MA 372 : Stochastic Calculus for Finance

July - November 2021

Department of Mathematics, Indian Institute of Technology Guwahati

TEST I

September 2, 2021

Duration: 35 min

- Answer **all** questions.
- Justify all your answers. Answers without justification carry no marks.

---

1. Find two sigma-algebras $\mathcal{F}_1$ and $\mathcal{F}_2$ on $\Omega = \{1, 2, 3, 4\}$ such that their union $\mathcal{F}_1 \cup \mathcal{F}_2$ is not a sigma-algebra. [2]

2. Let $\Omega = [0, 1]$ and $\mathcal{F} = \mathcal{B}([0, 1])$. Let $\mathbb{P}$ be the probability measure which assigns to each interval its length and

$$Z(w) = \begin{cases} 2 & \text{if } 0 \leq w \leq 1/2 \\ 0 & \text{if } 1/2 < w \leq 1 \end{cases}$$

For $A \in \mathcal{B}([0, 1])$, define $\tilde{\mathbb{P}}(A) = \int_A Z(w) d\mathbb{P}(w)$. Are $\tilde{\mathbb{P}}$ and $\mathbb{P}$ equivalent probability measure on $\mathcal{F}$? [3]

3. Let $\Omega = \mathbb{N}$ (the set of natural numbers) and let $\mathcal{F}$ be the family of all subset of Ω. Is

$$\mathbb{P}(A) = \liminf_{n \to \infty} \frac{\#(A \cap \{1, 2, \cdots, n\})}{n}$$

($\#B$ denotes the number of elements in B) a probability measure on $\mathcal{F}$? [5]

4. Let $\Omega = [0, 1]$ and $\mathcal{F} = \mathcal{B}([0, 1])$. Let $\mathbb{P}$ be the probability measure which assigns to each interval its length. Let $A \in \mathcal{B}([0, 1])$ and $\mathbb{P}(A) = 1/2$. Let $f : [0, 1] \to \mathbb{R}$ be the function defined by $f(x) = \mathbb{P}(A \cap [0, x])$.

   (a) Is this function f continuous on $[0, 1]$?

   (b) Is it possible to find a set $B \subset A$ such that $\mathbb{P}(B) = 1/4$?

   [4+3]

5. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and let $X(w) = 1$ for $w \in A$, $X(w) = -2$ for $w \in B$ and $X(w) = 2$ otherwise, where $A, B \in \mathcal{F}$, $\mathbb{P}(A) = 1/3$, $\mathbb{P}(B) = 1/2$ and $A \cap B = \phi$. Find the distribution function F_X of the random variable X. [3]