prime factorization method in order to find the square root of the given perfect square

- I. The prime factorisation of a number is as shown.

$$2 \times 2 \times 3 \times 3 \times 3 \times 3$$

What is the square root of the number?

Option 1: 6

Option 2: 8

Option 3: 18

Option 4: 22

Correct Answer: Option 3

2. A number d is such that it's prime factorisation consists of 12 prime numbers. Which of these shows the prime factorisation of the square root of the number?

Option 1: $2 \times 3 \times 5$

Option 2: $2 \times 2 \times 3 \times 5$

Option 3: $2 \times 2 \times 2 \times 3 \times 3 \times 5$

Option 4: $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$

Correct Answer: Option 3

LOB: Use prime factorization method in order to determine whether the given number is a perfect square or not

- I. Which of these is a prime factorisation of a square number?

Option 1: $2 \times 3 \times 5 \times 5$

Option 2: $3 \times 3 \times 4 \times 4 \times 5 \times 5$

Option 3: $2 \times 2 \times 3 \times 5 \times 5 \times 7 \times 7$

Option 4: $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 11 \times 11$

Correct Answer: Option 4

2. The prime factorization of an even square number is as shown.

$$3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 7 \times p$$

What is the smallest value of p ?

Option 1: 2

Option 2: 4

Option 3: 14

Option 4: 28

Correct Answer: Option 4

LOB: Use prime factorization method in order to find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect square and then find the square root of the new number

- I. Which number should be multiplied by 108 such that it becomes a perfect square number?

Option 1: 2

Option 2: 3

Option 3: 5

Option 4: 6

Correct Answer: Option 2

2. A number n is written as $p^2q^4r^3$, where p , q and r are the prime factors. Which expression represents a perfect square number and what is the square root of the number?

Option 1: $\frac{p^2q^4r^3}{r}$ and $\sqrt{\frac{p^2q^4r^3}{r}} = pq^2r$

Option 2: $\frac{p^2q^4r^3}{p}$ and $\sqrt{\frac{p^2q^4r^3}{p}} = pq^2r^2$

Option 3: $p^2q^4r^3 \times p$ and $\sqrt{p^2q^4r^3 \times p} = p^2q^2r$

Option 4: $p^2q^4r^3 \times q$ and $\sqrt{p^2q^4r^3 \times q} = pq^2r$

Correct Answer: Option 1

LOB: Use long division method in order to find the square root of the given perfect square number

- I. A part of work to find the square root of 1089 is shown below.

$$\begin{array}{r} 3 \\ 3 \overline{)1089} \\ -9 \\ \hline 1 \end{array}$$

Which of the following shows the next step?

$$\begin{array}{r} 32 \\ 3 \overline{)1089} \\ -9 \\ \hline 189 \\ -184 \\ \hline 5 \end{array}$$

Option 1:

$$\begin{array}{r} 32 \\ 3 \overline{)1089} \\ -9 \\ \hline 189 \\ -136 \\ \hline 53 \end{array}$$

Option 2:

$$\begin{array}{r} 32 \\ 3 \overline{)1089} \\ -9 \\ \hline 189 \\ -134 \\ \hline 55 \end{array}$$

Option 3:

$$\begin{array}{r} 33 \\ 3 \overline{)1089} \\ -9 \\ \hline 189 \\ -189 \\ \hline 0 \end{array}$$

Option 4:

Correct Answer: Option 4

2. The length and breadth of a rectangular field are 80 m and 39 m respectively. What is the length of the diagonal of the field, by using the long division method?

Option 1: 11 m

Option 2: 55 m

Option 3: 89 m

Option 4: 97 m

Correct Answer: Option 3

LOB: Use long division method in order to find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect square and then find the square root of the new number

- I. What is the least number that must be subtracted from 1170 to get a perfect square?

Option 1: 9

Option 2: 14

Option 3: 34

Option 4: 64

Correct Answer: Option 2

2. A square number is added to 2976 to make it a perfect square. What are the square roots of the square number that is added and the perfect square that is formed?

Option 1: 7, 55

Option 2: 7, 54

Option 3: 12, 56

Option 4: 12, 55

Correct Answer: Option 1

LOB: Use long division method in order to find the square root of the given decimal number

- I. Which of these shows the square root of 53.29?

$$\begin{array}{r} 7.5 \\ 7 \overline{)53.29} \\ -49 \\ \hline 429 \\ -375 \\ \hline 54 \end{array}$$

Option 1:

$$\begin{array}{r} 7.1 \\ \hline 53.29 \\ -49 \\ \hline 429 \\ -429 \\ \hline 0 \end{array}$$

Option 2:

$$\begin{array}{r} 7.3 \\ \hline 53.29 \\ -49 \\ \hline 493 \\ -493 \\ \hline 64 \end{array}$$

Option 3:

$$\begin{array}{r} 7.3 \\ \hline 53.29 \\ -49 \\ \hline 143 \\ -429 \\ \hline 0 \end{array}$$

Option 4:

Correct Answer: Option 4

2. The diagonals of a rhombus are 6.6 m and 11.2 m. What is the side length of the rhombus?

Option 1: 2.9 m

Option 2: 4.3 cm

Option 3: 6.5 m

Option 4: 8.9 m

Correct Answer: Option 3

LOB: Use estimation in order to approximate the value of the square root of the given number to the nearest whole

- I. What is the estimated square root of 750?

Option 1: 27

Option 2: 28

Option 3: 35

Option 4: 37

Correct Answer: Option 1

2. What is the greatest number whose estimated square root is 17?

Option 1: 273

Option 2: 289

Option 3: 306

Option 4: 307

Correct Answer: Option 3

Suggested Teacher Resources



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Lesson Plan

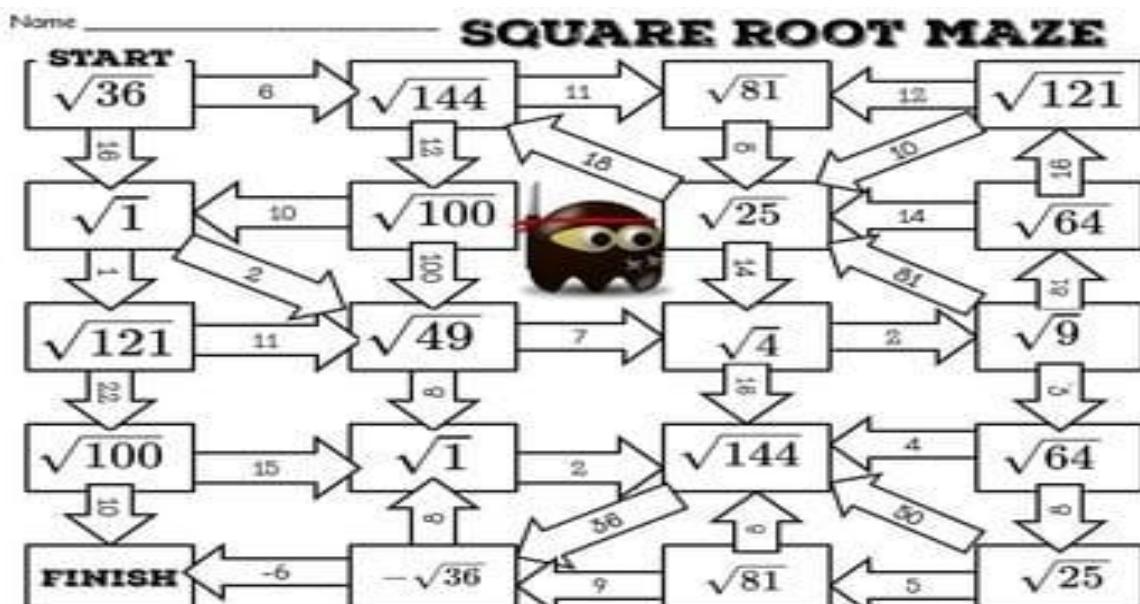


Objectives	Define perfect square in order to classify given number as perfect squares or non-perfect squares.																				
Prerequisite Knowledge	Multiplication and division																				
Material Required	Chart paper																				
Procedure	<ul style="list-style-type: none">Teacher will give small pieces of paper cut in square shape to the group of 4 students.Teacher will then display a table depicting the side of a square and the number of square pieces used to make that particular square.Then we will ask students to calculate how many square pieces it takes them to make the pattern of the given side of a square.Table will look like this. <table border="1"><thead><tr><th>Side of a Square</th><th>No. of square pieces used</th></tr></thead><tbody><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>9</td></tr><tr><td>4</td><td>16</td></tr><tr><td>5</td><td>25</td></tr></tbody></table> <p>Questions to be asked :-</p> <ol style="list-style-type: none">How is the number of square pieces related to its side of the square? Identify the pattern.Can you find the side of a square if it has 26 pieces of square? <p>Follow up questions:</p> <p>After students understand the concept then give the following question to solve:-</p> <ol style="list-style-type: none">Find the number of tiles if its side is 9cm?Find the length of the pool if its area is 100 m^2?Check whether 121 is perfect or no perfect square.Create a perfect square table till 13. <p>Expected Outcome:</p> <p>Students will be able to understand the concept of perfect and non-perfect squares and square root numbers</p> <p>Assessment:</p> <p>A society collected Rs 9216. Each number collected as many Rs as there were members. How many members were there and how much did each contribute? Find which one is perfect or non-perfect squares.</p> <table border="1"><tbody><tr><td>25</td><td>37</td><td>144</td><td>300</td></tr><tr><td>500</td><td>225</td><td>289</td><td>48</td></tr></tbody></table>	Side of a Square	No. of square pieces used	1	1	2	4	3	9	4	16	5	25	25	37	144	300	500	225	289	48
Side of a Square	No. of square pieces used																				
1	1																				
2	4																				
3	9																				
4	16																				
5	25																				
25	37	144	300																		
500	225	289	48																		



Objective: Students will be able to calculate the square and square roots of the numbers

Material Required: Activity Sheet-Square Root Maze



Time Required :- 40 Mins:-

Instructions:-

1. Teacher will divide the students into groups of 4.
 2. Each group will be given one activity sheet and a token.
 3. Each group will put their token at the starting point of their sheet. Their task will be to start from the starting point and keep on solving the square root so as to move their token to the finish line of the activity sheet.
 4. Teacher will hold a timer for each group. The group which reaches the finish point first will win.

Follow up questions:-

- Follow up questions :

 - Which of the following is a square of an even number?
a. 144 b. 169 c. 441 d. 62
 - Add the square of 12 and 5 and then find the square root of it.
 - Sum of squares of natural numbers from 1 to 20.

Source: Chalk lit application and Pinterest

7. CUBES AND CUBE ROOTS

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Cubes	Define perfect cube/cube number in order to classify the given numbers as cube numbers or non-cube numbers	Apply different methods in order to find the squares, cubes, square roots and cube roots of a given number
	Observe the pattern of cube of even numbers in order to generalise that cubes of even numbers are even	
	Observe the pattern of cube of numbers with one's digit as 1, 2, 3, 4... etc. in order to explore the one's digit of their perfect cubes and comment on it	
	Add n consecutive odd numbers in order to get the sum equal to n^3	
	Use prime factorization in order to rule out a number as a perfect cube	
	Use prime factorization on the given number in order to find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect cube	
Cube Roots	Use prime factorization in order to find the cube root of a given number	
	Use estimation in order to find the cube root of a given perfect cube	

Test items



LOB: Define perfect cube/cube number in order to classify the given numbers as cube numbers or non-cube numbers

1. Is 343 a perfect cube?

Option 1: Yes, because 3 is in one's place.
Option 2: No, because it is an odd number.
Option 3: No, because the sum of the digits is not a perfect cube.
Option 4: Yes, because all its prime factors can be grouped in triples.

Correct Answer: Option 4

2. Consider the given set of numbers.

{215, 625, 841, 1000, 1331, 2744, 12321, 27000, 35937, 64000}

Which option correctly classifies the given numbers as cube numbers and non-cube numbers?

Option 1: There are 6 cube numbers and 4 non-cube numbers in the given set.
Option 2: There are 5 cube numbers and 5 non-cube numbers in the given set.
Option 3: There are 4 cube numbers and 6 non-cube numbers in the given set.
Option 4: There are 3 cube numbers and 7 non-cube numbers in the given set.

Correct Answer: Option 1

LOB: Observe the pattern of cube of even numbers in order to generalize that cubes of even numbers are even

1. Observe the pattern shown below.

8, 64, 216, 512, ...

Based on the pattern, which statement is true?

Option 1: Cubes of all odd numbers are odd.
Option 2: Cubes of all even numbers are even.
Option 3: Cubes of all odd numbers end only in 1, 3, 5 and 7.
Option 4: Cubes of all even numbers end only in 2, 4, 6 and 8.

Correct Answer: Option 2

2. If c is an even number, which statement is correct about the value of the expression $[c + (c + 2) + (c + 4) + (c + 6) + \dots]^3$?

Option 1: For any value of c , the value of the expression will be an odd number.
Option 2: For any value of c , the value of the expression will be a prime number.
Option 3: For any value of c , the value of the expression will be an even number.
Option 4: For any value of c , the value of the expression will be an irrational number.

Correct Answer: Option 3

LOB: Observe the pattern of cube of numbers with one's digit as 1, 2, 3, 4, etc. in order to explore the one's digit of their perfect cubes and comment on it

1. Which digit comes in one's place in the cube of 17325483?

Option 1: 1
Option 2: 3
Option 3: 7
Option 4: 9

Correct Answer: Option 3

2. Which digit comes in one's place in the result of the expression $(21476 + 31520 + 41305 + 50697)^3$?

Option 1: 2 because $6 + 0 + 5 + 7 = 18$ and $8 \times 8 \times 8 = 512$
Option 2: 2 because $2 + 3 + 4 + 5 = 14$ and $4 + 4 + 4 = 12$
Option 3: 4 because $2 + 3 + 4 + 5 = 14$ and $4 \times 4 \times 4 = 64$
Option 4: 4 because $6 + 0 + 5 + 7 = 18$ and $8 + 8 + 8 = 24$

Correct Answer: Option 1

LOB: Add n consecutive odd numbers in order to get the sum equal to n^3

- I. How many consecutive odd numbers must be added to get the sum equal to 24^3 ?

Option 1: 2

Option 2: 4

Option 3: 20

Option 4: 24

Correct Answer: Option 4

2. Which of the following expressions can be used to find the value of $[(43 + 45 + 47 + 49 + 51 + 53 + 55) - (31 + 33 + 35 + 37 + 39 + 41)]$?

Option 1: $1 \times 6 \times 5 + 3$

Option 2: $1 \times 6 \times 5 \times 3$

Option 3: $1 \times 7 \times 6 + 3$

Option 4: $1 \times 7 \times 6 \times 3$

Correct Answer: Option 4

LOB: Use prime factorization in order to rule out a number as a perfect cube

- I. Which of the following expressions will not yield a perfect cube?

Option 1: $2 \times 2 \times 2 \times 3 \times 3 \times 3$

Option 2: $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 5 \times 5 \times 5$

Option 3: $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 3 \times 3 \times 3 \times 3$

Option 4: $8 \times 8 \times 8 \times 4 \times 4 \times 4 \times 5 \times 5 \times 5 \times 5 \times 5$

Correct Answer: Option 3

2. Nimish claims that -1728 is not a perfect cube number. Which of the following options justifies his claim?

Option 1: Nimish's claim is incorrect because each prime factor of -1728 appears three times.

Option 2: Nimish's claim is correct because 8 is in one's place which is not divisible by 3.

Option 3: Nimish's claim is incorrect because $1 + 7 + 2 + 8 = 18$ which is divisible by 3.

Option 4: Nimish's claim is correct because -1728 is a negative integer.

Correct Answer: Option 1

LOB: Use prime factorization on the given number in order to find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect cube

- I. Which smallest number should be multiplied to 2592 so that the product is a perfect cube?

Option 1: 2

Option 2: 3

Option 3: 12

Option 4: 18

Correct Answer: Option 4

2. If $k = (m \times n)^4$ where m and n are the prime factors of k , which expression will result in a perfect cube nearest to k with only factors m and n ?

Option 1: $k \div (m \times n)$

Option 2: $k + (m \times n)$

Option 3: $k \times (m \times n)^2$

Option 4: $k - (m \times n)^2$

Correct Answer: Option 1

LOB: Use prime factorization in order to find the cube root of a given number

- I. Which of the following options correctly shows the cube root of 13824 ?

Option 1: $13824 = \underline{2} \times \underline{3} \times \underline{3} \times \underline{3} = 2 \times 2 \times 2 \times 2 \times 6 \times 9 = 864$

Option 2: $13824 = \underline{2} \times \underline{3} \times \underline{3} \times \underline{3} = 2 + 2 + 2 + 6 + 9 = 23$

Option 3: $13824 = \underline{2} \times \underline{3} \times \underline{3} \times \underline{3} = 2 \times 2 \times 2 \times 3 = 24$

Option 4: $13824 = \underline{2} \times \underline{3} \times \underline{3} \times \underline{3} = 2 + 2 + 2 + 3 = 9$

Correct Answer: Option 3

2. If s and m are two perfect cube numbers such that $s = (a \times b)^6$ and $m = \frac{s}{8}$, which expression can be used to represent the cube root of m ?

Option 1: $\frac{ab}{2}$

Option 2: $\frac{ab}{4}$

Option 3: $\frac{(ab)^2}{2}$

Option 4: $\frac{(ab)^2}{4}$

Correct Answer: Option 3

LOB: Use estimation in order to find the cube root of a given perfect cube

1. What is the estimated cube root of 50653?

Option 1: 37

Option 2: 50

Option 3: 53

Option 4: 65

Correct Answer: Option 1

2. What is the cube root of 857375?

Option 1: Cube root of 857375 is 85 because 8 is the leftmost digit and 5 is the rightmost digit in 857375.

Option 2: Cube root of 857375 is 98 because 8 is the greatest digit in 857375 and $9^3 < 857 < 10^3$.

Option 3: Cube root of 857375 is 95 because 5 is the unit digit in 857375 and $9^3 < 857 < 10^3$.

Option 4: Cube root of 857375 is 75 because both 5 and 7 are repeated in 857375.

Correct Answer: Option 3

Suggested Teacher Resources



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Lesson Plan



Objective	Prime factorise a given number in order to find the smallest number to be operated (all the four arithmetic operations) on given number to get a perfect cube.						
Prerequisite	Prime factorisation, Multiplication and division						
Vocabulary words	Prime factorization						
Materials required	Cubical blocks of side 1 unit						
Procedure	<ul style="list-style-type: none">Teacher will introduce the cube root topic with the help of an activity of making a cube with the help of cubical blocks of side 1 unit.Teacher will divide the group accordingly with the given resources and the class strength.Each group will be given cube blocks and they have to make two to three different cubes with the help of those cubical blocks.Most of the students will be able to make a cube consisting of 8 and 27 cubical blocks.Then we will ask them to write its side(number of cubical blocks on one side) on the left column and its number of cube blocks used to make it on the right column.The table will look like this: <table border="1"><thead><tr><th>Cube of the Side</th><th>No. of cubical blocks used</th></tr></thead><tbody><tr><td>2units</td><td>8</td></tr><tr><td>3units</td><td>27</td></tr></tbody></table> <ul style="list-style-type: none">Students will be able to conclude that the meaning of a cube is multiplying the number itself three times.	Cube of the Side	No. of cubical blocks used	2units	8	3units	27
Cube of the Side	No. of cubical blocks used						
2units	8						
3units	27						

2

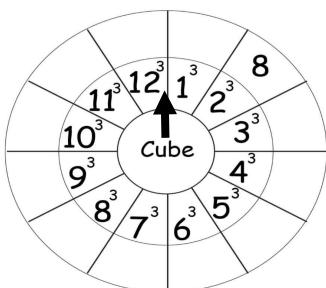
Activity



Objective: Students will be able to calculate the cubes and cube roots of the numbers

Material Required: Number Wheel

Cubes



Time Required: 45 minutes

Instructions:

1. Teacher will present the number wheel in front of the class and divide the students in 12 groups.
2. Each group will come and spin the wheel. Group will find the cube of the number which will be indicated by the arrow after the spin.
3. Teacher will hold a timer for each group. The group which find the cube earliest, will win.

Follow Up Questions:

1. Write cubes of all-natural numbers between 1 and 20 and verify the following statements:

- a) Cubes of all odd natural numbers are odd
- b) Cubes of all even natural numbers are even

2. Write cubes of 5 natural numbers which are multiples of 3 and verify the following:

'The cube of natural number, which is a multiple of 3 is a multiple of 27'

8. COMPARING QUANTITIES

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Recalling Ratios and Percentages	Convert ratios to percentage in order to solve the given questions	
Discount, Profit, Loss	Apply the formula for discount and discount percentage in order to solve the given problem on discount	Observe a given context in order to apply the concepts of profit and loss, discount, VAT, simple and compound interest
	Calculate the discount in given situations in order to comment whether the seller has made a profit/loss in the given transaction	
Simple Interest and Compound Interest	Define and compare simple interest and compound interest in order to comment on the situations where either of the two are applied	
	Calculate the simple interest in order to find the total amount to be paid by the debtor	
Deducing a Formula for Compound Interest	Use formula of simple interest in order to deduce the formula to calculate the compound interest	
	Calculate the compound interest in order to find the total amount to be paid by the debtor	
Rate Compounded Annually or Half Yearly (Semi Annually)	Define the terms 'compounded annually', 'compounded half yearly' and 'compounded quarterly' and give examples in order to differentiate between the three	
Applications of Compound Interest Formula	Use formula of compound interest in order to solve problems related to increase (or decrease) in population	
	Use formula of compound interest in order to solve problems related to increase (or decrease) in the price of an item in intermediate years	

Test items



LOB: Convert ratios to percentage in order to solve the given questions

- I. Which percentage is equivalent to $\frac{18}{25}$?

Option 1: 18%

Option 2: 36%

Option 3: 72%

Option 4: 78%

Correct Answer: Option 3

2. The table below shows the marks scored by Anuj in different subjects.

Subject	Marks scored	Total Marks
English	22	25
Science	52	60
Hindi	55	100
Mathematics	68	75

Of the marks given, in which subject he scored the highest percentage of marks?

Option 1: Hindi

Option 2: English

Option 3: Science

Option 4: Mathematics

Correct Answer: Option 4

LOB: Apply the formula for discount and discount percentage in order to solve the given problem on discount

- I. The marked price of a bottle is ₹100 and the discount percent is 25%. What is the sale price of the bottle?

Option 1: ₹125

Option 2: ₹105

Option 3: ₹75

Option 4: ₹25

Correct Answer: Option 3

2. The table below shows the total marked price of the books of different sections that Ria bought.

Genre	Marked Price (in ₹)
Science	₹1500
Horror	₹1850
Biography	₹500

If there is a discount of 28% on all the sections of books, what is the total amount that she has to pay?

Option 1: ₹1078

Option 2: ₹962.50

Option 3: ₹2772

Option 4: ₹2887.50

Correct Answer: Option 3

LOB: Calculate the discount in given situations in order to comment whether the seller has made a profit/loss in the given transaction

- I. The cost of making two tables is ₹4,500. A store owner marked the price of each table as ₹2,500 and sold each table at a discount 2%. Which of these is correct about the given situation?

Option 1: The store owner gave a discount of ₹100 on both the tables and incurred loss of ₹100.

- Option 2:** The store owner gave a discount of ₹50 on both the tables and incurred loss of ₹400.
Option 3: The store owner gave a discount of ₹50 on both the tables and incurred profit of ₹100.
Option 4: The store owner gave a discount of ₹100 on both the tables and incurred profit of ₹400

Correct Answer: Option 4

2. A shopkeeper bought 10 machines for ₹7,50,000 from a wholesale store. He marked the price of each machine as ₹80,000. If he sold each machine after a discount of 10%, which of these statements is correct?

- Option 1:** The shopkeeper has made a loss of ₹30,000 after selling the machines.
Option 2: The shopkeeper has made a loss of ₹80,000 after selling the machines.
Option 3: The shopkeeper has made a profit of ₹30,000 after selling the machines.
Option 4: The shopkeeper has made a profit of ₹80,000 after selling the machines.

Correct Answer: Option 1

LOB: Define and compare simple interest and compound interest in order to comment on the situations where either of the two are applied

1. Arvind borrows a sum of ₹2,50,000 at 6% p.a. simple interest for 3 years. On the same day, Asif borrows the same sum on the same rate of interest but compounded annually for 3 years. Who pays more interest and by how much?

- Option 1:** Asif will have to pay ₹2,754 more than Arvind as interest.
Option 2: Asif will have to pay ₹7,454 more than Arvind as interest.
Option 3: Arvind will have to pay ₹2,754 more than Asif as interest.
Option 4: Arvind will have to pay ₹7,454 more than Asif as interest.

Correct Answer: Option 1

2. Adil borrowed ₹4,00,000 from his friend Ankur at 5% per annum simple interest for 1 year 6 months. Had Adil borrowed this sum from a bank at 8% per annum compounded annually for 3 years, what extra amount would Adil have to pay?

- Option 1:** The extra amount that Adil have to pay is ₹30,000.
Option 2: The extra amount that Adil have to pay is ₹33,568.
Option 3: The extra amount that Adil have to pay is ₹45,557.40.
Option 4: The extra amount that Adil have to pay is ₹73,884.80.

Correct Answer: Option 4

LOB: Calculate the simple interest in order to find the total amount to be paid by the debtor

1. Arun lend ₹2,88,000 to his friend at the rate of 12% for 9 months. How much interest will he get?

- Option 1:** ₹25,920
Option 2: ₹34,560
Option 3: ₹3,11,040
Option 4: ₹3,13,920

Correct Answer: Option 1

2. Akash borrowed ₹18,000 at the rate of 3% p.a. for the first three years, 6% p.a. for the next two years and 9% p.a. for the period beyond five years. How much amount does Akash has to pay at the end of 7 years?

- Option 1:** ₹7,020
Option 2: ₹10,980
Option 3: ₹25,020
Option 4: ₹33,980

Correct Answer: Option 3

LOB: Use formula of simple interest in order to deduce the formula to calculate the compound interest

1. A principal gives ₹3,200 as simple interest at the rate of 10% p.a. in 2 years. What will be the compound interest at the rate of 5% p.a. for 3 years on the same principal?

- Option 1:** ₹800
Option 2: ₹2,522
Option 3: ₹18,522
Option 4: ₹25,056

Correct Answer: Option 2

2. The simple interest on a certain sum of money for 3 years at 5% per annum is ₹540. Which of these is the compound interest on that sum at the same rate for the same period?

Option 1: ₹56745

Option 2: ₹98245

Option 3: ₹312545

Option 4: ₹416745

Correct Answer: Option 1

LOB: Calculate the compound interest in order to find the total amount to be paid by the debtor

- I. Prakash borrows a sum of ₹35,000 at 5% per annum compounded annually for 24 months. How much amount will he pay?

Option 1: ₹1750

Option 2: ₹3587.50

Option 3: ₹37895

Option 4: ₹38587.50

Correct Answer: Option 4

2. What will be the amount of ₹35,000 after 2 years 6 months, compounded annually, the rate of interest being 2% per annum during the first year, 5% per annum during the second year and 1.5% per annum during the last 6 months of the period?

Option 1: ₹13930.40

Option 2: ₹35894.80

Option 3: ₹37766.14

Option 4: ₹187860.80

Correct Answer: Option 3

LOB: Define the terms 'compounded annually', 'compounded half yearly' and 'compounded quarterly' and give examples in order to differentiate between the three

- I. The table below shows 4 formulas to calculate compound interest.

Formula A	$C.I. = P \left[\left(1 + \frac{r}{100}\right)^{2n} - 1 \right]$
Formula B	$C.I. = P \left[\left(1 + \frac{r}{100}\right)^n - 1 \right]$
Formula C	$C.I. = P \left[\left(1 + \frac{r}{400}\right)^{4n} - 1 \right]$
Formula D	$C.I. = P \left[\left(1 + \frac{r}{200}\right)^{\frac{n}{2}} - 1 \right]$

If 'p' is the principal amount, 'r' is the rate of interest and 'n' is the number of years, which formula can be used to calculate compound interest, when compounded annually?

Option 1: Formula A

Option 2: Formula B

Option 3: Formula C

Option 4: Formula D

Correct Answer: Option 2

2. Akash wants to calculate the compound interest on ₹x for n years at r% p.a. He uses the formula $x \left[\left(1 + \frac{r}{100}\right)^n - 1 \right]$ to calculate the compound interest.

If n is replaced by 4n and 100 by 400, which of these is true?

Option 1: Interest is compound monthly

Option 2: Interest is compound quarterly

Option 3: Interest is compound half-yearly

Option 4: Interest is compound annually

Correct Answer: Option 2

LOB: Use formula of compound interest in order to solve problems related to increase (or decrease) in population

1. The population of a town was 50,000 in the year 1980. If the population of the town increased at a rate of 12% each year, what is the estimated population of the town after 3 years?

Option 1: 35,123

Option 2: 59,550

Option 3: 70,246

Option 4: 72,235

Correct Answer: Option 3

2. The population of a small town was 1,16,000 on the last day of the year 1959. During the next year it increased by 8.5% but due to swine flu it decreased by 10% in the following year. What was its population at the end of the year 1961?

Option 1: 2,726

Option 2: 59,550

Option 3: 1,13,274

Option 4: 1,18,726

Correct Answer: Option 3

LOB: Use formula of compound interest in order to solve problems related to increase (or decrease) in the price of an item in intermediate years

1. A machine was purchased 3 years ago at ₹ x and its value depreciated by 10% per annum. If its present value is ₹1,21,743, what is the value of x ?

Option 1: 91,467

Option 2: 1,62,039

Option 3: 1,67,000

Option 4: 1,81,006

Correct Answer: Option 3

2. The cost of an electronic gadget was ₹1,45,000 at the beginning of 1993. In the beginning of the year 1994 the price was hiked by 2%. But due to the decrease in demand, the cost was reduced by 5% in the beginning of the year 1995. What was the cost of the item in 1995?

Option 1: ₹1,34,850

Option 2: ₹1,39,350

Option 3: ₹1,40,505

Option 4: ₹1,40,650

Correct Answer: Option 3

Suggested Teacher Resources



I

Lesson Plan



Objective	Calculate the simple interest in order to find the total amount to be paid.
Prerequisite	Ratio, Selling Price, Cost Price and Percentage
Vocabulary words	Ratio, percentage
Materials required	Toys of different market objects like car, furniture, bike etc.
Procedure	<p>Teacher will divide all the students into 3 groups. Each group will have 1 bank manager who will provide the loan and 4 shopkeepers having different products like cars, furniture, bikes, T.V etc. Ask each shopkeeper to decide the cost of each article. Ask the bank manager to decide the time period as for how long he can give a particular loan and decide the rate of interest as well. Each shopkeeper will go to any of the bank managers to ask for the loan. Shopkeepers can go to more than one manager in order to fetch the minimum rate of interest on the loan amount.</p> <p>Then the teacher will tell the class the amount they got from the bank is their Principal. The duration in which they have to return the amount to the bank is the Time Period and the Interest Percentage which they will receive on the principal from the bank is the rate of interest.</p> <p>Then the teacher will introduce the formula of Simple Interest $=(P \times R \times T)/100$.</p> <p>Ask students to calculate their Interest and ask them how much money they will have to give to the bank manager at the end. Then the students will derive the Amount by the formula on their own i.e. Amount = Principal + Interest.</p> <p>Follow-Up Discussion: Have you ever heard about loans? Have you ever seen any loan related hoardings around? Find out various kinds of loans which banks give to customers and their rate of interests also.</p>

2

Activity



Objective: This small-group game will help students to identify the numeric values for the principal, rate, and interest located in word problems.

Material Required:

- Sets of cards with word problems written on them
- Chart paper

- Markers

Instructions:

1. Prior to the game, the teacher will create sets of cards with word problems involving simple interest written on them. Give each word problem a number.
2. Teacher will provide the class with a real-life scenario involving simple interest, such as interest in a savings account or a consumer loan.
3. Then, she will discuss the formula for simple interest and identify the different variables, such as I (interest), P (principal), R (rate), and T (time).
4. She will divide the class into teams, and provide each team with chart paper, markers, and a set of the word problem cards you made.
5. Each team should divide their chart paper into columns for the variables P, R, and T. Also, they should create a column for the number of each word problem card.
6. When the teacher says, 'go,' the teams will read each word problem and categorize the numbers in the word problems under the correct variable on their chart paper.
7. Students will write the number written on the word problem card and then write the numbers from the word problem that correspond to each variable.

For example, in the word problem:

45,000Rs is deposited into a savings account. After one year, 4 months and 20 days it totals 52,500Rs.

Calculate the simple interest rate for this account.

Students are expected to label 45,000Rs as P, One Year 4 months and 20 days as T and 52,500Rs as the amount. They need to determine the simple interest.

8. The first team to correctly categorize the variables from their word problems wins.

Discussion:

- Why is it important to identify what numbers represent different variables in a simple interest problem?
- Were there any problems in which it was difficult to figure out what numbers were the principal, rate, and time?

9. ALGEBRAIC EXPRESSIONS & IDENTITIES

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Monomials, Binomials and Polynomials	Count the number of terms in an algebraic expression in order to classify them as monomial, binomial, trinomial or polynomial in general	
Addition and Subtraction of Algebraic Expressions	Identify like and unlike terms in algebraic expressions in order to add or subtract the given algebraic expressions	
Multiplying a Monomial by a Monomial	Use rules of exponents and powers in order to multiply a monomial by a monomial	Apply distributive property in order to multiply two algebraic expressions
	Extend the multiplication of monomial by a monomial in order to obtain the product of any number of monomials	
Multiplying a Monomial by a Polynomial	Use distributive property of multiplication over addition and subtraction in order to obtain the product of a monomial and a binomial	
	Use distributive property of multiplication over addition and subtraction in order to obtain the product of a monomial and a trinomial	
Multiplying a Polynomial by a Polynomial	Simplify the algebraic expressions in order to find the value of expression for the given value of the variable	
	Use distributive law of multiplication in order to obtain the product of two binomials	
	Use distributive law of multiplication in order to obtain the product of a binomial and a trinomial	
What is an Identity?	Define and compare equation and identity in order to classify a given question into either of the two	Use various algebraic identities in order to solve problems of daily life
Standard Identities	Use multiplication of binomials in order to explore and verify the standard identities for squares of binomials	
Applying Identities	Use identities in order to simplify the given algebraic expressions	
	Use identities in order to find the product of the given numbers	

Test items



LOB: Count the number of terms in an algebraic expression in order to classify them as monomial, binomial, trinomial or polynomial in general

- I. Which of these expressions is an example of a binomial?

Option 1: $4xy^2$
Option 2: $4 \times y^2$
Option 3: $4(x + 2y^2)$
Option 4: $7x - 8y^2 - 5$

Correct Answer: Option 3

2. If $ax^n + bx - 1$ is a binomial, which of these must be true?

Option 1: $n = 1$ or $n = 0$
Option 2: $a = b$
Option 3: $n = 2$
Option 4: $a = b = -1$

Correct Answer: Option 1

LOB: Identify like and unlike terms in algebraic expressions in order to add or subtract the given algebraic expressions

- I. What is the sum of the expressions $4x + 3y^2$, $7xy - 4y^2$ and $6x^2 - x$?

Option 1: $6x^2 + 3x + 7xy + y^2$
Option 2: $6x^2 + 3x - 7xy - y^2$
Option 3: $6x^2 + 3x - 7xy + y^2$
Option 4: $6x^2 + 3x + 7xy - y^2$

Correct Answer: Option 4

2. The sum of polynomial ax^m and bx^n is $14x^4$. Which of these is correct about the values of a , b , m and n ?

Option 1: $a = b = 14$ and $m = n = 4$
Option 2: $a + b = 14$ and $m = n = 4$
Option 3: $a = b = 14$ and $m + n = 4$
Option 4: $a + b = 14$ and $m + n = 4$

Correct Answer: Option 2

LOB: Use rules of exponents and powers in order to multiply a monomial by a monomial

- I. What is the product of expressions $-4xy^2$ and $7xy^2z^3$?

Option 1: $28x^2y^4z^3$
Option 2: $28xy^2z^3$
Option 3: $-28xy^2z^3$
Option 4: $-28x^2y^4z^3$

Correct Answer: Option 4

2. The product of polynomial px^c and qx^d is $8x^6$. Which of these is correct about the values of p , q , c and d ?

Option 1: $p + q = 8$ and $c + d = 6$
Option 2: $p \times q = 8$ and $c \times d = 6$
Option 3: $p \times q = 8$ and $c + d = 6$
Option 4: $p + q = 8$ and $c \times d = 6$

Correct Answer: Option 3

LOB: Extend the multiplication of monomial by a monomial in order to obtain the product of any number of monomials

- I. The product of polynomials $4xy^2$, $-7yz$ and $0.5x^2yz^3$ is $kx^3y^4z^t$, where k and t are unknown constants. What are the values of k and t ?

Option 1: $k = 14$ and $t = 3$
Option 2: $k = -14$ and $t = 4$

Option 3: $k = 28$ and $t = 3$

Option 4: $k = -28$ and $t = 4$

Correct Answer: Option 2

2. The product of polynomials $-6tu^3$, $-t^3u^j$ and $-3t^k u^k$ is $-18t^7u^8$, where j and k are unknown constants. What are the values of j and k ?

Option 1: $j = 2$ and $k = 3$

Option 2: $j = 3$ and $k = 2$

Option 3: $j = 5$ and $k = 3$

Option 4: $j = 1$ and $k = 4$

Correct Answer: Option 1

LOB: Use distributive property of multiplication over addition and subtraction in order to obtain the product of a monomial and a binomial

1. What is the product of polynomials $(t^4u - 6tu^2)$ and $-3tu$?

Option 1: $3t^5u^2 + 18t^2u^3$

Option 2: $3t^5u^2 - 18t^2u^3$

Option 3: $-3t^5u^2 + 18t^2u^3$

Option 4: $-3t^5u^2 - 18t^2u^3$

Correct Answer: Option 3

2. Namrata's work to find the product of $7m^2n$ and $(-6m^3n - 3n^5)$ is shown.

Step 1: $7m^2n \times (-6m^3n - 3n^5)$

Step 2: $7m^2n \times (-6m^3n) - 7m^2n \times (-3n^5)$

Step 3: $-42m^5n^2 + 21m^2n^6$

In which step did she make her first error and what is the correct step?

Option 1: Step 2; $7m^2n \times (-6m^3n) + 7m^2n \times (3n^5)$

Option 2: Step 2; $7m^2n \times (-6m^3n) + 7m^2n \times (-3n^5)$

Option 3: Step 2; $7m^2n \times (6m^3n) + 7m^2n \times (-3n^5)$

Option 4: Step 3; $42m^5n^2 - 21m^2n^6$

Correct Answer: Option 2

LOB: Use distributive property of multiplication over addition and subtraction in order to obtain the product of a monomial and a trinomial

1. The product of $6x^2y$ and $xy - 4x^3y + 3xy^7$ is $6x^3y^2 + p + 18x^3y^8$, where p is a term. Which of these is equivalent to p ?

Option 1: $24x^5y^2$

Option 2: $-24x^5y^2$

Option 3: $24x^6y^2$

Option 4: $-24x^6y^2$

Correct Answer: Option 2

2. The product of $-4x^py$ and $xy^2 - 3x^qy + rxy$ is $-4x^4y^3 + 12x^8y^2 + 20x^4y^2$, here p , q and r are unknown constants. What are the values of p , q and r ?

Option 1: $p = 3$, $q = 5$ and $r = 5$

Option 2: $p = 3$, $q = 5$ and $r = -5$

Option 3: $p = 3$, $q = 3$ and $r = -5$

Option 4: $p = 4$, $q = 3$ and $r = -5$

Correct Answer: Option 2

LOB: Simplify the algebraic expressions in order to find the value of expression for the given value of the variable

1. What is the value of the expression $4x(3 - x) + 5x(x - 2)$ for $x = -3$?

Option 1: -15

Option 2: -3

Option 3: 3

Option 4: 15

Correct Answer: Option 3

2. The value of the expression $5x(4x - x^2) + 2x(2x^2 - 5x)$ is 0 for $x = t$, where t is a number. Given that $t \neq 0$, which of these can be the value of t ?

Option 1: -10

Option 2: -4

Option 3: 4

Option 4: 10

Correct Answer: Option 4

LOB: Use distributive law of multiplication in order to obtain the product of two binomials

- I. If the product of $(4x + 1)$ and $(x + 4)$ is $ax^2 + bx + c$, what is the value of $a + b + c$?

Option 1: 22

Option 2: 24

Option 3: 25

Option 4: 26

Correct Answer: Option 3

2. The length of a rectangle is $(4j^2 + 6jk)$ meters and the breadth of the rectangle is $(7jk - 3k^2)$ meters. What is the area of the rectangle?

Option 1: $28j^3k + 18jk^3$ square meters

Option 2: $28j^3k - 18jk^3$ square meters

Option 3: $28j^3k + 30j^2k^2 - 18jk^3$ square meters

Option 4: $28j^3k - 30j^2k^2 - 18jk^3$ square meters

Correct Answer: Option 3

LOB: Use distributive law of multiplication in order to obtain the product of a binomial and a trinomial

- I. The product of polynomial $(-7x + y)$ and $(x - 3y - 8xyz)$ is $-7x^2 + m - 3y^2 + 56x^2yz + k$, where m and k are terms. Which of these is equivalent to m and k ?

Option 1: $m = 22xy$ and $k = 8xy^2z$

Option 2: $m = 22xy$ and $k = -8xy^2z$

Option 3: $m = -22xy$ and $k = 8xy^2z$

Option 4: $m = -22xy$ and $k = -8xy^2z$

Correct Answer: Option 2

2. The area of the base of a cuboidal box is $(16l^2 + 4b^2 + 8lb)$ square meters and the height of the box is $(5l - 2b)$ meters. What is the volume of the cuboidal box?

Option 1: $80l^3 - 8l^2b - 4lb^2 - 8b^3$ cubic meters

Option 2: $80l^3 + 8l^2b - 4lb^2 - 8b^3$ cubic meters

Option 3: $80l^3 - 8l^2b + 4lb^2 - 8b^3$ cubic meters

Option 4: $80l^3 + 8l^2b + 4lb^2 - 8b^3$ cubic meters

Correct Answer: Option 4

LOB: Define and compare equation and identity in order to classify a given question into either of the two

- I. Which of the equalities is also an identity?

Equality A: $a^2 + 6a + 7 = 16$

Equality B: $a^2 + 2ab + b^2 = (a + b)^2$

Equality C: $a^2 - b^2 = (a + b)(a - b)$

Option 1: Only equality B

Option 2: Only equality C

Option 3: Both equalities B and C

Option 4: Equalities A, B and C

Correct Answer: Option 3

2. Consider the equality shown below:

Equality: $a^2 - b^2 = (a + b)(a - b)$

Which of these statements is true?

Option 1: The equality holds for all values of a and b .

Option 2: The equality holds for any value of a and only $b = 0$.

Option 3: The equality holds only for positive values of a and b .

Option 4: The equality holds for only positive values of a and negative value of b .

Correct Answer: Option 1

LOB: Use multiplication of binomials in order to explore and verify the standard identities for squares of binomials

1. Which option shows the correct steps to prove the identity of $(a - b)^2$?

Option 1: $(a - b)(a - b) = a^2 - ab + ba - b^2 = a^2 - b^2$

Option 2: $(a - b)(a - b) = a^2 - ab + ba + b^2 = a^2 + b^2$

Option 3: $(a - b)(a - b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$

Option 4: $(a - b)(a - b) = a^2 - ab - ba - b^2 = a^2 - 2ab - b^2$

Correct Answer: Option 3

2. Trisha's work to verify the identity $(a + b)(a - b)$ is shown below:

Step 1: $a(a - b) + b(a - b)$

Step 2: $a^2 - ab + ba + b^2$

Step 3: $a^2 - ab + ab + b^2$

Step 4: $a^2 + b^2$

Given that she made an error, in which step did she make her first error?

Option 1: Step 1

Option 2: Step 2

Option 3: Step 3

Option 4: Step 4

Correct Answer: Option 2

LOB: Use identities in order to simplify the given algebraic expressions

1. Which of these is equivalent to $(3x - 4y)^2$?

Option 1: $3x^2 - 4y^2$

Option 2: $9x^2 - 16y^2$

Option 3: $9x^2 - 24xy - 16y^2$

Option 4: $9x^2 - 24xy + 16y^2$

Correct Answer: Option 4

2. The expression $(9x^2 - 25)^2 - (9x^2 + 25)^2$ is equivalent to $-(t^2)$ where t is a monomial. Which of these is equivalent to t ?

Option 1: $15x$

Option 2: $30x$

Option 3: $300x$

Option 4: $900x$

Correct Answer: Option 2

LOB: Use identities in order to find the product of the given numbers

1. To find the value of 1003×997 , a student rewrite it as $1000^2 - k$. What is the value of k ?

Option 1: 3

Option 2: 6

Option 3: 9

Option 4: 81

Correct Answer: Option 3

2. To find the value of 196×206 , a student rewrite it as $200^2 + q \times 200 - 24$. What is the value of q ?

Option 1: -2

Option 2: 2

Option 3: 8

Option 4: 10

Correct Answer: Option 2

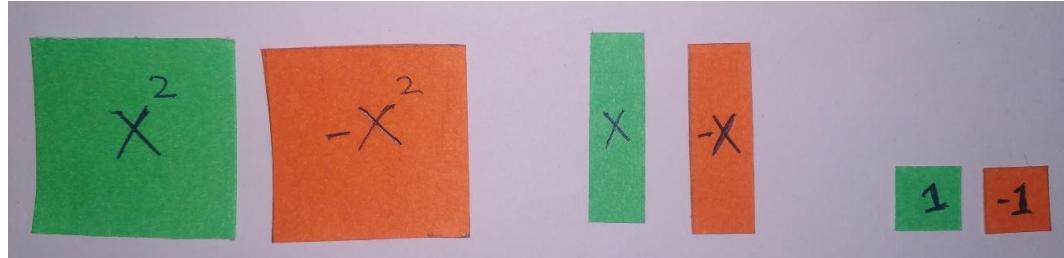
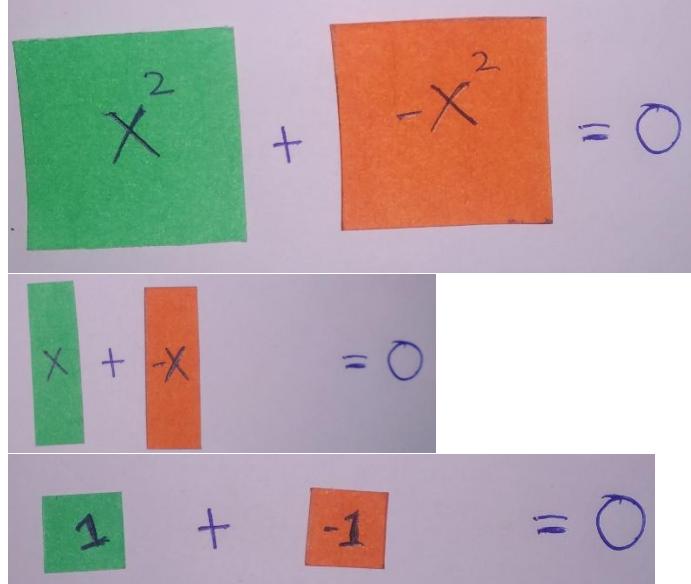
Suggested Teacher Resources



I

Lesson Plan

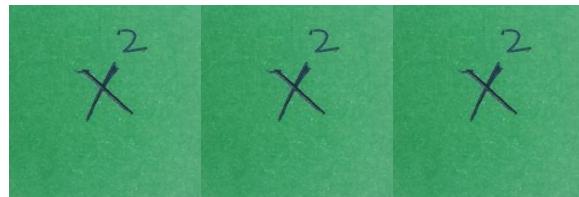
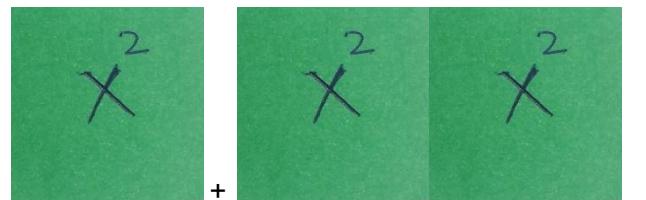


Objective	Identify like and unlike terms in algebraic expressions in order to add or subtract the given algebraic expressions
Prerequisite	Addition and subtraction of constants, basic understanding of variables
Vocabulary words	Algebraic expressions
Materials required	Algebraic tiles or coloured chalks
Procedure	<p>Teacher will start with an opening activity:</p> <p>Setup: Use a colour like green to represent + algebraic variable and red for negative variables as follows.</p>  <p>Rules:</p> <p>Zero pairs</p> <p>Say: We know $x^2 - x^2 = 0$ which can be represented as $x^2 + (-x^2) = 0$. Similarly, $x + (-x) = 0$ and $1 + (-1) = 0$.</p> <p>After representing the numbers using tiles as shown above figure we will get,</p> 

- Teacher will start with some arithmetic operations
- Addition:

Addition of $x^2 + 2x^2$

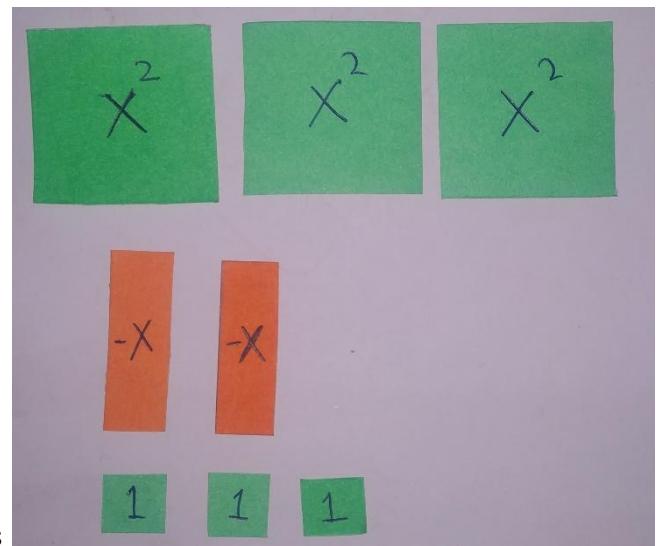
Will become



will give

- Teacher will show kids how to represent algebraic expressions using algebraic tiles.
- Students will use the same notations in their notebook

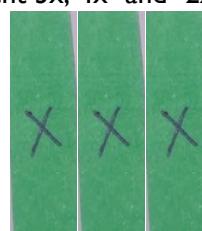
Example 1:



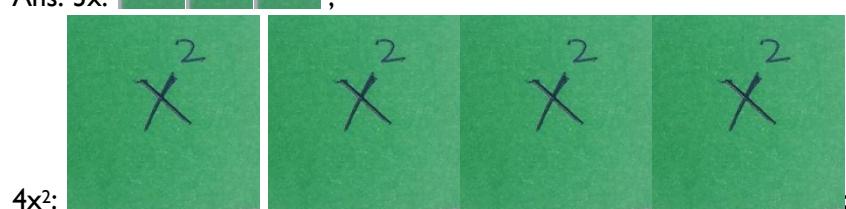
$3x^2 - 2x + 3$ will be represented as

Example 2:

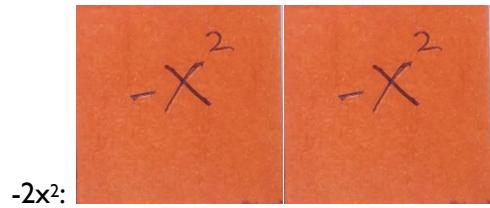
Represent $3x$, $4x^2$ and $-2x^2$ using algebraic tiles.



Ans: $3x$: ;

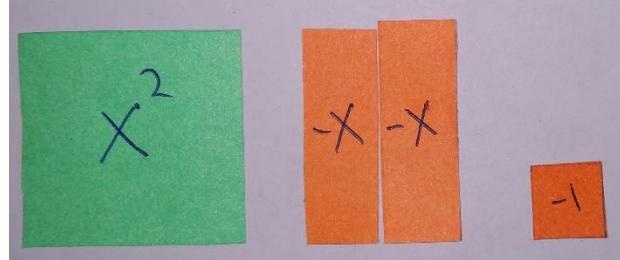


$4x^2$: ;



$-2x^2$:

Example 3:
What equation does



represent?

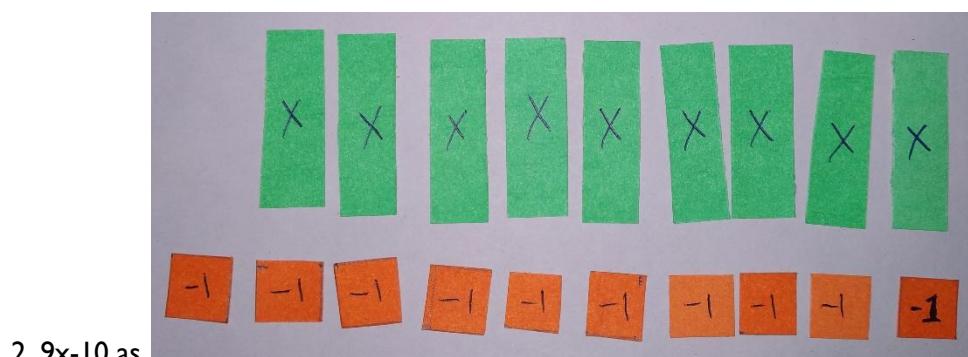
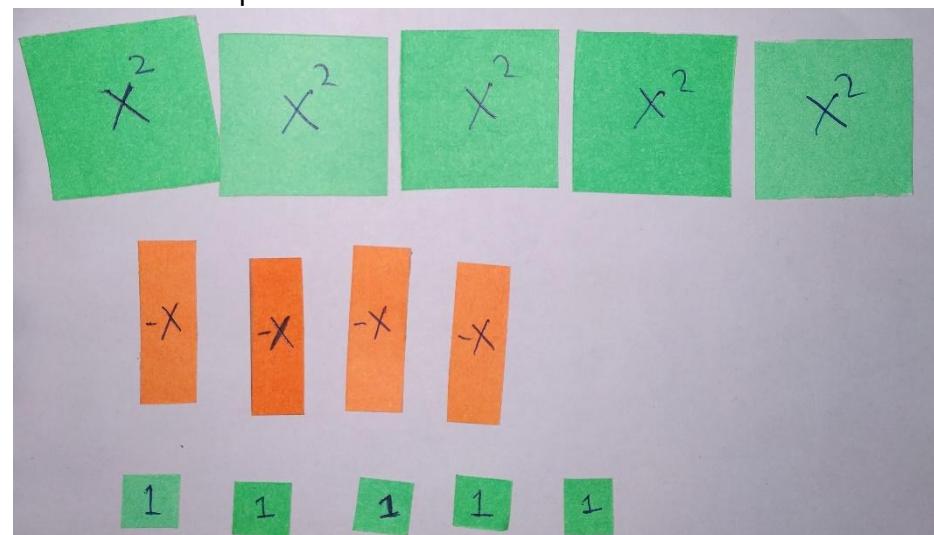
Ans: $x^2 - 2x - 1$

Addition:

Now the teacher will demonstrate addition of two algebraic expressions

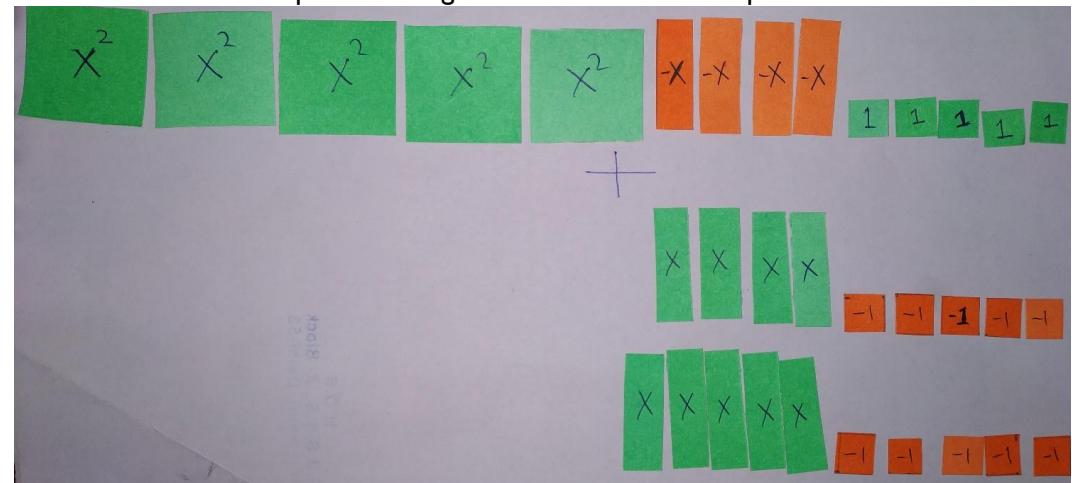
$(5x^2 - 4x + 5)$ and $(9x - 10)$

1. $5x^2 - 4x + 5$ is represented as follows

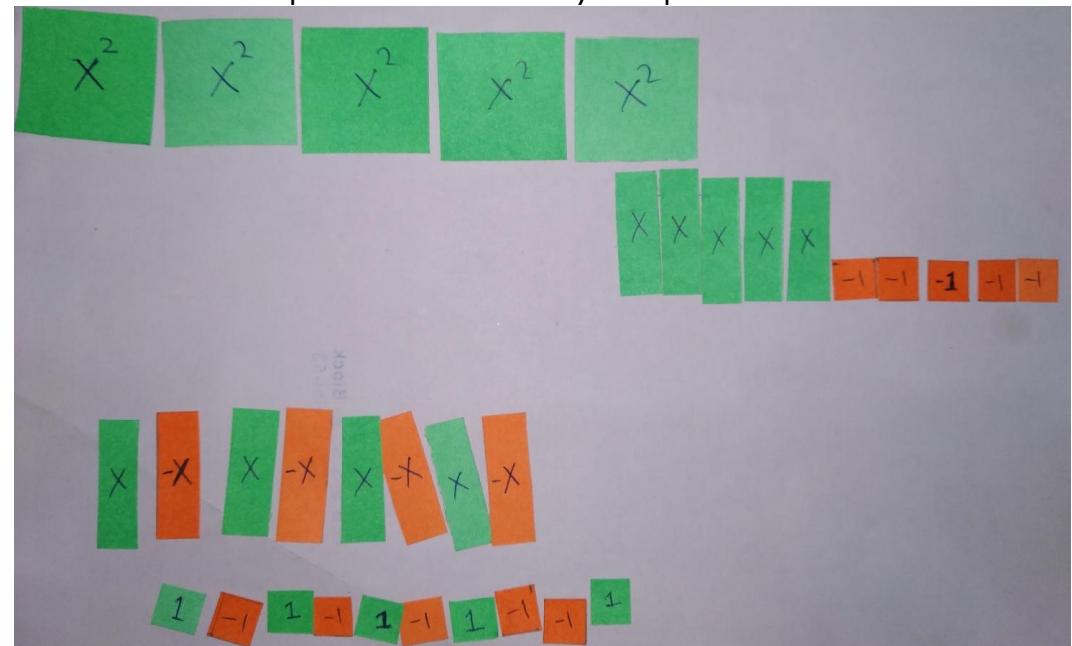


2. $9x - 10$ as

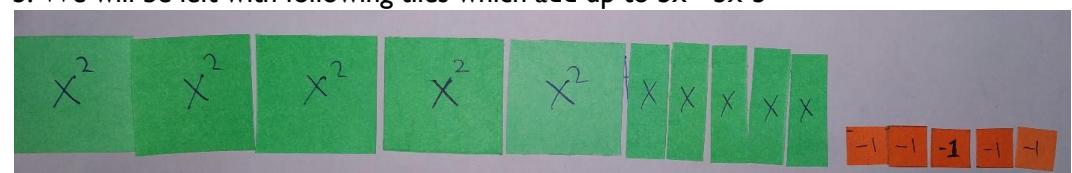
3. To add them we will put all the algebraic tiles in the same space as shown



4. Remove all the zero pairs since we know they add up to zero.



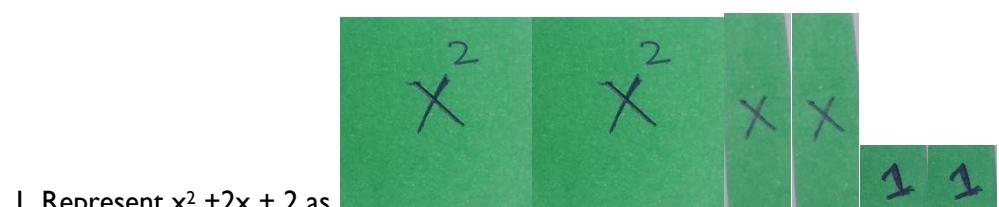
5. We will be left with following tiles which add up to $5x^2+5x-5$



Subtraction:

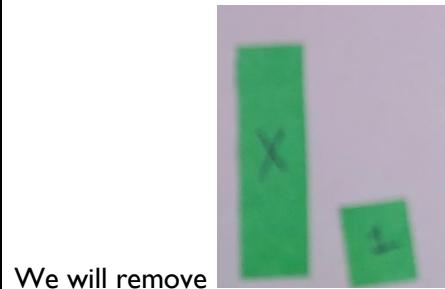
Now the teacher will demonstrate subtraction of algebraic expressions using algebraic tiles.

Subtract: (x^2+2x+2) and $(x+1)$



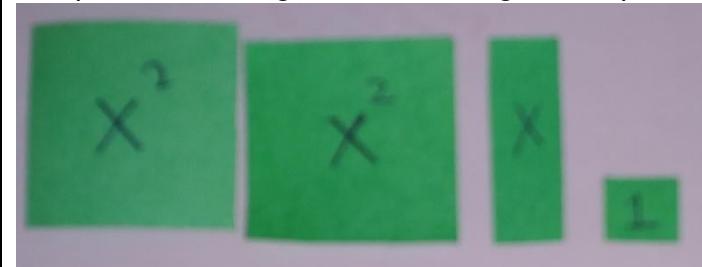
1. Represent x^2+2x+2 as

2. In subtraction we will take out $(x+1)$ from x^2+2x+2 .



We will remove

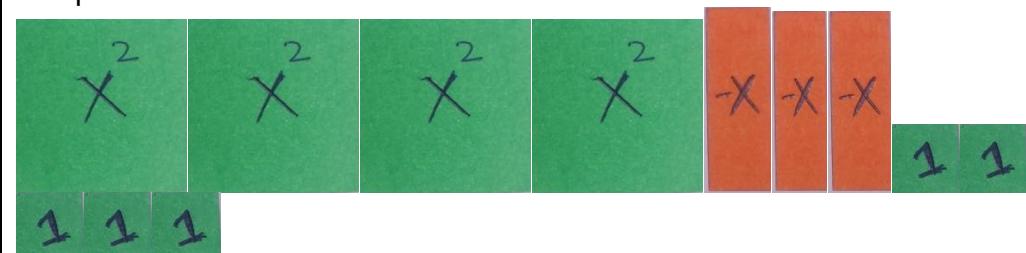
3. Represent remaining tiles in form of algebraic expression which is $x^2 + x + 1$



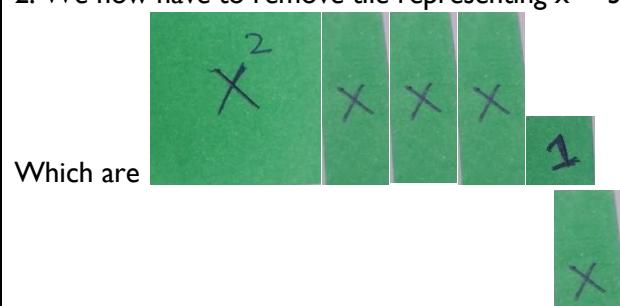
Example 1:

Subtract $4x^2 - 3x + 5$ and $x^2 + 3x + 1$

1. Represent $4x^2 - 3x + 5$ as



2. We now have to remove tile representing $x^2 + 3x + 1$



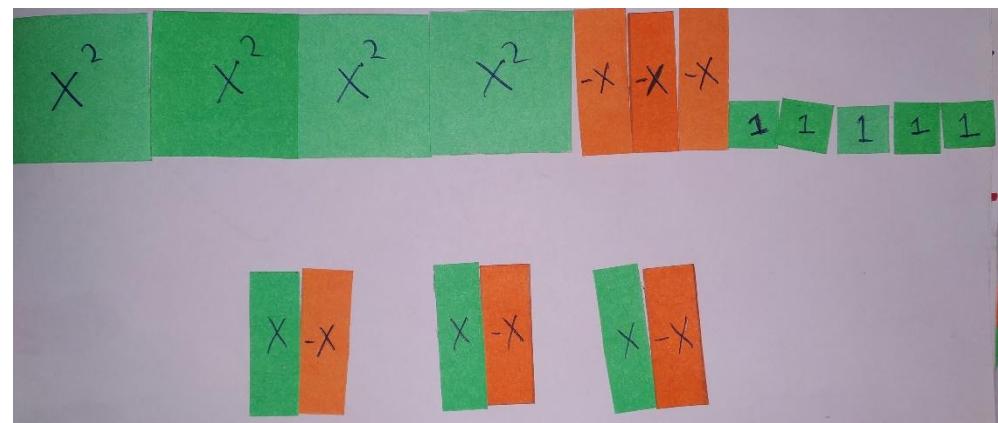
Note: students will struggle with removing 3 from $4x^2 - 3x + 5$ as there are only 3



tiles. Give some time for students to struggle.



3. Since we don't have any tiles to remove we will use the concept of zero pairs here. Since equations remain the same even if we add zero to it then we can add zero in this expression as follows.



4. Remove from the above space and we will get the required expression which is $3x^2 - 6x + 4$.

Follow Up Discussion:

Let students work in pairs and ask them to give 5 practice questions to each other.

Additional resources

Exemplar sheets, support material

2

Activity

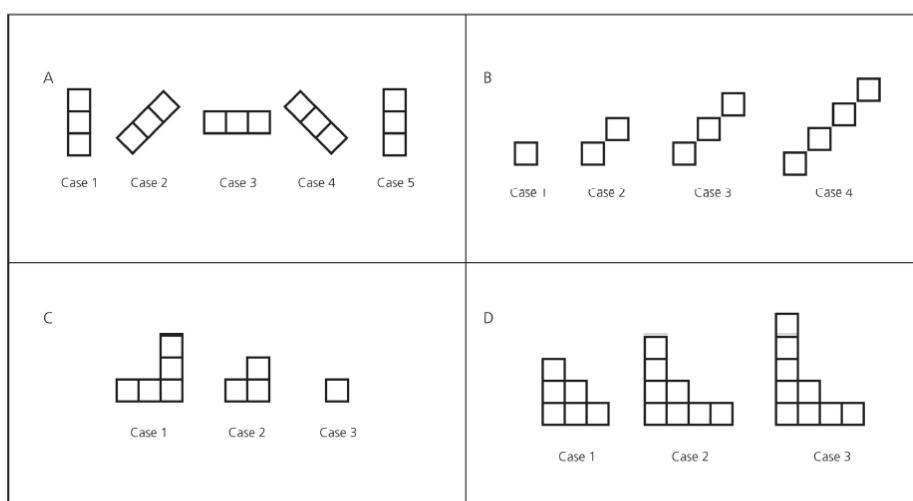


Objective: Students will be able to express growth of the square pattern in algebraic expressions

Material required: Coloured chalks to represent different patterns

Setup:

- There are 4 growth patterns A, B, C and D are shown below with each pattern having 4 cases.
- Students will explore what other cases such as 10th case, 100th case, 0 case or nth case will look like.
- Students will make an algebraic expression that will represent the number of squares in the pattern at any given case.



Steps:

- Teacher will divide the students into groups of 5.
- Show pattern B as shown above to each group. Give them 15 min to discuss the following questions:
 - How do they see the pattern growing?
 - How will the case 10 look like?
 - How many squares will case 100 have?
 - How many squares will case 0 have?
 - How many squares will case n have?
- After discussion the teacher will ask the students to fill the following table.

Here is an example of for case B

Make use of table using numbers		Note down your misconceptions/ important notes																		
<table border="1"><thead><tr><th>Case</th><th>Number of blocks</th></tr></thead><tbody><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>2</td></tr><tr><td>3</td><td>3</td></tr><tr><td>4</td><td>4</td></tr><tr><td>5</td><td>5</td></tr><tr><td>.</td><td>.</td></tr><tr><td>.</td><td>.</td></tr><tr><td>N</td><td>n</td></tr></tbody></table>		Case	Number of blocks	1	1	2	2	3	3	4	4	5	5	N	n	
Case	Number of blocks																			
1	1																			
2	2																			
3	3																			
4	4																			
5	5																			
.	.																			
.	.																			
N	n																			
Draw and describe the way the pattern is increasing or decreasing.		Describe your pattern using an algebraic expression that shows the number of blocks in any case number																		
		Algebraic expression – x Where x is number of blocks																		

- Teacher will ask the students to do the same for A, C and D.

Additional task: Ask students to create their own patterns and exchange them with their partners to solve.

Teacher's notes:

1. How are students reasoning about extending the pattern when the answers are less clear?
2. Are students seeing the relationship between case number and number of squares?
3. How are students moving from written descriptions of patterns to algebraic expressions?

References:

1. <https://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles/AlgebraTiles.html>
2. <https://www.youcubed.org/resource/k-8-curriculum>

10. VISUALIZING SOLID SHAPES

QR Code:



Learning outcome and Learning Objectives:

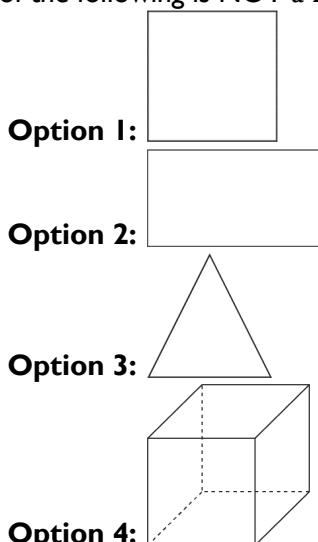
Content area / Concepts	Learning Objectives	Learning Outcome
Views of 3D-Shapes	Compare 2D shapes and 3D shapes in order to classify a given shape into either	Visualize 3-D shapes in order to represent them in a plane surface such as sheet of paper, black board, etc.
	Identify different shapes in nested objects in order to match the object with its shape	
	Visualize 3D objects in order to draw them from different perspectives	
	Discuss the given front, top and side view of an object in order to identify the object	
Mapping Space Around Us	Discuss the elements in a map in order to differentiate between a map and a picture	
	Read and interpret simple map in order to answer questions based on them	
	Choose appropriate scale and use symbols to denote landmarks in order to draw a simple map	
Faces, Edges and Vertices	Identify faces, edges and vertices in a given solid in order to classify it as a polyhedron or a non-polyhedron	Analyze patterns in order to verify Euler's relation
	Count vertices, edges and faces in 3D figures with flat faces in order to verify Euler's formula	

Test items



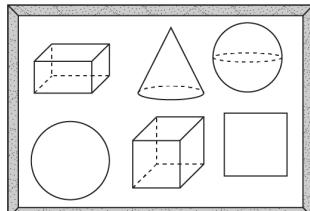
LOB: Compare 2D shapes and 3D shapes in order to classify a given shape into either

- I. Which of the following is NOT a 2D shape?



Correct Answer: Option 4

2. A picture frame is shown below.



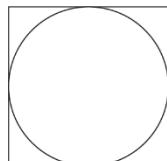
Which statement is true?

- Option 1:** There are 2 more 3D shapes in the picture frame than 2D shapes.
- Option 2:** There are 2 more 2D shapes in the picture frame than 3D shapes.
- Option 3:** There are 3 more 3D shapes in the picture frame than 2D shapes.
- Option 4:** There are 3 more 2D shapes in the picture frame than 3D shapes.

Correct Answer: Option 1

LOB: Identify different shapes in nested objects in order to match the object with its shape

- I. A figure is shown below.



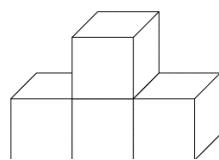
Which of the following is true about the figure?

- Option 1:** A cube is inside a circle.
- Option 2:** A circle is inside a cube.
- Option 3:** A square is inside a circle.
- Option 4:** A circle is inside a square.

Correct Answer: Option 4

LOB: Visualize 3D objects in order to draw them from different perspectives

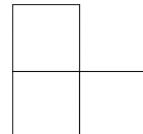
- I. An object is shown below.



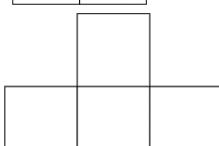
Which of the following is the top view of the object?



Option 1:



Option 2:



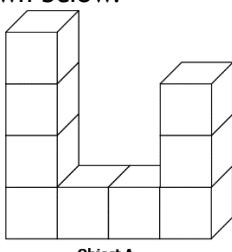
Option 3:



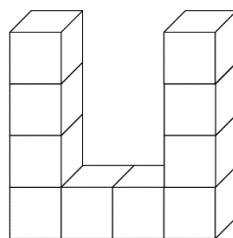
Option 4:

Correct Answer: Option 4

2. Two objects A and B are shown below.



Object A



Object B

Which of the following statement is true?

Option 1: Only the top view of objects A and B are the same.

Option 2: Only the side view of objects A and B are the same.

Option 3: Only the top and side view of objects A and B are the same.

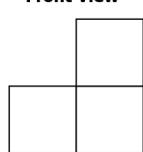
Option 4: The top, side and front view of objects A and B are the same.

Correct Answer: Option 3

LOB: Discuss the given front, top and side view of an object in order to identify the object

- I. The front, top and side view of an object is shown below.

Front View



Top View



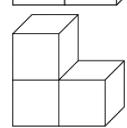
Side View



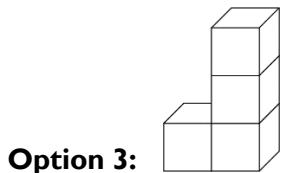
Which of these objects has the above views?



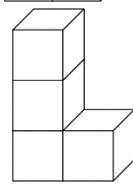
Option 1:



Option 2:



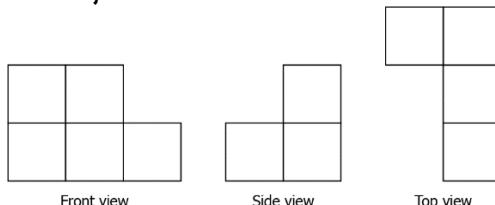
Option 3:



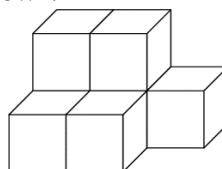
Option 4:

Correct Answer: Option I

2. The front, top and side view of an object X is shown below.



Kailash drew an object similar to X as shown.



Is he correct?

Option 1: Yes, because the top, side, front view is the same as the object X.

Option 2: No, because the top view is not the same as the object X.

Option 3: No, because the side view is not the same as the object X.

Option 4: Yes, because the side and front view is the same as the object X.

Correct Answer: Option 2

LOB: Discuss the elements in a map in order to differentiate between a map and a picture

- I. Hemanth says he can tell the distance between two buildings only using a map and not a picture and Amruta says she can tell if a lamp post is besides a building only using a map and not a picture.
Who is/are correct?

Option 1: Only Hemanth

Option 2: Only Amruta

Option 3: Both of them

Option 4: None of them

Correct Answer: Option I

2. Which of the following statement is NOT true?

Option 1: The symbols in a map denote the objects in a picture.

Option 2: The map is the same when viewed from any direction unlike a picture.

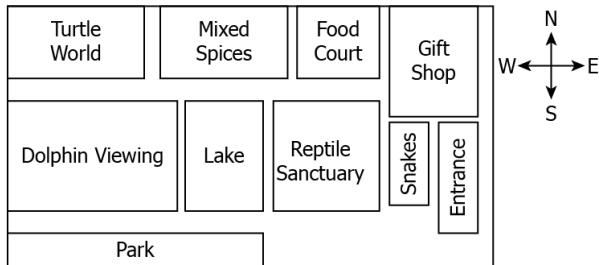
Option 3: The height of a building can be determined using a map and not a picture.

Option 4: The map of a city drawn to scale gives the distance between two locations unlike a picture.

Correct Answer: Option 3

LOB: Read and interpret simple map in order to answer questions based on them

- I. Consider the map of a water museum shown below.

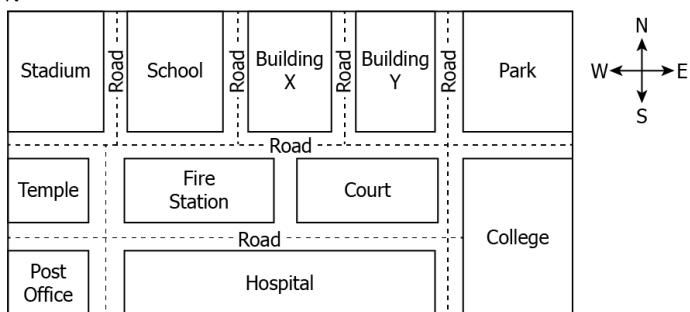


Which of the following is further south?

- Option 1:** Lake
- Option 2:** Park
- Option 3:** Gift Shop
- Option 4:** Turtle World

Correct Answer: Option 2

2. A map is shown below.



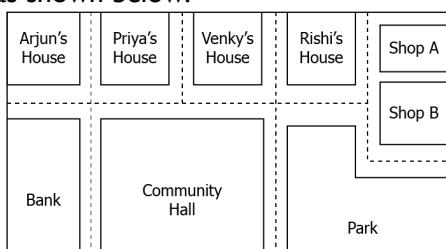
Which of the following statement is NOT true?

- Option 1:** A court is further south than a school.
- Option 2:** A temple is further west than a college.
- Option 3:** A stadium is further north than a hospital.
- Option 4:** A fire station is further east than a building Y.

Correct Answer: Option 4

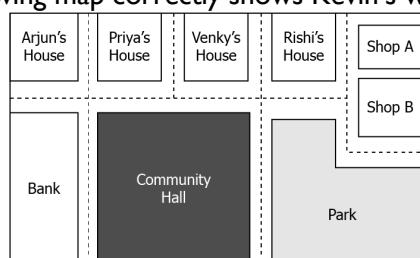
LOB: Choose appropriate scale and use symbols to denote landmarks in order to draw a simple map

I. Venky has a map of his locality as shown below.

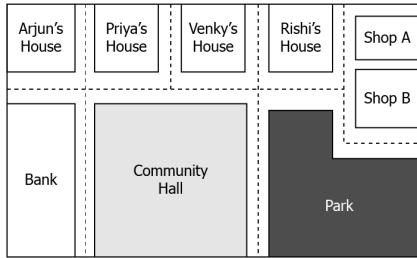


He wants to colour the map as shown. Community hall-dark grey and Park-light grey.

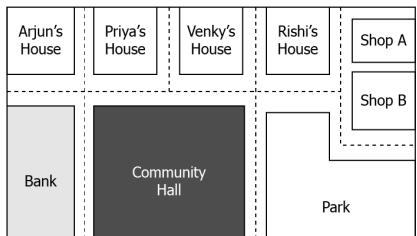
Which of the following map correctly shows Kevin's work?



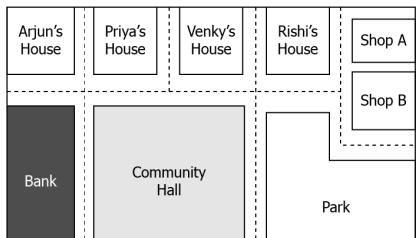
Option I:



Option 2:



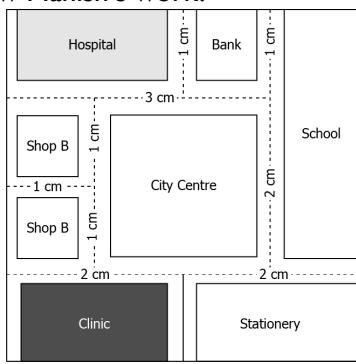
Option 3:



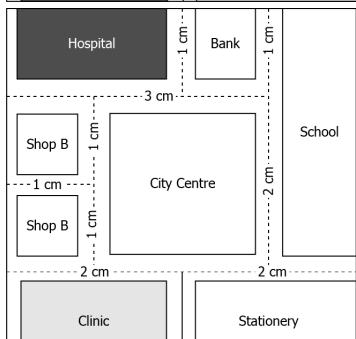
Option 4:

Correct Answer: Option 1

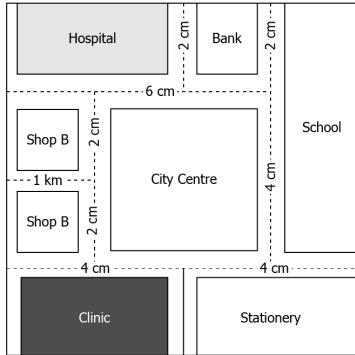
2. Manish drew a map where he highlighted a hospital as dark grey and clinic as light grey. The distance between the hospital and the clinic is 10 km. He drew a map with a scale of 1 cm = 5 km. Which of the following maps show Manish's work?



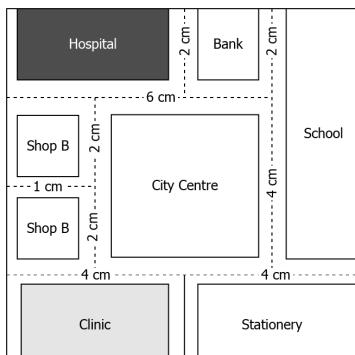
Option 1:



Option 2:



Option 3:

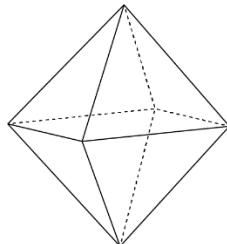


Option 4:

Correct Answer: Option 2

LOB: Identify faces, edges and vertices in a given solid in order to classify it as a polyhedron or a non-polyhedron

- Consider a solid shown below.



How many faces are there in this solid?

Option 1: 6

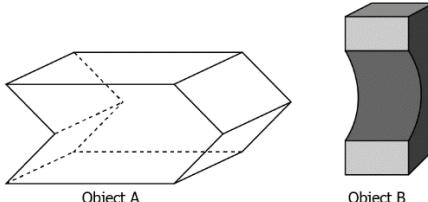
Option 2: 7

Option 3: 8

Option 4: 9

Correct Answer: Option 3

- Two objects are shown below.



Jasmine claims that object A is polyhedron whereas object B is not. Is her claim correct, why?

Option 1: Yes, because all faces of object A are polygons and object B has only some faces as polygons.

Option 2: Yes, because all faces of object A are rectangles and object B has only some faces as rectangles.

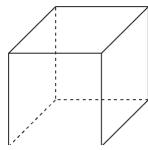
Option 3: No, because some faces of object A are convex polygons and object B has all faces as convex polygons.

Option 4: No, because some faces of object A are rectangles and object B has all faces as rectangles.

Correct Answer: Option 1

LOB: Count vertices, edges and faces in 3D figures with flat faces in order to verify Euler's formula

- I. A cube is shown below.



Which of the following is true about the cube?

- Option 1:** Faces: 6, Edges: 12, Vertices: 8
- Option 2:** Faces: 4, Edges: 8, Vertices: 8
- Option 3:** Faces: 6, Edges: 8, Vertices: 8
- Option 4:** Faces: 4, Edges: 12, Vertices: 8

Correct Answer: Option 1

2. Which of these shows the Euler's relation for a prism whose base has 5 sides?

- Option 1:** $6 + 6 = 10 + 2$
- Option 2:** $6 + 8 = 10 + 4$
- Option 3:** $10 + 7 = 15 + 2$
- Option 4:** $10 + 9 = 15 + 4$

Correct Answer: Option 3

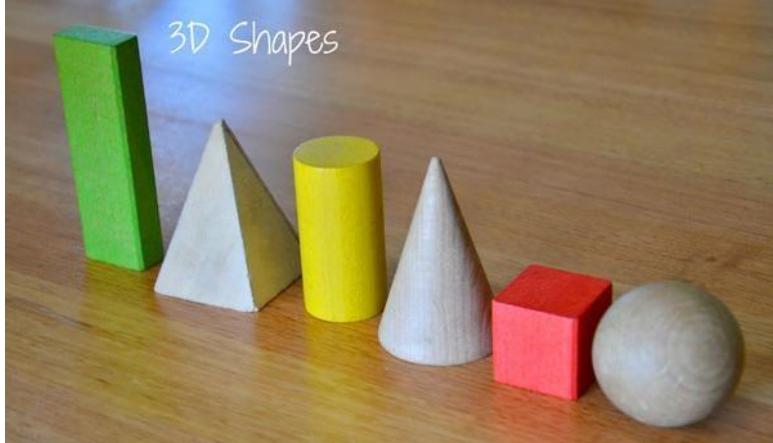
Suggested Teacher Resources



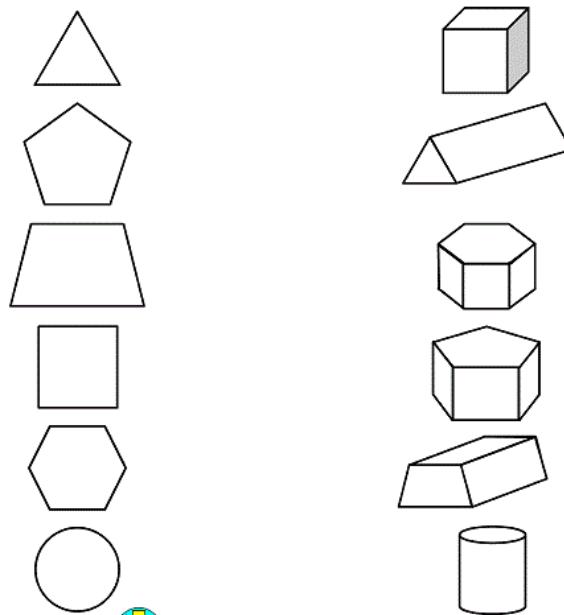
I

Lesson Plan

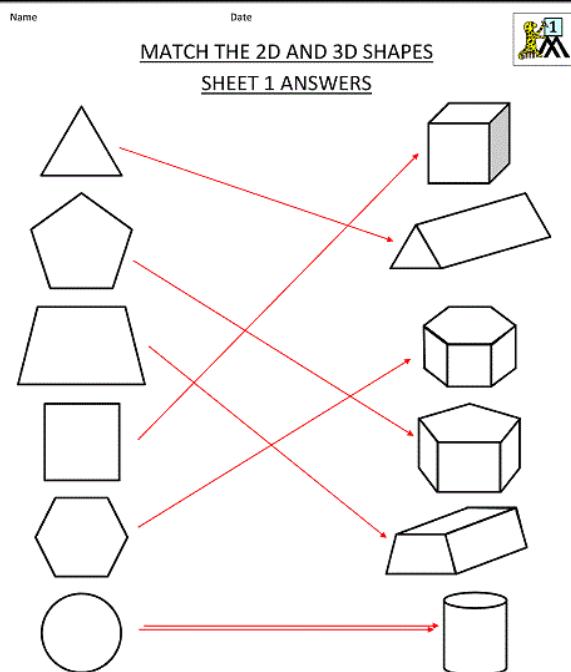


Objectives	<ul style="list-style-type: none">• Compare 2D shapes and 3D shapes in order to classify a given shape into either.• Identify different shapes in nested objects in order to match the object with its shape.• Visualize 3D objects in order to draw them from different perspectives.
Prerequisite Knowledge	Basic knowledge of common shapes (Triangle, Square, and Rectangle etc.)
Material Required	<ul style="list-style-type: none">• Pop sticks and 3D shapes.• Printed worksheets.• Pen/pencil and notebooks.
Procedure	<p>Teacher will show the following shapes to the students. She will ask them to draw these shapes in their notebooks.</p>  <p>Teacher will ask the students as to What difference do, they see in the drawings and the shapes they have in front of them?</p> <p>Teacher will help the students recognize the 2D shape and its corresponding 3D shapes by counter questioning and visualization.</p> <p>Note: Teacher can give hint to students by projecting shadows of 3D shapes on the board.</p> <p>Now teacher will provide the following worksheet to the students with the following instructions-</p> <p>Q) Match the 2D shapes with their following 3D shapes -</p>

Draw a line to match the 2d shape to the correct 3d shape.



Solution of this worksheet will look something like this –



Finally, teacher will ask the students to look at the available 3D shapes from different angles and note down what they have observed. Students can look at the shadows of the 3D shapes to relate them with corresponding 2D shapes.

Use Socratic Questioning to Make Thinking Visible of the students.

1. What connections can they see between a cube and a square?
2. Can you give some examples of 2D and 3D figures from this classroom or your home?

Teacher will write the common conclusions on the board.



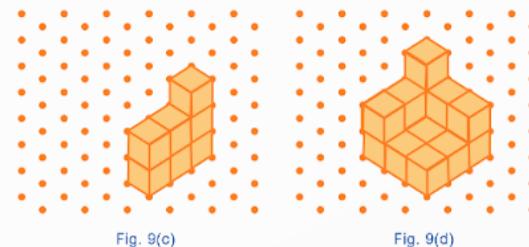
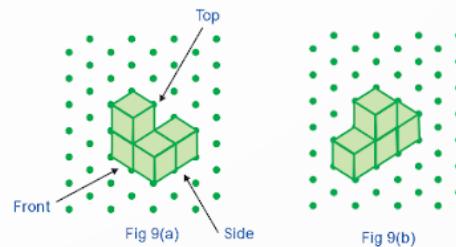
Objective: To draw objects as viewed from different positions.

Prerequisite: knowledge of making solid shaped using unit cubes.

Material required: Unit cubes, tapes

Procedure:

1. Take some unit cubes and make the shape shown in fig. 9(a)
2. Now draw shapes when viewed from the front, side and top.
3. Repeat the activity by making other shapes. Fig. 9(b) to 9(d)



(i) For Fig. 9(a) :	Front view is	side view is	top view is
(ii) For Fig. 9(b) :	Front view is	side view is	top view is
(iii) For Fig. 9(c) :	Front view is	side view is	top view is
(iv) For Fig. 9(d) :	Front view is	side view is	top view is

Observation:

Extension: you may extend this activity for drawing different views of some more 3D objects from real life.

III. MENSURATION

QR Code:



Learning outcome and Learning Objectives:

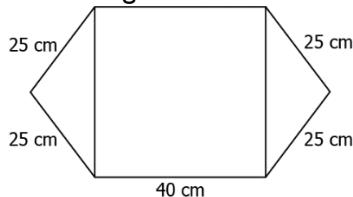
Content area / Concepts	Learning Objectives	Learning Outcome
Adjoint figures	Calculate area and perimeter of circle, square, rectangle, triangle in order to calculate area and perimeter of adjoint shapes	Use square grid/graph sheet in order to estimate the areas of various polygons
Area of Trapezium	Breakdown a given trapezium into known figures (triangles, squares, rectangles) in order to derive the formula for the area of a trapezium	
Area of a Polygon	Calculate the area of a given polygon after breaking down the polygon in multiple ways in order to compare the values and comment on it	Uses appropriate methods to find the area of a polygon
Surface Area of Cube, Cuboid and Cylinder	Illustrate 2-D representation of a cuboid, cube and cylinder in order to compute the surface areas by breaking them in to areas of known figures Calculate the surface area of a cube, cuboid and cylinder to determine the cost of painting/covering their surface	
Volume of Cube, Cuboid and Cylinder	Calculate the volume of a given cube, cuboid, cylinder in order to infer the quantity of any substance it can hold Modify the values of l, b, h in order to examine the effect it has on the value of the surface area/volume of a cuboid Modify the values of r, h in order to examine the effect it has on the value of the surface area/volume of a cylinder Calculate the volume of a given cuboid, cylinder in order to determine the time taken to fill it with a liquid at a given rate	Use appropriate formulae in order to find surface area and volume of cuboidal and cylindrical object

Test items



LOB: Calculate area and perimeter of circle, square, rectangle, triangle in order to calculate area and perimeter of adjoint shapes

- I. Raghav made a model using a square and triangles as shown.

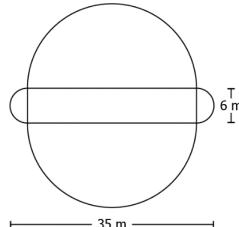


He wants to decorate the model by putting decorative tape along the model. How much tape, in centimetres, is required to decorate the model?

- Option 1:** 130
- Option 2:** 140
- Option 3:** 180
- Option 4:** 260

Correct Answer: Option 3

2. A garden is in the shape of a rectangle with four semi-circular flower beds as shown in the figure below.



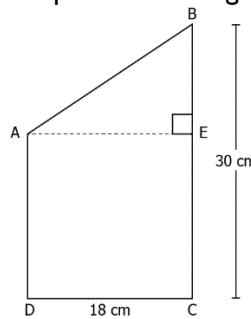
What is the total area of the garden, to the nearest whole number? (Use 3.14 for π)

- Option 1:** 518 m^2
- Option 2:** 862 m^2
- Option 3:** 1200 m^2
- Option 4:** 2928 m^2

Correct Answer: Option 2

LOB: Breakdown a given trapezium into known figures (triangles, squares, rectangles) in order to derive the formula for the area of a trapezium

- I. A trapezium ABCD is decomposed into a square and triangle as shown below.

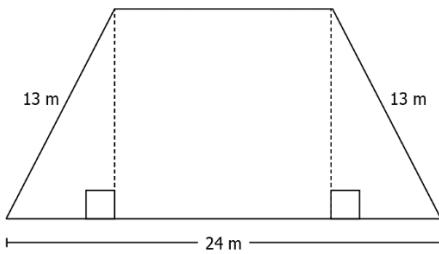


Which expression represents the area of the trapezium ABCD?

- Option 1:** $(18 \times 18) + (\frac{1}{2} \times 18 \times 12)$
- Option 2:** $(18 \times 18) + (18 \times 12)$
- Option 3:** $(18 \times 12) + (18 \times 12)$
- Option 4:** $(18 \times 12) + (\frac{1}{2} \times 18 \times 12)$

Correct Answer: Option 1

2. A park is in the shape of a trapezium, which is breakdown into 2 triangular parts and one rectangle, as shown below.



If the perpendicular distance between two parallel sides of the park is 12 m, which option correctly represents the area of the park?

Option 1: $\left(\frac{1}{2} \times 5 \times 12\right) + (14 \times 12) = 198 \text{ m}^2$

Option 2: $2\left(\frac{1}{2} \times 5 \times 12\right) + (14 \times 12) = 228 \text{ m}^2$

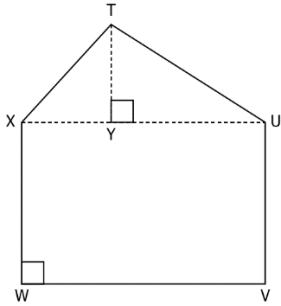
Option 3: $\left(\frac{1}{2} \times 13 \times 12\right) + (24 \times 12) = 366 \text{ m}^2$

Option 4: $2\left(\frac{1}{2} \times 13 \times 12\right) + (24 \times 12) = 444 \text{ m}^2$

Correct Answer: Option 2

LOB: Calculate the area of a given polygon after breaking down the polygon in multiple ways in order to compare the values and comment on it

- I. Consider a polygon shown.



Which expression shows the area of the polygon?

Option 1: $\left(\frac{1}{2} \times TY \times XU\right) + \left(\frac{1}{2} \times XW \times (XU + WV)\right)$

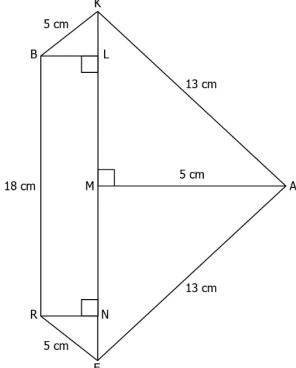
Option 2: $\left(\frac{1}{2} \times XT \times XU\right) + \left(\frac{1}{2} \times XW \times (XU + WV)\right)$

Option 3: $\left(\frac{1}{2} \times TY \times XU\right) + \left(\frac{1}{2} \times UV \times (XU + WV)\right)$

Option 4: $\left(\frac{1}{2} \times XT \times XU\right) + \left(\frac{1}{2} \times UV \times (XU + WV)\right)$

Correct Answer: Option 1

2. Consider a polygon shown.



If the length of KL and NE is the same, what is the area of the polygon?

Option 1: 116 cm²

Option 2: 138 cm²

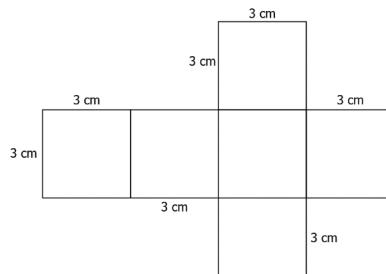
Option 3: 144 cm²

Option 4: 204 cm²

Correct Answer: Option 3

LOB: Illustrate 2-D representation of a cuboid, cube and cylinder in order to compute the surface areas by breaking them in to areas of known figures

- I. The 2-D representation of a cube is shown below.

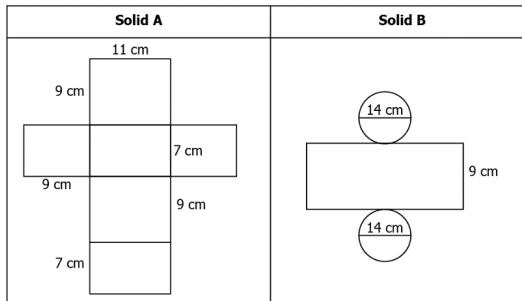


Which expression represents the total surface area of the cube?

- Option 1:** 3^2
- Option 2:** $3^2 + 3^2 + 3^2 + 3^2$
- Option 3:** $3^2 + 3^2 + 3^2 + 3^2 + 3^2 + 3^2$
- Option 4:** $3^2 + 3^2 + 3^2 + 3^2 + 3^2 + 3^2 + 3^2$

Correct Answer: Option 3

2. Consider the 2-D representation of 2 solids shown below.



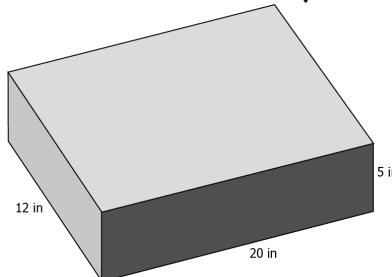
Which solid has greater total surface area and by how much? (Use $\frac{22}{7}$ for π)

- Option 1:** Solid B has greater total surface area than solid A by 11 cm^2
- Option 2:** Solid A has greater total surface area than solid B by 82 cm^2
- Option 3:** Solid B has greater total surface area than solid A by 226 cm^2
- Option 4:** Solid A has greater total surface area than solid B by 324 cm^2

Correct Answer: Option 3

LOB: Calculate the surface area of a cube, cuboid and cylinder to determine the cost of painting/covering their surface

- I. Urvashi wants to polish the exterior of a cuboidal wooden jewellery box as shown.



If the cost to polish is `0.75 per square inch, what is the cost to polish the jewellery box?

- Option 1:** `300
- Option 2:** `400
- Option 3:** `600
- Option 4:** `800

Correct Answer: Option 3

2. Divya makes two cylindrical models, A and B, of diameter 21 cm each for an art project. The height of model B is 2 more than the height of model A. She wants to paint the curved surface of the models. She paints a total of 396 square metres of model A.

If the cost to paint is 0.25/sq. m, what is the total cost to paint both the models? (Use $\frac{22}{7}$ for π)

Option 1: `132

Option 2: `231

Option 3: `264

Option 4: `363

Correct Answer: Option 2

LOB: Calculate the volume of a given cube, cuboid, cylinder in order to infer the quantity of any substance it can hold

- I. A cylindrical tank, of radius 0.35 m and height 0.70 m, is filled with water. How many cubic metres of water, to the nearest hundredth, is contained in the tank? (Use $\frac{22}{7}$ for π)

Option 1: 0.27

Option 2: 1.54

Option 3: 1.93

Option 4: 2.31

Correct Answer: Option 1

2. The areas of the three adjacent faces of a cuboidal container are $60,000 \text{ cm}^2$, $80,000 \text{ cm}^2$ and $1,20,000 \text{ cm}^2$. If the container is filled with liquid and the cuboid has whole number side lengths, how much litres of liquid is contained in the container?

Option 1: 9,000

Option 2: 24,000

Option 3: 26,000

Option 4: 52,000

Correct Answer: Option 2

LOB: Modify the values of l, b, h in order to examine the effect it has on the value of the surface area/volume of a cuboid

- I. If the length of a cuboid is reduced to its half and breadth become four times, how does the volume of the cuboid change?

Option 1: It will become 2 times

Option 2: It will remain the same

Option 3: It will reduce to half

Option 4: It will become four times

Correct Answer: Option 1

2. A cuboid is x metres long, y metres wide and z metres deep. If the length becomes one-fourth and breadth become three times, which expression represents the change in the lateral surface area of the cuboid?

Option 1: $(\frac{x}{2} + 6y)z$

Option 2: $(\frac{3x}{2} - 4y)z$

Option 3: $(\frac{3xy}{2} + 6yz + \frac{xz}{2})$

Option 4: $(\frac{xy}{2} - yz + \frac{3}{2}xz)$

Correct Answer: Option 2

LOB: Modify the values of l, b, h in order to examine the effect it has on the value of the surface area/volume of a cuboid

- I. If the height of a cylinder triples and its radius becomes one-third, how does the lateral surface area of the cylinder change?

Option 1: It becomes one-third

Option 2: It becomes triple

Option 3: It remains the same

Option 4: It will increase by 3

Correct Answer: Option 3

2. The radius and height of a cylinder is 6 cm and 9 cm respectively. If the radius becomes half and the height increases by 3 cm, which of these describes the new volume of the cylinder?

Option 1: The new volume will be half the original volume.

Option 2: The new volume will be twice the original volume.

Option 3: The new volume will be thrice the original volume.

Option 4: The new volume will be one-third the original volume.

Correct Answer: Option 4

LOB: Calculate the volume of a given cuboid, cylinder in order to determine the time taken to fill it with a liquid at a given rate

- I. Akriti is filling oil into a cylindrical container of radius 50 cm and height 105 cm, at the rate of 5 litres per minute. How many minutes will she take to fill the container completely? (Use $\frac{22}{7}$ for π)

Option 1: 41.25

Option 2: 45

Option 3: 125

Option 4: 165

Correct Answer: Option 4

2. A cuboidal tank of dimensions 3m, 4m, and 5m has 3 litres of water. Two pumps, P and Q, are used to fill the remaining volume of the tank completely with water. Pump P fills the tank at a rate of 1000 litres per 20 minutes and pump Q fills the tank at a rate of 240 litres per 3 minutes. First, the pump P fills 9 m^3 of the tank and then, pump Q fills the remaining volume. In how many hours will both the pumps completely fill the tank?

Option 1: 2.5 hours

Option 2: 3 hours

Option 3: 5.5 hours

Option 4: 6.125 hours

Correct Answer: Option 4

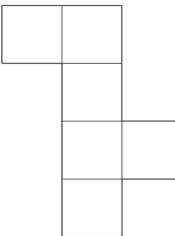
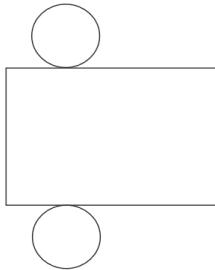
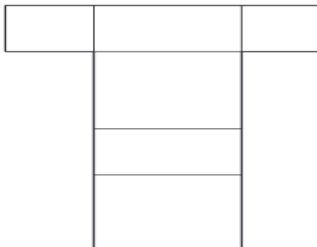
Suggested Teacher Resources



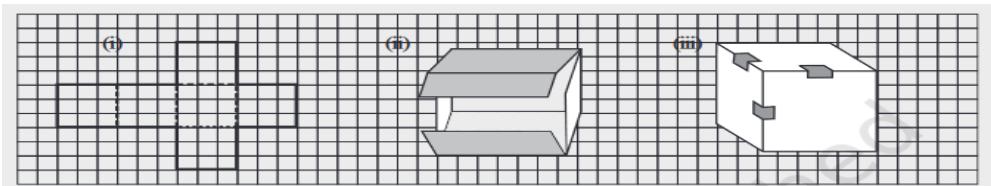
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Lesson Plan



Objective	Illustrate 2-D representation of a cuboid, cube and cylinder in order to compute the surface areas by breaking them in to areas of known figures
Prerequisite	Area of square, rectangle, circle
Vocabulary words	Total and lateral Surface area
Materials required	 <p>Net of Cube of Fig. 12.1</p>  <p>Net of Cylinder of Fig. 12.2</p>  <p>Nets of different solids, geometry box</p>
Procedure	<ol style="list-style-type: none"> 1. Teacher will start the class with the following activity of exploring nets of various objects. 2. She will ask the students to fold the paper cut outs of the given nets of different solid shapes on the graph paper and explore the possibility of formation of different solids like cubes, cuboids, cylinders. 3. By observation students will try to explore the formulae for the total surface area of the cube, cuboid as well as the curved surface area and total surface area of the cylinder. 4. Teacher will ask students to try to make some more solids of the above type using these nets indifferent ways. 5. How to make the net of a cube?

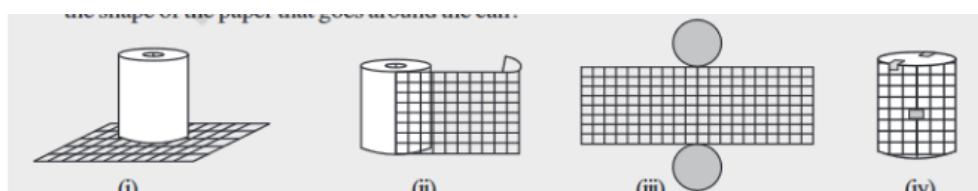
Draw the pattern shown on a squared paper and cut it out [Fig (i)]. (You know that this pattern is a net of a cube. Fold it along the lines [Fig (ii)] and tape the edges to form a cube



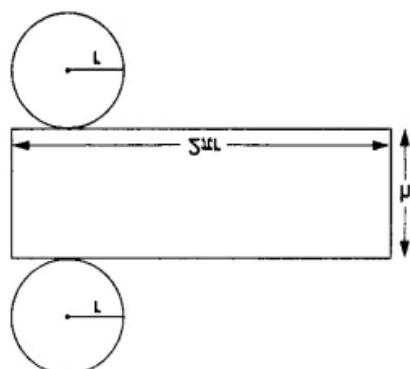
- Teacher will discuss the following points with students:
 - What is the length, width and height of the cube?
 - Observe that all the faces of a cube are square in shape. This makes length, height and width of a cube equal (Fig (i)).
 - Write the area of each of the faces. Are they equal?
 - Write the total surface area of this cube.
 - If each side of the cube is l , what will be the area of each face? (Fig (ii)). Can we say that the total surface area of a cube of side l is $6l^2$?

6. How to make the net of a cylinder:

- Take a cylindrical can or box and trace the base of the can on graph paper and cut it [Fig (i)].
- Take another graph paper in such a way that its width is equal to the height of the can.
- Wrap the strip around the can such that it just fits around the can (remove the excess paper) [Fig (ii)].
- Tape the pieces [Fig (iii)] together to form a cylinder [Fig(iv)].
- What is the shape of the paper that goes around the can? Of course, it is rectangular in shape.
- When the parts of this cylinder are taped together, the length of the rectangular strip is equal to the circumference of the circle.
- Record the radius (r) of the circular base, length (l) and width (h) of the rectangular strip. Is $2\pi r = \text{length of the strip}$.
- Check if the area of the rectangular strip is $2\pi r h$.
- Count how many square units of the squared paper are used to form the cylinder. Check if this count is approximately equal to $2\pi r (r + h)$.



- We can deduce the relation $2\pi r (r + h)$ as the surface area of a cylinder in another way. Imagine cutting up a cylinder as shown below.



7. Teacher will ask students to fill in the following table

Shape	Lateral Surface area	Total Surface area
Cube	_____	_____
Cuboid	_____	_____
Cylinder	_____	_____

8. Teacher can ask students to demonstrate their method for finding the required formula, help students understand the activity.

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Activity



Prerequisite: Total surface area and lateral surface area of cube and cuboid.

Materials required: Measuring tape or scale

Introduction:

- Teacher will introduce the activity to students by explaining a cube is made of many small unit cubes and will then show the following cases to students:
- Imagine that we paint a 4 by 4 by 4 cube blue on every side as shown here.



Case 1



Case 2



Case 3



Case 4

Answer the following questions:

- How many of the small cubes have 3 blue faces?
- How many have 2 blue faces?
- How many have 1 blue face?
- How many have not been painted at all?
- How many faces would be painted in a cube of any size?
- Answer the above questions for case 2 and case 3?

2. Teacher will give a rough estimation of the dimensions of the classroom to the students. Students in the groups of 5 can be tasked to find how much it will cost to paint the walls of their classroom at the rate of Rupees 7.50 per m².

Reference:

- <http://ncert.nic.in/textbook/textbook.htm?hemhl=11-16>
- http://www.ncert.nic.in/departments/nie/niew/school_kits/upper_prm_level/pdf/kit_manual_UP_math.pdf
- <https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2017/03/Painting-Youcubed-2016.pdf>

12. EXPONENTS AND POWERS

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Powers with Negative Exponents	Simplify powers with negative exponents in order to calculate the multiplicative inverse of a number	Apply rules of exponents in order to solve problems with integral exponents
Laws of Exponents	Give examples in order to show that is valid for all integer exponents.	
	Apply the first law of exponents () and principles of negative exponents in order to derive the rest of the laws of exponents	
	Apply laws of exponents in order to simplify a given expression	
Use of Exponents to Express Small Numbers in Standard Form	Express very large and very small numbers in the standard form in order to compare and estimate quantities	

Test items



LOB: Simplify powers with negative exponents in order to calculate the multiplicative inverse of a number

- I. What is the multiplicative inverse of 5^{-7} ?

Option 1: 5^{-7}

Option 2: 5^7

Option 3: $\frac{1}{5^7}$

Option 4: 7^{-5}

Correct Answer: Option 2

2. Which of these is the multiplicative inverse of the expression $\frac{4a^{-19}}{36a^{-13}}$?

Option 1: $9a^{-6}$

Option 2: $9a^6$

Option 3: $\frac{1}{9a^{-6}}$

Option 4: $\frac{1}{9}a^6$

Correct Answer: Option 2

LOB: Give examples in order to show that $(x^a)^b = x^{ab}$ is valid for all integer exponents.

- I. Which of the following is NOT true?

Option 1: $(3^3)^1 = 3^3$

Option 2: $(3^1)^{-1} = 3^{-1}$

Option 3: $(3^2)^4 = 3^6$

Option 4: $(3^{-1})^{-5} = 3^5$

Correct Answer: Option 3

2. Three expressions are shown below.

$(10^5)^{-2}$, $(10^2)^4$, $(10^{-3})^{-9}$

For which of these expressions can we use the formula, $(z^a)^b = z^{ab}$?

Option 1: Only $(10^2)^4$, as it holds true only for positive integer exponents.

Option 2: Only $(10^{-3})^{-9}$, as it holds true only for negative integer exponents.

Option 3: All $(10^5)^{-2}$, $(10^2)^4$ and $(10^{-3})^{-9}$, as it holds true for all integer exponents.

Option 4: Only $(10^5)^{-2}$ and $(10^{-3})^{-9}$, as it holds true only for negative integer exponents.

Correct Answer: Option 3

LOB: Apply the first law of exponents [$a^m \times a^n = a^{m+n}$] and principles of negative exponents in order to derive the rest of the laws of exponents

- I. Which of the following is equivalent to $5^8 \times 5^{-3}$?

Option 1: $5^{(8+3)}$

Option 2: $5^{(8-3)}$

Option 3: $5^{[8 \times (-3)]}$

Option 4: $5^{(-8-3)}$

Correct Answer: Option 2

2. If $(p^3 \times p^{-2}) \times (q^5 \times q^{-3}) = 50$, then what are the values of p and q ?

Option 1: $p = 2$ and $q = 5$

Option 4: $p = 4$ and $q = 5$

Option 2: $p = 2$ and $q = 25$

Option 3: $p = 4$ and $q = 25$

Correct Answer: Option 1

LOB: Apply laws of exponents in order to simplify a given expression

- I. Which of these is the simplified form of the expression $[(-7)^5 \times (-7)^2] \div (-7)^4$?

Option 1: $(-7)^3$
Option 2: $(-7)^6$
Option 3: $(-7)^7$
Option 4: $(-7)^{11}$

Correct Answer: Option 1

2. What is the simplified form of the expression $\left(\frac{p^r}{p^s}\right)^{r+s} \left(\frac{p^s}{p^t}\right)^{s+t} \left(\frac{p^t}{p^r}\right)^{r+t}$?

Option 1: 0
Option 2: 1
Option 3: $p^{2(r^2+s^2+t^2+rs+st+rt)}$
Option 4: $p^{2(r^2-s^2-t^2+rs+st+rt)}$

Correct Answer: Option 2

LOB: Express very large and very small numbers in the standard form in order to compare and estimate quantities

- I. The population of a bacteria in dish A is 3.2×10^8 and in dish B is 0.7×10^7 . What is the total population of bacteria in both the dishes?

Option 1: 3.9×10^8
Option 2: 3.27×10^7
Option 3: 3.27×10^8
Option 4: 1.02×10^7

Correct Answer: Option 3

2. Which option correctly describes the set of all possible values of j that make the given comparison true?

$$3.08 \times 10^{-4} > j \times 10^{-3}$$

Option 1: All numbers less than 3.08
Option 2: All numbers greater than 3.08
Option 3: All numbers less than 0.308
Option 4: All numbers greater than 0.308

Correct Answer: Option 3

Suggested Teacher Resources



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Lesson Plan



Objective	Students will be able to simplify powers with negative exponents in order to calculate the multiplicative inverse of a number																											
Prerequisite	Multiplication, exponential representation																											
Vocabulary words	Exponents																											
Materials required	Worksheet																											
Procedure	<p>Start the class with an activity. Teacher will group the students in pairs. Students have to represent 32 as power of 2 using factors.</p> <p>Answer: $2^5 = 2*2*2*2*2$</p> <p>Teacher will ask them to represent 2^5 divided by 2</p> <p>Answer: Students will take a 2 out and represent $2^4 = 2*2*2*2$</p> <p>Do the same till 2^1.</p> <p>Teacher will then ask students to represent 2^0. If students are stuck, ask them to do exactly what they have been doing till this point which is taking 2 out every time.</p> <p>Then students will represent 2^{-1} using the same steps they have been doing till now which is divided by 2.</p> <p>Teacher will note down the answers in the table below as represented:</p> <table border="1"><thead><tr><th>Exponent</th><th>Factors</th><th>Value</th></tr></thead><tbody><tr><td>2^5</td><td>$2*2*2*2*2$</td><td>32</td></tr><tr><td>2^4</td><td>$2*2*2*2$</td><td>16</td></tr><tr><td>2^3</td><td>$2*2*2$</td><td>8</td></tr><tr><td>2^2</td><td>$2*2$</td><td>4</td></tr><tr><td>2^1</td><td>2</td><td>2</td></tr><tr><td>2^0</td><td>_____</td><td>1</td></tr><tr><td>2^{-1}</td><td>_____</td><td>_____</td></tr><tr><td>2^{-2}</td><td>_____</td><td>_____</td></tr></tbody></table> <p>Students will be asked to find the value of 2^{-4} using the same method they followed above. Teacher will give following worksheet to students.</p> <p>The following worksheet will be given to students to demonstrate their learning:</p>	Exponent	Factors	Value	2^5	$2*2*2*2*2$	32	2^4	$2*2*2*2$	16	2^3	$2*2*2$	8	2^2	$2*2$	4	2^1	2	2	2^0	_____	1	2^{-1}	_____	_____	2^{-2}	_____	_____
Exponent	Factors	Value																										
2^5	$2*2*2*2*2$	32																										
2^4	$2*2*2*2$	16																										
2^3	$2*2*2$	8																										
2^2	$2*2$	4																										
2^1	2	2																										
2^0	_____	1																										
2^{-1}	_____	_____																										
2^{-2}	_____	_____																										

Exponential Notation	No Exponential Notation	Numeric Result	Visual	
3^{-3}		$\frac{1}{27}$		
3^{-2}	$\frac{1}{3 \cdot 3}$			
3^{-1}				
3^0				
3^1	3	3		Graph 3^x
3^2	$3 \cdot 3$	9		
3^3	$3 \cdot 3 \cdot 3$	27		

Once completed answers can be discussed in the large group.

Follow Up Questions:

1. What is the multiplicative inverse of 5^{-7} ?
2. What is the multiplicative inverse of the expression $4a^{-19}/36a^{-13}$?



Activity

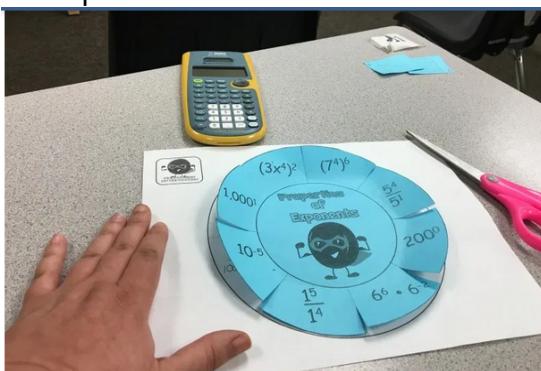


Objectives: Students will be able to simplify exponential numbers

Material Required: Chart Papers

Procedure:

1. The Teacher will divide the students in groups of 4.
2. Each group will cut a circle as shown from the cut-outs. In groups of 4 they write the questions on the top, solve those questions and write the answers under the leaf of the cut-out.



3. Each group will exchange their exponents circles and solve the questions and check the answers written underneath.

4. Once the time is over, answers will be discussed in the large group

Reference:

1. <https://www.teacherspayteachers.com/Product/Evaluating-Exponential-Expressions-Maze-1757743>
2. <https://s3.amazonaws.com/migleadmagnets/Properties+of+Exponents+Chart.pdf>

13. DIRECT & INVERSE PROPORTIONS

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Direct proportion and Inverse proportion	Observe the relationship between the given two quantities in order to solve for the constant of proportionality	Solve problems based on direct or inverse proportions in order to establish how one quantity depends on other
	Examine situations in order to decide whether two quantities are proportional to each other or not	
	Complete a given table showing two proportional quantities in order to answer questions based on them	
	Convert the given statement on relationship (directly or inversely proportional) between two quantities into a table in order to identify the missing quantity and solve for its value	
	Observe the table in order to determine which pair of variables are inversely proportional	
	Create their own scale by creating the best proportionality constant in order to draw a given figure with large dimensions	

Test items



LOB: Observe the relationship between the given two quantities in order to solve for the constant of proportionality

- I. The quantities A and B are in direct proportion. The table below shows the values of A and the corresponding values of B.

A	8	26
B	20	65

What is the constant of proportionality?

Option 1: $\frac{8}{65}$

Option 2: $\frac{4}{13}$

Option 3: $\frac{2}{5}$

Option 4: $\frac{10}{13}$

Correct Answer: Option 3

2. The quantities X and Y are in direct proportion. The table below gives the values of X and Y, where m is an unknown.

X	6	9
Y	$7m$	$10m + 1$

Which of these is the constant of proportionality?

Option 1: $\frac{1}{11}$

Option 2: $\frac{1}{3}$

Option 3: $\frac{3}{7}$

Option 4: $\frac{2}{3}$

Correct Answer: Option 3

LOB: Examine situations in order to decide whether two quantities are proportional to each other or not

- I. Which of the following situations is an example of direct proportion?

Option 1: The perimeter p of a square and its side length a .

Option 2: The length l and the width w of a rectangle, given the area remain constant.

Option 3: The number of units of a product that can be purchased given a fixed amount of money, and the unit price of the product.

Option 4: The number of rows m and the number of columns n in which a fixed number of marbles can be arranged.

Correct Answer: Option 1

2. Amrita is conducting an experiment to determine how the weight of a metal changes with its volume. Based on the data she collected from the experiment, she made the following observations.

Observation 1: As the volume of the metal increases, the weight of the metal also increases.

Observation 2: The weight of the metal per cubic centimetre remains constant.

Which observation(s) is/are sufficient to conclude that the weight and the volume of the metal are in direct proportion?

Option 1: Only observation 1 is sufficient.

Option 2: Only observation 2 is sufficient.

Option 3: Both observations are required.

Option 4: Neither observation is sufficient.

Correct Answer: Option 3

LOB: Complete a given table showing two proportional quantities in order to answer questions based on them

- I. The table below shows the price and quantity of product A.

Price (in ₹)	17	51
Quantity (in kg)	a	b

Which of these can be the values of a and b ?

- Option 1:** $a = 6$ and $b = 18$
- Option 2:** $a = 18$ and $b = 6$
- Option 3:** $a = 16$ and $b = 50$
- Option 4:** $a = 3$ and $b = 289$

Correct Answer: Option 1

2. The table below shows the number of units of a product that can be bought for a given amount.

Number of units	8
Amount (in ₹)	24

Given that the unit cost is same, if the number of units bought is increased by $2p$, what will be the change in the total cost?

- Option 1:** the total cost will decrease by $6p$
- Option 2:** the total cost will increase by p
- Option 3:** the total cost will increase by $6p$
- Option 4:** the total cost will decrease by p

Correct Answer: Option 3

LOB: Convert the given statement on relationship (directly or inversely proportional) between two quantities into a table in order to identify the missing quantity and solve for its value

- I. If 30 cupcakes are prepared using 500 grams of flour, how many cupcakes can be prepared by using 750 grams of flour assuming each cupcake is equal in size?

- Option 1:** 15
- Option 2:** 25
- Option 3:** 36
- Option 4:** 45

Correct Answer: Option 4

2. A dish is prepared using some quantity of wheat flour and semolina as shown in the table.

Wheat flour	300 grams
Semolina	100 grams

How many grams of semolina are needed to prepare the same dish if the quantity of wheat flour is increased by 50%?

- Option 1:** 50 grams
- Option 2:** 100 grams
- Option 3:** 150 grams
- Option 4:** 250 grams

Correct Answer: Option 3

LOB: Observe the table in order to determine which pair of variables are inversely proportional

- I. In which of these tables X and Y are in inverse proportion?

Option 1:

X	15	3
Y	20	4

Option 2:

X	15	30
Y	20	10

Option 3:

X	15	20
Y	30	10

Option 4:

X	15	20
Y	4	3

Correct Answer: Option 2

2. Kaya and Navya have the same number of marbles. Kaya arranges the marbles in a rectangular array. The table below shows the number of rows and columns of the array.

Number of rows	15
Number of columns	20

How many marbles are there in a column, if Navya arranged all the marbles in 5 more columns than Kaya?

Option 1: 10

Option 2: 12

Option 3: 20

Option 4: 30

Correct Answer: Option 2

LOB: Create their own scale by creating the best proportionality constant in order to draw a given figure with large dimensions

- I. To draw a model of a residential society, Jaya used a scale factor of 1:300. If the height of the building in model is 12 cm, which of these can be the actual height of the building?

Option 1: 20 m

Option 2: 25 m

Option 3: 30 m

Option 4: 36 m

Correct Answer: Option 4

2. The floor plan drawing of a square and a rectangular room is drawn using the same scale.

The plan shows the area of the floor of the rectangular room as 12 cm^2 for the actual area of 24 m^2 . What is the area of the floor of the square room in the plan if the actual side of the floor of the square room is 6 m?

Option 1: 10 cm^2

Option 2: 18 cm^2

Option 3: 20 cm^2

Option 4: 25 cm^2

Correct Answer: Option 2

Suggested Teacher Resources



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Lesson Plan



Objectives	Students will be able to examine quantities in order to decide whether two quantities are proportional to each other or not
Prerequisite Knowledge	Basic mathematical operations.
Material Required	<ul style="list-style-type: none"> • Chalk and blackboard. • Pen/Pencil and notebook. • Empty Juice box. • Paper slips to write questions.
Procedure	<ol style="list-style-type: none"> 1. Teacher will start the class by displaying a packet of juice and will ask a student to come and read its nutritional value. 2. All the students will write the information in their notebooks. For example: Apple juice 440ml, each 440ml provides: <ol style="list-style-type: none"> i) Energy 226 calories, ii) Carbohydrate 6.6g, iii) Vitamin c 20 mg 3. Teacher will ask the following questions to students: <ul style="list-style-type: none"> • If 440 ml of juice provides 226 calories how many calories 100ml of juice will provide? • How many grams of Carbohydrates would there be in 100 ml of juice? • How many grams of Vitamin C would be there in 100 ml of juice? • How many calories, grams carbohydrates and Vitamin C would be there in 2 litres of the juice? 4. Now the teacher will ask students to analyse are these direct or inverse proportion questions, can they show the working? If they are unable, show working on the board.
Solution:	<p>440 ml of juice contains calories =226 1 ml of juice contains calories = $226 / 440$ 100 ml contain = $226 / 440 \times 100 = 56.5$ Cal</p> <p>440 ml of juice contains carbohydrates=6.6g 1 ml of juice contains carbohydrates= $6.6 / 440$ 100 ml of juice contains carbohydrates= $6.6 / 440 \times 100 = 1.5$ g</p> <p>440 ml of juice contains Vitamin c= 20 mg = 0.02 g 1 ml of juice contains Vitamin c= $0.02 / 440$ 100 ml of juice contains Vitamin c= $0.02 / 440 \times 100 = 0.0045$ g</p> <p>Using the above equations, 1 ml of juice contains calories= $226 / 440$ 2 L of juice contains calories = $226 / 440 \times 2000 = 1027$ Cal</p> <p>1 ml of juice contains carbohydrates = $6.6 / 440$ 2 L of juice contains carbohydrates= $6.6 / 440 \times 2000 = 30$ g</p> <p>1 ml of juice contains Vitamin c = $0.02 / 440$</p>

	<p>2L of juice contains Vitamin c= $0.02/440 \times 2000 = 0.09\text{g}$</p> <ol style="list-style-type: none"> 5. Teacher will then ask students to mention various examples of direct and inverse quantities from their immediate environment. 6. Follow up Discussion: <ul style="list-style-type: none"> • Teacher will ask the students to bring packets of different products and compare their nutritional value/quantities of the ingredients in order to understand direct and inverse ratios.
Reference	https://aminghori.blogspot.com/2017/03/lesson-plan-of-direct-and-inverse.html



Activity

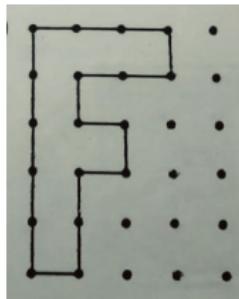


Objective: To develop intuitive understanding of proportion through enlargement. Materials: Square dot paper

Time Required: 30 minutes

Procedure:

1. Teacher will make pairs of students and will give a square dot paper to each pair.
2. Give a drawing of a letter of the alphabet or a numeral, as shown in the figure.



3. She will ask the students to double it. Were they able to do it correctly?
4. Teacher will evaluate the student's drawing basis on these questions:
 - Did their drawing seem right to them?
 - What kind of errors did they make?
 - What could have caused the errors?
5. Then she will ask them to triple it. Did they find any difficulty?

Extension of the activity:

Teacher can then extend the concept of proportion or scaling up/down in terms of ratios by using various examples.

Source: At Right Angles, Azim Premji University, July 2019

14. FACTORIZATION

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Factors of algebraic expressions	Express each term as a product of irreducible factors in order to find the common factors of the given terms	
Method of common factors	Use the method of common factors in order to factorize the given algebraic expression	
Factorisation by regrouping terms	Regroup the terms in order to factorize the given algebraic expressions	
Factorisation using identities	Apply the standard algebraic identities in order to factorize the given algebraic expressions (for perfect squares)	
Factors of the form	Factorize algebraic expressions in the form in order to express it as a product of its irreducible factors of the form	
Division of Algebraic Expressions	Use the common factor method in order to divide a monomial by a monomial	
	Use the common factor method in order to divide a polynomial by a monomial	
	Divide each term in the numerator by the denominator in order to divide a polynomial by a monomial	
	Use the common factor method in order to divide a polynomial by a polynomial	
Find the Error	Check the given mathematical statements in order to find and give reasons for the possible errors in them	

Test items



LOB: Express each term as a product of irreducible factors in order to find the common factors of the given terms

- I. Which of the following shows the product of irreducible factors of each term of the algebraic expression, $20xy + 28y^2$?

- Option 1:** $4 \times 5 \times x \times y + 4 \times 7 \times y \times y$
Option 2: $4 \times 5 \times x \times y + 4 \times 7 \times x \times y \times y$
Option 3: $2 \times 2 \times 5 \times x \times y + 2 \times 2 \times 7 \times y \times y$
Option 4: $2 \times 2 \times 5 \times x \times y + 2 \times 2 \times 7 \times x \times y \times y$

Correct Answer: Option 3

2. An algebraic expression is $35pq^2r$ minus the sum of $45p^2r^2$ and $40qr^2$.

Which of the following is equivalent to the above algebraic expression?

- Option 1:** $5 \times 7 \times p \times q \times q \times r - 3 \times 3 \times 5 \times p \times p \times r \times r + 2 \times 2 \times 2 \times 5 \times q \times r \times r$
Option 2: $5 \times 7 \times p \times q \times q \times r - 3 \times 3 \times 5 \times p \times p \times r \times r - 2 \times 2 \times 2 \times 5 \times q \times r \times r$
Option 3: $5 \times 7 \times p \times p \times q \times q \times r - 3 \times 3 \times 5 \times p \times p \times r \times r + 2 \times 2 \times 2 \times 5 \times q \times q \times r \times r$
Option 4: $5 \times 7 \times p \times p \times q \times q \times r - 3 \times 3 \times 5 \times p \times p \times r \times r - 2 \times 2 \times 2 \times 5 \times q \times q \times r \times r$

Correct Answer: Option 2

LOB: Use the method of common factors in order to factorize the given algebraic expression

- I. Which of the following is equivalent to $63abc - 42b^2c$?

- Option 1:** $7 \times b \times c \times (3 \times a - 2 \times b)$
Option 2: $3 \times 7 \times b \times c \times (3 \times a - 2 \times b)$
Option 3: $3 \times 7 \times b \times c \times (3 \times a - 2 \times 3 \times b)$
Option 4: $3 \times 7 \times b \times c \times (3 \times a \times c - 2 \times b)$

Correct Answer: Option 2

2. Which of the following is equivalent to the sum of $90lm^2 - 25m^2n$ and $10l^2m - 5mn$?

- Option 1:** $5m(18l - 5n + 10l^2 - n)$
Option 2: $5lm(18m - 5n + 10l - n)$
Option 3: $5m(18lm - 5mn + 10l^2 - n)$
Option 4: $5lm(18lm - 5mn + 10l^2 - n)$

Correct Answer: Option 3

LOB: Regroup the terms in order to factorize the given algebraic expressions

- I. Which of these is equivalent to $20pq - 4pr + 10q - 2r$?

- Option 1:** $4p(5q - r) + (5q - r)$
Option 2: $4p(5q - r) - (5q - r)$
Option 3: $4p(5q - r) + 2(5q - r)$
Option 4: $4p(5q - r) - 2(5q - r)$

Correct Answer: Option 3

2. Two expressions are shown below.

A: $(6a^2b + 56) - (16ab + 21a)$
B: $(6ab^2 - 56) - (16ab - 21b)$

What are the factorized forms of expression A and B?

- Option 1:** A: $(2ab - 7)(3a - 8)$ B: $(2ab + 7)(3b - 8)$
Option 2: A: $(2ab - 7)(3b - 8)$ B: $(2ab + 7)(3a - 8)$
Option 3: A: $(2ab + 7)(3a - 8)$ B: $(2ab - 7)(3b - 8)$
Option 4: A: $(2ab + 7)(3b - 8)$ B: $(2ab - 7)(3a - 8)$

Correct Answer: Option I

LOB: Apply the standard algebraic identities in order to factorize the given algebraic expressions (for perfect squares)

1. Which of the following is equivalent to $81k^2 - 64$?

Option 1: $(9k + 4)(9k - 4)$

Option 2: $(9 + 4k)(9 - 4k)$

Option 3: $(9 + 8k)(9 - 8k)$

Option 4: $(9k + 8)(9k - 8)$

Correct Answer: Option 4

2. What is the factorized form of the expression $9x^2 + 14y - 1 - 49y^2$?

Option 1: $(3x + 7y - 1)(3x - 7y + 1)$

Option 2: $(3x + 7y + 1)(3x - 7y - 1)$

Option 3: $(7y - 1 - 3x)(1 - 7y + 3x)$

Option 4: $(7y - 1 + 3x)(1 - 7y - 3x)$

Correct Answer: Option 1

LOB: Factorize algebraic expressions in the form $x^2 + px + q$ in order to express it as a product of its irreducible factors of the form $(x + a)(x + b)$

1. What are the factors of $k^2 + 4k - 32$?

Option 1: $(k + 4)$ and $(k - 8)$

Option 2: $(k - 4)$ and $(k + 8)$

Option 3: $(k + 4)$ and $(k + 8)$

Option 4: $(k - 4)$ and $(k - 8)$

Correct Answer: Option 2

2. If $(x + a)$ and $(x + b)$ are factors of the expression $x^2 + x - 56$, where $a > b$, then what is the factorized form of the expression $ax^2 + ax + ab$?

Option 1: $8(x + 4)(x - 3)$

Option 2: $8(x - 4)(x + 3)$

Option 3: $4(x + 4)(x - 3)$

Option 4: $4(x - 4)(x + 3)$

Correct Answer: Option 1

LOB: Use the common factor method in order to divide a monomial by a monomial

1. Which of these is equivalent to $6m^4 \div 2m$?

Option 1: $3 \times m$

Option 2: $3 \times m \times m$

Option 3: $3 \times m \times m \times m$

Option 4: $3 \times m \times m \times m \times m$

Correct Answer: Option 3

2. If $A = (80p^2q \div 5p) \div 2q$ and $B = (100p^2q^2 \div 10q^2) \div 2p$, what is the value of $A \div B$?

Option 1: $\frac{5}{4}$

Option 2: $\frac{4}{5}$

Option 3: $\frac{5}{8}$

Option 4: $\frac{8}{5}$

Correct Answer: Option 4

LOB: Use the common factor method in order to divide a polynomial by a monomial

1. Which of these is equivalent to $(14a - 6a^3 + 2a^5) \div 2a$?

Option 1: $(7 - 3a^2 + a^3)$

Option 2: $(7 - 6a^2 + 2a^3)$

Option 3: $(7 + 3a^2 + a^3)$

Option 4: $(7 + 6a^2 + 2a^3)$

Correct Answer: Option I

2. Which of the following expressions when divided by $9mn^2$ does not give a polynomial?

Option 1: $3(3m^2n + 18m^2n^2 - 60mn^4)$

Option 2: $2(45m^4n^2 - 9mn^2 - 36mn^3)$

Option 3: $3(21m^3n^2 + 15m^2n^5 - 6mn^2)$

Option 4: $2(56m^4n^3 - 18mn^3 + 54mn^2)$

Correct Answer: Option I

LOB: Divide each term in the numerator by the denominator in order to divide a polynomial by a monomial

- I. Which of these is equivalent to $(18h^3 - 9h^2 + 6h) \div 3h$?

Option 1: $\frac{3h(6h^2)}{3h} - \frac{3h(3h)}{3h} + \frac{3h(2)}{3h}$

Option 2: $\frac{3h(3h^2)}{3h} - \frac{3h(3h)}{3h} - \frac{3h(2)}{3h}$

Option 3: $\frac{3h(6h^2)}{3h} + \frac{3h(3h)}{3h} + \frac{3h(2)}{3h}$

Option 4: $\frac{3h(3h^2)}{3h} - \frac{3h(h)}{3h} + \frac{3h(2)}{3h}$

Correct Answer: Option I

2. Which of these is obtained by dividing $(36abc + 24a^2b^2c - (18a^2b^3c + 30a^2bc^2))$ by $3abc$?

Option 1: $-6ab^2 - 10ac + 8c + 12$

Option 2: $6ab^2 - 10ac + 8c + 12$

Option 1: $-6ab^2 - 10ac + 8c - 12$

Option 2: $6ab^2 - 10ac + 8c - 12$

Correct Answer: Option I

LOB: Use the common factor method in order to divide a polynomial by a polynomial

- I. What is the quotient obtained when $(18r^2 - 32d^2)$ is divided by $(3r + 4d)$?

Option 1: $2 \times (3r - 4d)$

Option 2: $2 \times (3r + 4d)$

Option 3: $4 \times (3r - 4d)$

Option 4: $4 \times (3r + 4d)$

Correct Answer: Option I

2. An expression is shown below: $22(x^4 - x^3 - 20x^2)$

Vinod claims that when the expression is divided by $11x^2(x + 4)$ we get $2(x - 5)$.

Varun claims that when the expression is divided by $11x(x + 5)$ we get $2x(x + 4)$.

Who is/are correct?

Option 1: Only Vinod

Option 2: Only Varun

Option 3: Both of them

Option 4: None of them

Correct Answer: Option I

LOB: Check the given mathematical statements in order to find and give reasons for the possible errors in them

- I. The steps to solve the equation $3(y + 1) + 2y = 20$ are shown below.

$$3(y + 1) + 2y = 20$$

Step 1: $3y + 3 + 2y = 20$

Step 2: $5y = 20$

Step 3: $y = \frac{20}{5}$

Step 4: $y = 5$

Given that the solution is incorrect, in which step was the first mistake made?

Option 1: Step 1

Option 2: Step 2

Option 3: Step 3

Option 4: Step 4

Correct Answer: Option 2

2. The steps to solve the equation $(x + 5)^2 - 3x = x^2 - 3$ are shown below.

$$(x + 5)^2 - 3x = x^2 - 3$$

Step 1: $x^2 - 10x + 25 - 3x = x^2 - 3$

Step 2: $-13x + 25 = -3$

Step 3: $-13x = -28$

Step 4: $x = \frac{28}{13}$

Given that the solution is incorrect, in which step was the first mistake made and what is the correct solution to the equation?

Option 1: Step 1; -4

Option 2: Step 2; 4

Option 3: Step 3; $-\frac{22}{13}$

Option 4: Step 4; $-\frac{28}{13}$

Correct Answer: Option 1

Suggested Teacher Resources



I

Lesson Plan



Objectives	Use the method of common factors in order to factorize the given algebraic expression
Prerequisite Knowledge	Meaning of factorization, common factors.
Material Required	Chalk, duster, blackboard, Pen/Pencil and notebook
Procedure	<p>The teacher will start with a question: The area of a rectangle, in square units, is represented by $3n^2 + 3n$ for a natural number n. Find the length and width of the rectangle. The students will be given 2 minutes to list the answer(s) they find.</p> <div style="border: 1px solid black; padding: 10px; text-align: center;">$(3n^2 + 3n)$ square units</div> <p>The following discussion should include a similar approach as below: We know that the area of a rectangle is a product of its length and breadth, i.e., $3n^2 + 3n = \text{length} \times \text{breadth (of the rectangle)}$ That means, we need to factorize the polynomial given for area to find the algebraic expressions for length and breadth. First, we need to check the Greatest Common factor (GCF) in $3n^2 + 3n$. When factoring a polynomial, we first look for a monomial that is the GCF of all the terms of the polynomial. Then, we reverse the distribution process by factoring the GCF out of each term and writing it on the outside of the parentheses. We can express $3n^2 + 3n = 3 \times n \times n + 3 \times n$, we can see that $3n$ is the GCF. We can take it as common and hence factorize the expression as $3n^2 + 3n = 3n(n + 1)$, where $3n$ and $(n + 1)$ are the two factors. Here, the width of the rectangle could be $3n$ units and the length could be $(n + 1)$ units OR the width of the rectangle could be $(n + 1)$ units and the length could be $3n$ units.</p> <p>Note: If students try to use 1 as the common factor for two or more numbers, point out that, while 1 is indeed a factor, factoring out a 1 does not help in finding the factors of an expression. If this issue arises, it may be necessary to discuss the results when factoring out a 1. Students may opt to factor only n or 3 or even $\frac{1}{2}n$: $n(3n + 3)$ or $3(n^2 + n)$ or $\frac{1}{2}n(6n + 6)$.) There are infinite representations for the dimensions of the rectangle.</p> <p>Now give students a few questions to attempt with their partners. A few have been suggested below:</p> $\begin{aligned} & 30pq + 15p \\ & ax^3 - bx^2 + cx \\ & l^2m^2n - lm^2n^2 - l^2mn^2 \end{aligned}$

$$\frac{18x^3y - 54x^2y + 72xy}{a^3 + a^2 + a + 1}$$

The teacher will then discuss all the expressions on the board by encouraging the students to volunteer.

Reference

<https://www.engageny.org/resource/algebra-i-module-4-topic-lesson-2>
<https://www.engageny.org/resource/algebra-i-module-4-topic-lesson-3>
<https://www.engageny.org/resource/algebra-i-module-4-topic-lesson-4>

2

Activity



Instructions:

- 1) The teacher will divide the class in two groups.
- 2) Group 1 will have to randomly choose one chit from bowl 1 and Group 2 will do the same from bowl 2. The teacher has to ensure that all the students have got the chit and no one is left.
- 3) After every student has got a chit, the teacher will give 1 minute for them to expand/ factorize the given algebraic expression. This will be done in their notebooks.
Those who got the expanded form of the expression, will need to factorize it.
Those who got the factored form, will need to expand the expression.
- 4) After one minute, the teacher will instruct everyone to stop.
- 5) The teacher will then ask volunteers from each group to collect the chits from their group members and return it in the respective bowls.
- 6) The teacher will randomly choose one chit from Bowl 2, and write down the factorized form on the board. The student from Group 1 who has the corresponding expanded form of the factorized form on board will need to raise hand and tell the expression.
- 7) Similarly, the teacher will now randomly choose one chit from Bowl 1, and write down the expanded form on the board. The student from Group 2 who has the corresponding factor form on her chit will raise a hand and tell the factor form.
- 8) Both the groups will get only 40 seconds to raise hand and answer.
- 9) The chits picked by the teacher will not go back in the bowls.
- 10) If the expression is correct then the respective Group will get 5 points, if a wrong expression is given then there will be a penalty of 3 points. If no answer is given then no points will be given.
- 11) The teacher will note the scores of both the teams on the board and will announce the winner after all the chits have been exhausted.

A sample of expressions has been given below:

S. No.	Expanded Form	Factorised form
1	$21x^2y^3 + 27x^3y^2$	$3x^2y^2(7y + 9x)$
2	$4x^2 - 20x + 25$	$(2x - 5)(2x - 5)$
3	$23xy - 46x + 54y - 108$	$(y - 2)(23x + 54)$
4	$6x^2y^3 + 9xy^4 + 18y^5$	$3y^3(2x^2 + 3xy + 6y^2)$
5	$16h^2 - 36k^2$	$(4h - 6k)(4h + 6k)$
6	$r^4 - 16s^4$	$(r^2 + 4s^2)(r + 2s)(r - 2s)$
7	$x^2 - 2x + 15/16$	$(x - 5/4)(x - 3/4)$
8	$-2x^3 - 2x^2 + 112x$	$-2x(x - 7)(x + 8)$
9	$y^8 - 81x^4$	$(y^4 + 9x^2)(y^2 + 3x)(y^2 - 3x)$
10	$9x^2 - 3x - 12$	$3(x + 1)(3x - 4)$

15. INTRODUCTION TO GRAPHS

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
A line graph	Draw a line graph in order to represent the given data that changes continuously over periods of time	-
	Interpret the given line graph in order to answer the given questions	
Linear graph and Location of a point/coordinates	Plot a point on the graph in order to describe its coordinates	-
	Plot the given points on the graph in order to verify if they lie on the same line or not	
	Choose an appropriate scale in order to plot a graph for the given data	
Some applications	Construct the line graph in order to discuss the relationship between independent and dependent variable in a given mathematical situation	

Test items

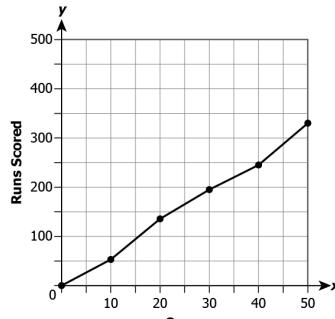


LOB: Draw a line graph in order to represent the given data that changes continuously over periods of time

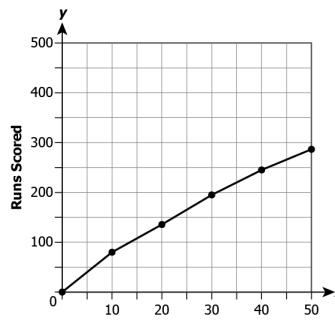
- I. The table below shows the run scored by a team in a 50 over match.

Over	10	20	30	40	50
Run	80	135	195	245	330

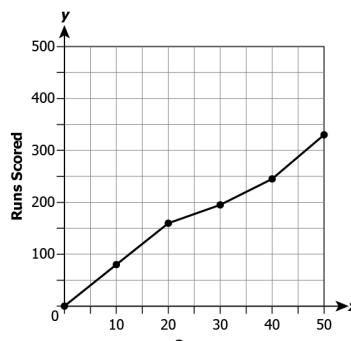
Which of these line plots shows the runs scored by the team?



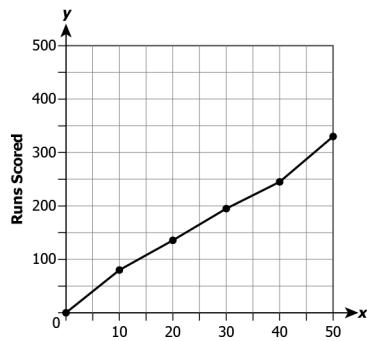
Option 1:



Option 2:



Option 3:



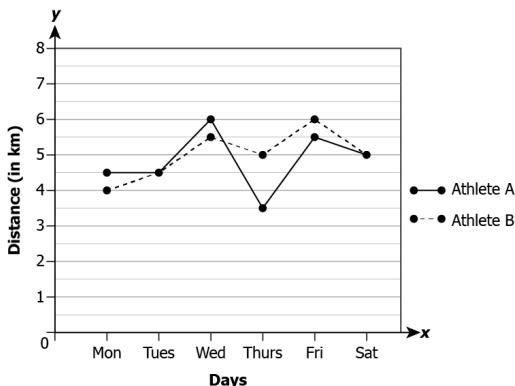
Option 4:

Correct Answer: Option 4

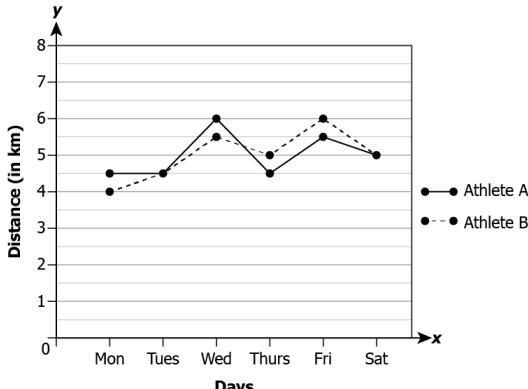
2. The table below shows the distance, in km, covered by two athletes on each day of last week.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Athlete A	4.5	4.5	6.0	3.5	5.5	5.0
Athlete B	4.0	4.5	5.5	5.0	6.0	5.0

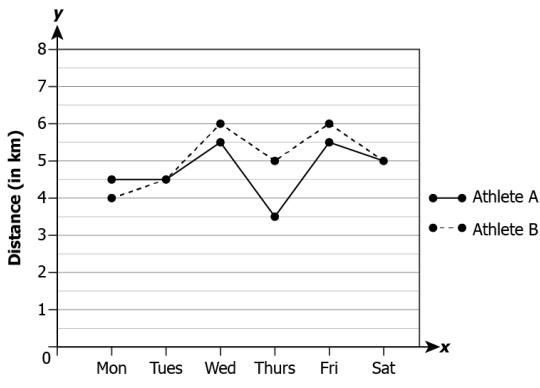
Which of these line graphs correctly represents the data?



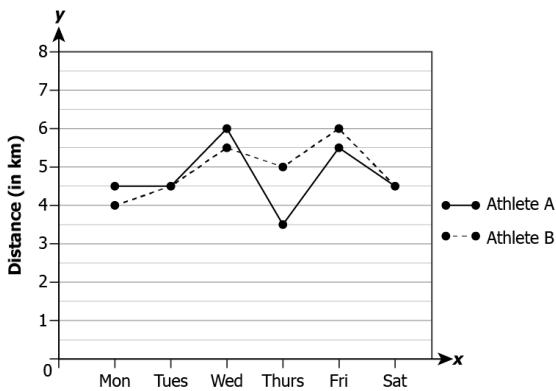
Option 1:



Option 2:



Option 3:

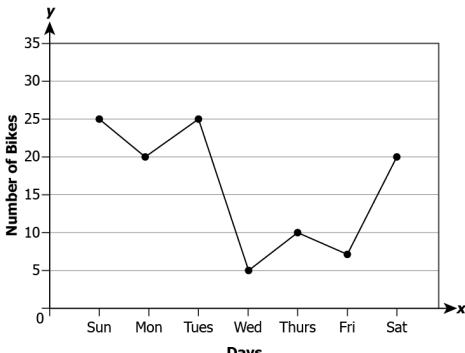


Option 4:

Correct Answer: Option 1

LOB: Interpret the given line graph in order to answer the given questions

- I. The line graph below shows the number of bikes sold at a store on each day of last week.

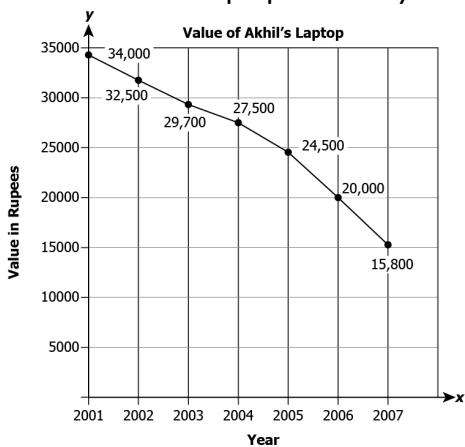


On which two days' equal number of bikes was sold?

- Option 1:** Sunday and Monday
- Option 2:** Sunday and Tuesday
- Option 3:** Wednesday and Friday
- Option 4:** Monday and Wednesday

Correct Answer: Option 2

2. The line graph below shows the value of Akhil's laptop over the years.



Which of the following statements is not correct?

- Option 1:** The value of laptop decreases by ₹3000 in 2004-05.
- Option 2:** The value of laptop decreases by ₹1500 in 2001-02.
- Option 3:** The value of laptop decreases by ₹5800 in 2006-07.
- Option 4:** The value of laptop decreases by ₹4500 in 2005-06.

Correct Answer: Option 3

LOB: Plot a point on the graph in order to describe its coordinates

- I. Which of the following is correct about the point R (4, 1) when plotted on a coordinate grid?

- Option 1:** The point R lies on the x-axis.
- Option 2:** The point R lies on the y-axis.
- Option 3:** The point R lies 4 units to the right of y-axis and 1 unit up the x-axis.
- Option 4:** The point R lies 1 unit to the right of y-axis and 4 units up the x-axis.

Correct Answer: Option 3

2. A point T lies 6 units to the right of the y-axis. Which of these can be the coordinates of the point T?

- Option 1:** (0, 6)
- Option 2:** (6, 0)
- Option 3:** (3, 3)
- Option 4:** (12, 12)

Correct Answer: Option 2

LOB: Plot the given points on the graph in order to verify if they lie on the same line or not

- I. Of the given points A (0, 8), B (5, 5), C (10, 2) and D (8, 4), which one is not collinear?

- Option 1:** (0, 8)

Option 2: (5, 5)

Option 3: (10, 2)

Option 4: (8, 4)

Correct Answer: Option 4

2. If the points (4, 0), (7, 3) and (k, 5) lies on the same line, what is the value of k ?

Option 1: 5

Option 2: 8

Option 3: 9

Option 4: 10

Correct Answer: Option 3

LOB: Choose an appropriate scale in order to plot a graph for the given data

- I. The table below shows the cost of parking for different hours.

Hours	2	4	6	8	10
Amount (in ₹)	160	320	480	640	800

Which of following options shows the appropriate scale to plot a graph of the given data?

Option 1: On horizontal axis, 1 unit = 1 hour; on vertical axis, 1 unit = 1

Option 2: On horizontal axis, 1 unit = 1 hour; on vertical axis, 1 unit = 10

Option 3: On horizontal axis, 1 unit = 1 hour; on vertical axis, 1 unit = 160

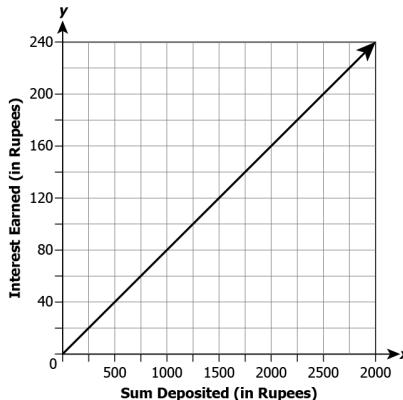
Option 4: On horizontal axis, 1 unit = 1 hour; on vertical axis, 1 unit = 800

Correct Answer: Option 3

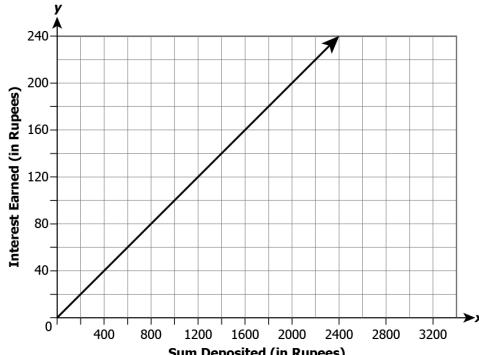
2. The table below shows the simple interest earned for different amount of sum deposited.

Sum Deposited (in ₹)	1000	1500	2000	2500	3000
Interest Earned (in ₹)	80	120	160	200	240

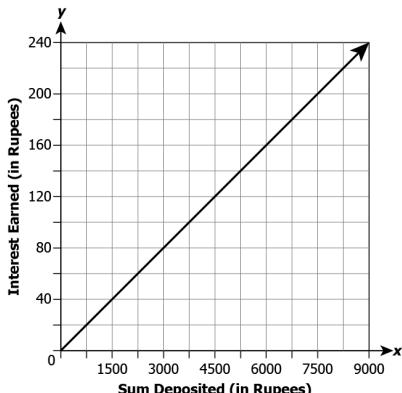
Which of following graphs best represents the given data?



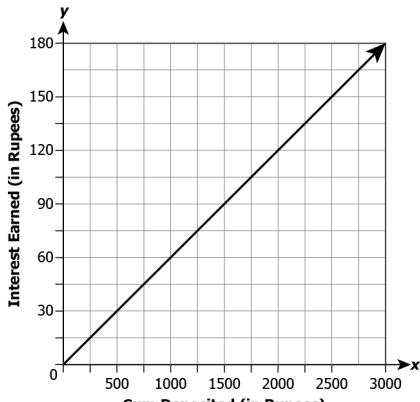
Option 1:



Option 2:



Option 3:



Option 4:

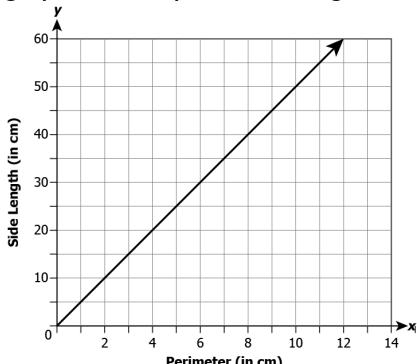
Correct Answer: Option I

LOB: Construct the line graph in order to discuss the relationship between independent and dependent variable in a given mathematical situation

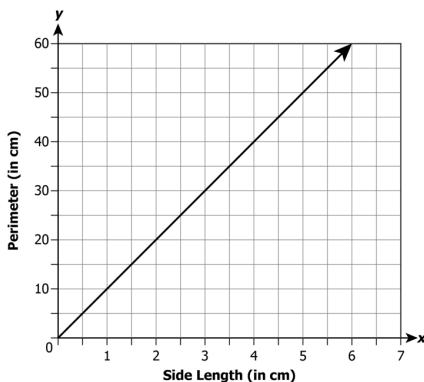
- I. The table below shows the perimeter of a regular pentagon for different side lengths.

Side length (in cm)	3	5	7	8
Perimeter (in cm)	15	25	35	40

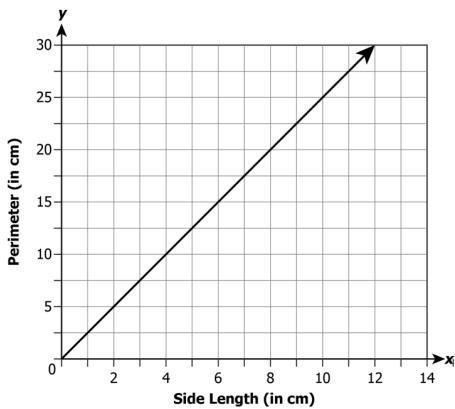
Which of following graphs best represents the given data?



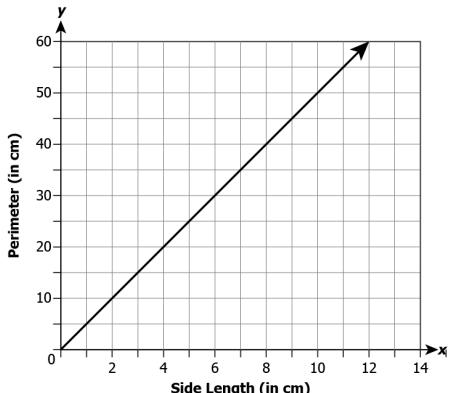
Option 1:



Option 2:



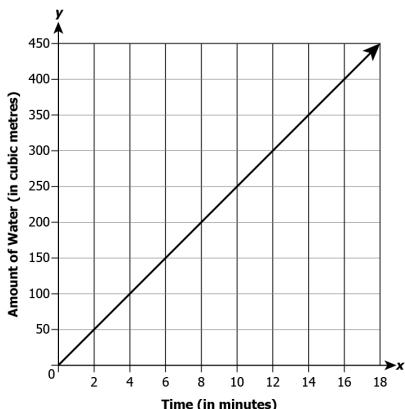
Option 3:



Option 4:

Correct Answer: Option 4

2. A swimming pool has a capacity of 375 cubic metres. A pump fills the empty pool at the constant rate shown in the graph.



Which of the following statements is correct for the given data?

- Option 1:** The pump fills 25 cubic metres of water each minute.
- Option 2:** The pump fills 1 cubic metre of water in 25 minutes.
- Option 3:** The pump fills 50 cubic metres of water each minute.
- Option 4:** The pump will completely fill the tank in 25 minutes.

Correct Answer: Option 1

Suggested Teacher Resources



I

Lesson Plan

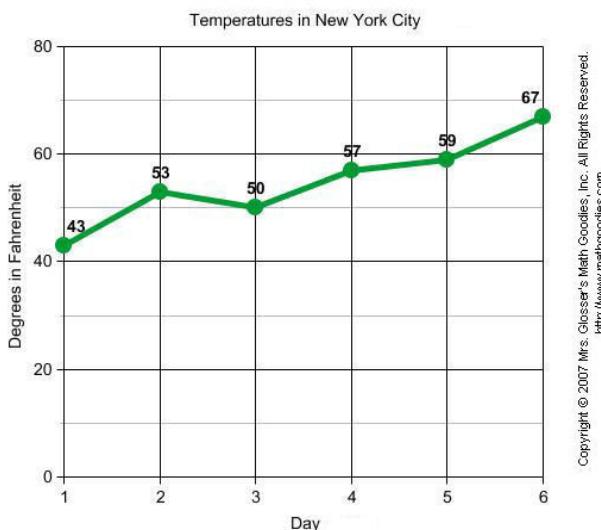


Objectives	Draw a line graph in order to represent the given data that changes continuously over periods of time. Interpret the given line graph in order to answer the given questions.																				
Prerequisite Knowledge	Interpreting the given data. Basic knowledge of Cartesian plane.																				
Material Required	Chalk and blackboard. Pen/pencil and notebooks. Graph Papers.																				
Procedure	<p>Draw the following table on blackboard to represent the given data-</p> <p>The table below shows daily temperatures for New York City, recorded for 6 days, in degrees Fahrenheit.</p> <p>Temperatures In New York City</p> <table><thead><tr><th>Day</th><th>Temperature</th></tr></thead><tbody><tr><td>1</td><td>43° F</td></tr><tr><td>2</td><td>53° F</td></tr><tr><td>3</td><td>50° F</td></tr><tr><td>4</td><td>57° F</td></tr><tr><td>5</td><td>59° F</td></tr><tr><td>6</td><td>67° F</td></tr></tbody></table> <p>Now, draw the line graph of the given data with the help of the students using the following instructions-</p> <p>Step 1: Draw and label the X-axis and the Y-axis on the board. Tell students that they will be learning about line graphs in this activity. Explain that a line graph uses points and lines to examine changes over time. Line graphs are often used when examining relationships between two types of information.</p> <p>Step 2: Tell students that, like the bar graph, the line graph has an X- and a Y-axis. The dependent variable is plotted on the Y-axis and usually measures quantity (percentage, Rupees, litres, etc.). The independent variable is plotted on the X-axis and usually measures time periods. Use the given data to complete your line graph on the board.</p> <p>Let's define the various parts of a line graph.</p> <table><tbody><tr><td>title</td><td>The title of the line graph tells us what the graph is about.</td></tr><tr><td>labels</td><td>The horizontal label across the bottom and the vertical label along the side tells us what kinds of facts are listed.</td></tr><tr><td>scales</td><td>The horizontal scale across the bottom and the vertical scale along the side tell us how much or how many.</td></tr></tbody></table>	Day	Temperature	1	43° F	2	53° F	3	50° F	4	57° F	5	59° F	6	67° F	title	The title of the line graph tells us what the graph is about.	labels	The horizontal label across the bottom and the vertical label along the side tells us what kinds of facts are listed.	scales	The horizontal scale across the bottom and the vertical scale along the side tell us how much or how many.
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scales	The horizontal scale across the bottom and the vertical scale along the side tell us how much or how many.																				

points	The points or dots on the graph show us the facts.
lines	The lines connecting the points give estimates of the values between the points. Now that we are familiar with the parts of a line graph, we can answer some questions about each of the graphs from the examples above.

Step 3:

Write the title “Temperature in New York City” above the graph. Ask students where the first point in the graph should go if the temperature was 43° F on Day 1. Mark this point on the graph. Have students point out where other points should be marked. When done, connect the points with a line.



Now ask the students to interpret the given line graph in order to answer the given questions. –

QUESTION

1. What is the title of this line graph?
2. What is the range of values on the horizontal scale?
3. What is the range of values on the vertical scale?
4. How many points are in the graph?
5. What was the lowest temperature recorded?
6. What was the highest temperature recorded?
7. At what point did the temperature dip?

ANSWER

- Temperatures in New York City**
- I to 6**
- 0 to 80**
- 6**
- 43° F**
- 67° F**
- Day 3: 50° F**

References

<https://www.mathgoodies.com/lessons/graphs/line>

<https://www.scholastic.com/teachers/sponsored-content/unexpected-math/17-18/looking-through-line-graphs/>

Activity



Objective: To collect, organise and represent the data graphically.

Step 1: Ask a question to the class- how many first cousins do you have?

Step 2: Collect the responses. Suppose the responses are:

6,2,1,3,4,3,2,6,5,6,1,2,3,3,2,3,2,1,2,4,4,5,2,1,2,3,2,3,2,3,4,1,4,5,4,5,2,1,3,2,1,2,1.

Step 3: Ask the students- Do you find it difficult to identify which student has given which response? (Discuss)

Step 4: Now, we will prepare a table in which the responses are mentioned in front of the student's name.

S.N.o.	Student Name	Number of cousins
1.	Ramesh	6
2.	Shipra	2
3.	Anuj	1
4.	Ankit	3
5.	Anit	4
6.	Mahima	3
7.	Priya	2
8.	Shikha	6
9.	Reema	5
10.	Sonu	6
11.	Monu	1
12.	Shweta	2
13.	Silpa	3
14.	Renu	3
15.	Ritu	2

S.N.o.	Student Name	Number of cousins
16.	Pooja	3
17.	Shantnu	2
18.	Ekansh	3
19.	Dimple	2
20.	Richa	1
21.	Amit	2
22.	Vinit	4
23.	Sunil	4
24.	Dinesh	5
25.	Yash	2
26.	Ashish	1
27.	Sohan	2
28.	Mohan	3
29.	Rekha	2
30.	Rajesh	3

S.N.o.	Student Name	Number of cousins
31.	Shreya	2
32.	Vani	3
33.	Amita	4
34.	Pankaj	1
35.	Anu	4
36.	Om	5
37.	Jaya	4
38.	Sushma	5
39.	Sunita	2
40.	Neha	1
41.	Atul	3
42.	Akash	2
43.	Alok	1
44.	Vinod	2
45.	Sharda	1

From this table we can clearly see the responses of each student. For instance, Ritu has only two cousins whereas Ramesh has six cousins.

Now ask the following questions from the students –

1. What is the number of students who have the least number of cousins?
2. What is the number of students who have the highest number of cousins?
3. What is the number of students who have no cousins?
4. How many students have cousins more than 3 but less than 6?

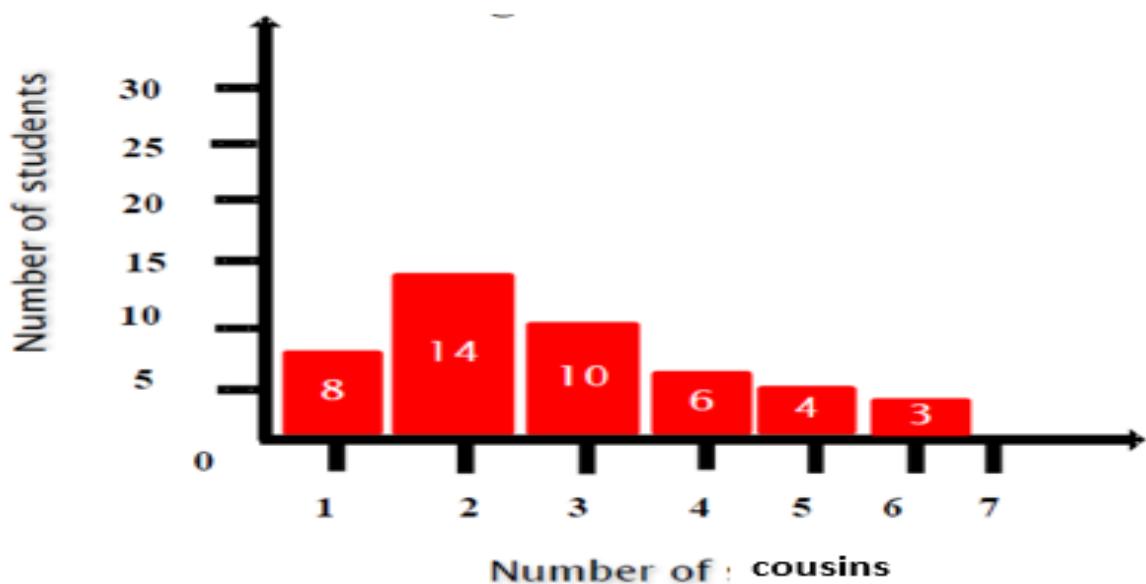
The students would have to count for each of the above questions. To avoid any errors, the teacher and the students should make a frequency distribution table using tally marks for this data.

No. of Cousins	Number of students	Frequency
1		8
2		14
3		10
4		6
5		4
6		3
		45

The students can now discuss the previous questions based on this table and also record other observations.

Step 5: To prepare a better visual representation, we can construct a bar graph for this data using the table.

The students will then construct a bar graph for this data which would look as below. The teacher will clear queries while the students attempt this.



Step 6: The teacher could now also ask the students to make a pie chart for this data.

Now again ask the same questions from the students.

This activity helps to understand the importance of graphs and give students the general understanding of how graphs are made out of the given data.

16. PLAYING WITH NUMBERS

QR Code:



Learning outcome and Learning Objectives:

Content area / Concepts	Learning Objectives	Learning Outcome
Numbers in General Form	Use the concepts of place value in order to express the given numbers in their generalised form	
Games with Numbers	Apply the divisibility rule of 11 in order to check whether a given number is divisible by 11 or not	Observe patterns using algebraic operations in order to derive the divisibility rules of 2,3,4,5,6,9 & 11
	Add or subtract a two-digit number and its reverse in order to check whether it is divisible by 9 or not	
	Subtract a three-digit number and its reverse in order to verify that it is divisible by 99	
	Form all possible three-digit numbers using the given 3 digits in order to verify that the sum of these numbers will be divisible by 37	
Letters for Digits	Use addition and multiplication in order to find the values of the letters in the given puzzles	
Tests of Divisibility	Apply the divisibility rule of 10 in order to check whether a given number is divisible by 10 or not	
	Apply the divisibility rule of 5 in order to check whether a given number is divisible by 5 or not	
	Apply the divisibility rule of 2 in order to check whether a given number is divisible by 2 or not	
	Apply the divisibility rule of 3 and 9 in order to check whether a given number is divisible by them	
	Apply the divisibility rule of 2,5 and 10 in order to check whether a given number is divisible by all of them or not.	

Test items



LOB: Use the concepts of place value in order to express the given numbers in their generalised form ($22 = 2 \times 10 + 2 \times 1$)

1. Which out of these is a generalized form of 405?

Option 1: $4 \times 10 + 5 \times 1$
Option 2: $4 \times 100 + 0 \times 10 + 5 \times 1$
Option 3: $4 \times 1000 + 0 \times 100 + 5 \times 10$
Option 4: $4 \times 100 + 5 \times 10$

Correct Answer: Option 2

2. Which of the following statements are correct?

a) The generalized form of a number xyz is $100x + 10y + z$
b) The generalized form of a number and its reverse is the same.

Option 1: Only a) is correct
Option 2: Only b) is correct
Option 3: Both a) and b) are correct
Option 4: Neither a) nor b) is correct

Correct Answer: Option 1

LOB: Apply the divisibility rule of 11 in order to check whether a given number is divisible by 11 or not

1. Which of the following numbers is divisible by 11?

Option 1: 710098
Option 2: 741889
Option 3: 759066
Option 4: 771124

Correct Answer: Option 3

2. What value of k would make the number $75k077$ divisible by 11?

Option 1: 0
Option 2: 6
Option 3: 8
Option 4: 9

Correct Answer: Option 4

LOB: Add or subtract a two-digit number and its reverse in order to check whether it is divisible by 9 or not.

1. Which of the following options make the statement below true?

If $a > b$, then $ab - ba$ is divisible by ___ as $ab - ba$ can be written as ___

Option 1: 9; $9(a - b)$
Option 2: 9; $9(a + b)$
Option 3: 6; $9(a - b)$
Option 4: 6; $9(a + b)$

Correct Answer: Option 1

2. Which of the following numbers obtained as a result of these operations are divisible by 9?

a) 76–67
b) 98–89
c) 21–12

Option 1: c and a
Option 2: b and c
Option 3: a and b

Option 4: a, b, and c

Correct Answer: Option 4

LOB: Subtract a three-digit number and its reverse in order to verify that it is divisible by 99.

- I. Which of the following options make the statement below true?

If $a > c$, then $abc - cba$ is divisible by _____ $abc - cba$ can be written as _____

Option 1: 99; $99(a - c)$

Option 2: 99; $99(a + c)$

Option 3: 100; $100(a - c)$

Option 4: 100; $100(a + c)$

Correct Answer: Option 1

2. By which number would the numbers obtained as a result of the following operations are divisible?

a) 756–657

b) 908–809

c) 921–129

Option 1: 99

Option 2: 22

Option 3: 66

Option 4: 33

Correct Answer: Option 1

LOB: Form all possible three-digit numbers using the given 3 digits in order to verify that the sum of these numbers will be divisible by 37

- I. By what number is the sum of $678+867+786$?

Option 1: 36

Option 2: 37

Option 3: 46

Option 4: 47

Correct Answer: Option 2

2. Consider the number abc , where a,b and c numbers from 1 to 9 and . Which of these is divisible by 111?

Option 1: $abc + bca + cab$

Option 2: $abc + bac + cab$

Option 3: $acb + bca + cab$

Option 4: $acb + bac + cab$

Correct Answer: Option 1

LOB: Use addition and multiplication in order to find the values of the letters in the given puzzles

- I. Which of the following options make the puzzle below true?

$$\begin{array}{r} Z \\ Z \\ + Z \\ \hline 65Z \\ \hline yy6 \end{array}$$

Option 1: $Z=9, Y=6$

Option 2: $Z=4, Y=9$

Option 3: $Z=4, Y=6$

Option 4: $Z=6, Y=6$

Correct Answer: Option 3

2. What values of A and B will make the puzzle below true?

$$\begin{array}{r}
 A B A \\
 \times 5 \\
 \hline
 4 4 B 0
 \end{array}$$

Option 1: A=9, B=8

Option 2: A=8, B=9

Option 3: A=4, B=2

Option 4: A=4, B=2

Correct Answer: Option 2

LOB: Apply the divisibility rule of 10 in order to check whether a given number is divisible by 10 or not

- I. Which of the following numbers is divisible by 10?

Option 1: 3390

Option 2: 3445

Option 3: 4039

Option 4: 7601

Correct Answer: Option 1

2. What value of X and Y would make the number 90Y77X divisible by 10?

Option 1: X=0 and Y can be any number from 1 to 9

Option 2: Y=0 and X can be any number from 1 to 9

Option 3: X=5 and Y =0

Option 4: X=7 and Y =5

Correct Answer: Option 1

LOB: Apply the divisibility rule of 5 in order to check whether a given number is divisible by 5 or not

- I. Which of the following numbers is divisible by 5?

Option 1: 1355

Option 2: 2411

Option 3: 3198

Option 4: 4466

Correct Answer: Option 1

2. What value of X and Y would make the number Y77X divisible by 5?

Option 1: Y=0 and X can be any number from 1 to 9

Option 2: X=6 and Y =9

Option 3: X=7 and Y =5

Option 4: X=0 or 5 and Y can be any number from 1 to 9

Correct Answer: Option 4

LOB: Apply the divisibility rule of 2 in order to check whether a given number is divisible by 2 or not

- I. Which of the following options show a list of numbers which are divisible by 2?

Option 1: 67; 896789; 9087

Option 2: 67543; 98; 98456

Option 3: 98456; 870956; 854780

Option 4: 67453; 898; 9076

Correct Answer: Option 3

2. What value of X and Z would make the number 98ZX divisible by 2?

Option 1: Z=0 and X can be any number from 1 to 9

Option 2: X=9 and Z =9

Option 3: X=4 and Z =5

Option 4: X=7 and Z=6

Correct Answer: Option 3

LOB: Apply the divisibility rule of 3 and 9 in order to check whether a given number is divisible by them.

1. Which of the following options show a list of numbers which are all divisible by 3?

Option 1: 672; 96789; 9087
Option 2: 60043; 98; 98456
Option 3: 98456; 870956; 854780
Option 4: 67453; 898; 9076

Correct Answer: Option I

2. What value of digits A and B would make the number 894A5B divisible by 3?

Option 1: A=3 and B=1
Option 2: A=2 and B=1
Option 3: A=3 and B=4
Option 4: A=4 and B=2

Correct Answer: Option I

LOB: Apply the divisibility rule of 2,5 and 10 in order to check whether a given number is divisible by all of them or not.

1. Which of the following options make the statement below true?

A number divisible by 10 would always have its one's digit as ___ and a number divisible by 5 always has the one's digit as ___.

Option 1: 0; 0 or 5
Option 2: 0; 5
Option 3: 0 and 5; 0 and 5
Option 4: 0; 0

Correct Answer: Option I

2. By what number would a sum of two even numbers be always divisible?

Option 1: 2
Option 2: 4
Option 3: 5
Option 4: 6

Correct Answer: Option I

Suggested Teacher Resources



I

Lesson Plan



Objectives	Apply the divisibility rule of 3 and 9 in order to check whether a given number is divisible by them																				
Prerequisite Knowledge	Simple Division, divisibility rules of 2, 4, 5, 8 and 10.																				
Material Required	Chalk and blackboard. Pen/Pencil and notebook																				
Procedure	<p>Start the class with the following activity-</p> <p>Write down a few numbers on the board as below:</p> <p style="text-align: center;">5, 10, 24, 25, 115, 975, 4646, 625, 216, 920, 5280, 44420</p> <table border="1" style="margin-left: auto; margin-right: auto;"><tr><td style="text-align: center;">2</td><td style="text-align: center;">4</td><td style="text-align: center;">5</td><td style="text-align: center;">8</td><td style="text-align: center;">10</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></table> <p>Create a table separately mentioning numbers 2, 4, 5, 8 and 10 as column headers. Ask the students to place each number from the list under the column(s) that is a factor of the number. For example, if 32 was in the list, it would be placed under 2, 4 and 8, because they are all its factors.</p> <p>Give 5 minutes to the students to attempt this task. After 5 minutes, the teacher can ask the students to raise hands to fill the table on the board and also give reasons while doing so. The table would now look as below:</p> <p style="text-align: center;">5, 10, 24, 25, 115, 975, 4646, 625, 216, 920, 5280, 44420</p> <table border="1" style="margin-left: auto; margin-right: auto;"><tr><td style="text-align: center;">2</td><td style="text-align: center;">4</td><td style="text-align: center;">5</td><td style="text-align: center;">8</td><td style="text-align: center;">10</td></tr><tr><td>10, 24, 4646, 216, 920, 5280, 44420</td><td>24, 216, 920, 5280, 44420</td><td>5, 10, 25, 115, 975, 625, 920, 5280, 44420</td><td>24, 216, 920, 5280,</td><td>10, 920, 5280, 44420</td></tr></table> <p>Note: During this task, most students would have used the divisibility rules of 2, 4, 5, 8 and 10 to do this. Discuss them in case students have any doubt. The teachers can also take different numbers in their class.</p> <p>Now, move on to discuss the divisibility rules of 3 and 9. Write a few numbers which students already know have 3 or 9 as their factors.</p> <p>What do the numbers 12, 15, 18, 27, 30, 66, and 93, 102 all have in common? Each is divisible by 3. Calculate the sum of the digits for each given number. For example, the sum of the digits in the number 12 is 3 because $1 + 2 = 3$.</p>	2	4	5	8	10						2	4	5	8	10	10, 24, 4646, 216, 920, 5280, 44420	24, 216, 920, 5280, 44420	5, 10, 25, 115, 975, 625, 920, 5280, 44420	24, 216, 920, 5280,	10, 920, 5280, 44420
2	4	5	8	10																	
2	4	5	8	10																	
10, 24, 4646, 216, 920, 5280, 44420	24, 216, 920, 5280, 44420	5, 10, 25, 115, 975, 625, 920, 5280, 44420	24, 216, 920, 5280,	10, 920, 5280, 44420																	

Give students time to find the sums. Record the sums on the board.

Number	Sum of its digits
12	1+2=3
15	1+5=6
18	1+8=9
27	2+7=9
30	3+0=3
66	6+6=12
93	9+3=12
114	1+1+4=6

Now based on the table ask the following questions:

What do these sums in column 2 have in common?

They are divisible by 3.

When the sum of a number's digits is divisible by 3, the entire number is divisible by 3. **This is the divisibility rule for 3.**

Now let's examine a different set of numbers: 27, 36, 45, 54, 63, 72, 90, 108. What do these numbers have in common?

They are divisible by 9.

Now ask the students to calculate the sum of the digits for each given number as done earlier. Record the sums on the board.

Number	Sum of its digits
27	2+7=9
36	3+6=9
45	4+5=9
54	5+4=9
63	6+3=9
72	7+2=9
108	1+0+8=9
144	1+4+4=9

We can observe that all the sums in column 2 are divisible by 9.

When the sum of a number's digits is divisible by 3 and 9, the entire number is divisible by 9.

This is the divisibility rule for 9.

Now let us consider a big number like 1503. This number is divisible by both 3 and 9.

We can represent 1503 in the expanded form as

$$1 \times 1000 + 5 \times 100 + 0 \times 10 + 3 \times 1$$

Let's decompose 1000 to 999+1 and 100 to 99+1 (999 and 99 are divisible by 3 and 9).

$$1 \times 1000 + 5 \times 100 + 0 \times 10 + 3 \times 1 = 1(999 + 1) + 5(99 + 1) + 3 \times 1$$

Now use the distributive property,

$$1(999 + 1) + 5(99 + 1) + 3 \times 1 = 1 \times 999 + 1 \times 1 + 5 \times 99 + 5 \times 1 + 3 \times 1$$

Since we know that 9 is divisible by both 3 and 9, we will now factor the 9 out of the expression. We can use the commutative and associative properties for this.

$$999 + 5 \times 99 + 5 + 3 + 1$$

$$111(9) + 5 \times 11(9) + 5 + 3 + 1$$

$$9 \times 111 + 9 \times 5 \times 11 + 5 + 3 + 1$$

$$9(111 + 5 \times 11) + 5 + 3 + 1$$

	<p>We know that the product of $9(111 + 5 \times 11)$ is divisible by 9 since 9 is already factored out. Since the first term is divisible by 3 and 9, the number 1503 is divisible by 3 and 9 if and only if $5 + 3 + 1$ is also divisible by 3 and 9.</p> <p>The sum of $5 + 3 + 1$ is 9. That means it is also divisible by 3 and 9 as both 3 and 9 are factors of 9.</p> <p>$5 + 3 + 1$, these three digits are part of our original number.</p> <p>Also, explain to the students that if a number is divisible by 9, it is divisible by 3 as well as $9 = 3 \times 3$. But if any number is divisible by 3, it is not necessarily divisible by 9. For example, 12, 24, 42 are divisible by 3 but are not divisible by 9.</p>
Reference	https://www.engageny.org/sites/default/files/downloadable-resources/math-g6-m2-topic-d-lesson-17-teacher.pdf

2

Activity



The following puzzle can be given to the students to solve.

The digits are distinct whole numbers. Solve for A, B and C, given that:

$$\begin{array}{r}
 \text{AA} \\
 \text{BB} \\
 + \text{CC} \\
 \hline
 \text{ABC}
 \end{array}$$

Students should be given some time to explore and try to solve it on their own.

There could be different ways to solve this. Two of them have been discussed below:

Method I:

We have been given three two-digit numbers which are being added and their sum is a three-digit number. Any two-digit number XY could be expressed as $10X+Y$ and any three digit number XYZ could be expressed as $100X + 10Y + Z$.
So, we can write,

$$\begin{aligned}
 AA &= 10A + A = 11A \\
 BB &= 10B + B = 11B \\
 CC &= 10C + C = 11C \\
 ABC &= 100A + 10B + C
 \end{aligned}$$

We have been given that

$$\begin{aligned}
 AA + BB + CC &= ABC \\
 AA + BB + CC &= 100A + 10B + C
 \end{aligned}$$

We can write it as

$$11A + 11B + 11C = 100A + 10B + C$$

We can take 11 common in the left-hand side of the equation, which could be shown as below:

$$11(A + B + C) = 100A + 10B + C$$

This means that the number ABC is divisible by 11.

By the divisibility rule of 11, we know that any whole number is divisible by 11 if the sum of the digits at even position = sum of the digits at the odd position.

So,

$$A + C = B$$

Substituting this in above,

$$\begin{aligned}11(A + B + C) &= 100A + 10B + C \\11(A + A + C + C) &= 100A + 10(A + C) + C \\11 \times 2(A + C) &= 100A + 10A + 10C + C \\22(A + C) &= 110A + 11C \\22A + 22C &= 110A + 11C \\11C &= 88A \\C &= 8A\end{aligned}$$

Since A, B and C are distinct whole numbers, the only possible value for A here is 1.

$$\begin{aligned}A &= 1, C = 8(1) = 8 \\B &= A + C = 1 + 8 = 9\end{aligned}$$

Therefore, the number ABC is 189.

Method 2:

$$\begin{array}{r} \text{AA} \\ \text{BB} \\ + \text{CC} \\ \hline \text{ABC} \end{array}$$

We have been given that three two-digit numbers are added and the sum is a three-digit number.

The largest possible value for ABC could be $99 + 88 + 77 = 264$.

(Even if we were not given the condition that A, B and C are distinct, then also the largest possible value for ABC would be less than 300. Think about this!)

Considering the largest possible value of ABC is 264, there are only two choices left for A, i.e., A=1 or A=2.

Also, if we look at the ones place of the resultant, we can see that

$$A + B + C = C$$

Which implies that $A + B = 10$, so that a carryover can be taken and C remains in the ones place of ABC.

The two possible cases are:

Case(i)

If A = 1, then B = 9.

So, AA + BB + CC = ABC

$$11 + 99 + CC = 19C$$

$$110 + CC = 19C$$

Now, if we focus on the one's column, we have

$$0 + C = C$$

If we focus on the ten's column, we have

$$1 + C = 9$$

$$i.e., C = 8$$

$$110 + 88 = 198$$

This way, we get the number ABC as 198.

Case(ii)

If A = 2, then B = 8.

So, AA + BB + CC = ABC

$$22 + 88 + CC = 28C$$

$$110 + CC = 28C$$

Now, if we focus on the one's column, we have

$$0 + C = C$$

If we focus on the ten's column, we have

$$1 + C = 8$$

$$i.e., C = 7$$

If C = 7, then

$$AA + BB + CC = 110 + 77 = 187$$

Which is not possible as we have considered A=2?
Hence this solution is incorrect.

This way, we get the number ABC as 198.

Reference: <https://mindyourdecisions.com/2016/05/22/aa-bb-cc-abc-sunday-puzzle/>

