1. Customers arrive to a single-server queueing system according to a Poisson process at a rate of 10 customers per hour. A customer upon arrival is designated as a type-A customer or a type-B customer with equal probability. While type-A customers have an exponential service time with a mean of 3 minutes , type-B customers have an exponential service time with a mean of 6 minutes. If W_q^{AB} is the overall mean waiting time in queue when type-A is assigned a nonpreemptive higher priority over type-B and W_q^{BA} is the overall mean waiting time in queue when type-B is assigned a nonpreemptive higher priority over type-A, then which one of the following is correct?

   (A) $W_q^{AB} > W_q^{BA}$
   (B) $W_q^{AB} < W_q^{BA}$
   (C) $W_q^{AB} = W_q^{BA}$

   Solution:

   Answer: (B)

   The system is a two-class (denoting 1 for type-A and 2 for type-B) non-preemptive priority queueing model with $\lambda_1 = 1/12$/min., $\mu_1 = 1/3$/ min., $\lambda_2 = 1/12$/ min., $\mu_2 = 1/6$/ min. Thus $\rho_1 = 1/4$, $\rho_2 = 1/2$.

   First case: Type-A has priority

$$W_q^{(1)} = \frac{\sum_{i=1}^{2} \rho_i/\mu_i}{1-\rho_1} = 5 \;\text{ min}, \qquad W_q^{(2)} = \frac{\sum_{i=1}^{2} \rho_i/\mu_i}{(1-\rho_1)(1-\rho_1-\rho_2)} = 20 \;\text{ min.}$$

   The average waiting time for all customers is

$$W_q^{AB} = \sum_{i=1}^{2} \frac{\lambda_i}{\lambda} W_q^{(i)} = 12.5 \;\text{ min}, \qquad \lambda = \lambda_1 + \lambda_2$$

   Second case: Type-B has priority

$$W_q^{(2)} = \frac{\sum_{i=1}^{2} \rho_i/\mu_i}{1-\rho_2} = 7.5 \;\text{ min.} \qquad W_q^{(1)} = \frac{\sum_{i=1}^{2} \rho_i/\mu_i}{(1-\rho_2)(1-\rho_1-\rho_2)} = 30 \;\text{ min.}$$

$$\text{Therefore,} \qquad W_q^{BA} = \sum_{i=1}^{2} \frac{\lambda_i}{\lambda} W_q^{(i)} = 18.75 \;\text{ min.}$$

2. State whether the following statement is TRUE or FALSE:
   In a nonpreemptive priority queueing system, the waiting time distribution for the customers of highest priority will be the same as that of a corresponding $M/M/1$ queueing system.

   Solution:

   Answer: FALSE.

   Referring to the lectures, it can be observed that the presence of lower priority customers creates more delay for the higher priority customer (even the mean value is higher) and hence the distribution would not be the same.

3. State whether the following statement is TRUE or FALSE:
   In a two-class nonpreemptive priority system, the average number of lower priority customers

waiting in the queue is always greater than the average number of higher priority customers waiting in the queue.

Solution:

Answer: FALSE

Referring to the lecture, it can be noted that while it is always true that $W_q^{(2)} > W_q^{(1)}$, it is not always the case that $L_q^{(2)} > L_q^{(1)}$.

4. Suppose that a car servicing facility has a single mobile number to which the calls are made for booking. Calls to the facility are modelled by an $M/M/1$ retrial queue with arrival rate $\lambda = 10$ per hour, service rate $\mu = 15$ per hour, and retrial rate $\gamma = 6$ per hour. Then the average waiting time (in minutes) in the orbit equals

   (A) 14
   (B) 18
   (C) 24
   (D) 28

Solution:

Answer: (D)

Since $\lambda = 10$ per hour, $\mu = 15$ per hour, and $\gamma = 6$ per hour, we have $\rho = 2/3$ and

$$L_o = \frac{(2/3)^2}{1-2/3}\frac{15+6}{6} = \frac{14}{3} \quad \text{and} \quad W_o = \frac{L_o}{\lambda} = \frac{7}{15} \text{ hours} = 28 \text{ minutes.}$$

5. Consider the car servicing facility as given in Question 4 above. What is the average rate (per hour) that call attempts are made to the service facility? (Call attempts include calls that are answered and calls that receive a busy signal).

   (A) 36
   (B) 37
   (C) 38
   (D) 39

Solution:

Answer: (C)

The system is an $M/M/1$ retrial queue. The average rate of calls due to newly arriving customers is λ and the average rate of calls due to customers in orbit is γL_o. Therefore, the total rate of call attempts is $\lambda + \gamma L_o = 10 + 6(14/3) = 38$ per hour.

6. Consider the car servicing facility as given in Question 4 above. What is the fraction of call attempts that receive a busy signal?

   (A) $\frac{4}{18}$

(B) $\frac{5}{19}$

(C) $\frac{13}{18}$

(D) $\frac{14}{19}$

Solution:

Answer: (D)

The rate of successful call attempts is given by (using the balance equations and the fact that $P_1(1) = \rho$ from the lectures)

$$\sum_{n=0}^{\infty}(\lambda + n\gamma)p_{0,n} = \sum_{n=0}^{\infty}\mu p_{1,n} = \mu\rho = \lambda.$$

The total rate of call attempts is $\lambda + \gamma L_o$ and hence the proportion of call attempts that receive a busy signal is $\frac{\gamma L_0}{\lambda + \gamma L_0} = \frac{28}{38} = \frac{14}{19}$.

7. In a $Geo/Geo/1$ (LAS) queueing system with $\lambda = 0.5$ and $\mu = 0.7$, the steady state mean number of customers in the queue equals

(A) $\frac{5}{4}$

(B) $\frac{25}{28}$

(C) $\frac{5}{7}$

(D) $\frac{15}{4}$

Solution:

Answer: (B)

For the $Geo/Geo/1$ (LAS) queueing system, we have that the mean queue length is given by

$$\frac{\lambda\bar{\lambda}}{\mu - \lambda}\frac{\lambda}{\mu} = \frac{5}{4}\frac{5}{7} = \frac{25}{28}.$$

8. In a $Geo/Geo/1$ (EAS) queueing system with $\lambda = 0.4$ and $\mu = 0.6$, the steady state probability of an empty system equals

(A) $\frac{5}{9}$

(B) $\frac{2}{3}$

(C) $\frac{5}{7}$

(D) $\frac{1}{2}$

Solution:

Answer: (A)

For the $Geo/Geo/1$ (EAS) queueing system, we have that probability of an empty system is given by

$$\pi_0 = 1 - \alpha = 1 - \frac{4}{9} = \frac{5}{9}, \text{ where } \alpha = \frac{\lambda\bar{\mu}}{\mu\bar{\lambda}} = \frac{4}{9}.$$