MA 323 | End-Sem Lab Exam
Monte Carlo Simulation | 09:00–12:00 IST
IIT GUWAHATI | November 17, 2021

Instructions

1. Attempt **all** the questions.
2. There is **no credit** for a solution if the appropriate work is not shown, even if the answer is correct.
3. Notations are standard and same as used during the lectures.
4. No question requires any clarification from the instructor. Even if a question has an error or incomplete data, the students are advised to write answer according to their understanding or write reasons for why it is not possible to solve the question partially or completely by citing errors/insufficient data.
5. Write a report (one pdf file for all the questions) carefully. **It should contain all the steps, figures, numerical values and conclusions, if any. Don't need to provide the generated random numbers in the report.** The pages of the report must be numbered. Upload the file through Microsoft Teams against the assignment. The portal will remain active till 12:05 hours and you need to complete the submission procedure by 12:05 hours. If you submit through any other means, **a penalty of 15 marks will be imposed**.
6. Also, **submit codes** (one file for each question). Code should be well commented for easy understanding. Code should run as a single file.
7. **You can use inbuilt random number generator**, if not otherwise mentioned.
8. The question paper has **2** pages. This examination has **3** questions, for a total of **30** points.

Questions

1. (4 points) Generate 10 sample paths for the standard Brownian Motion in the time interval $[0, 1]$ with 5000 generated values for each of the paths. Plot all the sample paths in a single figure. Write all the steps in the report.
2. Suppose that we are interested to calculate the probability of $X > 4$, where $X \sim N(0,\, 1)$.
   (a) (6 points) Use simple Monte Carlo technique to find $P(X > 4)$ taking 10,000 replications. Write the steps in the report. Compute and report the estimated standard error and 99% confidence interval.
   (b) (7 points) Use importance sampling technique to find $P(X > 4)$ taking 10,000 replications. Write the steps in the report. Write the importance density and provide the reason of choosing it. Compute and report the estimated standard error and 99% confidence interval.
   (c) (3 points) Compare the gain in variance of the estimator. Compare the confidence intervals obtained in parts (a) and (b).
3. Consider the expectation of $h(x, y) = x \ln(1 + y)$, where the joint distribution of X and Y is given by

$$f(x,\, y) = \begin{cases} \frac{9}{8} y^2 e^{-(\frac{3}{2}+x)y} & \text{if } x > 0,\, y > 0 \\ 0 & \text{otherwise.} \end{cases}$$

(a) (4 points) Compute the $E(h(X, Y)|Y)$ analytically. Report all the steps.

(b) (6 points) Construct and implement a Monte Carlo using conditioning technique. Report the number of replications along with other necessary information. Report the estimate of the expectation of $h(X, Y)$ and estimated variance. Write all the steps clearly in the report.