Indian Institute of Technology Guwahati

MA311M: Scientific Computing **(End Semester Examination)**

Duration: **120** Minutes (2:00 PM to 4:00 PM) Total Marks: **40** Marks

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Important Instructions:

- There are **Nine** questions in this paper. Answer **all** questions.
- **Write your Name and Roll Number on all pages.**
- **Submit your answer in a single PDF file with the file name as your Roll Number**.
- No clarification will be given during the examination.

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1. Determine values for A, B, C that make the formula
$$\int_0^2 xf(x)\,dx \approx Af(0) + Bf(1) + Cf(2)$$
exact for all polynomials of degree as high as possible. What is the maximum degree? **[4]**

2. Show that the iteration defined by
$$y_{n+1}^{(k)} = y_n + \frac{h}{24}\left[9f(x_{n+1}, y_{n+1}^{(k-1)}) + 19f_n - 5f_{n-1} + f_{n-2}\right], \quad k = 1, 2, \ldots,$$
with x_n fixed, will converge provided that $\left|\frac{9h}{24}\left(\frac{\partial f}{\partial y}\right)\right| < 1.$ **[4]**

3. Consider the multistep method
$$y_{n+1} = y_{n-1} + \frac{h}{3}(f_{n+1} + 4f_n + f_{n-1}), \quad n \geq 1,$$
approximating the initial value problem $\frac{dy}{dx} = \lambda y, \quad y(0) = 1$. Discuss the stability of the numerical method. Here $f_k = f(x_k, y_k)$. **[4]**

4. Set up a forward-time central-space(FTCS) scheme for
$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\left(b(x,t)\frac{\partial U}{\partial x}\right)$$
without simplifying the right hand term. **[4]**

5. Investigate the stability of an explicit scheme
$$u_{i,j+1} = 2u_{i+1,j} - 3u_{i,j} + 2u_{i-1,j}, \quad i = 1, 2, 3 \text{ and } j \geq 0.$$
approximating the heat equation $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$ by matrix method. **[4]**

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6. Investigate the stability of the Crank-Nicolson scheme

$$-ru_{i-1,j+1} + (2+2r)u_{i,j+1} - ru_{i+1,j+1} = ru_{i-1,j} + (2-2r)u_{i,j} + ru_{i+1,j}, \quad r = \frac{k}{h^2}$$

approximating the heat equation $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$ by von-Neumann method. **[5]**

7. Let the equation $\dfrac{\partial U}{\partial t} = \dfrac{\partial^2 U}{\partial x^2}$ be approximated at the point (ih, jk) by the difference equation

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{u_{i+1,j} - (u_{i,j+1} + u_{i,j-1}) + u_{i-1,j}}{h^2}.$$

Discuss the consistency of this scheme with the given partial differential equation when $k = rh$, where r is a positive constant. **[5]**

8. Suppose a finite difference approximation leads to the linear equations $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & \alpha & 1 \\ \alpha & 1 & \alpha \\ -\alpha^2 & \alpha & 1 \end{bmatrix}, \quad \alpha \in \mathbb{R}, \quad \alpha \neq 0.$$

Find the range of values for α for which the Gauss-Seidel iteration converges. Will the Jacobi iteration converges for $\alpha = 2$? **[5]**

9. Solve the Laplace equation $\dfrac{\partial^2 U}{\partial x^2} + \dfrac{\partial^2 U}{\partial y^2} = 0$ at the mesh points P_1, P_2, P_3 and P_4 shown in Figure 1. The exact solution $U = 3x$ along the boundary AB, $U = 9 - 3y$ along the boundary BC, $U = x^2 - 1.5x$ along the sloping portion of the boundary CD, $U = 0$ along the boundary DE and $U = 0$ along the boundary AE. Note that, for solving the differential equation at the mesh point P_3, choose $\theta_1 = \theta_2 = \frac{1}{2}$. Determine the resulting system of linear equations. **[5]**

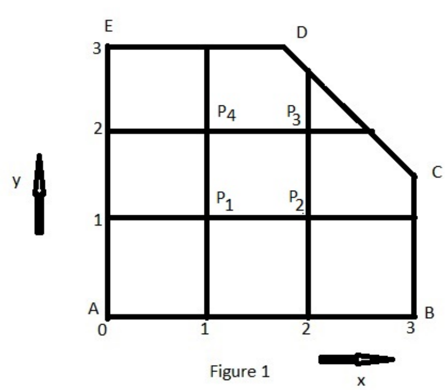

Figure 1

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