MA597 QTA (Jan-May 2022): Mid-Semester Examination

The duration of this mid-semester examination is 110 minutes (including the uploading time). To avoid congestion, scan and upload as soon as one question is completed.

For the first two questions, neatly write the solutions to each question separately on a paper, scan each question separately and upload at the appropriate place. Your roll number should be written in each page on the top right corner. Similarly, write the question number also on each page. Start a new question on a new page.

The naming convention for the files MUST be 'Your Roll Number', followed by an underscore '_', followed by 'MSQ1' or 'MSQ2', appropriately. For example, '190101099_MSQ1'. Files not following the specification may be missed. Hence, please follow the convention.

For the remaining six questions, enter the answers online.

Sign-off the Honor Pledge and also mention the question numbers for which you have uploaded the scanned answers.

1

[Neatly write your solution in a sheet with precise explanations, take a good quality scan, create a pdf file and upload it.] [File name: **RollNumber_MSQ1**]

(Non-anonymous question ⓘ)

(6 Points)

Customers arrive to a service facility according to a Poisson process with rate λ. There are two servers in the facility and the service times with server i follow an exponential distribution with parameter $\mu_i, i = 1, 2$. Each arriving customer is assigned to the next available server out of two servers, but a customer who finds an empty system on arrival is assigned to the one of the servers with equal probability. The facility can accommodate any number of customers to queue for service and the customers are served on an FCFS basis.

Draw the state transition diagram, write down the steady state balance equations and solve them. What is the condition for the stability of the system? What is the probability of positive delay in queue? Also, determine L_q.

190123066_queueing theory_ABHISHEK AGRAHARI 19.pdf

2

[Neatly write your solution in a sheet with precise explanations, take a good quality scan, create a pdf file and upload it.] [File name: **RollNumber_MSQ2**]

(Non-anonymous question ⓘ)

(4 Points)

Consider an $M/M/1$ queueing system with parameters $\lambda = 5$ and $\mu = 3$. Additionally, assume that the customers are impatient and will leave system if the service is not completed within a certain time of their arrival according to an exponential time with parameter $\theta = 3$.

State the transition rates of the CTMC representing the number in the system. Write down the global and local balance equations. Are they equivalent? Solve them. What is the condition for stability of the system? Let Q be the probability that an arrival abandons the system without completing his/her service. Is it true that $5Q = 3L$ and $Q = 3W$ (where L and W are defined as usual)? Why or why not?

3

[Type your solution in the space provided. It should be precise and but sufficient information should be provided. You may work out separately but 3 or 4 main steps should be given.]
(4 Points)

Consider an $M/M/1$ queueing system with parameters λ and μ.

(a) Suppose that a customer is in service. What is the distribution of his/her age of service time?

(b) What is the answer to the question in part-(a) above, if nobody arrived during his/her past service time?

(c) What is the answer to the question in part-(a) above, if the queue is empty?

(d) What is the probability that the queue is empty if a customer is currently in service and his/her past service time is t?

a) Exp(u)
b) Exp(u) because service time and arrival time are independent
c) Exp(u) because service time and arrival time are independent
d) p0 = 1 - lambda/mu because service time and arrival time are independent

4

[Type your solution in the space provided. It should be precise and but sufficient information should be provided. You may work out separately but 3 or 4 main steps should be given.]
(2 Points)

Let X and Y be exponential random variables with parameters λ and μ, respectively. Suppose that $\lambda < \mu$. Let Z be equal to X with probability λ/μ and equal to $X + Y$ with probability $1 - \lambda/\mu$. Determine the distribution of Z.

PDF of X + Y = (lambda*mu)/(lambda - mu)*(e^(-mu*x) - e^(-lamda*x))
CDF of X+Y = can be obtained by integrating above expression from 0 to t.
CDF of X(t) = 1- e^(-lambda*t)
CDF of Y(t) = 1 - e^(-mu*t)

by total probability law -
P(Z <= t) = (lambda/mu)*P(X <= t) + (1-lambda/mu)*P(X + Y <= t)
CDF of Z = (lambda/mu)*(CDF of X) + (1-lambda/mu)*(CDF of X + Y)

Final answer -
CDF of Z = (1 + lambda/mu)*(CDF of X) - (lambda/mu)*(CDF of Y)

5

[Type your solution in the space provided. It should be precise and but sufficient information should be provided. You may work out separately but 3 or 4 main steps should be given.]
(3 Points)

Show that the Erlang loss formula $B(c, r)$ satisfies $B(c, r) \geq \max\{0, 1 - c/r\}$. Is $B(c, r)$, for a fixed r, increasing or decreasing in c as c runs through the positive integers? Justify.

Probability of not blocking a system = 1-B(c,r)
Offered load = r
Therefore carried load = r*(1-B(c,r))
carried load should be less than number of available servers therefore it implies
r*(1-B(c,r)) <= c
which gives B(c,r) >= 1-c/r
Also B(c,r) is positive therefore,
B(c,r) >= max{0, 1-c/r}
B(c,r) represents the probability of finding all c servers busy. Therefore keeping r fixed and increasing c (no. of servers) would decrease the probability of finding all c servers busy. Hence B(c,r) would decrease.

6

[Enter only the final answers in the space provided.]
(2 Points)

A PBX was installed to handle the voice traffic generated by the 500 employees in IITG. Each employee on average makes 2 calls per hour with an average call duration of 5 minutes. The PBX has 90 outgoing links. What is the offered load to the PBX? What is the utilization of the outgoing links? Assume that calls arriving when all the links are busy are queued up.

6a) 250/3 (b) 25/27

7

[Enter only the final answers for the three parts in the space provided.]
(3 Points)

In a restaurant, two servers are on duty. The owner notices the percent time idle of each server to be 1%.

(a) If the owner decides to add a third server, how much percent idle time would each server have then?

(b) Suppose that by adding the third server, the pressure on the servers is reduced and hence they can work more carefully, but their service output rate is reduced by 20%. What now is the percent time each would be idle?

(c) Suppose, instead, that the owner decies to provide a helper to the two servers (rather than hiring an additional server). This allows the two servers to decrease their average service time by 20% (relative to the original). What now is the percent idle time of each of the two servers?

7a) 34% (b) 17.5% (c) 17.5%

8

[Enter only the final answer in the space provided.]
(1 Point)

Assume that taxis pass through a station (where people wait to board a taxi) at a mean rate of 10 per hour. Only an empty taxi stops at the station to take exactly one passenger. Assume further that a taxi would be empty (and hence would stop at the station) with probability $1/10$. You are second in line waiting for a taxi. What is the probability that you will have to wait for more than 2 hours?

3/(e^2)

9

Honor Pledge:

I pledge on my honor that I have neither given nor received any unauthorized assistance on this examination and I affirm that all the work are my own only. *

Enter 'Yes' or 'No' to the Pledge.

Additionally, mention the question numbers (out of Q1 and Q2) for which you have uploaded the scanned answers.

Yes.q1