Assignment 11 - Report

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Question 1

- In the Vasicek Model the risk neutral dynamics of r can be expressed as –

$$dr = (b - ar)dt + \sigma dW$$

- On comparing with the model we get a = β and b = βμ.
- Price of the bond is calculated using following formulas –

$$B(t,T) = \frac{\left(1 - e^{-a(T-t)}\right)}{a}$$

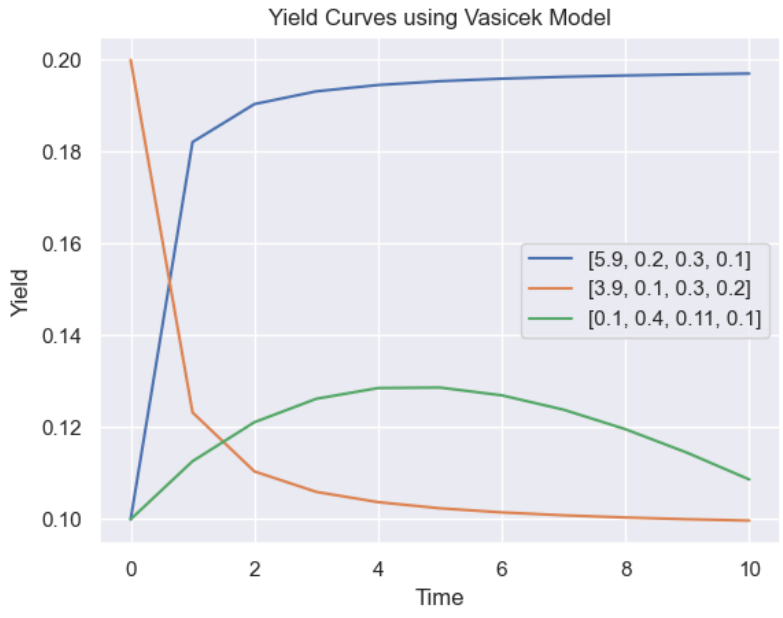
$$A(t,T) = \frac{(B(t,T) - T + t)(ab - \frac{\sigma^2}{2})}{a^2} - \frac{\sigma^2 B^2(t,T)}{4a}$$

$$p(t,T) = e^{A(t,T) - B(t,T)\, r(t)}$$

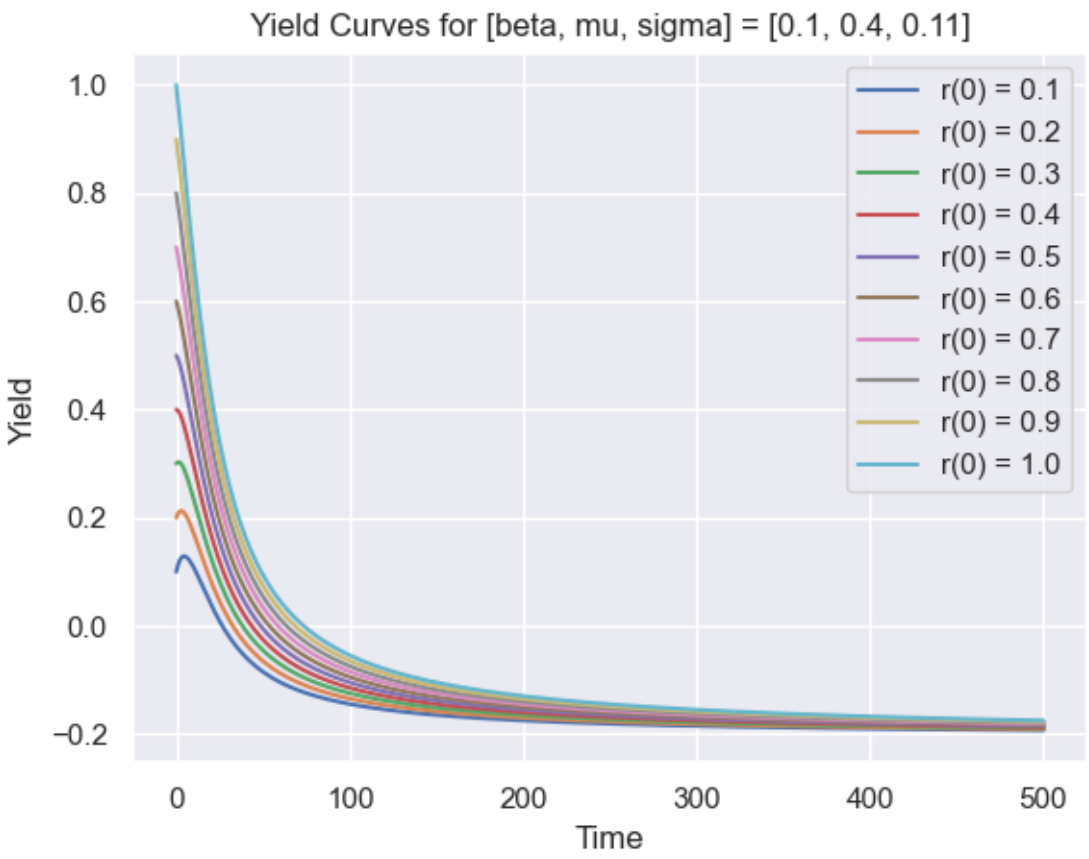
- Yield can be calculated from the price using following formula–

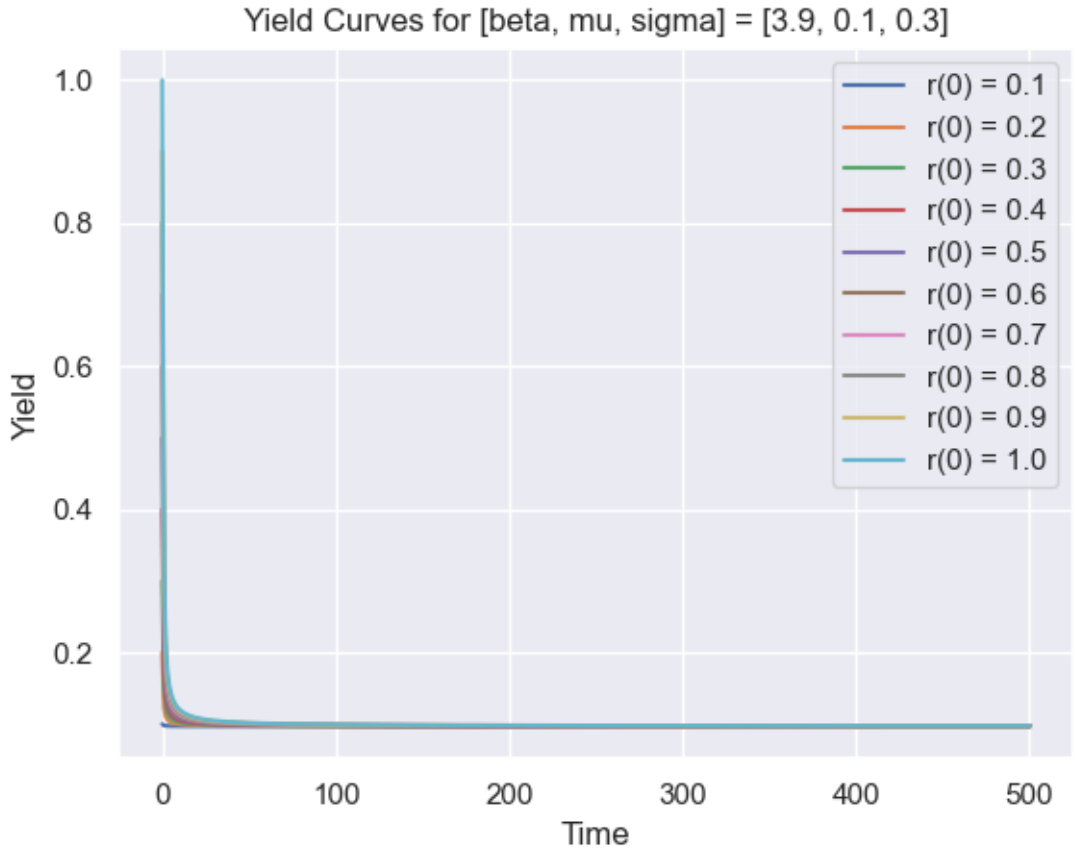
$$Yield = \frac{-log\ p(t,T)}{T - t}$$

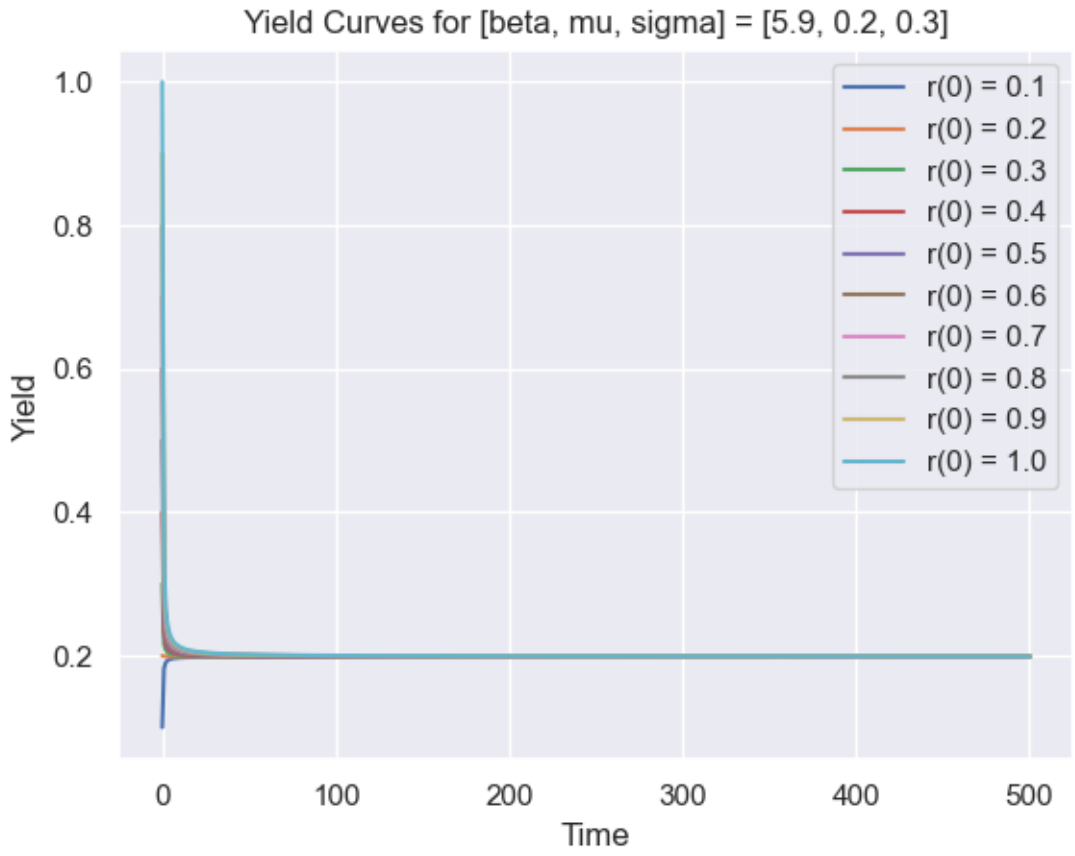
- t = 0 in our case.
- Term structure for the given parameters is plotted using 10 time units.



- Now, yield curves versus maturity up to 500 time units for 10 different values of r(0) is plotted for all the three set of parameters.


Yield Curves for [beta, mu, sigma] = [0.1, 0.4, 0.11]
Yield
Time
r(0) = 0.1
r(0) = 0.2
r(0) = 0.3
r(0) = 0.4
r(0) = 0.5
r(0) = 0.6
r(0) = 0.7
r(0) = 0.8
r(0) = 0.9
r(0) = 1.0
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
100
200
300
400
500


Yield Curves for [beta, mu, sigma] = [3.9, 0.1, 0.3]
Yield
Time
r(0) = 0.1
r(0) = 0.2
r(0) = 0.3
r(0) = 0.4
r(0) = 0.5
r(0) = 0.6
r(0) = 0.7
r(0) = 0.8
r(0) = 0.9
r(0) = 1.0
1.0
0.8
0.6
0.4
0.2
0
100
200
300
400
500



Observations

- For higher r(0), yield is higher.
- Yield converges to a limit for all the parameters.
- Yield can increase or decrease with time to maturity. It depends on the prediction made using the current parameters about the future interest rates.

Question 2

- In the CIR(Cox-Ingersoll-Ross) model the risk neutral dynamics of r can be expressed as –

$$dr = a(b - r)dt + \sigma\sqrt{r}dW$$

- On comparing with the model we get a = β and b = μ.
- Price of the bond is calculated using following formulas –

$$p(t,T) = A_0(T-t)\, e^{-B(T-t)\, r}$$

$$B(x) = \frac{2(e^{\gamma x} - 1)}{(\gamma + a)(e^{\gamma x} - 1) + 2\gamma}$$

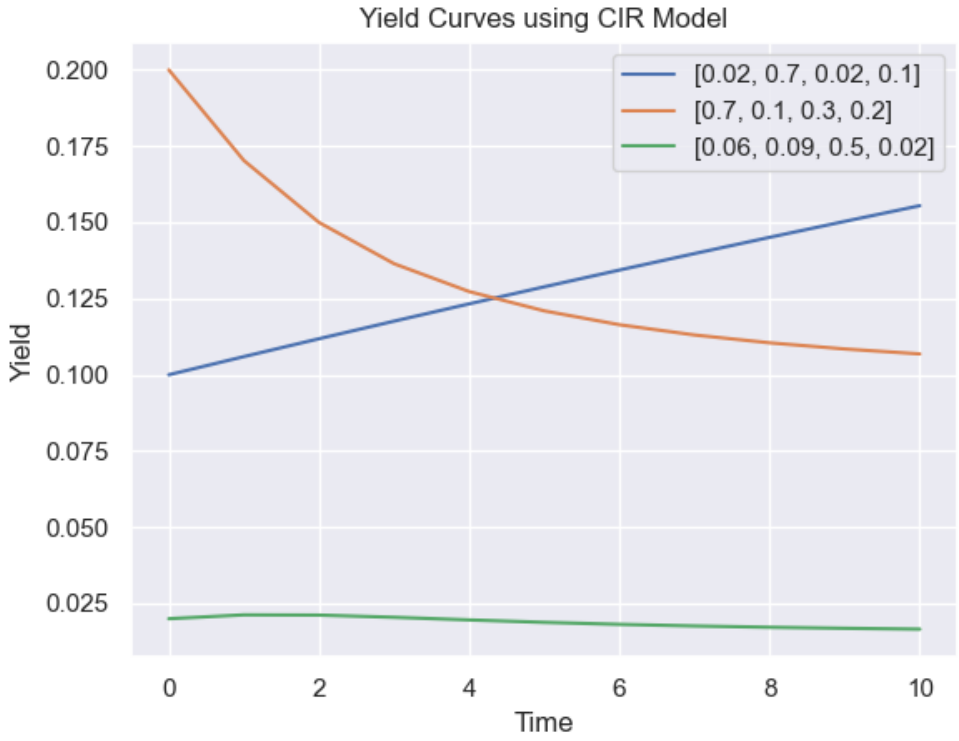
$$A_0(x) = \frac{2\gamma e^{(a+\gamma)(x/2)}}{(\gamma + a)(e^{\gamma x} - 1) + 2\gamma}$$

$$\gamma = \sqrt{a^2 + 2\sigma^2}$$

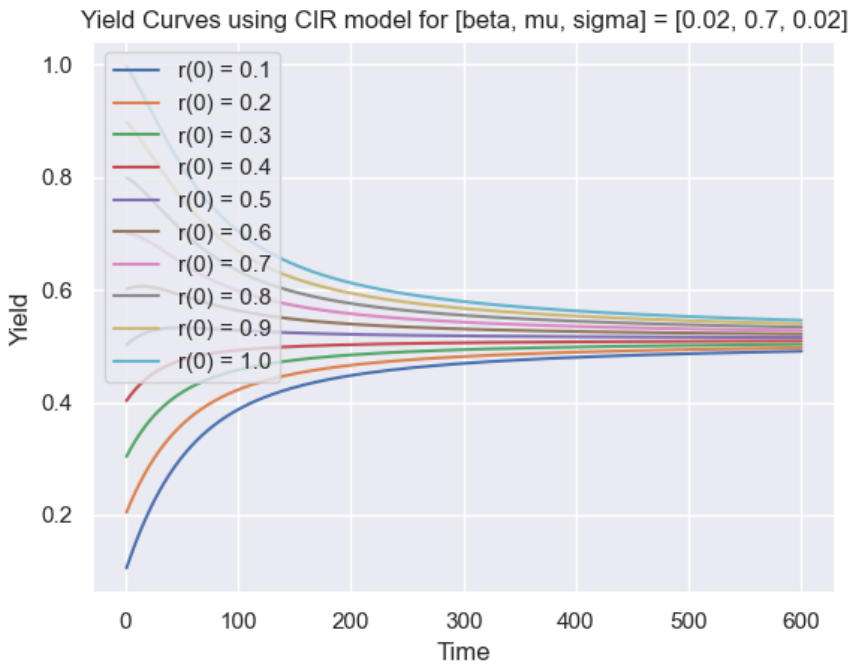
- Yield can be calculated from the price using following formula–

$$Yield = \frac{-log\ p(t,T)}{T-t}$$

- t = 0 in our case.
  - Term structure for the given parameters is plotted using 10 time units.



- Now, yield curves versus maturity up to 600 time units for r(0) = 0.1:0.1:1 is plotted for [beta, mu, sigma] = [0.02, 0.7, 0.02].



Obeservations

- For higher r(0), yield is higher.
- Yield converges to a limit.
- Yield can increase or decrease with time to maturity. It depends on the prediction made using the current parameters about the future interest rates.