

$$(6) X(t) = 2t + 3W(t)$$

$$Y(t) = 2t + W(t)$$

Comparing with $X(t) = \alpha t + \sigma W(t)$ we get

$$\alpha \text{ of } X(t) = 2$$

$$\sigma \text{ of } X(t) = 3$$

Similarly $\alpha \text{ of } Y(t) = 2$

$$\sigma \text{ of } Y(t) = 1$$

~~We know that~~

$$E[e^{-s\tau}] = e^{\frac{\alpha}{\sigma^2}(\mu - \sqrt{2s\sigma^2 + \mu^2})}$$

$$E[\tau] = \left(-\frac{d}{ds} E[e^{-s\tau}] \right)_{s=0}$$

$$= \frac{\alpha}{\sigma^2}$$

$$E[\tau^2] = \left(\frac{d^2}{ds^2} E[e^{-s\tau}] \right)_{s=0}$$

$$= \frac{\alpha^2}{\sigma^2} + \frac{\sigma^2}{\alpha^3} \alpha$$

$$\text{var}(\tau) = \frac{\alpha^2}{\sigma^2} + \frac{\sigma^2}{\alpha^3} \alpha - \frac{\alpha^2}{\sigma^2} = \frac{\sigma^2}{\alpha^3}$$

$$E[\tau]_{X_0} = E[\tau]_Y = \frac{\alpha}{\sigma^2}$$

$$\text{var}(\tau)_X = \frac{9\alpha}{8}$$

$$\text{var}(\tau)_Y = \frac{\alpha}{8}$$

Because for reaching any barrier x
Expected time is ~~same~~ in both cases
but Y offers more certainty on reaching
the barrier ~~in~~ in time close to the
expected value because of less
volatility. ~~So~~ Therefore I would
prefer Y .