Department of Mathematics
Indian Institute of Technology Guwahati
MA322: Lab Assignment 10

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1. Consider the following initial-boundary problems

$$U_t = U_{xx} + U_{yy}, \;\; 0 < x < 1;\; 0 < y < 1; \;\; t \in (0;1]$$

satisfying the initial condition

$$U(x;y;0) = \sin(x)\sin(y); \;\; 0 \le x \le 1;\; 0 \le y \le 1,$$

and the boundary conditions

$$U(0;y;t) = U(1;y;t) = U(x;0;t) = U(x;1;t) = 0; \;\; \forall t \in (0;1].$$

Solve this problem numerically using the explicit method with $r = k/h^2 = 1/4$ and plot the numerical solution.

2. Solve the above problem using Peaceman-Rachford ADI scheme:

$$\begin{aligned}\left(1 - \frac{r}{2}\delta_x^2\right)u_{i,j}^{n+\frac{1}{2}} &= \left(1 + \frac{r}{2}\delta_y^2\right)u_{i,j}^{n},\\ \left(1 - \frac{r}{2}\delta_y^2\right)u_{i,j}^{n+1} &= \left(1 + \frac{r}{2}\delta_x^2\right)u_{i,j}^{n+\frac{1}{2}},\end{aligned}$$

where $r = k/h^2$. The intermediate boundary values for $u_{i,j}^{n+\frac{1}{2}}$ can be obtained from

$$u_{i,j}^{n+\frac{1}{2}} = \frac{1}{2}\left[(1 + \frac{r}{2}\delta_y^2)u_{i,j}^{n} + (1 - \frac{r}{2}\delta_y^2)u_{i,j}^{n+1}\right].$$

Solve this problem numerically with $k = h = 0.1$ and plot the numerical solution.