Assignment 3

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Data used for the computation of initial stock price –

$S(0) = 100$, $T = 1$, $r = 8\%$, $\sigma = 20\%$

Formulas used –

- $u = e^{\sigma\sqrt{\Delta t}+(r-\frac{\sigma^2}{2})\Delta t}$
- $d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{\sigma^2}{2})\Delta t}$

where $\Delta t = T/M$ with M being the number of subintervals in the time interval [0,T].

- Payoff of loopback (European option) –

$$V = \max_{0 \le i \le M} S(i) - S(M)$$

Question 1

(a) Initial price of the option for –

$M = 5$ is 9.119299

$M = 10$ is 10.080583

Using binomial algorithm, calculating option price for $M = 25$ and $M = 50$ is computationally infeasible.

(b)

M	Loopback Option Price
5	9.119299
6	9.415434
7	9.609088
8	9.806368
9	9.936758
10	10.080583
11	10.175899
12	10.286896
13	10.367182
14	10.452999
15	10.519165

We can see from the above table that initial price of the loopback option increases with increase in M.

(c)

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Values of the options for M = 5 is -
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9.119299  9.50484   12.168664  17.582062  25.051229  32.1054
          9.027951  7.147916   7.148418   10.680904  18.8059
                    9.799119   8.324615   10.680904  18.8059
                    8.548076   6.201917   3.846929    2.90135
                               13.712863  13.071381  18.8059
                               6.201917   3.846929    2.90135
                               9.955271   8.003614    7.81842
                               7.416771   4.60048     0
                                          21.188089  21.235
                                          6.680843    5.33038
                                          8.003614    7.81842
                                          4.60048     0
                                          15.631852  16.2664
                                          4.60048     0
                                          9.571392    9.34992
                                          5.501639    0
                                                     29.4826
                                                     13.578
                                                     13.578
                                                      0
                                                     16.2664
                                                      0
                                                      9.34992
                                                      0
                                                     25.3946
                                                      6.37452
                                                      9.34992
                                                      0
                                                     19.4527
                                                      0
                                                     11.1814
                                                      0
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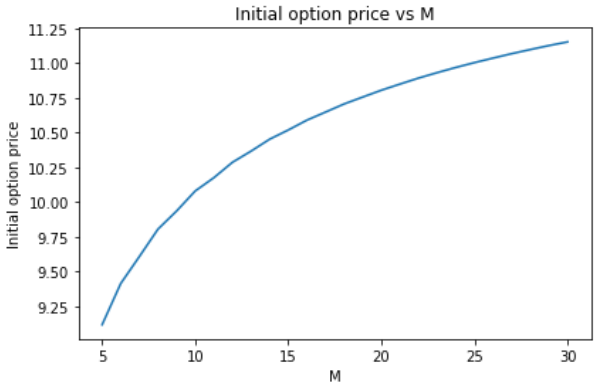
Question 2

Initial option price for given M using Markov Algo is given below -

M	Initial Option Price
5	9.119299
10	10.080583
25	11.003495
50	11.510862

Initial option price increases with M which is evident from the following curve.



Time Complexity

- Time complexity for binomial algorithm is $O(2^M)$ because we are exploring every path of the binomial tree.
- Markov algorithm depends on two states the current stock price and maximum stock price encountered along the path till now. No. of unique values for stock price is bounded by $O(M^2)$. Therefore maximum stock price encountered along the path will also be bounded by $O(M^2)$. Therefore number of states in markov algorithm would be bounded by $O(M^4)$. In each state we are doing $O(1)$ work. Therefore time complexity of markov algorithm would be $O(M^4)$.

Maximum M allowed

Maximum value of M that can be handled in reasonable time for –

- Binomial algorithm – 15
- Markov algorithm - 50.

Computation time for various M

Time required by both the algorithm for various values of M is given below –

(these values might vary slightly with every run of the code)

M	Time required for Binomial Algorithm (in ms)	Time required for Markov Algorithm(in ms)
5	0.996351	0.000000
10	12.995958	0.001000
25	Computationally infeasible	0.117999
50	Computationally infeasible	7.160001

Question 3

Similar to above questions, pricing of European call option (assuming strike price(K) = 100) is being performed using binomial algorithm and markov algorithm and following computational difference is observed –

Time Complexity

- for binomial algorithm it is $O(2^M)$, because we are exploring every path.
- for markov algorithm we are using step number and count of up step taken in the path till now. Step number in bounded by O(M) and count of up step taken would also be bounded by O(M). Therefore number of unique states would be bounded by $O(M^2)$. In each state we are doing O(1) work, therefore time complexity of the algorithm is $O(M^2)$.

Maximum M allowed

Maximum value of M that can be handled in reasonable time for –

- Binomial algorithm – 20
- Markov algorithm – around 1000 (My python code ran successfuly in reasonable time for M till 995, after that it gives maximum recursion depth exceeded error)

Computation time for various M

- Time required by both the algorithm for various values of M is given below (these values might vary slightly with every run of the code) -

M	Time required for Binomial Algorithm (in ms)	Time required for Markov Algorithm(in ms)
5	0.0	0.0
10	3.0069351196289062	0.0
20	2816.011428833008	0.0
25	Computationally infeasible	0.99945068359375
50	Computationally infeasible	3.0002593994140625
100	Computationally infeasible	13.997077941894531

Output of the Code