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1. Generate 5000 random numbers from the following Gaussian mixture PDF:

$$f(x) = \sum_{k=1}^{3} \pi_i \frac{1}{\sigma_i} \phi\left(\frac{x-\mu_i}{\sigma_i}\right), \quad x \in \mathbb{R},$$

   where ϕ is the PDF of the standard normal distribution, $(\pi_1, \pi_2, \pi_3) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$, $(\mu_1, \mu_2, \mu_3) = (-1, 0, 1)$, and $(\sigma_1, \sigma_2, \sigma_3) = \left(\frac{1}{4}, 1, \frac{1}{2}\right)$. Find the average of the generated random numbers.

2. Generate 10 sample paths for the standard Brownian Motion in the time interval $[0, 5]$ using the recursion

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i}\, Z_{i+1},$$

   with 5000 generated values for each of the paths. Plot all the sample paths in a single figure. Also estimate $E[W(2)]$ and $E[W(5)]$ from the 10 paths that you have generated.

3. Repeat the above exercise with the following Brownian motion ($BM(\mu, \sigma^2)$) discretization

$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}\, Z_{i+1}.$$

   Take $X(0) = 5$, $\mu = 0.06$ and $\sigma = 0.3$.

Submission Deadline: November 09, 2021, 11:50 PM