MA322 Lab 07
Abhishek Agrahari
190123066

Question 1

Code

```
clc;
clear;


solve(-1, 0, 1);
fprintf('\n\n');
solve(0,-1,-10);
function solve(a1, a2, a3)
fprintf('For a1 = %f, a2 = %f, a3 = %f\n', a1, a2, a3);
fprintf('N \t\t EM \t\t\t\t RK4 \t\t\t Error(EM)   \t\t Error(RK4)\n')
    for N = [10, 20, 40, 80]
        h = 1/N;
        A = 1/2*[a2+a3 a3-a1 a2-a1; a3-a2 a1+a3 a1-a2; a2-a3 a1-a3 a1+a2];
        y1 = [1;1;1];
        y2 = [1;1;1];
        for i = 1:N
            y1_prev = y1;
            y1 = y1_prev + h*f(A, y1_prev);
        end
        for i = 1:N
            y2_prev = y2;
            k1 = h*f(A, y2_prev);
            k2 = h*f(A, y2_prev + k1/2);
            k3 = h*f(A, y2_prev + k2/2);
            k4 = h*f(A, y2_prev + k3);
            y2 = y2_prev + 1/6*(k1 + 2*k2 + 2*k3 + k4);
        end
        act_val = act(a1,a2,a3);
        err1 = error(act_val, y1);
        err2 = error(act_val, y2);
        fprintf('%d  [%f %f %f] [%f %f %f] [%f %f %f] [%f %f %f] \n', N, y1(1), y1(2), y1(3), y2(1),
y2(2),  y2(3), err1(1),err1(2),err1(3),err2(1),err2(2),err2(3) );
    end
end

function y_prime = f(A, y)
    y_prime = A*y;
end

function err = error(a, b)
    err = abs(a-b);
end

function val = act(a1, a2, a3)
    y1 = -exp(a1) + exp(a2) + exp(a3);
    y2 = exp(a1) - exp(a2) + exp(a3);
    y3 = exp(a1) + exp(a2) - exp(a3);
    val = [y1; y2; y3];
end
```

Output

```
Command Window
For a1 = -1.000000, a2 = 0.000000, a3 = 1.000000
N               EM                              RK4                             Error(EM)                       Error(RK4)
10  [3.245064 1.942421 -1.245064] [3.350400 2.086160 -1.350400] [0.105338 0.143740 0.105338] [0.000002 0.000002 0.000002]
20  [3.294812 2.011784 -1.294812] [3.350402 2.086161 -1.350402] [0.055591 0.074378 0.055591] [0.000000 0.000000 0.000000]
40  [3.321831 2.048296 -1.321831] [3.350402 2.086161 -1.350402] [0.028571 0.037865 0.028571] [0.000000 0.000000 0.000000]
80  [3.335917 2.067053 -1.335917] [3.350402 2.086161 -1.350402] [0.014486 0.019108 0.014486] [0.000000 0.000000 0.000000]


For a1 = 0.000000, a2 = -1.000000, a3 = -10.000000
N               EM                              RK4                             Error(EM)                       Error(RK4)
10  [-0.651322 0.651322 1.348678] [-0.632065 0.632175 1.367825] [0.019246 0.019156 0.019156] [0.000010 0.000009 0.000009]
20  [-0.641513 0.641515 1.358485] [-0.632075 0.632166 1.367834] [0.009438 0.009349 0.009349] [0.000000 0.000000 0.000000]
40  [-0.636758 0.636778 1.363222] [-0.632075 0.632166 1.367834] [0.004682 0.004612 0.004612] [0.000000 0.000000 0.000000]
80  [-0.634409 0.634455 1.365545] [-0.632075 0.632166 1.367834] [0.002334 0.002289 0.002289] [0.000000 0.000000 0.000000]
```

Observations

1. On increasing n, error decreases.
2. Error in RK4 is less compared to Euler method because order of error in euler method is O(h^2) while order of error in RK4 method is O(h^5),

Question 2

Code

```
clc;
clear;

h = 0.1;
y_prev = 1;
x_prev = 0;
TOL = 0.0001;
fprintf('x \t\t y(x)\n');
for i = 1:2
   x = x_prev + h;
   y0 = y_prev + h*f(x_prev, y_prev);
   while(1)
      y1 = y_prev + h/2*(f(x_prev, y_prev) + f(x, y0));
      if abs(y1 - y0)/abs(y1) < TOL
         break;
      end
      y0 = y1;
   end
      y_prev = y1;
   x_prev = x;
   fprintf('%f \t %f\n', x, y1)
end

function val = f(x, y)
   val = x-1/y;
end
```

Output

```
x               y(x)
0.100000        0.899410
0.200000        0.796006
```

Observations

1. To verify the convergence of the iterations we have to check that

$del(f)/del(y) = 1/y^2 < 2/h$. Here $h = 0.1$. Therefore to converge $y^2 > 1/20 = 0.05$ or $y > 0.22$ which is being followed as shown in the output.