MA 323 | Mid-Sem Lab Exam
Monte Carlo Simulation | 11:00–13:00 IST
IIT GUWAHATI | September 27, 2021

Instructions

1. Attempt **all** the questions.
2. There is **no credit** for a solution if the appropriate work is not shown, even if the answer is correct.
3. Notations are standard and same as used during the lectures.
4. No question requires any clarification from the instructor. Even if a question has an error or incomplete data, the students are advised to write answer according to their understanding or write reasons for why it is not possible to solve the question partially or completely by citing errors/insufficient data.
5. Write the report carefully. **It should contain all the steps, figures, numerical values and conclusions, if any. Don't need to provide the generated random numbers in the report.** The pages of the report must be numbered. Upload the file through Microsoft Teams against the assignment. The portal will remain active till 13:03 hours and you need to complete the submission procedure by 13:03 hours. If you submit through any other means, **a penalty of 15 marks will be imposed**.
6. Also, **submit codes** (one file for each question). Code should be well commented for easy understanding. Code should run as a single file.
7. **You can use inbuilt uniform generator**, if not otherwise mentioned. Don't use inbuilt function to generate random number from any other distribution.
8. The question paper has **2** pages. This examination has **3** questions, for a total of **30** points.

Questions

1. Suppose that there is a 20% chance of rain on a given day. If it does rain then the amount of rain has an exponential distribution with mean $\frac{1}{2}$.
   (a) (7 points) Using inverse transformation technique, generate sample of size 1000 on the amount of rain. Draw the histogram and comment on it. Write the cumulative distribution function and other steps clearly in the report.
   (b) (3 points) Based on the sample generated in part (a), what is the average amount of rain?
2. (10 points) Let X and Y be two random variables such that $X \sim N(0,\, 1)$ and $Y|X = x \sim N(2x+1,\, 4)$ for all $x \in \mathbb{R}$. Generate 1000 random numbers from the bivariate distribution of $(X,\, Y)$. Calculate the sample means, sample variances and sample correlation coefficient of the bivariate data. Write the steps in the report clearly.
3. Let X be a continuous random variable with probability density function

$$f(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi\nu}\,\Gamma(\frac{\nu}{2})}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}} \qquad \text{for } x \in \mathbb{R},$$

   where $\nu > 0$.

(a) (7 points) Taking $\nu = 3.2$, construct and implement a acceptance-rejection algorithm to generate random number form the distribution of X taking an appropriate proposal distribution. Write the steps (including proposal probability density function, the steps to find the bounding constant, method of generation from the proposal probability density function) clearly in the report.

(b) (1 point) Report the exact value of acceptance probability.

(c) (2 points) Calculate the proportion of acceptance, when you generate 1000 sample and compare it with the answer in part (b).