Date of Submission: 22/02/2022

1. Write a program to approximate the function

$$f(x) = \frac{1}{1+x^2}, \quad -5 \le x \le 5, \quad \text{(Runge's example)},$$

   using the points $x_i = -5 + 10\dfrac{i}{8}$, $i = 0, 1, 2, \ldots, 8$ by natural cubic spline. Plot the polynomial against the exact function.

2. Approximate the derivative of $f(x) = x \log(1+x^2)$ at $x = 1$ numerically by using forward, backward, and central difference formulae i.e.,

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}, \quad f'(x) \approx \frac{f(x) - f(x-h)}{h}, \quad f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

   with the step size $h = 0.1, 0.01$ and 0.001. Compute the error between the numerical approximations and exact derivatives. Plot the computational error versus the step size.

3. From the following table of values of $f(x) = \sinh(x)$, find $f'(0.400)$ using the formulae

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h},$$

   and

$$f'(x) \approx \frac{1}{12h}\left[-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)\right]$$

   with $h = 0.001$. Which of these is the more accurate? The correct result is $f'(0.4) = \cosh 0.4 = 1.081072$.

x	f(x)
0.398	0.408591
0.399	0.409671
0.400	0.410752
0.401	0.411834
0.402	0.412915