DEPARTMENT OF MATHEMATICS

Indian Institute of Technology, Guwahati

Quiz 2 | MA321

01-11-2021 | Instructor : Sukanta Pati

Time : 16.10am–19.10pm | Maximum Score : 20

Write appropriate and precise justifications, with readable handwriting. Use pencils for convenience. Submit a single pdf (rollnumber.pdf) to my email pati@iitg.ac.in by 19.20pm.

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1. `(Profit maximizing)`

A company has four factories and four outlets. The factories (F_1, F_2, F_3, F_4) produce $60, 70, 80, 90$ units (of some goods) in a month, respectively. The demands at outlets (O_1, O_2, O_3, O_4) is $70, 50, 80, 60$. For each unit of goods sent from factory F_i to outlet O_j the profit made by the company is the (i, j)th entry of the matrix P given below.

$$P = \begin{bmatrix} 3 & 2 & 3 & 2 \\ 2 & 4 & 1 & 2 \\ 4 & 3 & 3 & 4 \\ 1 & 1 & 2 & 1 \end{bmatrix}$$

The company has to transport all the goods from the factories to the outlets while meeting the demands. The company wants to maximize its profit.

a) Make an appropriate reduction to one of our known problems. (Justification required.) $\boxed{4}$

b) Solve it. $\boxed{4}$

Answer. a) There are two ways to approach this. Treat it as a maximization problem. Then you must target the most positive entry in the relative cost matrix. Or take the matrix $-P$ and proceed with the minimization. (If you do not like a matrix with negative entries, just add 5 to each entry. That will not change the solution.)

We will proceed with the first one. This is an unbalanced problem. This falls under the case where we meet the demands and we can send more units. (Imagine storage cost is infinite.) So the transportation array for the corresponding btp is

3	2	3	2	3	60
2	4	1	2	4	70
4	3	3	4	4	80
1	1	2	1	2	90
70	50	80	60	40	

.

b)

60 3	2	3	2	3	60 2
10 2	50 4	10 1	2	4	70 1
4	3	70 3	10 4	4	80 3
1	1	2	50 1	40 2	90 0
70 1	50 3	80 0	60 1	40 2	

.

$$\overline{c} = \begin{bmatrix} 0 & -3 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -3 & 0 & 0 & -1 \\ 0 & -2 & \boxed{2} & 0 & 0 \end{bmatrix}$$

Cycle: $x_{43}, x_{44}, x_{34}, x_{33}$; $\theta = 50$.

60 3	2	3	2	3	60 0
10 2	50 4	10 1	2	4	70 −1
4	3	20 3	60 4	4	80 1
1	1	50 2	1	40 2	90 0
70 3	50 5	80 2	60 3	40 2	

.

$$\overline{c} = \begin{bmatrix} 0 & -3 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & \boxed{3} \\ 0 & -3 & 0 & 0 & 1 \\ -2 & -4 & 0 & -2 & 0 \end{bmatrix}$$

Cycle: $x_{25}, x_{45}, x_{43}, x_{23}$; $\theta = 10$.

60 3	2	3	2	3	60 3
10 2	50 4	1	2	10 4	70 2
4	3	20 3	60 4	4	80 1
1	1	60 2	1	30 2	90 0
70 0	50 2	80 2	60 3	40 2	

.

$$\overline{c} = \begin{bmatrix} 0 & -3 & -2 & -4 & -2 \\ 0 & 0 & -3 & -3 & 0 \\ \boxed{3} & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & -2 & 0 \end{bmatrix}$$

Cycle: $x_{31}, x_{33}, x_{43}, x_{45}, x_{25}, x_{21}$; $\theta = 10$.

60 3	2	3	2	3	60 0
2	50 4	1	2	20 4	70 2
10 4	3	10 3	60 4	4	80 1
1	1	70 2	1	20 2	90 0
70 3	50 2	80 2	60 3	40 2	

.

$$\overline{c} = \begin{bmatrix} 0 & 0 & \boxed{1} & -1 & 1 \\ -3 & 0 & -3 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -2 & -1 & 0 & -2 & 0 \end{bmatrix}$$

Cycle: $x_{31}, x_{33}, x_{13}, x_{11}$; $\theta = 10$.

50 / 3	2	10 / 3	2	3	60 / 1
2	50 / 4	1	2	20 / 4	70 / 2
20 / 4	3	3	60 / 4	4	80 / 2
1	1	70 / 2	1	20 / 2	90 / 0
70 / 2	50 / 2	80 / 2	60 / 2	40 / 2	

.

$$\overline{c} = \begin{bmatrix} 0 & -1 & 0 & -1 & 0 \\ -2 & 0 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ -1 & -1 & 0 & -1 & 0 \end{bmatrix}$$

Final solution:

50 / 3	2	10 / 3	2	60
2	70 / 4	1	2	70
20 / 4	3	3	60 / 4	80
1	1	90 / 2	1	90
70	50	80	60	

. Profit 960.

2. There are three men who can do any of the six jobs. The time (in days) taken by each man to do various jobs is given in the table. All the jobs have to be completed and each person gets at least one job. Minimize the total time taken.

	J_1	J_2	J_3	J_4	J_5	J_6
M_1	10	9	7	6	8	5
M_2	7	4	8	6	5	9
M_3	3	6	9	8	4	7

Reduce it to some known problem. (Justification required.) $\boxed{4}$

Answer. Imagine any such minimum solution. Suppose that J_1, J_2 has gone to M_1. It is easy to see that M_1 must have bidded the lowest for job J_1, otherwise, we could easily shift J_1 to the lowest bidder.

So, consider a minimum solution. Take away any extra job from men who have more than one jobs. Remember that those were done with lowest bidding price (time). Now what remains must be a minimum cost assignment of the remaining three jobs to the three persons.

Our idea then, is to consider solutions in which three jobs are given to lowest bidders and the rest three are distributed as a minimum cost assignment. Out of such solution we shall find the cheapest one.

To do that, you can select a set of three jobs (these will go to the lowest bidders) and do the minimum assignment for the remaining. Evaluate the total cost. Minimize over all $\binom{6}{3}$ cases. (Don't do this.)

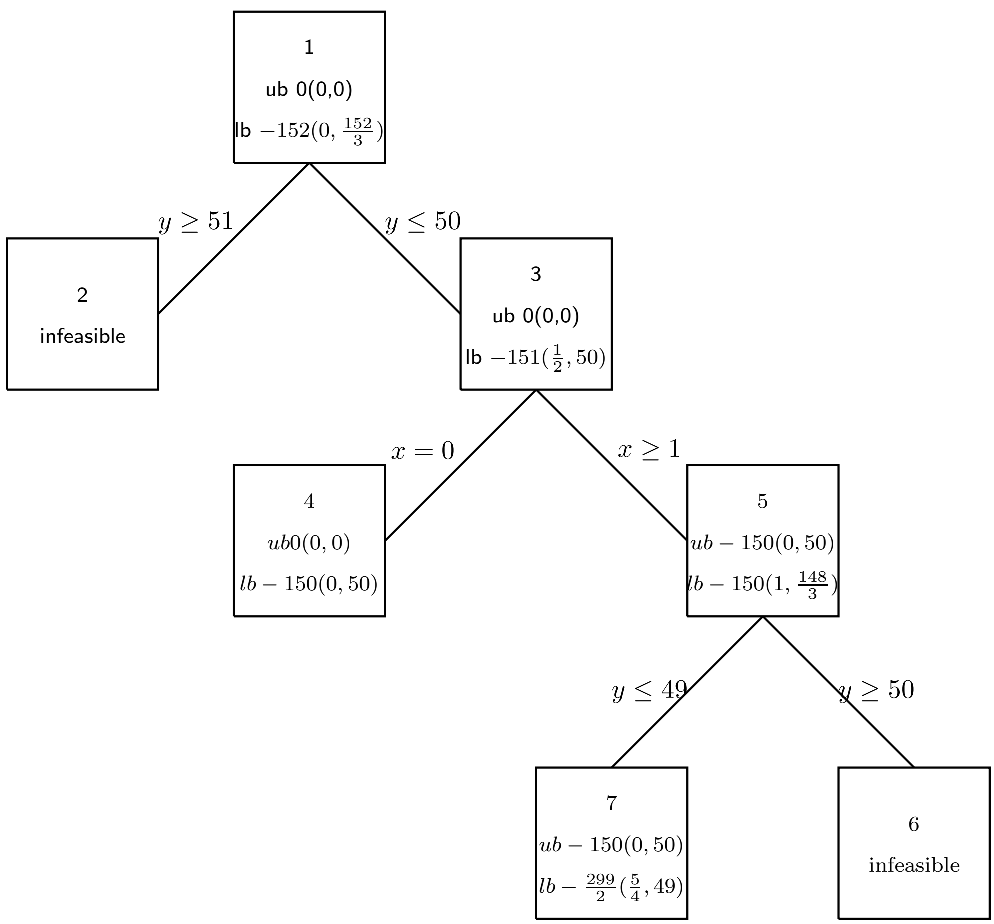
Alternately, introduce 3 virtual men M_4, M_5, M_6 bidding the minimum for each job. This problem considers all possible assignments where three jobs are assigned to the cheapest bidders and the remaining is an assignment. And continue.

So our problem is to find a minimum assignment of
$$\begin{matrix} 10 & 9 & 7 & 6 & 8 & 5 \\ 7 & 4 & 8 & 6 & 5 & 9 \\ 3 & 6 & 9 & 8 & 4 & 7 \\ 3 & 4 & 7 & 6 & 4 & 5 \\ 3 & 4 & 7 & 6 & 4 & 5 \\ 3 & 4 & 7 & 6 & 4 & 5 \end{matrix}$$

3. Solve using branch and bound method (only show the final picture). $\boxed{8}$

$$\begin{array}{ll} \min & \underline{-2x - 3y \qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\ \text{s.t.} & 2x + y \leq 150,\ \ x + 2y \leq 151,\ \ 4x + 3y \leq 152,\ \ x, y \in \mathbb{Z}_+. \end{array}$$

Answer.



As the minimum from node 5 is -150, we see that node 4 already gives us a minimum solution. Whether there are more minimum solutions? For that we branched to nodes 6 and 7. After that we conclude that the minimum is -150 attained only at $(0, 50)$.