MODULE 8: Queueing Networks

LECTURE 28

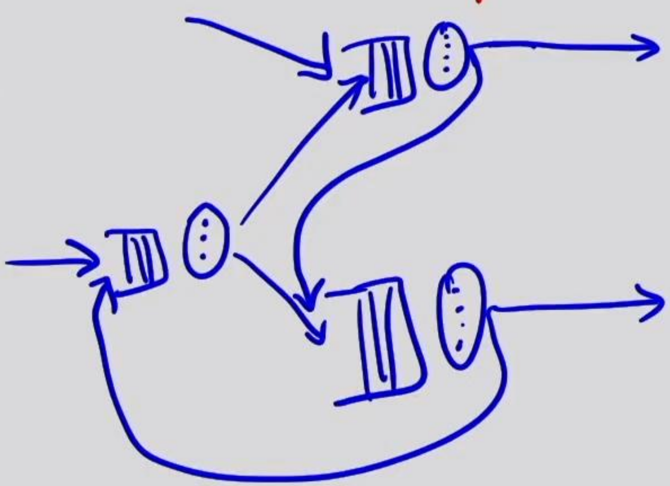
Introduction to Queueing Networks, Two-Node Network

Queueing Networks

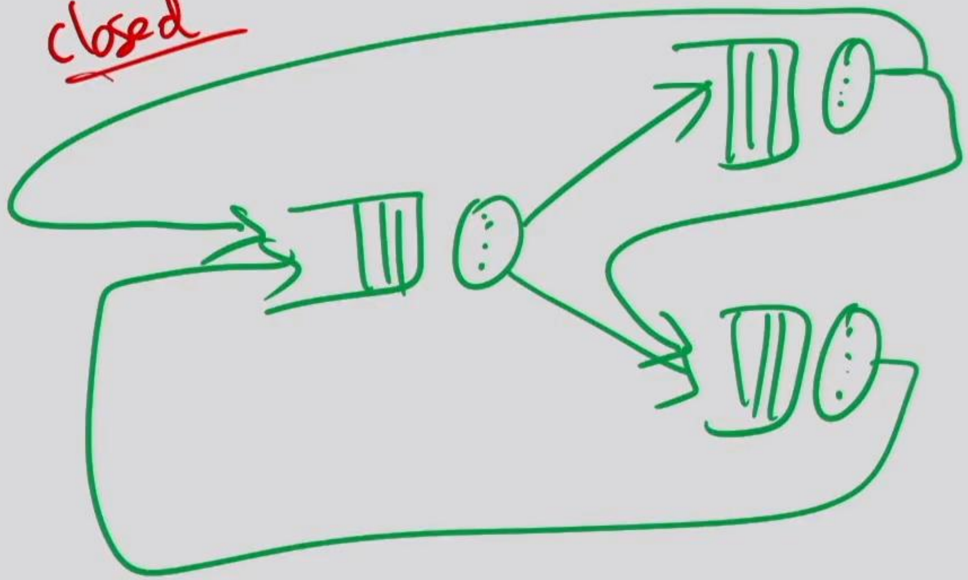
- We have so far studied queueing systems which could be termed as ‘single-node’ queues.
- ‘Networks’ of queues is an important area with a variety of quite natural application areas and with extremely difficult problems.
- While a (single-node) queues represent situations where the customer demands one service, a queueing network represent situations where a customer may need more than one service (or different kind of service) from different servers and may be required to wait before each of the different service channels for service.
  - ♦ For example, bank counters, car repair facility, etc.
- Queueing networks have diverse application areas including production and assembly lines, maintenance operations, airport terminals, computer/communication sytems/networks and health care facilities.
- We give some basic concepts and results that are quite useful in their own right and are important in the design of many manufacturing/production facilities and computer/communication networks.
- Bolch et al. (2006) and Gelenbe and Pujolle (1998), and a host of others, can be looked into for a detailed look into the topic.

- Main types of queueing networks:
  - ⋆ **Open queueing network**: Customers (of one or more classes) enter the system from outside and eventually leave the system, after service at one or more nodes.
  - ⋆ **Closed queueing network**: A fixed number of customers (of one or more classes) circulate in the system, moving from one queue to the next, getting served at individual nodes. No external arrivals or departures.
  - ⋆ **Mixed queueing network**: Open for one class of customers and closed for another class of customers.

open



closed



- Just as in the case of single queues, performance measures of a queueing networks can be obtained.
- One important measure is the time taken to serve one customer in the system.
- Analysis would give us measures for the number of customers in each node so that estimates of buffer requirements can be made.
- Bottlenecks may be identified leading to better design (in terms of more servers/waiting spaces).
- It may be easier to model a complicated service scenario as a queueing network in order to capture better the way the service is actually provided.
- Networks of queues requires some additional specification such as
  - Interconnection between the queues
  - Routing strategy (deterministic, class based or probabilistic)
  - Strategy to handle blocking if the destination queue is a finite-capacity queue
- Possible that a customer returns to a queue for another service (i.e., feedback).
  ▶ Feedforward networks are open networks with no feedback (a queue will be visited at most once).

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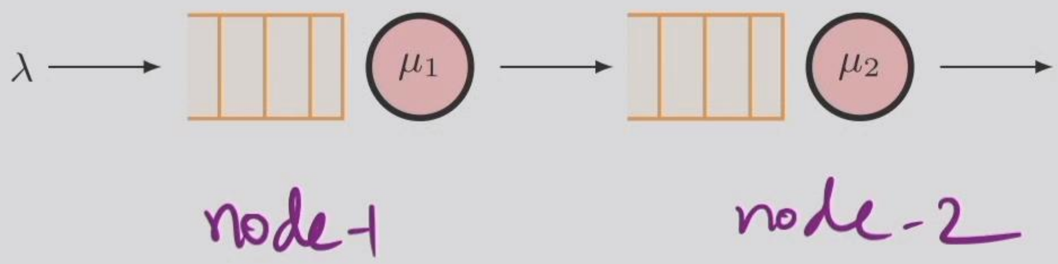
Two-Node Queueing Network

- Consider the following simple network of two queues (or nodes):
  - ▶ Customers arrive at the first node according to a Poisson process with rate λ.
  - ▶ The fist node is a single-server system with exponentially distributed service times (with parameter μ_1) having infinite queueing capacity.
  - ▶ Once the service is completed in the first node, the customer moves to the second node which is also a single-server system with exponentially distributed service times (with parameter μ_2) having infinite queueing capacity.
  - ▶ Once the service is completed in the second node, the customer departs the system.
  - ▶ No feedback or departure at the first node. No arrival at the second node.
  - ▶ The system can be represented as $\boxed{M/M/1 \to \bullet/M/1}$



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- The system state can be represented as a two-dimensional CTMC with state space $S = \{(n_1, n_2) : n_1, n_2 = 0, 1, 2, \ldots\}$.
- Denote the probability of n_1 customers in the first node and n_2 customers in the second node in steady state by p_{n_1,n_2}.
- Then, the steady state solution for this system exists under the condition that $\rho_1 = \lambda/\mu_1 < 1$ and $\rho_2 = \lambda/\mu_2 < 1$ and can be obtained from the balance equations given by

$$
\begin{aligned}
(\lambda + \mu_1 + \mu_2)p_{n_1,n_2} &= \lambda p_{n_1-1,n_2} + \mu_1 p_{n_1+1,n_2-1} + \mu_2 p_{n_1,n_2+1}, \quad n_1 \geq 1, n_2 \geq 1 \\
(\lambda + \mu_1)p_{n_1,0} &= \lambda p_{n_1-1,0} + \mu_2 p_{n_1,1}, \quad n_1 \geq 1 \\
(\lambda + \mu_2)p_{0,n_2} &= \mu_1 p_{1,n_2-1} + \mu_2 p_{0,n_2+1}, \quad n_2 \geq 1 \\
\lambda p_{0,0} &= \mu_2 p_{0,1}
\end{aligned}
$$

It can be shown that

$$p_{n_1,n_2} = \rho_1^{n_1}\rho_2^{n_2} p_{0,0} \quad \text{and} \quad p_{0,0} = (1-\rho_1)(1-\rho_2).$$

Thus,

$$p_{n_1,n_2} = \left[(1-\rho_1)\rho_1^{n_1}\right]\left[(1-\rho_2)\rho_2^{n_2}\right], \; n_1, n_2 \geq 0.$$

- A queueing network of this type where the joint distribution of the number of customers in each node can be written as a product of terms involving the number in individual nodes is referred to as a **product-form network**.
  ■ In many situations and under some conditions, this type of solution is observed to hold.
- The form of the solution indicates that in steady state each node behaves independently of the other (the joint distribution factors into product of marginals).
- The second stage behaves like a system with an input process that is Poisson with rate λ. That is, it behaves as an $M/M/1$ queue independent of the behaviour of the first stage.
- This can be proved if we can characterize the output process of the first node.
- The output process (distribution of times between successive departures) of the first node is the input process to the second node.
- Burke's theorem determines the output process of $M/M/c$ queues.
  ♦ Of all the systems with FCFS, $M/M/c$ is the only system with the stated property.