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(1) $x_{n+1} = g(x_n)$

$\because$ α & β are the roots

$\alpha + \beta = -p$

$\alpha\beta = q$

(i) $g(x) = -p - q/x$

$g'(x) = q/x^2$

For convergence to α,

$|g'(\alpha)| < 1$

$\therefore \left|\frac{b}{\alpha^2}\right| = \left|\frac{\alpha\beta}{\alpha^2}\right| = \left|\frac{\beta}{\alpha}\right| < 1$

$\Rightarrow |\beta| < |\alpha|$

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② $g(x) = x - \frac{f(x)}{f'(x_0)}$

$g(\xi) = \xi - \frac{f(\xi)}{f'(x_0)} = 0 \quad \because f(\xi) = 0$

$g'(x) = 1 - \frac{f'(x)}{f'(x_0)}$

$g'(\xi) = 1 - \frac{f'(\xi)}{f'(x_0)} \neq 0$

$\therefore$ Order of convergence = 1

$\therefore \lim_{n \to \infty} \left| \frac{e_{n+1}}{e_n} \right| = c$ where

$c = \left| \frac{g^{(1)}(\xi)}{1!} \right|$

$\Rightarrow \cancel{c = \frac{1 - f'(\xi)}{}}$

$\Rightarrow c = \left| 1 - \frac{f'(\xi)}{f'(x_0)} \right|$

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(3) Let

$$S(x) = \begin{cases} S_1 = x^3 & x \in [0,1] \\ S_2 = \frac{1}{2}(x-1)^3 + a(x-1)^2 + b(x-1) + c & x \in [1,3] \end{cases}$$

$S_1' = 3x^2$ $\quad S_1'' = 6x$

$S_2' = \frac{3(x-1)^2}{2} + 2a(x-1) + b$ $\quad S_2'' = 3(x-1) + 2a.$

For cubic spline

$S_1(1) = 1 = S_2(1) = c \Rightarrow c = 1$

$S_1'(1) = 3 = S_2'(1) = b \Rightarrow b = 3$

$S_1''(1) = 6 = S_2''(1) = 2a \Rightarrow \underline{a = 3}$

For natural cubic spline -

$S_1''(0) = 0$ and $S_2''(3) = 0$

$S_1''(0) = 6(0) = 0$

$S_2''(3) = 3(3-1) + 2(3) = 3 \times 2 + 6 = 12$

$\because S_2''(3) \neq 0$ $\therefore$ it is not a natural cubic spline

4.

~~$f(x) \in C^3(x_0, x_2)$~~

$$f(x) = P_2(x) + f[x_0, x_1, x_2, x][x - x_0](x - x_1)(x - x_2)$$

where P_2 is the quadratic polynomial interpolating $f(x)$

$$f'(x) = P_2'(x) + f[x_0, x_1, x_2, x, x](x - x_0)(x - x_1)(x - x_2)$$
$$+ f[x_0, x_1, x_2]\big((x - x_1)(x - x_2) + (x - x_0)(x - x_1) + (x - x_0)(x - x_2)\big)$$

~~$Err = |f'(x) - P_2'(x)| =$~~

$$f'(x) = P_2'(x) + \frac{f^4(\xi)}{4!}(x - x_0)(x - x_1)(x - x_2)$$
$$+ \frac{f^3(\eta)}{3!}\big((x - x_1)(x - x_2) + (x - x_0)(x - x_1) + (x - x_0)(x - x_2)\big)$$

for some ξ and $\eta \in [x_0 x_2]$

$$|f'(x_0) - P_2'(x_0)| = \left|\frac{f^4(\xi)}{4!}(0) + \frac{f^3(\eta)}{3!}\big((x_0 - x_1)(x_0 - x_2)\right|$$

$$= \left|\frac{f^3(\eta)}{3!}(x_0 - x_1)(x_0 - x_2)\right|$$

$$= \left|\frac{f^3(\eta)}{3!}(-h)(-2h)\right|$$

$$= \frac{f^3(\eta)}{3!}(2h^2) \leq \frac{10^{-3} \times 2h^2}{6} = \frac{h^2 \times 10^{-3}}{3}$$

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Similarly —

$$\left|f'(x_1) - P_2'(x_1)\right| = \left|\frac{f^3(\eta)}{3!}(x_1 - x_0)(x_1 - x_2)\right|$$

$$= \left|\frac{f^3(\eta)}{6}(h)(-h)\right|$$

$$\leq \frac{10^{-3}}{6} h^2$$

$$\left|f'(x_2) - P_2'(x_2)\right| = \left|\frac{f^3(\eta)}{6}(x_2 - x_0)(x_2 - x_1)\right|$$

$$= \left|\frac{f^3(\eta)}{6} \times 2h \times h\right|$$

$$\leq \frac{10^{-3}}{3} h^2$$

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(6) ~~Let take f is to be~~

As ~~it is~~ diff. rule is true for all pol. with deg ≤ 2

Let take $f = \text{constant} = p$

$\therefore \quad 0 = c_1 p + c_2 p + c_3 p$

$\Rightarrow c_1 + c_2 + c_3 = 0$ —— (1)

Let take $f = \text{linear} = px$.

$\Rightarrow \cancel{p} = c_1 \cancel{p} x_0 + c_2 \cancel{p}(x_0 + h) + c_3 \cancel{p}(x_0 + 2h)$

$x_0(c_1 + c_2 + c_3) + h(c_2 + 2c_3) = 1$

$\Rightarrow c_2 + 2c_3 = \frac{1}{h}$ —— (2) — using (1)

Let take $f = \text{quad.} = x^2$

$2x_0 = c_1 x_0^2 + c_2(x_0 + h)^2 + c_3(x_0 + 2h)^2$

$\cancel{2x_0} = \cancel{+ 2x_0 h(c_2 + 2c_3)} + h^2(c_2 + 4c_3)$

$\cancel{2x_0 =} \quad \therefore c_2 + 4c_3 = 0$ —— (3)

$\cancel{c_2 = -4c_3}$

Solving (1), (2), (3) gives.

$c_1 = \frac{-3}{2h} \qquad c_2 = \frac{2}{h} \qquad c_3 = -\frac{1}{2h}$.

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(7) E_c^T = error in composite trapezoidal Rule

$$= \frac{-f''(\eta)(h)^2(b-a)}{12} \quad \text{for some } \eta \in (a,b)$$

$$= \frac{-f''(\eta)(x_N - x_0)^3}{12N^2}$$

Here $x_0 = 0$ and $x_N = 1$

~~for~~ $f(x) = e^{-x^2}$

$f'(x) = -2x e^{-x^2}$

$f''(x) = -2e^{-x^2} + 4x^2 e^{-x^2}$

$= 2e^{-x^2}(2x^2 - 1)$

$\therefore$ ~~$E_c^T =$~~

$$|f''(\eta)| = |2e^{-\eta^2}(2\eta^2 - 1)| \leq 2$$

Hence $$|E_c^T| \leq \frac{2(1-0)^3}{12 \times N^2} < 10^{-6}$$

for answer to be correct to 6 decimal places.

$$\frac{1}{6 \times 10^{-6}} \leq N^2$$

$$\frac{10^3}{\sqrt{6}} \leq N$$