Department of Mathematics
Indian Institute of Technology Guwahati
MA322: Lab Assignment 8

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1. Consider the one-dimensional heat equation
$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} \text{ for } (x,t) \in (0,1) \times (0,0.1].$$
Use classical explicit scheme to solve the above equation, where the initial and boundary data are taken from the exact solution $U(x,t) = \exp(-\pi^2 t) \sin \pi x$. Perform the following experiments: (i) Plot the numerical solutions with $h = 0.1$, $k = .005$ against the exact solution; (ii) Study the convergence of numerical solutions when $k/h^2 > 1/2$.

2. For the heat equation
$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} \text{ for } (x,t) \in (0,1) \times (0,0.1],$$
apply the Crank-Nicolson scheme to solve the above equation, where the initial and boundary data are taken from the exact solution $U(x,t) = \exp(-\pi^2 t) \sin \pi x$. Use mesh parameters $h = 0.1$ and $k = 0.05$. Plot both numerical and exact solutions.

3. Solve the heat equation
$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} \text{ for } (x,t) \in (0,1) \times (0,0.1],$$
using the Crank-Nicolson scheme satisfying the following boundary and initial conditions:
$$\begin{aligned} U(0,t) &= U(1,t) = 0, \quad t > 0, \\ U(x,0) &= 2x, \quad 0 \le x \le 1/2, \\ &= 2(1-x), \quad 1/2 \le x \le 1. \end{aligned}$$
Use mesh parameters $h = 0.1$ and $k = 0.01$. Plot both numerical and exact solutions.