Department of Mathematics
Indian Institute of Technology Guwahati
MA322: Lab Assignment 6

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1. Use the Gaussian quadrature rule

$$\int_{-1}^{1} f(x)dx \approx \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)$$

to evaluate the integral $\displaystyle\int_0^4 \frac{\sin t}{t}\, dt$.

2. Consider two-point Gauss quadrature rule ($n = 2$)

$$\int_0^{\infty} e^{-x} f(x)dx \approx W_1 f(x_1) + W_2 f(x_2).$$

Note that, here the weight function is $w(x) = e^{-x}$, so the orthogonal polynomial family that one needs to use is the Laguerre family. The Gauss points x_1 and x_2 are the roots of the second-order Laguerre polynomial $L_2(x) = \frac{1}{2}\left(x^2 - 4x + 2\right)$. The roots (the Gauss points) are $x_1 = 2 - \sqrt{2} = 0.5857864376$, $x_2 = 2 + \sqrt{2} = 3.414213562$. The weights are $W_1 = 0.8535533903$, $W_2 = 0.1464466092$ (Can you verify these two values?). Compute the approximate value of the integral for $f(x) = x^3$.

3. Solve the following initial-value problem

$$\frac{dy}{dx} = x^{-2}(xy - y^2), \quad y(1) = 2$$

on the interval $[1, 2]$ using the Euler's method (EM), second-order Runge Kutta method(RK2) and fourth-order Runge Kutta method(RK4). Choose the step size $h = 0.1$ and $h = 0.01$. Compare the numerical values with the exact solution $y(x) = (\frac{1}{2} + \ln x)^{-1} x$ at $x = 1.2, 1.4, 1.6, 1.8, 2$. Print the solutions as per the following format.

Table 1: With $h = 0.1$

x	EM	$Error(EM)$	$RK2$	$Error(RK2)$	$RK4$	$Error(RK4)$
1.2						
⋮						
2						

Table 1: With $h = 0.01$

x	EM	$Error(EM)$	$RK2$	$Error(RK2)$	$RK4$	$Error(RK4)$
1.2						
⋮						
2						