Assignment 10

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Question 1

Taking $t_i = i/5000$ generated W(t) for time interval [0,1] from the following formula-

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i}\ Z_{i+1}$$

where Zi are independent standard normal N(0,1) and W (0) = 0

Question 2

(a) Using simple monte carlo –

Because there were no points which are greater than 4 therefore simple monte carlo was unable to capture the actual distribution.

P(X > 4) = 0

Standard Error = 0

CI = [0,0]

(b) Using importance sampling –

I generated from N(4,1).

q(X) = PDF_Of_Normal(4,1)

p(X) = PDF_Of_Normal(0,1)

f(X) = I(X > 4)

h(X) = f(X)*p(X)/q(X), where X is N(4,1)

$\hat{\mu}_{\text{imp}} = \frac{1}{n}\sum_{i=1}^{n} h(Xi)$

P(X > 4) = 1.6303722361504857e-05
Standard Error = 3.4200776836168436e-05
CI = [3.1453817866343184e-05,3.145985348292596e-05]
Variance = 1.1696931361973954e-09

(c) On calculating variance by importance sampling variance is increased to 1.1696931361973954e-09 from 0 in simple monte carlo.

On running the same code Confidence interval of the simple monte carlo changes more rapidly than importance sampling. This is because barely 1 point came out 10,000 points which is greater than 4. Therefore results from simple monte carlo are not reliable.

Question 3

③ a) $$f(x,y) = \begin{cases} \frac{9}{4} y^2 e^{-\left(\frac{3}{2}+x\right)y} & \text{if } x>0,\ y>0 \\ 0 & \text{otherwise} \end{cases}$$

$$h(x,y) = x \ln(1+y)$$

$$E[h(X,Y) \mid Y] = \int_0^\infty x \ln(1+y) f_{X|Y}(x|y)\, dx$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{\frac{9}{4} y^2 e^{-\left(\frac{3}{2}+x\right)y}}{f_Y(y)}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx = \int_0^\infty \frac{9}{4} y^2 e^{-\left(\frac{3}{2}+x\right)y} dx$$

$$= \frac{9}{4} y^2 e^{-\frac{3y}{2}} \int_0^\infty e^{-xy}\, dx$$

$$= \frac{9}{4} y^2 e^{-\frac{3y}{2}} \frac{\left[e^{-xy}\right]_0^\infty}{-y}$$

$$= \frac{9}{4} y^2 e^{-\frac{3y}{2}} \frac{[0-1]}{-y}$$

$$= \frac{9}{4} y e^{-\frac{3y}{2}}$$

$$f_{X|Y}(x|y) = \frac{\frac{9}{4} y^2 e^{-\left(\frac{3}{2}+x\right)y}}{\frac{9}{4} y e^{-\frac{3y}{2}}} = y e^{-xy}$$

$$E[h(X,Y) \mid Y] = \int_0^\infty x \ln(1+y)\, y e^{-xy}\, dx$$

$$= \ln(1+y)\, y \int_0^\infty \left[x e^{-xy}\right] dx$$

$$= \ln(1+y)\, y \left[\frac{1}{y^2}\right]$$

$$= \frac{\ln(1+y)}{y}.$$

Distribution of y is Gamma(α= 2, β = 1.5)

Gamma(α, β) can be generated by following algorithms –

n = α

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Y = 0
while n > 0 do
        generate U from uniform [0,1] distribution
        set X = -ln(U)
        Y = Y + X
        n = n-1
return Y/ β
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f(Y) = E[h(X,Y)|Y = ln(1+y)/y

E[h(X,Y)] = E[E[h(X,Y)|Y] = E[f(Y)]

After generating Y from the gamma distribution, I calculated

μ_cond = E[h(X,Y)] = $\frac{1}{n}\sum_{i=1}^{n} f(Yi)$
Estimated E[h(X,Y)] = 0.6737996806270716
Estimated variance = 0.018594256330639537