Department of Mathematics
Indian Institute of Technology Guwahati
MA322: Lab Assignment 7

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1. Consider the initial-value problem

$$y' = Ay, \quad 0 \le x \le 1, \;\; y(0) = \mathbf{y_0},$$

where

$$A = \frac{1}{2}\begin{bmatrix} \lambda_2+\lambda_3 & \lambda_3-\lambda_1 & \lambda_2-\lambda_1 \\ \lambda_3-\lambda_2 & \lambda_1+\lambda_3 & \lambda_1-\lambda_2 \\ \lambda_2-\lambda_3 & \lambda_1-\lambda_3 & \lambda_1+\lambda_2 \end{bmatrix}, \quad \mathbf{y_0} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

The exact solution is

$$y(x) = \begin{bmatrix} y^1 \\ y^2 \\ y^3 \end{bmatrix}, \quad \begin{aligned} y^1 &= -e^{\lambda_1 x} + e^{\lambda_2 x} + e^{\lambda_3 x}, \\ y^2 &= e^{\lambda_1 x} - e^{\lambda_2 x} + e^{\lambda_3 x}, \\ y^3 &= e^{\lambda_1 x} + e^{\lambda_2 x} - e^{\lambda_3 x}. \end{aligned}$$

Integrate the initial value problem with constant step size $h = 1/N$ with $N = 10, 20, 40, 80$ by (i) Euler Method; (ii) the classical fourth order Runge-Kutta method. To debug the program suggested λ values: (i) $\lambda_1 = -1,\;\; \lambda_2 = 0,\;\; \lambda_3 = 1$; (ii) $\lambda_1 = 0,\;\; \lambda_2 = -1,\;\; \lambda_3 = -10$. Compare the results with the exact solution at $y(1)$ and print the results as per the following format.

N	EM	$RK4$	$Error(EM)$	$Error(RK4)$
10				
$\vdots$				
80				

2. Solve the initial-value problem

$$y' = x - \frac{1}{y}, \quad y(0) = 1$$

from $x = 0$ to $x = 0.2$ with step size $h = 0.1$ using the following algorithm

- Step 1: Compute $y_{n+1}^{(0)}$ by $y_{n+1}^{(0)} = y_n + hf(x_n, y_n)$
- Step 2: Compute $y_{n+1}^{(k)} (k = 1, 2, \ldots)$, using $y_{n+1}^{(k)} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^{(k-1)})]$ until

$$\frac{|y_{n+1}^{(k)} - y_{n+1}^{(k-1)}|}{|y_{n+1}^{(k)}|} < .0001$$

Verify that inner iterations for this problem will converge for the given choice of h.