

Mid-Sem Examination

Marks: 30 + 30 = 60

24 February 2022

Duration: 2 hours 30 mins

Instructions

- Your program should be written in such a way that there is only one program for each question and all the outputs for each question should be displayed by running the program once only.
- Put down all your observations and outputs of the questions asked in a single Word/LaTeX document. Finally create a pdf file from the Word/LaTeX file.
- The file names should be your roll number and name separated by “_”. If your roll number is 190123099 and your name is xyz then file names should be 190123099_xyz for output files (in pdf) and 190123099_xyz_q1 and 190123099_xyz_q2 etc for programs. Write your full name and roll number at the top of the output file.
- All your programs (executable) and output files (in pdf format) must be submitted as Microsoft Teams assignment.

1. Write a program to determine the initial price of an European option with payoff $V(T) = (K - S_{\min})^+$, where S_{\min} is the minimum of the stock prices in $[0, T]$, in the binomial model with the following data: $S(0) = 100$; $T = 1$; $r = 0.04$; $\sigma = 0.25$; $K = 95$. Use the following values of u and d for your program: $u = e^{\sigma\sqrt{\Delta t}}$, $d = e^{-\sigma\sqrt{\Delta t}}$, where $\Delta t = \frac{T}{M}$ (M being the number of subintervals of the time interval $[0, T]$). Use the continuous compounding convention in all your calculations (i.e., both in \tilde{p} and in the pricing formula). Note that $r = 0.04$ is the interest rate for the period of $T = 1$.
 - (a) Obtain and tabulate the initial price of the option for different values of M starting from $M = 1$ to $M = 10$, in steps of 1.
 - (b) Tabulate the values of the option at all intermediate time points for $M = 5$.
 - (c) Study the sensitivity of the initial option price by varying the parameters K , S_0 , r and σ one-at-a-time. Present your results in the form of figures and write down your observations in the report (take $M = 5$ for all cases).
2. Consider the data given in the file `midsemdata.csv`/`midsemdata.xls` showing the yearly prices of three stocks along with the values of the market index.
 - (a) Find the mean return vector and covariance matrix for returns of all the three stocks. Also find the mean return and variance of the market index return.
 - (b) Compute the beta, alpha, the systematic risk and the unsystematic (diversifiable) risk of each of the stocks.
 - (c) Compute and plot the values of the mean return of all possible portfolios (attainable set or feasible set) constructed using the three stocks (with short sales allowed). Now, compute and plot the values of the standard deviation of the returns of all possible portfolios constructed using the three stocks (with short sales allowed). Note that the plots are 3-D plots with two of the axes are the weights and the third axis is the expected return or the standard deviation.
 - (d) Determine the minimum variance portfolio (i.e., weights, return and risk). How does this return and risk compare with that of the market?

Note: No computations in Excel sheet. Your program should read the file and do all the calculations.