MA 323 | Quiz II
Monte Carlo Simulation | 17:00–18:00 IST
IIT GUWAHATI | November 08, 2021

Instructions

1. Attempt **all** the questions.
2. There is **no credit** for a solution if the appropriate work is not shown, even if the answer is correct.
3. Notations are standard and same as used during the lectures.
4. No question requires any clarification from the instructor. Even if a question has an error or incomplete data, the students are advised to write answer according to their understanding or write reasons for why it is not possible to solve the question partially or completely by citing errors/insufficient data.
5. Write the answers on blank papers (preferably white). You must write your name and roll number on the first page. Every page (both sides) must be self-attested and numbered. Please scan all the pages and make a single PDF file. It is your responsibility to check quality of the PDF file so that it is easily readable. Upload the file through Microsoft Teams against the assignment. The portal will remain active till 18:03 hours and you need to complete the submission procedure by 18:03 hours. If you submit through any other means, **a penalty of 15 marks will be imposed**.
6. The question paper has **2** page. This examination has **3** questions, for a total of **15** points.

Questions

1. (5 points) Let $y_1, \ldots, y_n\,(n \geq 2)$ be observed realization of a random sample. Suppose that we need to calculate $s^2 = \displaystyle\sum_{i=1}^{n}(y_i - \overline{y})^2$, where $\overline{y} = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} y_i$. Show that the following algorithm can be used to compute s^2.

   $i = 1$;
   $\widehat{\mu}_1 = y_1$;
   $s_1 = 0$;
   for $i = 2$ **to** n **do**
       $\delta_i = y_i - \widehat{\mu}_{i-1}$;
       $\widehat{\mu}_i = \widehat{\mu}_{i-1} + \frac{\delta_i}{i}$;
       $s_i = s_{i-1} + \frac{i-1}{i}\delta_i^2$;
   end
   return s_n

2. (5 points) Consider the importance sampling estimate of $\mu = E(X)$, where X has an exponential distribution with mean $\theta > 0$. Let the importance sampling probability density function is of the form

$$q(x;\nu) = \begin{cases} \frac{1}{\nu}e^{-\frac{x}{\nu}} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$

   where $\nu > 0$. Find the optimum choice of ν such that $\widehat{\mu}_{\text{imp}}$ has minimum variance.

3. (5 points) Let $a \in [0,\, 0.99]$ be a real number. Consider the function $f : \mathbb{R} \to \{0, 10\}$ defined by

$$f(x) = \begin{cases} 10 & \text{if } a < x \leq a + 0.01 \\ 0 & \text{otherwise.} \end{cases}$$

Consider the quantity of interest is $E(f(X))$, where $X \sim U(0,\, 1)$. Determine for what values of a antithetic sampling is helpful and harmful, respectively. You may do this by comparing the variances of $\widehat{\mu}$ under simple Monte Carlo and $\widehat{\mu}_{\text{anti}}$ under antithetic sampling.