MA 373 : Financial Engineering II

January - May 2022

Department of Mathematics, Indian Institute of Technology Guwahati

TEST I

February 08, 2022

Duration: 50 min

- Answer **all** questions.
- Justify all your answers. Answers without justification carry no marks.

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1. Use Ito's-formula to write the stochastic process $Y(t) = e^{2t+4W(t)}$ on the standard form

$$dY(t) = b(t, W(t))dt + \sigma(t, W(t))dW(t).$$

[5]

2. Let X be the solution of the SDE, for $t < 1$

$$dX(t) = -\frac{1}{2(1-t)}X(t)dt + \sqrt{1-t}\, dW(t),\ X(0) = 0$$

   (i) Find the solution $X(t)$ of this equation.

   (ii) Is $\{X(t), t \geq 0\}$ a Gaussian process?

   (iii) Compare the variance of $X(t)$ with the corresponding variance of a Brownian bridge from 0 to 0 on [0,1] at time t.

   (iv) Is $X(t)$ a Brownian bridge from 0 to 0 on [0,1] ? (i.e., the process $X(t)$ has the same distribution as the Brownian bridge from 0 to 0 on [0,1])

[6+3+3+2]

3. Use Feynman-Kac stochastic representation result in order to solve the following boundary value problem in the domain $[0, T] \times \mathbb{R}$.

$$\begin{aligned} \frac{\partial u}{\partial t}(t,x) + \frac{1}{2}\sigma^2\frac{\partial^2 u}{\partial x^2}(t,x) &= 0 \\ u(T,x) &= x^2, \end{aligned}$$

   where σ is a constant.
   (Find an explicit expression for u(t,x) in terms of σ, T, t and x) [6]