Assignment 7

Name – Abhishek Agrahari

Roll Number 190123066

Question 1

Formula Used-

$\hat{I}_M = \hat{\mu}_{anti} = \frac{2}{M}\sum_{i=1}^{M/2}\hat{Y}_i$, where $\hat{Y}_i = (e^{\sqrt{U_i}} + e^{\sqrt{1-U_i}})/2$

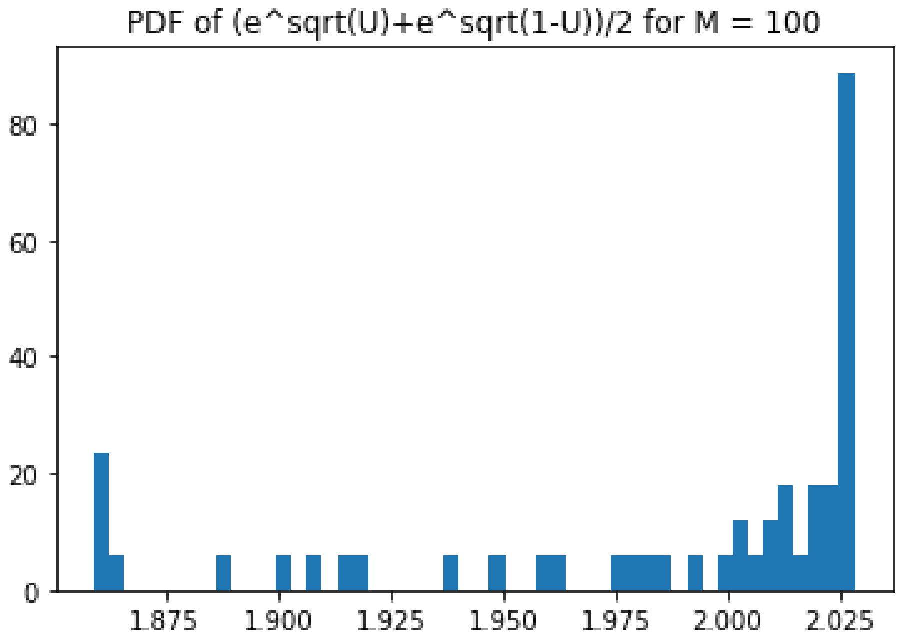
$s_{anti}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\hat{Y}_i - \hat{\mu}_{anti})^2$ where n = M/2.

95% confidence interval = ($\hat{\mu}_{anti} - 1.96\frac{s_{anti}}{\sqrt{n}}$, $\hat{\mu}_{anti} + 1.96\frac{s_{anti}}{\sqrt{n}}$) where n = M/2 and s_{anti} as defined above.

For different values of M, I_M and 95% confidence interval obtained from the generated sample are as below –

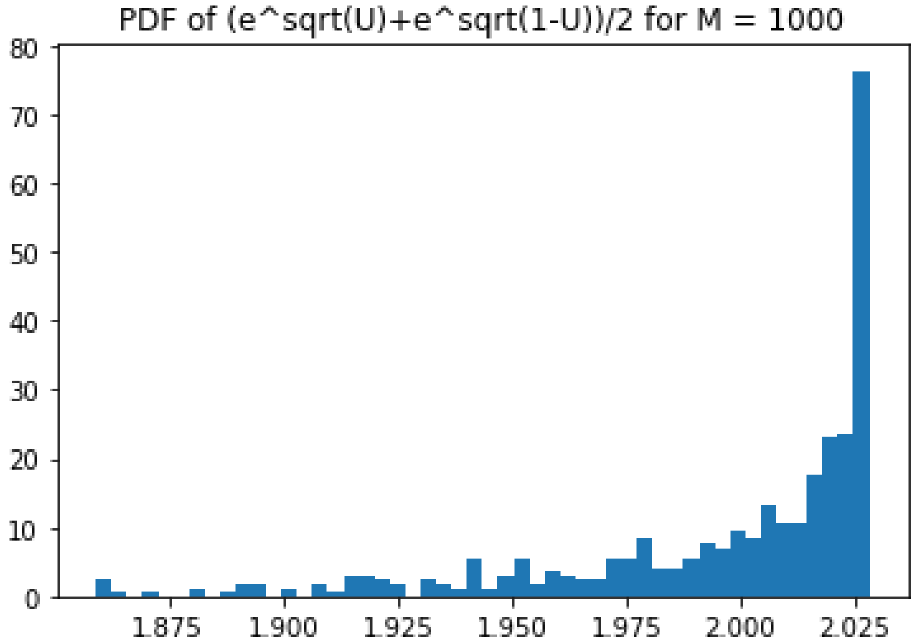
i) M = 100

I_M = 1.9835514858163157
95% confidence interval =
(1.9680162496788438, 1.9990867219537876)



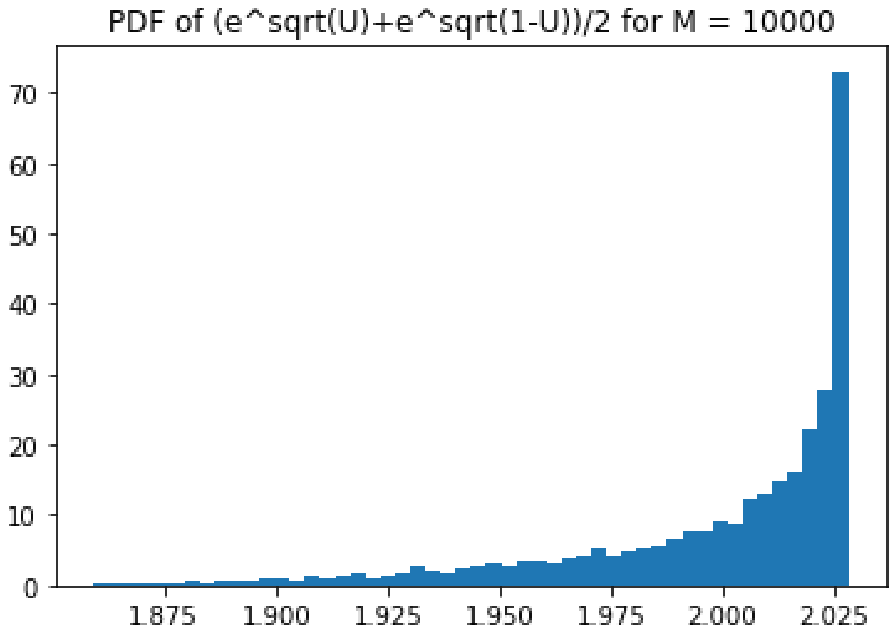
ii) M = 1000

I_M = 1.9970388257694407
95% confidence interval =
(1.993776847093539, 2.0003008044453425)



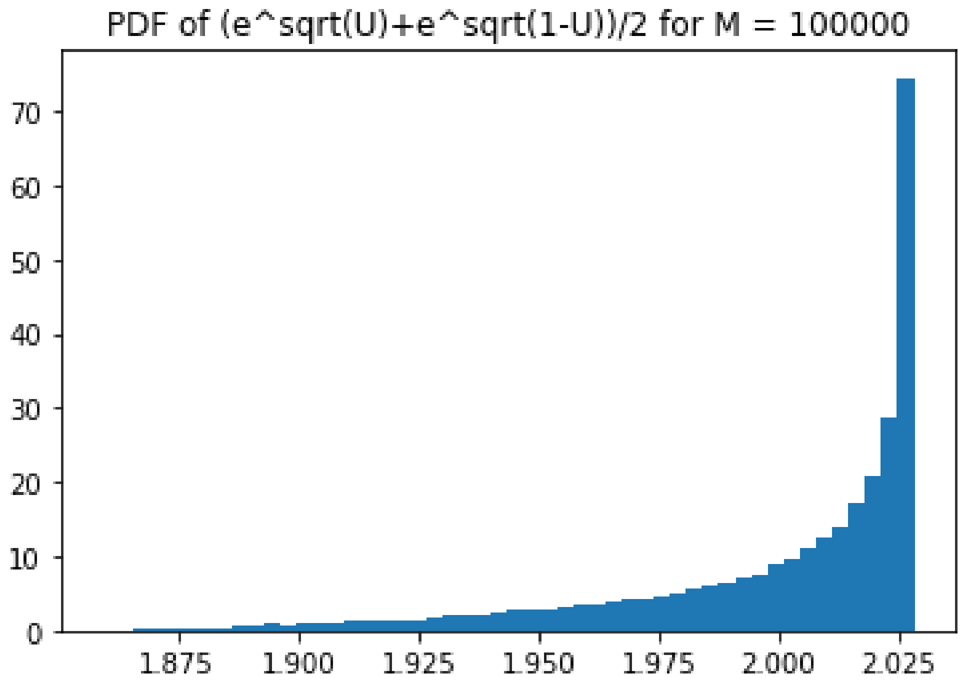
iii) M = 10000

I_M = 1.9993787032361658,
95% confidence interval =
(1.998451217156572, 2.0003061893157597)



iv) M = 100000

I_M = 1.9997835287779493,
95% confidence interval =
(1.9994935248575483, 2.0000735326983503)



Question 2

M	Im-hat	Im	95% CI using Antithetics	95% CI using simple MC	Ratio of width of intervals
100	1.98355149	1.9996175	[1.96801625,1.99908672]	[1.90262461,2.09694244]	6.25409961
1000	1.99703883	1.99321038	[1.99377685,2.0003008]	[1.97152457,2.02804248]	8.66313318
10000	1.9993787	2.00643207	[1.99845122,2.00030619]	[1.991072,2.00849506]	9.3926226
100000	1.99978353	2.00292144	[1.99949352,2.00007353]	[1.99704475,2.00252231]	9.44394544

Ratio of width of interval = $\frac{Width\ of\ 95\%\ CI\ using\ simple\ monte\ carlo}{Width\ of\ 95\%\ CI\ using\ antithetic\ method}$

Actual value of I = $\int_0^1 e^{\sqrt{x}}\,dx = 2$

As value of M increases, $\hat{I}_M$ and I_M both gets closer to actual value of I.

For same value of M, $\hat{I}_M$ is closer to I than I_M for all M except for M = 100. For M = 100, it is just luck due to small sample space.

Width of confidence intervals in antithetic method are smaller as compared to simple monte carlo and this ratio increases with increase in M.