1. State whether the following statement is TRUE or FALSE:
In a queueing system with priority queueing discipline, any ongoing service of a lower priority customer is always interrupted on arrival of a higher priority customer.

Solution:

Answer: FALSE

Only in the case of preemptive priorities, the given statement is true. It is not true in case of nonpreemptive priority queueing discipline.

2. State whether the following statement is TRUE or FALSE:
Every queueing system always has a fixed (nonrandom) number of servers.

Solution:

Answer: FALSE

A queueing system can have a random number of servers, as can happen in a taxi service facility where the number of taxis available at a time could vary.

3. State whether the following statement is TRUE or FALSE:
In a queueing system, a served customer cannot queue for service again.

Solution:

Answer: FALSE

In a queueing system with feedback, a customer can queue for service again, as can happen in the case of production of a defective item in a manufacturing process.

4. Which of the following is/are assumed default in Kendall's notation for a queueing system (and hence may not be specified explicitly)?

(A) Exponentially distributed service times.
(B) FIFO queue discipline.
(C) Infinite number of servers.
(D) Availability of space for waiting of infinite number of customers.

Solution:

Answer: (B) and (D)

Of the given four, only infinite system capacity and FIFO queue discipline are assumed default.

5. Vehicles arrive at a petrol pump being manned by a single worker at a rate of 24 per hour. On an average, it takes 2 minutes to fill petrol for one vehicle. Then, the proportion of time the worker is free equals _____.

Solution:

Answer: 0.2 [Hint: Enter the answer as a fractional value between 0 and 1 and in two decimals]

From the given data, we have that $\lambda = 24$ per hour, $\mu = 1/E(S) = 30$ per hour and there is a single server. By Little's law, the proportion (or fraction) of time the worker is free equals $p_0 = 1 - \lambda E(S) = 1 - \lambda/\mu = 1 - 24/30 = 0.2$.

6. During peak hours of a day, vehicles arrive at a toll booth at a rate of 140 vehicles per hour. The toll booth has two parallel lanes through any of which the vehicles can pass through and the average time taken to pass through is 45 seconds. Then, the offered load to the system equals _____.

Solution:

Answer: 1.75 [Hint: Enter the answer in two decimals]

From the given data, we have that $\lambda = 140$ per hour, $\mu = 1/E(S) = 80$ per hour and there are two servers. The offered load is given by $r = \lambda/\mu = 140/80 = 1.75$.

7. In a single channel service system, suppose that the customers arrive at a rate of 250 per hour and are served at a rate of 5 customers per minute. If the expected waiting time in the queue for a customer is one minute, then the expected number of customers in the system equals _____.

Solution:

Answer: 5 [Hint: Enter the answer in two decimals]

From the given data, we have that $\lambda = 250$ per hour, $\mu = 300$ per hour, there is a single server and $W_q = 1$ minute or $W_q = 1/60$ hour. We then have that $W = W_q + 1/\mu = 1/50$ hour and therefore $L = \lambda W = 5$ customers.

8. At a barber shop with a single barber, customers arrive on an average at a rate of one customer for every 12 minutes. If the average number of customers waiting for hair cut at any point of time equals 25/6, then the average waiting time (in minutes) of a customer for a hair cut equals _____.

Solution:

Answer: 50 [Hint: Enter the answer as an integer]

From the given data, we have that $\lambda = 5$ per hour and $L_q = 25/6$ customers. We then have that $W_q = L_q/\lambda = 5/6$ hour and equals 50 minutes.

9. State whether the following statement is TRUE or FALSE:
For the stability of a service system (i.e., to be in steady-state), the offered load must be less than the number of servers.

Solution:

Answer: TRUE

TRUE, as we require $r < c$ for the service system to be a stable one.

10. If the Laplace-Stieltjes transform (LST) of a random variable X is given by $\dfrac{20}{s^3 + 9s^2 + 24s + 20}$, then $Var(X)$ equals _____.

Solution:

Answer: 0.54 [Hint: Enter the answer in two decimals]

Note that the given LST can be written as $\dfrac{20}{s^3 + 9s^2 + 24s + 20} = \left(\dfrac{2}{s+2}\right)^2 \left(\dfrac{5}{s+5}\right)$. This implies that the X is the sum of two independent random variables, a $Gamma(2, 2)$ and an $Exp(5)$ random variables, and hence $Var(X)$ is the sum of variances of these two random variables and equals 0.54.