1. Suppose that there are only two stations in a closed cyclic network, and the mean service time at the two stations are 20 minutes and 10 minutes, respectively. If there are two customers in the network, and for $n_1 = 0, 1, 2$, which one of the following is the steady state distribution of the system?

(A) $p_{n_1,2-n_1} = \frac{4}{7}\left(\frac{1}{4}\right)^{2-n_1}$

(B) $p_{n_1,2-n_1} = \frac{7}{4}\left(\frac{1}{2}\right)^{2-n_1}$

(C) $p_{n_1,2-n_1} = \frac{4}{7}\left(\frac{1}{2}\right)^{2-n_1}$

(D) $p_{n_1,2-n_1} = \frac{7}{4}\left(\frac{1}{4}\right)^{2-n_1}$

Solution:

Answer: (C)
Here, $\mu_1 = 1/20/$ min, $\mu_2 = 1/10/$ min. The traffic equation for closed cyclic is $\rho_2 = \frac{\mu_1}{\mu_2}\rho_1 = \frac{1}{2}\rho_1$. Setting $\rho_1 = 1$, we get $\rho_2 = 1/2$. The limiting distribution is

$$p_{n_1,2-n_1} = \frac{1}{G(2)}\left(\frac{1}{2}\right)^{2-n_1}, \quad 0 \le n_1 \le 2$$

$$\text{since} \quad \sum_{n_1=0}^{2} p_{n_1,2-n_1} = 1, \quad \text{then} \quad G(2) = \frac{1-(\frac{1}{2})^3}{1-\frac{1}{2}} = \frac{7}{4}$$

Therefore, $p_{n_1,2-n_1} = \frac{4}{7}\left(\frac{1}{2}\right)^{2-n_1}$.

2. Consider the two-station closed cyclic queueing network problem given in Question 1 above. The steady state probability that both the stations are non-empty is equal to

(A) $\frac{6}{7}$

(B) $\frac{5}{7}$

(C) $\frac{2}{7}$

(D) $\frac{1}{7}$

Solution:

Answer: (C)
Since $p_{n_1,2-n_1} = \frac{4}{7}\left(\frac{1}{2}\right)^{2-n_1}$, the required probability is $p_{1,1} = 2/7$.

3. Assuming usual notations, which one of the following conditions when imposed on a general Jackson network lead to a closed Jackson network?

(A) $\gamma_i = 1,\ r_{i0} = 1,\ \forall i$

(B) $\gamma_i = 1,\ r_{i0} = 0,\ \forall i$

(C) $\gamma_i = 0,\ r_{i0} = 1,\ \forall i$

(D) $\gamma_i = 0,\ r_{i0} = 0,\ \forall i$

Solution:

Answer: (D)

Directly follows from the lectures.

4. State whether the following statement is TRUE or FALSE.
The routing matrix in a closed Jackson network need not be a stochastic matrix.

Solution:

Answer: FALSE

Referring to lectures, since $r_{i0} = 0$ for all i, the routing matrix R must be a stochastic matrix. Note that this need not be the case with open Jackson networks.

5. Consider a two-node single-server-at-each-node closed Jackson network with three customers and with usual parameters $\mu_1 = 1, \mu_2 = 2, r_{12} = 1, r_{21} = 1$. The normalization constant is equal to

(A) 0.75

(B) 1.875

(C) 2.225

(D) 2.75

Solution:

Answer: (B)

The given network is a cyclic queueing network and hence (from lectures) $G(K) = \frac{1-\rho_2^{K+1}}{1-\rho_2}$. Here, $K = 3$ and $\rho_2 = \mu_1/\mu_2 = 1/2$. Substituting, we get $G(3) = 15/8 = 1.875$.

6. Consider a two-node single-server-at-each-node closed Jackson network with three customers and with usual parameters $\mu_1 = 1, \mu_2 = 2, r_{12} = 1, r_{21} = 1$. The expected number of customers at node 1 is equal to

[*Note: There is a typo in the corresponding quantity in the lecture and so be careful.*]

(A) $\frac{34}{15}$

(B) $\frac{11}{15}$

(C) $\frac{17}{5}$

(D) $\frac{1}{5}$

Solution:

Answer: (A)

The given network is a cyclic queueing network and hence (from lectures) $G(K) = \frac{1-\rho_2^{K+1}}{1-\rho_2}$. Here, $K = 3$ and $\rho_2 = \mu_1/\mu_2 = 1/2$. Substituting, we get $G(3) = 15/8 = 1.875$.

The steady state solution is $p_{3-m,m} = \rho_2^m/G(3)$ for $m = 0, 1, 2, 3$. Thus, we have $p_{0,3} = 1/15 = 0.0667, p_{1,2} = 2/15 = 0.1333, p_{2,1} = 4/15 = 0.2667, p_{3,0} = 8/15 = 0.5333$.

The expected number of customers at node 1 is given by (with S denoting the state space of the model)

$$L_1 = \sum_{j=0}^{K} j \sum_{(n_1,n_2)\in S, n_1=j} p_{n_1,n_2} = 1p_{1,2} + 2p_{2,1} + 3p_{3,0} = 34/15 = 2.2667.$$

7. Consider a two-node single-server-at-each-node closed Jackson network with 3 customers and with usual parameters $\mu_1 = 1, \mu_2 = 2, r_{12} = 1, r_{21} = 1$. The probability that node 2 server is busy is equal to

(A) $\frac{7}{15}$

(B) $\frac{14}{15}$

(C) $\frac{8}{15}$

(D) $\frac{11}{15}$

Solution:

Answer: (A)

The given network is a cyclic queueing network and hence (from lectures) $G(K) = \frac{1-\rho_2^{K+1}}{1-\rho_2}$. Here, $K = 3$ and $\rho_2 = \mu_1/\mu_2 = 1/2$. Substituting, we get $G(3) = 15/8 = 1.875$.

The steady state solution is $p_{3-m,m} = \rho_2^m/G(3)$ for $m = 0, 1, 2, 3$. Thus, we have $p_{0,3} = 1/15 = 0.0667, p_{1,2} = 2/15 = 0.1333, p_{2,1} = 4/15 = 0.2667, p_{3,0} = 8/15 = 0.5333$.

The probability that node 2 is busy equals $1 - p_{3,0} = 7/15$. Note that this is also the actual or effective server utilization at node 2.

8. State whether the following statement is TRUE or FALSE:
For a closed Jackson network of M nodes and K customers, the number of balance equations that need to be solved to get the steady state joint system size probabilities is equal to $\binom{K+M-1}{K}$.

Solution:

Answer: TRUE.

The number of balance equations equals the number of states and hence the value is $\binom{K+M-1}{K}$.