



As warm-up for $2 + 2 \neq 5$ let's prove $0 \neq 1$. To do this we need to introduce Peano's last axiom `zero_ne_succ n`, a proof that $0 \neq \text{succ } n$. To learn about this result, click on it in the list of lemmas on the right.

Start with `intro h`.

Now change `1` to `succ 0` in `h`.

Now you can `apply zero_ne_succ at h`.

Level

Next

False

`rw[add_comm 1 0] at h` ⌕ Retry

Active Goal

Assumptions:

`h : 0 = 0 + 1`

Goal:

False

`rw[<-succ_eq_add_one 0] at h` ⌕ Retry

Active Goal

Assumptions:

`h : 0 = succ 0`

Goal:

False

`apply zero_ne_succ at h` ⌕ Retry

Active Goal

Assumptions:

`h : False`

Goal:

False

`exact h` ⌕ Retry

level completed! 🎉



zero_ne_succ

$(a : \mathbb{N}) : 0 \neq \text{MyNat.succ } a$

`zero_ne_succ n` is the proof that $0 \neq \text{succ } n$.

In Lean, $a \neq b$ is *defined to mean* $a = b \rightarrow \text{False}$. Hence `zero_ne_succ n` is really a proof of $0 = \text{succ } n \rightarrow \text{False}$. Here `False` is a generic false statement. This means that you can `apply zero_ne_succ at h` if `h` is a proof of $0 = \text{succ } n$.