

$$(ii) E[X|Y] = \begin{cases} \frac{2}{3} + \frac{1}{3} \\ X=2 \end{cases} \quad X=2$$

$$X=4$$

$$(i) P(\{a\}) = 1/8 \quad P(\{b\}) = 1/8 \quad P(\{c\}) = 1/2 \quad P(\{d\}) = 1/4$$

$$X(a) = X(d) = 2, \quad X(b) = X(c) = 4$$

$$Y(a) = Y(c) = 1, \quad Y(b) = Y(d) = 2.$$

$$E[X|Y] = \begin{cases} \frac{1}{P(\{a\}) + P(\{d\})} [X(a)Y(a) \times P(a) + X(d)Y(d) \times P(d)] & Y=1 \\ \frac{1}{P(\{b\}) + P(\{c\})} [X(b)Y(b) \times P(b) + X(c)Y(c) \times P(c)] & Y=2 \end{cases}$$

$$w \in \{a, d\}$$

$$w \in \{b, c\}$$

$$\frac{X(\{b\})Y(\{b\}) + X(\{c\})Y(\{c\})P(\{c\})}{P(\{b\}) + P(\{c\})} \quad w \in \{b, c\}$$

$$= \begin{cases} 2 \times 1 \times \frac{1}{8} + 2 \times 2 \times \frac{1}{4} \\ \frac{1}{8} + \frac{1}{4} \end{cases} \quad w \in \{a, d\}$$

$$\begin{cases} 4 \times 2 \times \frac{1}{8} + 4 \times 1 \times \frac{1}{2} \\ \frac{1}{8} + \frac{1}{2} \end{cases} \quad w \in \{b, c\}$$

$$\begin{cases} \frac{\frac{5}{4}}{\frac{3}{8}} \\ \frac{3}{\frac{5}{8}} \end{cases} \quad w \in \{a, d\}$$

$$w \in \{b, c\}$$

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$$E[X \cdot Y | X] = \begin{cases} \frac{5 \cdot 8}{4 \cdot 3} \\ \frac{24}{5} \end{cases}$$

$$\cancel{X=2} \text{ we } \{a, d\}$$

$$\cancel{X=4} \text{ we } \{b, c\}$$

$$= \begin{cases} \frac{10}{3} \\ \frac{24}{5} \end{cases}$$

$$\cancel{X=2} \text{ we } \{a, d\}$$

$$\cancel{X=4} \text{ we } \{b, c\}$$

$$\therefore E[X \cdot Y | X] = 5$$

$$\frac{10}{3} I\{a, d\} + \frac{24}{5} I\{b, c\}$$

$$Y=1$$