1. Cars arrive at a parking garage according to a Poisson process at a rate of 6 per hour and park there if there is space, or else they go away. The garage has a capacity to hold 10 cars. A car stays in the garage on average for 45 minutes. Assume that the times spent by the cars in the garage are IID exponential random variables. Suppose that the parking fee charged for each car is based on the exact duration of parking in the garage at a rate of Rs. 25 per hour. What is the long-run revenue rate per hour (in rupees, rounded to nearest integer value) for the garage?

   (A) 241
   (B) 41
   (C) 91
   (D) 191

   Solution:

   Answer: Since the none of the options match the answer, the question is dropped.

   Solution: This is an $M/M/c/c$ queue with $\lambda = 6/\text{hr.}, \mu = 4/3/\text{hr.}, c = 10$, thus $r = \lambda/\mu = 4.5$. By the iteration scheme of $B(c, r)$,

   $$B(0, 4.5) = 1, B(1, 4.5) = 0.81818, B(2, 4.5) = 0.648, B(3, 4.5) = 0.4929,$$
   $$B(4, 4.5) = 0.35671, B(5, 4.5) = 0.24302, B(6, 4.5) = 0.15417, B(7, 4.5) = 0.09017,$$
   $$B(8, 4.5) = 0.04827, B(9, 4.5) = 0.02357, B(10, 4.5) = 0.01049.$$

   Using the formula

   $$C(10, 4.5) = 0.01891, L_q = 0.015472, L = r + L_q = 4.515472$$

   Since each car pays Rs.25 per hour, the long run revenue rate per hour is given by

   $$25 \times L = 112.89 \approx 113 \quad \text{rupees/hr..}$$

2. A telephone switch can handle K calls at any point of time. Calls arrive according to a Poisson process with rate 4 per minute. If the switch is already serving K calls when a new call arrives, then the new call is lost. If a call is accepted, it lasts for an exponentially distributed duration with a mean of 2 minutes and then terminates. All call durations are independent of each other. What should be the optimal switch capacity K such that at most only 10 percent of the incoming calls are lost?

   (A) 10
   (B) 11
   (C) 12
   (D) 13

   Solution:

   Answer: (B)

This is an $M/M/c/c$ queue with $\lambda = 4/\text{min.}, \mu = 0.5/\text{min.}, c = K$, thus $r = \lambda/\mu = 8$. The fraction of calls lost is given by $p_K = B(K, 8)$. By the iteration scheme of $B(c, r)$, we compute p_K iteratively until the desired K as

$$
\begin{aligned}
&B(0,8) = 1, B(1,8) = 0.8889, B(2,8) = 0.7805, B(3,8) = 0.6755, B(4,8) = 0.5746,\\
&B(5,8) = 0.4789, B(6,8) = 0.3896, B(7,8) = 0.3082, B(8,8) = 0.2356,\\
&B(9,8) = 0.1731, B(10,8) = 0.1217, B(11,8) = 0.0813.
\end{aligned}
$$

Since $p_{11} = 0.0813$, we conclude that the telephone switch needs at least 11 lines to lose less than 10 percent of the traffic.

3. Visitors arrive at an amusement park in cars according to a Poisson process at a rate of 40 cars per hour. They (the visitors of a car) stay in the park for a random amount of time that is exponentially distributed with mean of 3 hours and leave. Assuming that the parking lot of the park is sufficiently big so that nobody is turned away, the expected number of cars in the lot in the long run equals _____.

Solution:

Answer: 120 [Hint: Enter the answer as an integer.]

This is an $M/M/\infty$ queue with $\lambda = 40, \mu = 1/3$. Hence in steady state the number of cars in the lot is a Poisson random variable with mean $40/(1/3) = 120$.

4. Students arrive at a bookstore according to a Poisson process at a rate of 8 per hour. Each student spends on average 15 minutes in the store, independently of each other. The time in the checkout counter is negligible, since most of them come just to browse! Let α denote the probability that there are more than 4 students in the store at equilibrium. Then, $100\,\alpha$ equals _____.

Solution:

Answer: Range: 5.10 **to** 5.40 [Hint: Enter the answer in two decimals.]

This is an $M/G/\infty$ queue with $\lambda = 8, \mu = 4$. Hence in steady state the number of customers in the store is a Poisson random variable with parameter $\rho = 8/4 = 2$. Then

$$
\alpha = 1 - \sum_{i=0}^{4} e^{-\rho}\frac{\rho^i}{i!} = 0.052653 \quad \text{and} \quad 100\alpha = 5.2653.
$$

5. Consider an $M/M/1/1$ queueing system with arrival rate λ and service rate μ. An arriving customer is arriving (i.e., entering) to the system if the server is idle; otherwise the arrival is lost. The number in the system is given by the process $\{N(t), t \geq 0\}$ and modelled as a CTMC with state space $\{0, 1\}$. At equilibrium, let X_n be the number of customers left behind by the nth departing customer, X_n^* be the number of customers as seen by the nth entry (not including the entering customer), $\hat{X}_n$ is the number of customers as seen by nth outsider (at an arbitrary time). Define, for $j = 0, 1$ and $n \geq 1$,

$$
\pi_j = \lim_{n\to\infty} P\left(X_n = j\right), \pi_j^* = \lim_{n\to\infty} P\left(X_n^* = j\right), \hat{\pi}_j = \lim_{n\to\infty} P\left(\hat{X}_n = j\right).
$$

Then, (π_0, π_1), (π_0^*, π_1^*) and $(\hat{\pi}_0, \hat{\pi}_1)$, respectively, are given by

(A) $(1,0)$, $(0,1)$ and $\left(\frac{\mu}{\lambda+\mu}, \frac{\lambda}{\lambda+\mu}\right)$

(B) $(0,1)$, $(0,1)$ and $\left(\frac{\lambda}{\lambda+\mu}, \frac{\mu}{\lambda+\mu}\right)$

(C) $(1,0)$, $(1,0)$ and $\left(\frac{\mu}{\lambda+\mu}, \frac{\lambda}{\lambda+\mu}\right)$

(D) $(0,1)$, $(1,0)$ and $\left(\frac{\lambda}{\lambda+\mu}, \frac{\mu}{\lambda+\mu}\right)$

Solution:

Answer: (C)

The steady state distribution can be computed from $Q = \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$ as $p_0 = \frac{\mu}{\lambda+\mu}, p_1 = \frac{\lambda}{\lambda+\mu}$. Since the arrival is Poisson, by PASTA property, $\hat{\pi}_0 = \frac{\mu}{\lambda+\mu}, \hat{\pi}_1 = \frac{\lambda}{\lambda+\mu}$. Since departing customer always leaves an empty system, we have $X_n = 0$ for all $n \geq 0$, and hence $\pi_0 = 1, \pi_1 = 0$. Similarly, since a customer can enter only if the system is empty, and $\pi_0^* = 1, \pi_1^* = 0$.

6. State whether the following statement is TRUE or FALSE:
For an $M/M/1$ queueing system with parameters $\lambda = 3$ and $\mu = 2$ and assuming an empty system initially, we can obtain the time-dependent (i.e., transient) system size probabilities $p_n(t)$ such that $\sum_{n=0}^{\infty} p_n(t) = 1$ for all $t \geq 0$ but $p_n(t)$ tends to zero for all n as $t \to \infty$.

Solution:

Answer: TRUE

The transient solution exists for all values of λ and μ, and for all $t \geq 0$ and they form a proper probability distribution. But, the steady state solution exists only under the condition $\lambda/\mu < 1$ which is not the case here (as $\lambda > \mu$).

7. Which one of the following describes the relationship between Erlang-C formula $C(c, r)$ and Erlang-B formula $B(c, r)$, for all $c \geq 1$ and for all $0 < r < c$?

(A) $C(c, r) < B(c, r)$

(B) $C(c, r) > B(c, r)$

(C) $C(c, r) = B(c, r)$

(D) No specific relationship exists and it depends on the parameter values.

Solution:

Answer: (B).

$$C(c, r) = \frac{cB(c, r)}{c - r + rB(c, r)} = \frac{B(c, r)}{1 - (r/c)(1 - B(c, r))} > B(c, r).$$

8. State whether the following statement is TRUE or FALSE:
   For a c-server Engset loss system, the time blocking probability in a system with M customers equals the call blocking probability in a system with $M-1$ customers.

   Solution:

   Answer: FALSE

   For the Engset loss system, we have that $a_c(M) = p_c(M-1)$ and hence the reverse is true, i.e., the call blocking probability in a system with M customers equals the time blocking probability in a system with $M-1$ customers.