Assignment 1

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Question 1-

I have taken initial left point a = 0 as we have to find the smallest positive root. I have taken initial right point as b = π/2. We can observe that f(0) f(π/2) < 0 and f'(x) is negative between 0 to π/2. Therefore there is only one root between 0 and π/2, that we want to find.

For Newton Raphson method we have to take a starting point close to the root. I have chosen π/4 as the starting point. It is because we know that root lies in 0 to π/2 and by taking their midpoint as the starting point we can ensure that the distance between the root and the starting point would be less than π/4.

Code for Bisection Method-

```
clc;
clear all;
TOL = 10^(-5);
y = @(x) exp(-x) - sin(x);
a = 0;
b = pi/2;
c = b + (a - b)/2;
x = zeros(1,100);
i = 1;
x(i) = c;

while abs(y(c)) >= TOL
   if (y(a)*y(c)) < 0
       b = c;
   else
       a = c;
   end
   c = b + (a - b)/2;
   i = i + 1;
   x(i) = c;
end

fprintf('  Iteration   Bisection Method\n')
for k = 1:i
    fprintf('   %5d          %8.7f\n',k,x(k))
end
```

Code for Newton Raphson Method

```
clc;
clear all;
f = @(x) exp(-x)-sin(x);
f_prime = @(x) -exp(-x)-cos(x);
TOL = 10^(-5);
x = zeros(1,100);
t = 1;
x(t) = pi/4;

while abs(f(x(t))) >= TOL
    t = t + 1;
    x(t) = x(t-1) - f(x(t-1))/f_prime(x(t-1));
end

fprintf('  Iteration   Newton Raphson Method\n')
for k = 1:t
    fprintf('    %5d          %8.8f\n',k,x(k))
end
```

Ouput of Bisection Method

Iteration	Bisection Method
1	0.7853982
2	0.3926991
3	0.5890486
4	0.4908739
5	0.5399612
6	0.5645049
7	0.5767768
8	0.5829127
9	0.5859807
10	0.5875146
11	0.5882816
12	0.5886651
13	0.5884734
14	0.5885693
15	0.5885213
16	0.5885453
17	0.5885333

Ouput of Newton Raphson Method

Iteration	Newton Raphson Method
1	0.78539816
2	0.56944033
3	0.58838948
4	0.58853274

We can observe that number of iteration in Newton Raphson Method is very less when compared to Bisection Method. The reason for this is that the order of convergence is 1 in Bisection Method and 2 for Newton Raphson Method (for simple roots as in this case(as $f'(x) < 0$ in 0 to $\pi/2$)).

Question 2

By taking $g(x) = \sqrt{x+2}$ and $I = [0,7]$ we can observe that

- g : I -> I
- g is continuous
- $g'(x) = \frac{1}{2\sqrt{x+2}}$ is between 0 to 1 in I.

Therefore g is an iteration function and x_n generated by $x_{n+1} = g(x_n)$ would converge to the root.

Code

```
clc;
clear all;
TOL = 10^(-3);
g = @(x) sqrt(x + 2);
f = @(x) x^(2)-x-2;
x = zeros(1,100);
k = 2;
x(k) = g(x(k-1));

while abs(x(k)-x(k-1))/abs(x(k)) >= TOL
    k = k + 1;
    x(k) = g(x(k-1));
end

fprintf('       Xn                f(Xn)            Error \n')
for i = 2:k
    fprintf('   %5.9f       %5.9f      %5.9f\n',x(i),f(x(i)),abs(x(i)-x(i-1))/abs(x(i)))
end
```

Ouput

Xn	f(Xn)	Error
1.414213562	-1.414213562	1.000000000
1.847759065	-0.433545503	0.234633135
1.961570561	-0.113811496	0.058020597
1.990369453	-0.028798893	0.014469119
1.997590912	-0.007221459	0.003615084
1.999397637	-0.001806725	0.000903635