1. If $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space, then $P(\Omega)$ equals _______

Solution:

Answer: Range: 1 **to** 1

2. Let X be a random variable on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ such that $P\{X = i\} = \dfrac{k}{i+1}$ for $i = 1, 2, 3, 4$ (and 0 otherwise), where k is a suitable constant. Then the expectation $E(X)$ equals _______

Solution:

Answer: Range: 2.05 **to** 2.20

We have that $k\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right) = 1 \Rightarrow k = \frac{60}{77}$.

Therefore, $E(X) = \sum_i iP\{X = i\} = \frac{60}{77}\left(\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}\right) = \frac{163}{77} \approx 2.1169$.

3. Let $(\Omega, \mathcal{F}, P)$ be a probability space, where Ω is a finite set. Then which one of the following is always true?

(A) $P\Big(\bigcup_{n=1}^{20} A_n\Big) = \sum_{n=1}^{20} P(A_n)$, for all $A_1, A_2, \ldots, A_{20} \in \mathcal{F}$.

(B) For any random variable X that takes a finite number of values, the expectation of X is finite.

(C) For any two random variables X and Y, we have $E(XY) = E(X)E(Y)$.

Solution:

Answer: (B)

(A) is true only if they are disjoint. (B) is true always, as $E(X)$ is a finite sum. (C) is true only if X and Y are uncorrelated.

4. Let $\rho_{X,Y}$ be the correlation coefficient between two random variables X and Y defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Then $|\rho_{X,Y}|$ is less than or equal to _______

Solution:

Answer: Range 1 **to** 1

5. State whether the following statement is TRUE or FALSE:
If X and Y are two uncorrelated random variables defined on a probability space $(\Omega, \mathcal{F}, P)$, then they are independent.

Solution:

Answer: FALSE

FALSE, in general, as uncorrelation need not imply independence (while independence always implies uncorrelation).

6. The literacy rate for a state measures the proportion of people aged 15 and over who can read and write. The literacy rate for a particular state is 40%. Let X denote the number of people you ask until one says that he/she is literate. Assume that X has a geometric distribution. Then the minimum number of people you must ask so that the probability of finding a literate is at least 0.9 equals _______

Solution:

Answer: Range 5 **to** 5

The CDF of a $Geo(p)$ random variable is $F(x) = 1 - q^x$. Here, $p = 0.4$ and we want x such that $F(x) \geq 0.9$. That means solving for x such that $1 - (0.6)^x \geq 0.9 \;\Rightarrow x = 5$ (By observing $F(4) = 0.8702$ and $F(5) = 0.9222$).

7. Let the random variable X have the moment generating function $M_X(t) = e^{2t(1+t)}$, $t \in \mathbb{R}$. Then $P\{X \leq 2\}$ equals _______

Solution:

Answer: Range: 0.49 **to** 0.51

As the given MGF corresponds to a $\mathcal{N}(2, 4)$ random variable and since the normal distribution is symmetric about its mean, we have that $P\{X \leq 2\}$ equals 0.5.

8. The time between two consecutive insurance claims can be modelled by an exponential distribution with the average amount of time equal to 2 hours. Then the probability that there is a claim within the next 100 minutes from now equals _______

Solution:

Answer: Range: 0.55 **to** 0.58

Since $X \sim Exp(1/2)$, the required probability is $P\{X < 5/3\} = 1 - e^{-5/6} \approx 0.5654$.

9. For a Poisson random variable X with mean λ, if $P\{X = 6\} = \dfrac{1}{5}P\{X = 4\}$, then λ equals _______

Solution:

Answer: Range: 2.40 **to** 2.50

Since $P\{X = n\} = \frac{\lambda^2}{n(n-1)}P\{X = n - 2\}$ for a Poisson random variable, the required answer is $\lambda = \sqrt{30/5} = \sqrt{6} \approx 2.4495$.