DEPARTMENT OF MATHEMATICS
Indian Institute of Technology, Guwahati

Quiz1 MA321
26-08-2021 Instructor : Sukanta Pati
Time : 11.10am–12.10pm Maximum Score : 20 of 30

Write appropriate and precise justifications. Use pencils for convenience. Submit by 12.20pm. Try to submit using the teams app. If that does not work, only then send it to my email pati@iitg.ac.in

1. `(The story of four techniques)` Let $T = \mathsf{conv}(0, e_1, 3e_2, 2e_3) \subseteq \mathbb{R}^3$.

   a) What are the vertices of T? [1]

   b) Assume that initially each point in T was given a value 0. On the first day the value of each point was increased by the amount of its first coordinate. On the second day the value of each point was decreased by twice the amount of its second coordinate and on the third day the value of each point was increased by thrice the amount of its third coordinate. On the final day we are supposed to find the maximum of the value at each point on T.

   c) Formulate the problem as an LPP. [1]

   d) Write its slpp in matrix form. [1]

   e) Can we use FTLP to tell the maximum solution for c)? [2]

   f) Write the dual lpp of the lpp in c). [2]

   g) Verify the primal-dual theorem for c) and f) and hence show that one can find the maximum value of the function in c) in this way too. [1]

   h) You can justify your answer to c) in a third way. Write the form of all linear functions that are strictly maximized at your vertex and continue. [3]

   i) You can answer your question in a fourth way. Compute the corresponding bfs and compute the relative cost vector. [3]

   j) Suppose that we had to maximize the square of the value at each point on the final day. Can that be dealt with given our limited knowledge of lpp? Show that it can be. Do the points of maximum form a convex set here? [3]

2. `(Proper writing)` Prove/disprove. Let S be a nonempty compact convex subset of $\mathbb{R}^5$ and $a \in \mathbb{R}^5$ be an arbitrary fixed point. Then a point of S which is at the maximum distance from a must be a vertex of S. [3]

3. `(Proper writing)` Let S and T be nonempty disjoint convex sets in $\mathbb{R}^n$. Then show that there exists a hyperplane which separates S and T. [3]

4. `(Proper writing)` In $\mathbb{R}^5$, consider the sets $A = \mathsf{conv}(0, e_1, 3e_2, 2e_3)$ and $B = \mathsf{conv}(0, e_4, e_5, e_4 + e_5)$. Is $A + B$ a polytope? [7]