Assignment 9
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Code

```
clc;
clear;

L = 0.5;
T = 1;
h = 0.1;
r = 0.25;
k = 0.0025;
Nx = 5;
Nt = 400;
u = zeros(Nx+1, Nt+1);
err = zeros(Nx+1, Nt+1);
pos = zeros(1,Nx+1);
time = zeros(1,Nt+1);
for n = 1:Nx+1
    u(n,1) = 0;
    pos(n) = (n-1)*h;
end
for n = 1:Nt+1
    time(n) = (n-1)*k;
end
MMl = zeros(Nx+1, Nx+1);
for i = 1:Nx+1
    MMl(i,i) = 1;
end

aar = zeros(1,Nx);
bbr = zeros(1,Nx+1);
ccr = zeros(1,Nx);
aar(1:Nx-1) = r;
aar(Nx) = 2*r;
bbr(1:Nx+1) = 1.-2.*r;
ccr(2:Nx) = r;
ccr(1) = 2*r;
MMr = diag(bbr, 0) + diag(aar, -1) + diag(ccr, 1);

Cr = zeros(1,Nx+1);
Cr(Nx+1) = 2*h*r;
for j = 2:Nt+1
    u(1:Nx+1,j) = MMl\MMr*u(1:Nx+1,j-1) + MMl\transpose(Cr);
end

for j = 1:Nt+1
    for i = 1:Nx+1
        err(i, j) = abs(act(i,j,h,k)-u(i,j));
    end
end

figure(1);
surf(time,pos,u);
shading interp;
xlabel('Time');
ylabel('Position');
title('Explicit Scheme: U(x,t)');
```

```matlab
figure(2);
surf(time, pos, err);
shading interp;
xlabel('Time');
ylabel('Position');
title('Explicit Scheme: Error value');

fprintf('Time \t    x=0 \t  x=0.1 \t   x=0.2 \t  x=0.3 \t  x=0.4 \t  x=0.5\n')
for t = 1:Nt+1
    actt = (t-1)*k;
    if actt == 0.01 || actt == 0.05 || actt == 0.5 || actt == 1
        fprintf('%.2f \t %10f \t %5f \t %5f \t %5f \t %5f \t %5f \n',actt, u(1,t), u(2,t),u(3,t), u(4,t),
u(5,t), u(6,t));
    end
end

function val = act(i,j,h,k)
    t = (j-1)*k;
    x = (i-1)*h;
    val = 2*t;
    val = val + 1/2*((12*x*x-1)/6);
    sum = 0;
    TOL = 10^(-10);
    n = 1;
    while term(x,t,n) > TOL
        sum = sum + term(x,t,n);
        n = n + 1;
    end
    val = val - sum;
end

function val = term(x,t,n)
    val = ((-1)^n*exp(-4*pi*pi*n*n*t)*cos(2*n*pi*x))/n^2;
    val = val/(pi*pi);
end
```
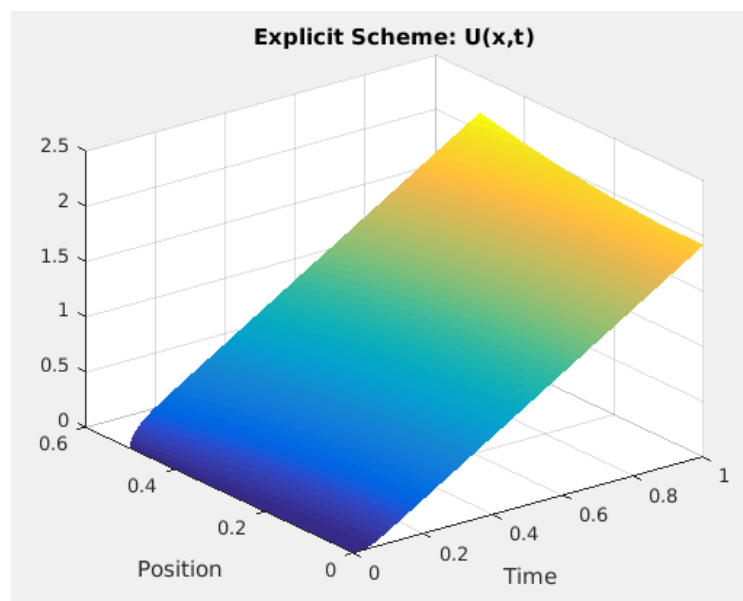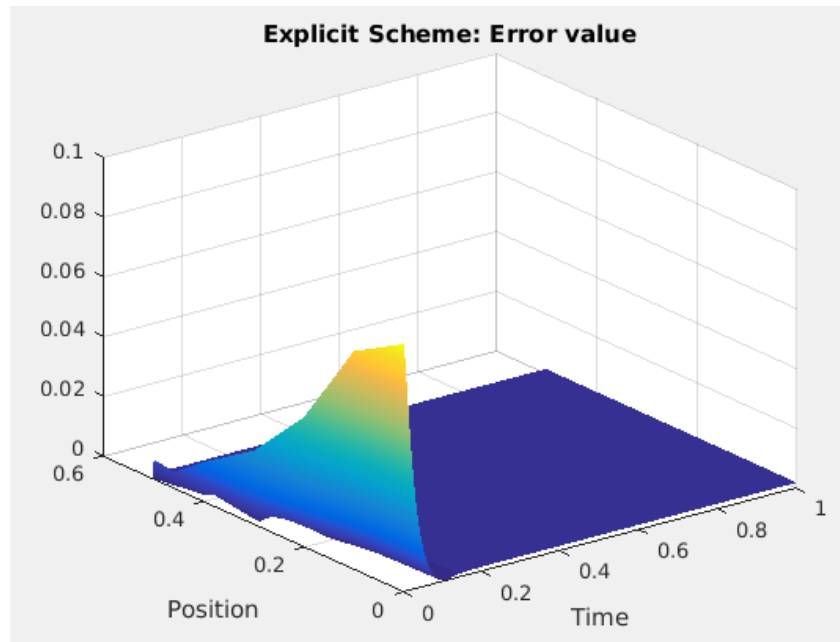
Output

Time	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5
0.01	0.000000	0.000000	0.000781	0.007812	0.036719	0.109375
0.05	0.029064	0.036381	0.059353	0.100657	0.163615	0.250924
0.50	0.915000	0.925000	0.955000	1.005000	1.075000	1.165000
1.00	1.915000	1.925000	1.955000	2.005000	2.075000	2.165000

Explicit Scheme: U(x,t)

Explicit Scheme: Error value

Observations

Values of the error is more for lower valus of time and x. As time increases error goes to zero.

Code

```
clc;
clear;

L = 0.5;
T = 1;

Nt = 100;
k = 0.01;
Nx = 5;
h = 0.1;
r = 1;
u = zeros(Nx+1, Nt+1);
err = zeros(Nx+1, Nt+1);
pos = zeros(1,Nx+1);
time = zeros(1,Nt+1);
for n = 1:Nx+1
    u(n,1) = 0;
    pos(n) = (n-1)*h;
end
for n = 1:Nt+1
    time(n) = (n-1)*k;
end
aal = zeros(1,Nx);
bbl = zeros(1,Nx+1);
ccl = zeros(1,Nx);
aal(1:Nx-1) = -r;
aal(Nx) = -2*r;
bbl(1:Nx+1) = 2.+2.*r;
ccl(2:Nx) = -r;
ccl(1) = -2*r;
MMl = diag(bbl,0) + diag(aal,-1) + diag(ccl,1);

aar = zeros(1,Nx);
bbr = zeros(1,Nx+1);
```

```matlab
ccr = zeros(1,Nx);
aar(1:Nx-1) = r;
aar(Nx) = 2*r;
bbr(1:Nx+1) = 2.-2.*r;
ccr(2:Nx) = r;
ccr(1) = 2*r;
MMr = diag(bbr, 0) + diag(aar, -1) + diag(ccr, 1);

Cr = zeros(1,Nx+1);
Cr(Nx+1) = 4*h*r;
for j = 2:Nt+1
    u(1:Nx+1,j) = MMl\MMr*u(1:Nx+1,j-1) + MMl\transpose(Cr);
end

for j = 1:Nt+1
    for i = 1:Nx+1
        err(i, j) = abs(act(i,j,h,k)-u(i,j));
    end
end

figure(1);
surf(time,pos,u);
shading interp;
xlabel('Time');
ylabel('Position');
title('Crank Nicolson Scheme: U(x,t)');

figure(2);
surf(time, pos, err);
shading interp;
xlabel('Time');
ylabel('Position');
title('Crank Nicolson Scheme: Error value');

fprintf('Time \t    x=0 \t  x=0.1 \t  x=0.2 \t  x=0.3 \t  x=0.4 \t  x=0.5\n')
for t = 1:Nt+1
    actt = (t-1)*k;
    if actt == 0.01 || actt == 0.05 || actt == 0.5 || actt == 1
        fprintf('%.2f \t %10f \t %5f \t %5f \t %5f \t %5f \t %5f \n',actt, u(1,t), u(2,t),u(3,t), u(4,t),
u(5,t), u(6,t));
    end
end

function val = act(i,j,h,k)
    t = (j-1)*k;
    x = (i-1)*h;
    val = 2*t;
    val = val + 1/2*((12*x*x-1)/6);
    sum = 0;
    TOL = 10^-7;
    n = 1;
    while term(x,t,n) > TOL
        sum = sum + term(x,t,n);
        n = n + 1;
    end
    val = val - sum;
end

function val = term(x,t,n)
    val = ((-1)^n*exp(-4*pi*pi*n*n*t)*cos(2*n*pi*x))/n^2;
    val = val/(pi*pi);
end
```
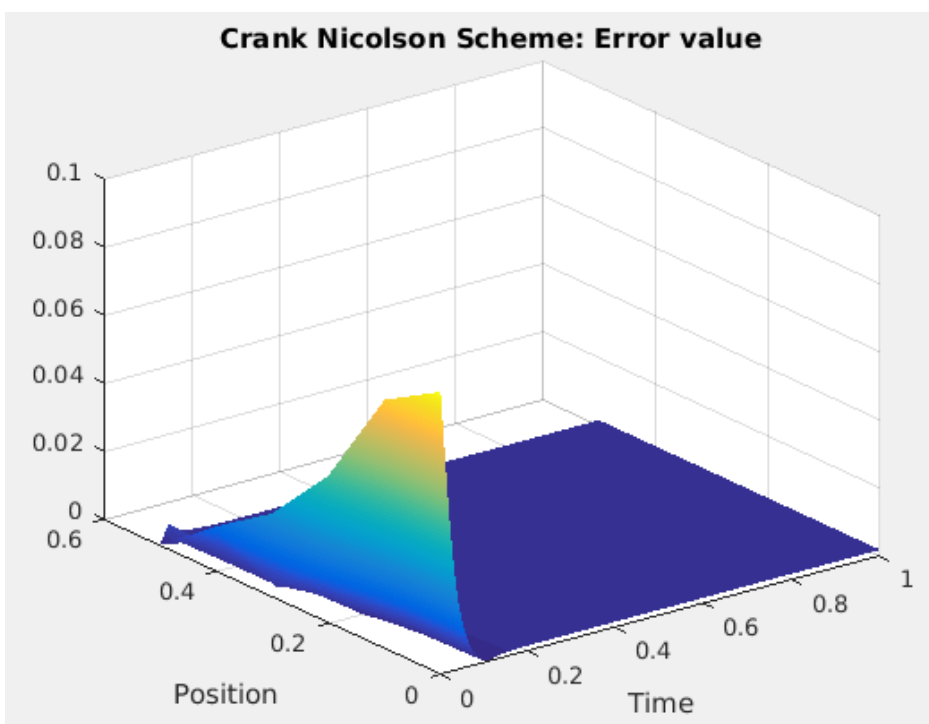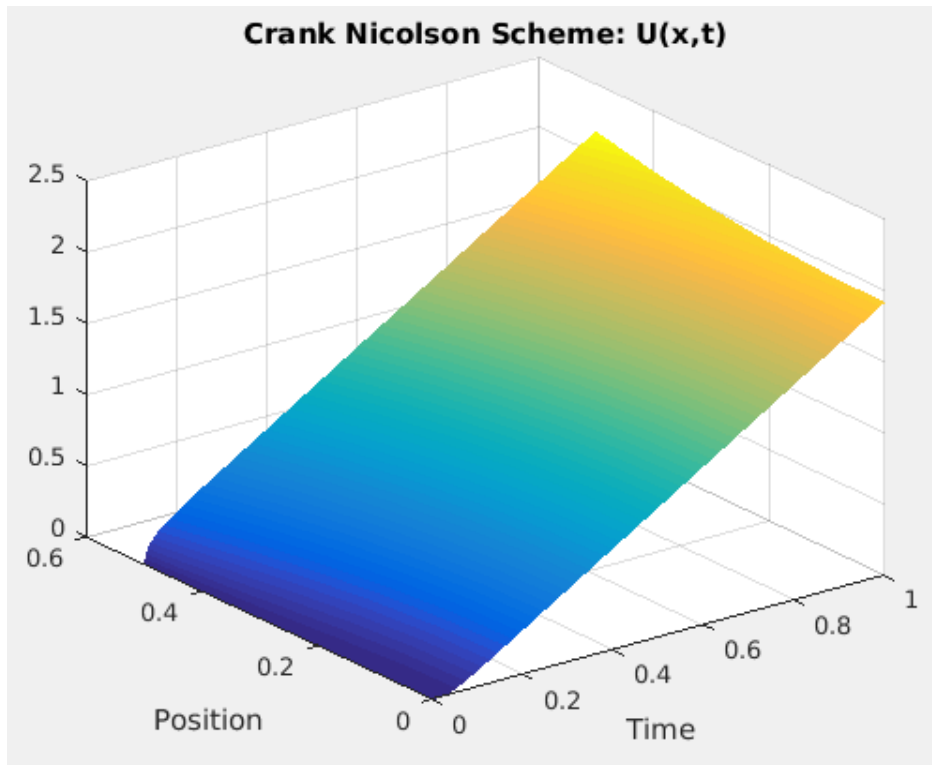
Output

Time	x=0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5
0.01	0.000319	0.000638	0.002233	0.008293	0.030941	0.115470
0.05	0.030141	0.037255	0.059672	0.100351	0.162706	0.249891
0.50	0.915000	0.925000	0.955000	1.005000	1.075000	1.165000
1.00	1.915000	1.925000	1.955000	2.005000	2.075000	2.165000

Crank Nicolson Scheme: U(x,t)

Crank Nicolson Scheme: Error value

Observations

Values of the error is more for lower valus of time and x. As time increases error goes to zero. Although the order for error in Explicit scheme is $O(k + h^2)$ and Crank nicolson is $O(k^2 + h^2)$, the order of error comes out to be same in both.