Assignment 7

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Question 1

The solution to the Black-Scholes-Merton equation with terminal condition and boundary conditions is -

$$c(t,x) = x.N(d_+(T-t,x)) - K.e^{-r(T-t)}.N(d_-(T-t,x)) \text{ where } 0 \leq t < \text{T and x} > 0.$$

where

$$d_\pm(\tau,x) = \frac{1}{\sigma\sqrt{\tau}}\left[\log\frac{x}{K} + (r \pm \frac{\sigma^2}{2})\right]$$

and N is the cumulative standard normal distribution

$$N(y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} e^{\frac{-z^2}{2}}dz = \frac{1}{\sqrt{2\pi}}\int_{-y}^{\infty} e^{\frac{-z^2}{2}}dz$$

By put call parity we know that -

$$f(t,x) = x - e^{-r(T-t)}K = c(t,x) - p(t,x)$$

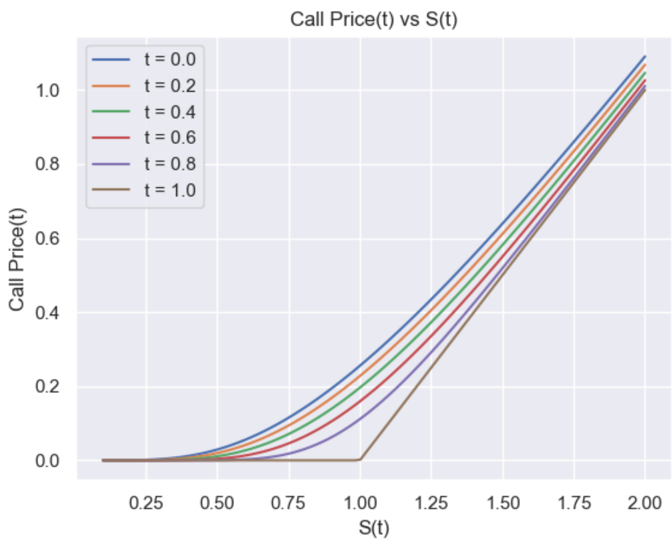
where f is the value of a forward quantity at time $t \in [0,T]$ where stock price at time t is $S(t) = x$. Therefore,

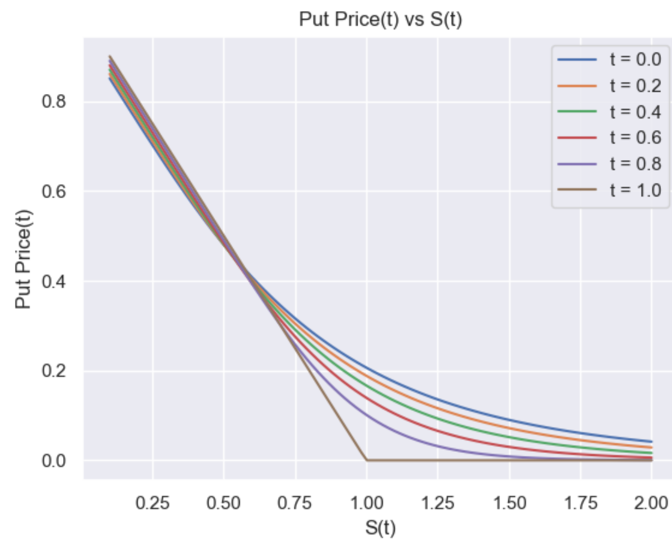
$$\begin{aligned} p(t,x) &= x.(N(d_+(T-t,x)) - 1) - K.e^{-r(T-t)}.(N(d_-(T-t,x)) - 1) \\ &= K.e^{-r(T-t)}.N(-d_-(T-t,x)) - x.N(-d_+(T-t,x)) \end{aligned}$$

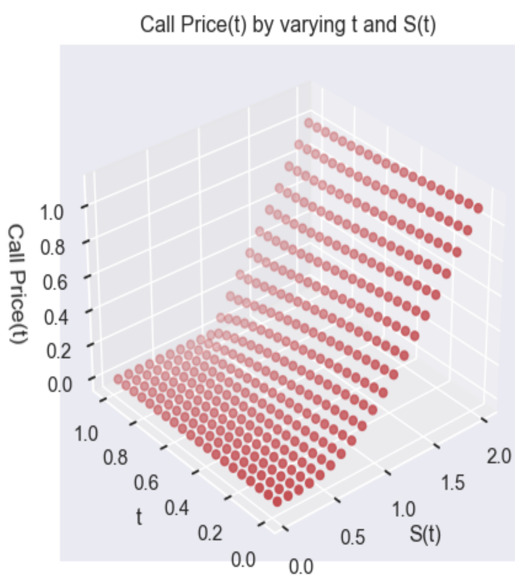
Using above formulas for $c(t,x)$ and $p(t,x)$, prices for European call and put option is computed for $0 \leq t <$ T. For $t = T$, $c(t,x) = max(x-K,0)$ and $p(t,x) = max(K-x,0)$

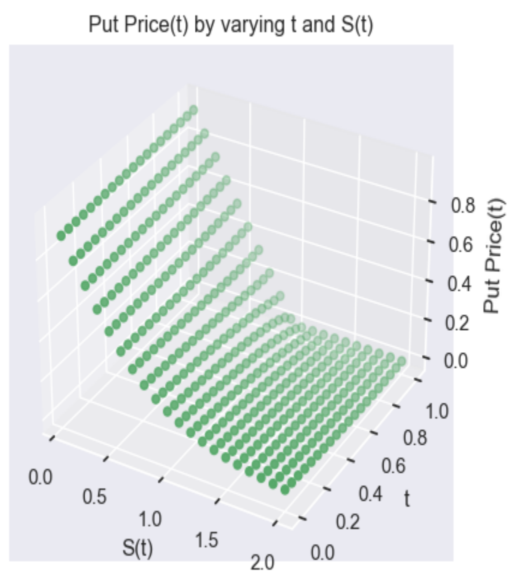
Question 2

- 2d plots for the Value of options ($c(t,x)$) is plotted by varying price of the underlying at a given time. This plot is plotted for $t =$ [0.0, 0.2, 0.4, 0.6, 0.8, 1.0]
  - By increasing the price of the underlying, call prices increases while put prices decreases.
  - As time to maturity $(T-t)$ decreases value of the options decreases.
- 3d scatter plots are plotted by varying time and stock price at that time simultaneously.
  - Plots are suitably rotated to get a better view.



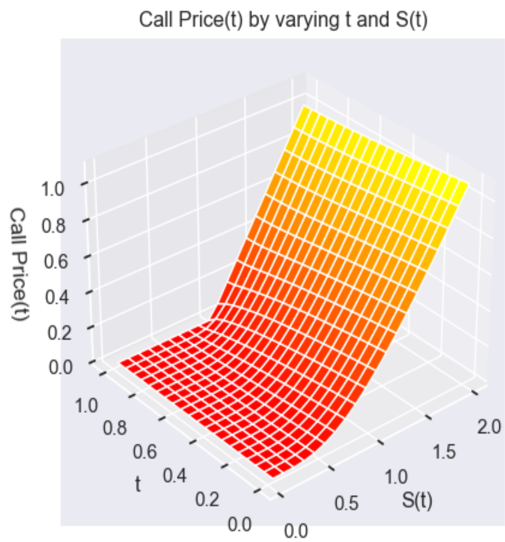


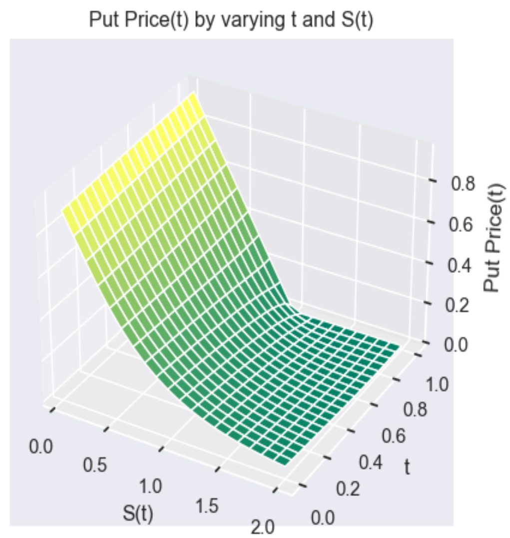




Question 3

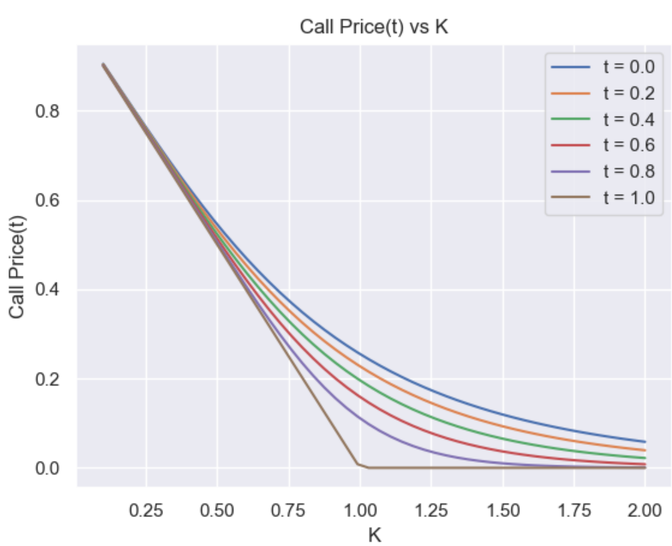
- Surface plots are plotted by varying time and stock price at that time simultaneously.
- These plots are the surface plot version of the 3d scatter plots obtained in question 2.
- Plots are suitably rotated to get a better view.

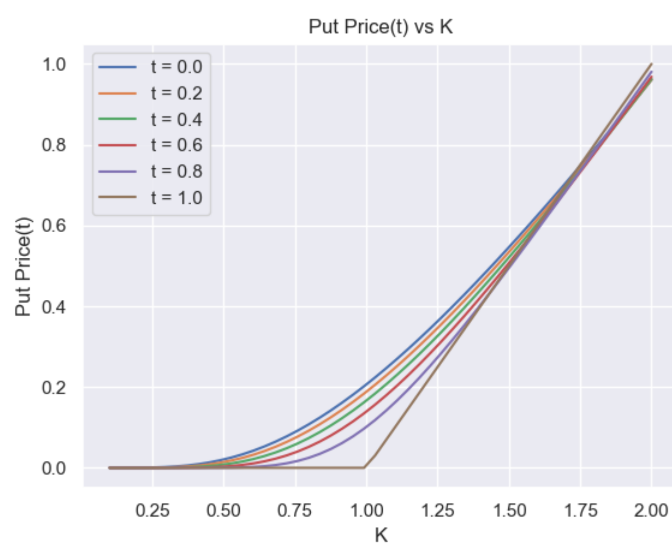


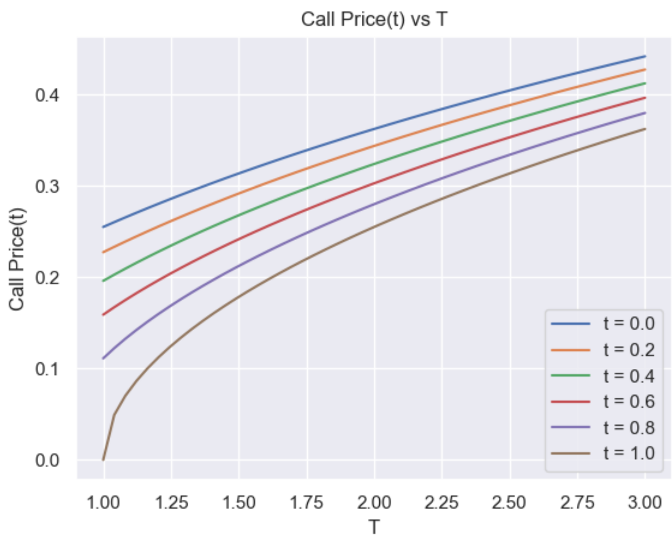


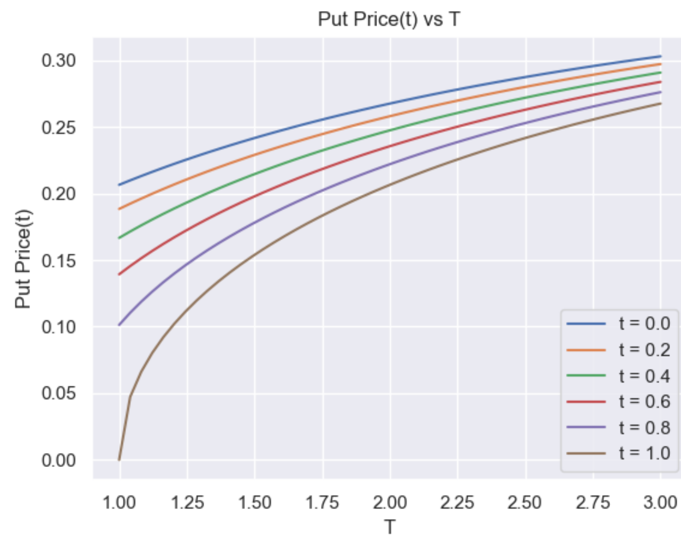
Question 4

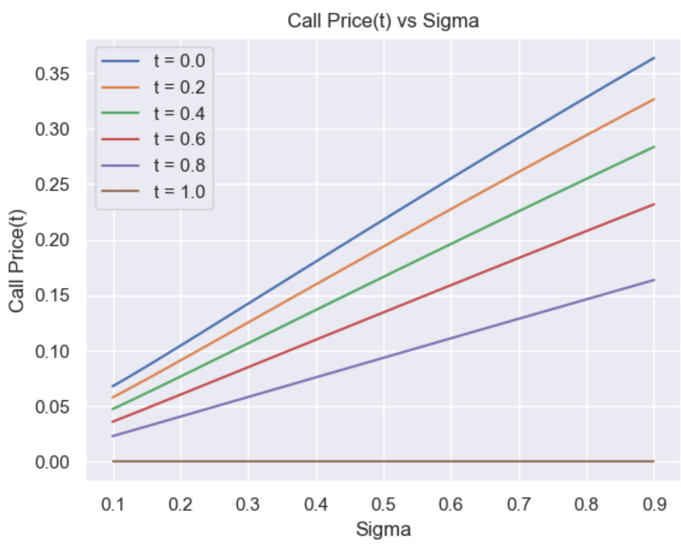
- I have done the senstivity analysis for the value of European call and put option as a function of following model parameters
  - * Strike Price (K)
  - * Risk Free Return (r)
  - * Sigma (σ)
  - * Time of expiration (T)
- 2d plots are plotted assuming price of the underlying asset as 1 at different time points. All other values are taken as given in question 2, i.e. $K = 1$, $T = 1$, $r = 0.05$, $\sigma = 0.6$.
- One by one all these model parameters are varied and options price at given point of time is plotted.

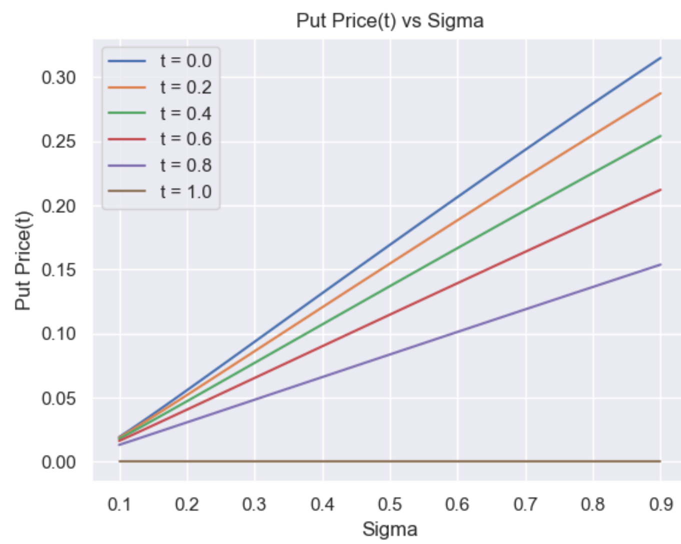


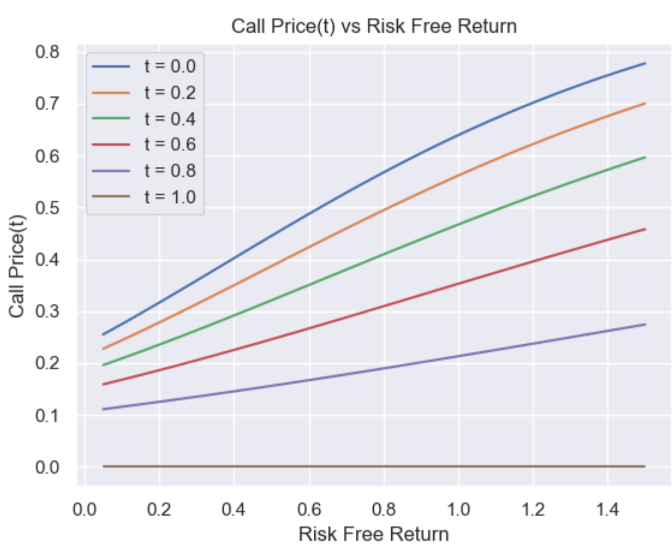



Call Price(t) vs T
Call Price(t)
T
t = 0.0
t = 0.2
t = 0.4
t = 0.6
t = 0.8
t = 1.0


Put Price(t) vs T
Put Price(t)
T
t = 0.0
t = 0.2
t = 0.4
t = 0.6
t = 0.8
t = 1.0


Call Price(t) vs Sigma
Call Price(t)
Sigma
t = 0.0
t = 0.2
t = 0.4
t = 0.6
t = 0.8
t = 1.0


Put Price(t) vs Sigma
Put Price(t)
Sigma
t = 0.0
t = 0.2
t = 0.4
t = 0.6
t = 0.8
t = 1.0


Call Price(t) vs Risk Free Return
Call Price(t)
Risk Free Return
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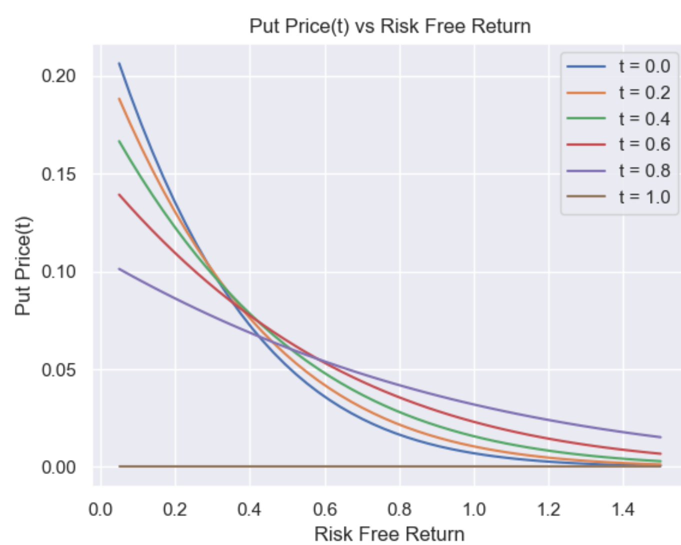
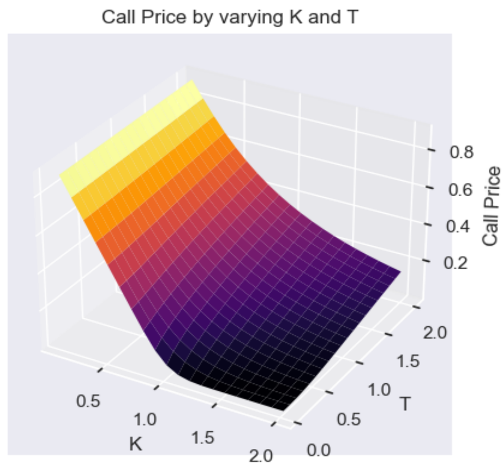
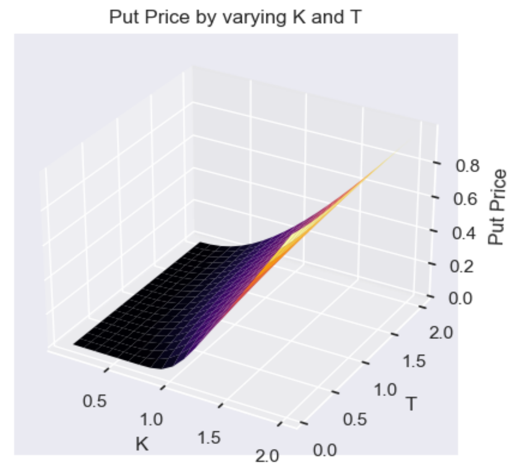
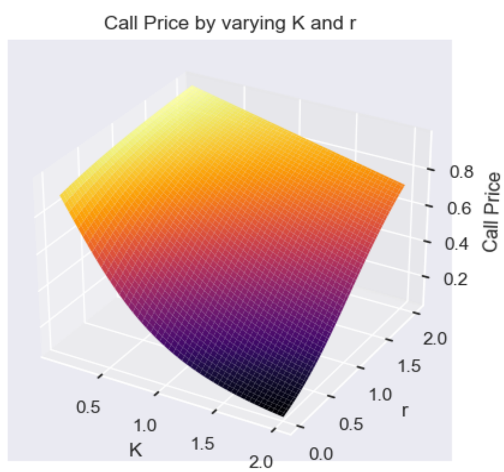
t = 0.2
t = 0.4
t = 0.6
t = 0.8
t = 1.0

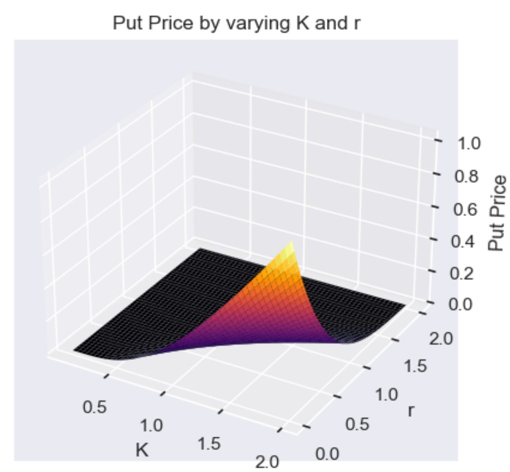

Put Price(t) vs Risk Free Return
Put Price(t)
Risk Free Return
t = 0.0
t = 0.2
t = 0.4
t = 0.6
t = 0.8
t = 1.0

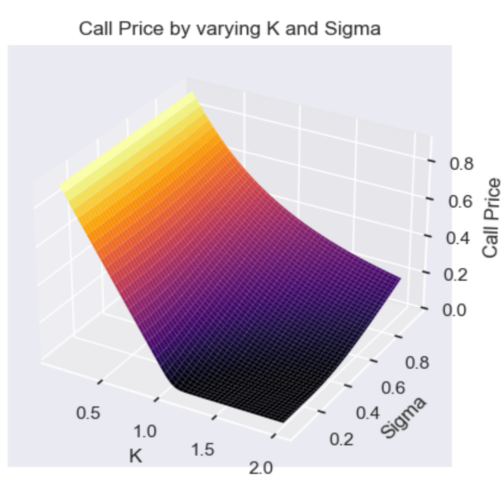
- Options price at t = 0 is plotted by varying two model parameters at a time.
- Price of the underlying at t = 0, is assumed to be 1.

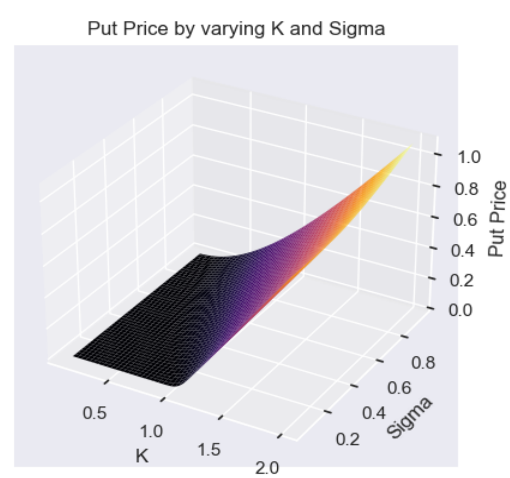




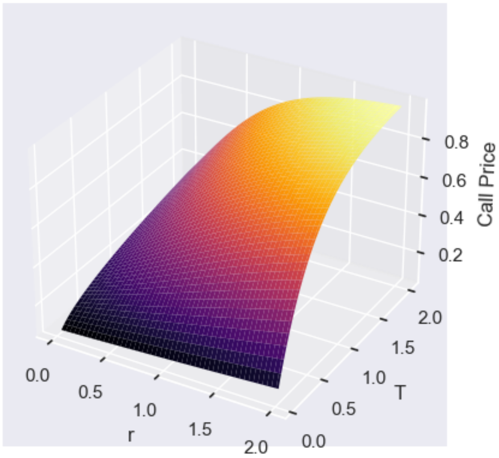




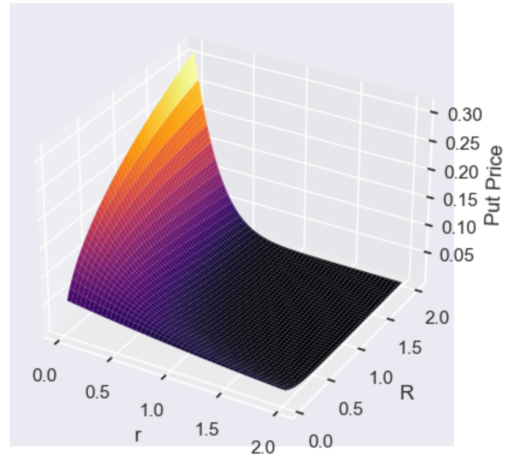




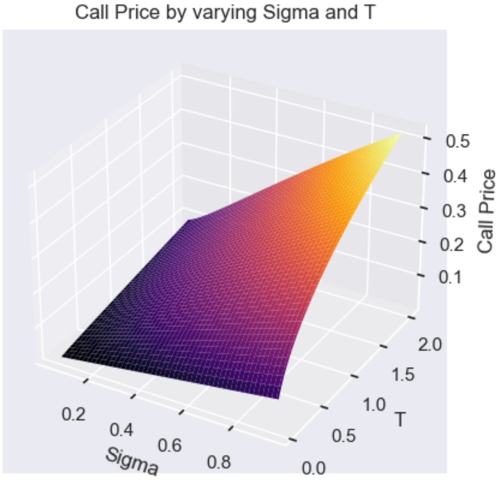
Call Price by varying r and T



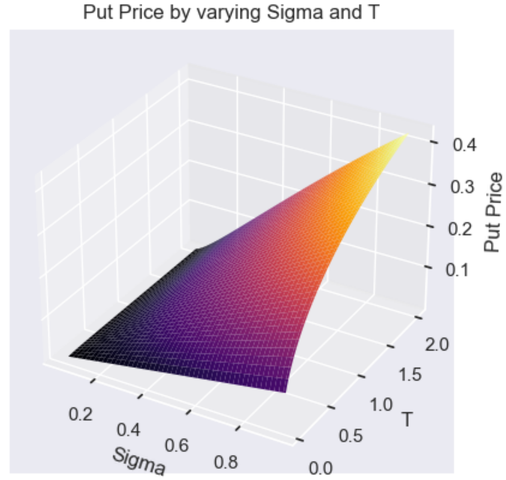
Put Price by varying r and T



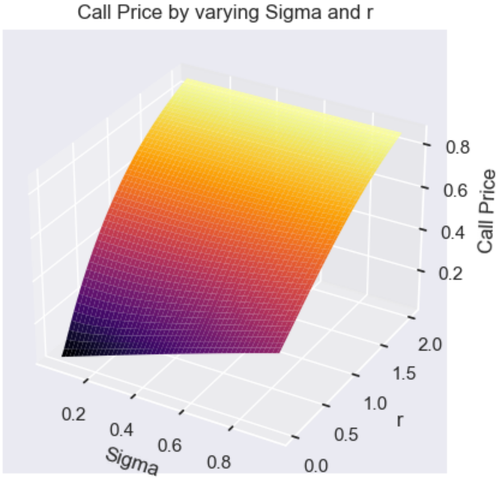
Call Price by varying Sigma and T



Put Price by varying Sigma and T



Call Price by varying Sigma and r



Put Price by varying Sigma and r