DEPARTMENT OF MATHEMATICS
Indian Institute of Technology, Guwahati

Write appropriate and precise justifications. Draw neatly. Use pencils for convenience. Submit in the portal. If that does not work, only then send it to my email pati@iitg.ac.in before 16:05.

1. Consider the problem table

bv	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b
$*$	1	1	1	0	1	0	0	0	1
$*$	1	1	0	1	0	1	0	0	1
$*$	1	0	1	1	0	0	1	0	1
$*$	0	1	1	1	0	0	0	1	1
$*$	-1	-1	-1	-1	0	0	0	0	$*$

Write the simplex table for the basis (x_2, x_1, x_5, x_6). $\boxed{3}$

2. Continue from the previous table. Taking x_5 as the outgoing variable, use dual simplex method to reach the next simplex table. $\boxed{3}$

3. Consider the problem table given above. Write the simplex table at the ordered basis (x_4, x_5, x_6, x_7). What is the current vertex? What is the direction given by the nonbasic variable x_1? What is the next ordered basis to consider using Bland's rule? $\boxed{2+1+2+1}$

4. Consider optimizing $x^2 + y$ over the set $P = \{(x, y) \mid \frac{x^2}{4} + \frac{y^2}{16} = 1\}$. Use graphical method to solve it. $\boxed{4}$

5. Consider a 4×5 btp with the cost matrix $C = \begin{bmatrix} 1 & 2 & 1 & 3 & 1 \\ 3 & 2 & 1 & 2 & 2 \\ 2 & 1 & 1 & 3 & 1 \\ 1 & 1 & 3 & 3 & 3 \end{bmatrix}$, where the availabilities at the sources S_1, S_2, S_3, S_4 are $50, 60, 60, 70$, respectively and the demands at the sinks $T_1, \ldots, T_5$ are $40, 60, 30, 30, 80$, respectively.

  a) Write the corresponding transportation array. $\boxed{1}$

  b) Is $\{x_{11}, x_{12}, x_{21}, x_{23}, x_{24}, x_{25}, x_{31}, x_{45}\}$ a basis? Argue using any of the three methods. $\boxed{2}$

  c) What is the corresponding basic solution to the above basis? Is it feasible? $\boxed{1}$

  d) Select the initial bfs using nw-corner rule. $\boxed{1}$

e) Verify whether the bfs in d) is a minimal bfs. $\boxed{3}$

f) Find a better bfs. $\boxed{2}$

6. (`Write properly`) Consider the set

$$P = \{x \in \mathbb{R}^4 \mid x \geq 0,\ x_1 + x_2 + x_3 \leq 1,\ x_1 + x_2 + x_4 \leq 1,\ x_1 + x_3 + x_4 \leq 1,\ x_2 + x_3 + x_4 \leq 1\}.$$

Which of the following statements are correct? $\boxed{7}$

a) It has a vertex with no coordinates 0.

b) It has a vertex with exactly one coordinate 0.

c) It has a vertex with exactly two coordinates 0.

d) It has a vertex with exactly three coordinates 0.

e) It has a vertex with all coordinates 0.