DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA 597 Queueing Theory and Applications January - May 2019

Exercise Sheet - 3

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1. For a homogeneous CTMC (with infinitesimal matrix Q), show that the Laplace transform of the transition probability matrix $P(t)$, denoted by $\bar{P}(s)$, is given by $\bar{P}(s) = (sI - Q)^{-1}$. Also show that 0 is an eigenvalue of Q.

2. Consider the following pure birth process, known as the Yule process, in which each individual in the population independently gives birth at rate λ, that is, $\lambda_k = k\lambda, k \geq 1$. If we start with a single individual at time 0, what is the population size at time $t > 0$? What if we start with $i > 1$ individuals?

3. Consider two independent series of events E and F occurring in accordance with Poisson processes with rates λ and μ respectively. Show that the number N of occurrences of E between two successive occurrences of F has a geometric distribution.

4. There are m different types of coupons. Each time a person collects a coupon it is, independently of ones previously obtained, a type j coupon with probability p_j, $\sum_{j=1}^{m} p_j = 1$. Let N denote the number of coupons one needs to collect in order to have a complete collection of at least one of each type. Find $E(N)$.

5. Suppose that items arrive at a processing unit in accordance with a Poisson process with intensity λ. At a fixed time T, all items are dispatched from the system. Choose an intermediate time $t \in (0, T)$ at which all items in the system are dispatched, so as to minimize the total expected wait of all items.

6. Suppose that families migrate to an area at a Poisson rate $\lambda = 2$ per week. If the number of people in each family is independent and takes on the values $1, 2, 3, 4$ with respective probabilities $1/6, 1/3, 1/3, 1/6$, then what is the expected value and variance of the number of individuals migrating to this area during a fixed five-week period.

7. An insurance company feels that each of its policyholders has a rating value and that a policyholder having rating value λ will make claims at times distributed according to a Poisson process with rate λ, when time is measured in years. The firm also believes that rating values vary from policyholder to policyholder, with the probability distribution of the value of a new policyholder being uniformly distributed over $(0, 1)$. Given that a policyholder has made n claims in his or her first t years, what is the conditional distribution of the time until the policyholder's next claim?