Department of Mathematics
Indian Institute of Technology Guwahati
MA322: Lab Assignment 9

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1. A uniform solid rod of one-half a unit of length is thermally insulated along its length and its initial temperature at zero time is 0^oC. One end is thermally insulated and the other supplied with heat at a steady rate. The temperature at points within the rod satisfy the equation

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} \quad \text{for all} \;\; x \in (0, 1/2) \text{ and } \; t > 0,$$

satisfying the initial condition

$$U(x, 0) = 0, \quad 0 \leq x \leq 1/2,$$

and the boundary conditions

$$\frac{\partial U}{\partial x} = 0 \;\; \text{at } x = 0, \;\; t > 0, \quad \frac{\partial U}{\partial x} = 1 \;\; \text{at } x = 1/2, \;\; t > 0.$$

Solve the problem numerically using (i) an explicit method with $h = 0.1$ and $r = 1/4$; (ii) the Crank-Nicolson method with $h = 0.1$ and $r = 1$. The analytical solution is given by

$$U(x, t) = 2t + \frac{1}{2}\left\{\frac{12x^2 - 1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-4\pi^2 n^2 t}\cos(2n\pi x)\right\}.$$

Print the solution at the time $t = 0.01, 0.05, 0.5, 1$ for all $x = 0, 0.1, 0.2, 0.3, 0.4, 0.5$.