Assignment 1

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- By first fundamental theorem of asset pricing, if a market model has a risk neutral measure, then it does not admit arbitrage. Therefore, we will check for the existence of risk neutral measure for checking no-arbitrage condition in the model.
- Under risk neutral measure, the discounted portfolio is a martingale. Therefore

$$E[e^{-rt_1}S(t_1)|S(t_0)] = e^{-rt_0}S(t_0)$$

- For one step binomial model taking $t_0 = 0$,

$$E[S(t_1)|S(0)] = e^{rt_1}S(0) \qquad \text{...eq1}$$

- In one step binomial model,

$$E[S(t_1)|S(0)] = quS(0) + (1-q)dS(0) \qquad \text{...eq2}$$

where q is the risk neutral probability
- By eq1 and eq2 –

$$quS(0) + (1-q)dS(0) = e^{rt_1}S(0)$$

$$q = \frac{e^{rt_1} - d}{u - d}$$

- q will be valid risk neutral probability if $0 \le q \le 1$. Therefore, for a binomial model no arbitrage condition boils down to checking if $0 \le q \le 1$. The case of multistep binomial model is equivalent to applying the same single step binomial model multiple times. Therefore checking for $0 \le q \le 1$ is sufficient.

Question 1

Price of the option can be calculated by using multistep binomial model.

Given Data –

S(0) = Initial stock price = 100

K = Strike price = 105

T = Time to maturity = 5 years

r = Risk free rate = 0.05%

σ = Volatility of the stock.

At each time step prices of the stock can go up by a factor of u or go down by a factor of d.

Formula used for u and d –

$$u = e^{\sigma\sqrt{\Delta t}+(r-\frac{\sigma^2}{2})\Delta t}$$

$$d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{\sigma^2}{2})\Delta t}$$

where $\Delta t = T/M$ with M being the number of subintervals in the time interval [0,T]. Final payoff can be calculated using following formula –

for call option – max(S(T) - K, 0)

for put option – max(K - S(T),0)

After calculating the final payoff, we have to discount it to time 0 to get the price of the option. Discounting at each step i can be done using following formula –

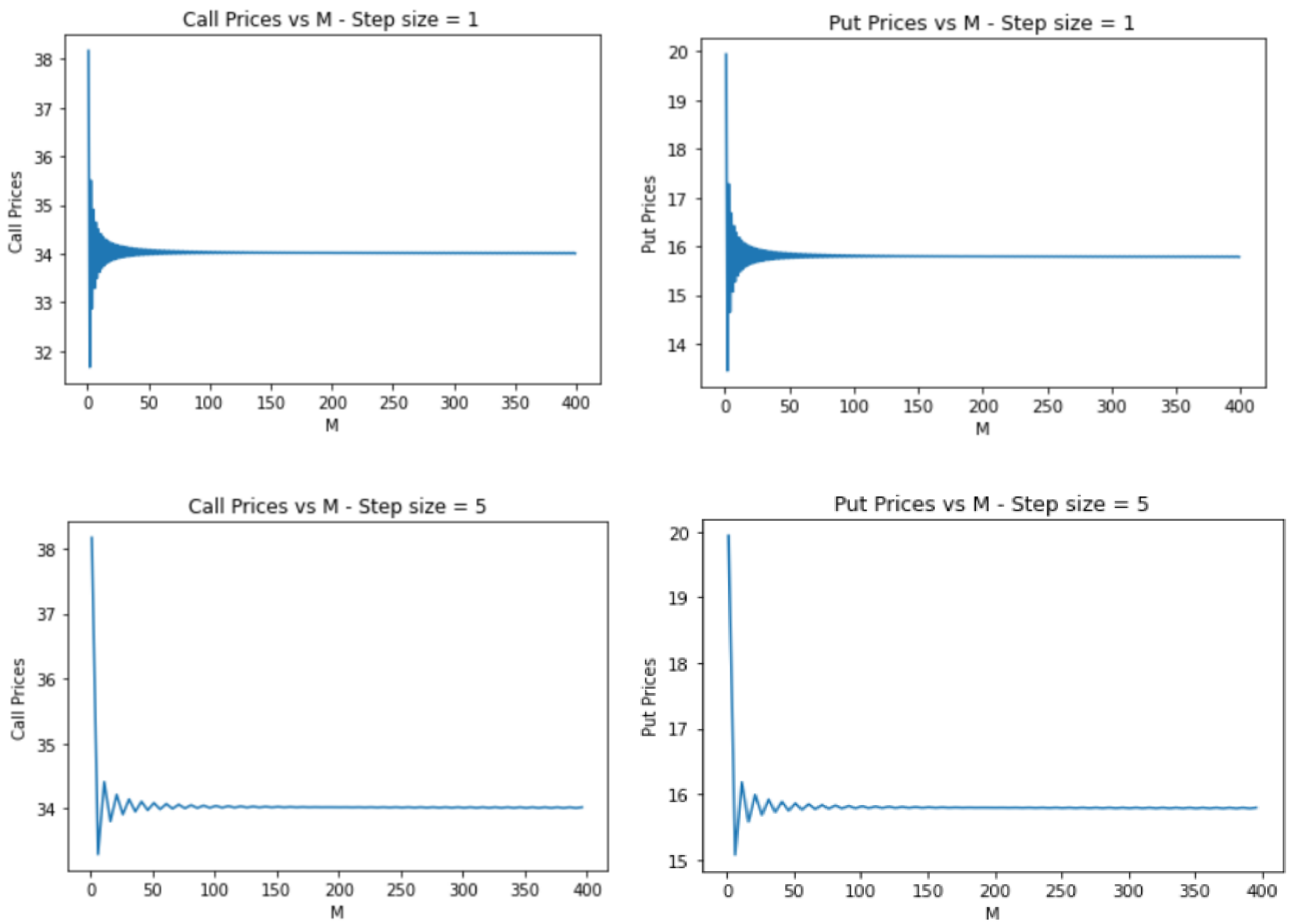
$$C(i) = e^{-r\Delta t}(qC_u + (1-q)C_d)$$

where C(i) is value of the option in step i, C_u is payoff in up state and C_d is payoff in down state.

On running the program for various values of M, following initial option prices were obtained.

Step Size	Call Option Price	Put Option Price
1	38.167635	19.941717
5	34.906533	16.680615
10	33.625022	15.399104
20	33.859449	15.633532
50	33.981184	15.755267
100	34.011161	15.785243
200	34.019579	15.793661
400	34.019132	15.793214

Question 2



Observations

1. Value of the European call option converges to 34.0 and value of the put option converges to 15.7.

2. As the value of M increases deviation in the options prices decreases.

3. Option prices oscillates around the final value of convergence.

Question 3

Depending on number of up steps and down steps taken till the ith step we have (i+1) possible values of the options. Values of the options at different t values are tabulated below -

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For t = 0 (time step = 0) -
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	No. of up steps	No. of down steps	Call Option Values	Put Option Values
0	0	0	33.859449	15.633532

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For t = 0.5 (time step = 2) -
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	No. of up steps	No. of down steps	Call Option Values	Put Option Values
0	0	2	15.095873	24.672817
1	1	1	31.893253	15.487143
2	2	0	59.958769	8.479204

```
For t = 1 (time step = 4) -
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	No. of up steps	No. of down steps	Call Option Values	Put Option Values
0	0	4	5.154831	35.965304
1	1	3	13.469716	24.983287
2	2	2	29.803955	15.269432
3	3	1	57.699995	8.004223
4	4	0	100.662666	3.504174

```
For t = 1.5 (time step = 6) -
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	No. of up steps	No. of down steps	Call Option Values	Put Option Values
0	0	6	1.125003	48.304951
1	1	5	4.121405	36.970072
2	2	4	11.767497	25.270960
3	3	3	27.573204	14.963372
4	4	2	55.295356	7.436262
5	5	1	98.438869	2.998250
6	6	0	160.611388	0.942427

```
For t = 3 (time step = 12) -
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	No. of up steps	No. of down steps	Call Option Values	Put Option Values
0	0	12	0.000000	78.228223
1	1	11	0.000000	72.357695
2	2	10	0.000000	64.433311
3	3	9	0.118330	53.854842
4	4	8	1.235971	40.533314
5	5	7	6.148520	25.955024
6	6	6	19.725206	13.221829
7	7	5	46.976188	4.958186
8	8	4	91.193433	1.235702
9	9	3	154.841699	0.172103
10	10	2	242.030183	0.008705
11	11	1	359.934184	0.000000
12	12	0	519.099689	0.000000

For t = 4.5 (time step = 18) -

	No. of up steps	No. of down steps	Call Option Values	Put Option Values
0	0	18	0.000000	95.534063
1	1	17	0.000000	93.129316
2	2	16	0.000000	89.883248
3	3	15	0.000000	85.501514
4	4	14	0.000000	79.586791
5	5	13	0.000000	71.602751
6	6	12	0.000000	60.825424
7	7	11	0.000000	46.277554
8	8	10	0.000000	26.639984
9	9	9	8.149174	8.281211
10	10	8	36.251494	0.601546
11	11	7	83.950577	0.000000
12	12	6	149.149606	0.000000
13	13	5	237.159089	0.000000
14	14	4	355.959465	0.000000
15	15	3	516.323199	0.000000
16	16	2	732.791598	0.000000
17	17	1	1024.993373	0.000000
18	18	0	1419.424512	0.000000