Indian Institute of Technology Guwahati

MA322: Scientific Computing **(Mid Semester Examination)**

Duration: **90** Minutes (2:00 PM to 3:30 PM) Total Marks: **20** Marks

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Important Instructions:

- There are **SEVEN** questions in this paper. Answer **all** questions.
- **Write your Name and Roll Number on all pages.**
- **Submit your answer in a single PDF file with the file name as your Roll Number**.
- No clarification will be given during the examination.

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1. Let $p, q \in \mathbb{R}$, the equation $x^2 + px + q = 0$ has two real roots α and β. Show that the iteration method $x_{n+1} = -(px_n + q)/x_n$ is convergent near $x = \alpha$ if $|\alpha| > |\beta|$. **[2 marks]**

2. Consider a variation of Newton's method in which only one derivative is needed, that is,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_0)}, \quad n \geq 0.$$

Let $e_n = x_n - \xi$, where ξ is a simple root of $f(x) = 0$. Find C and s such that $e_{n+1} = Ce_n^s$. Here, f' denotes the derivative of f. **[3 marks]**

3. Determine the values of a, b and c for which the function

$$S(x) = \begin{cases} x^3, & x \in [0, 1] \\ \frac{1}{2}(x-1)^3 + a(x-1)^2 + b(x-1) + c, & x \in [1, 3] \end{cases}$$

is a cubic spline. Is it a natural cubic spline? **[3 marks]**

4. Let $f(x)$ be three times continuously differentiable in $[x_0, x_2]$ with $f'''(x) \leq 10^{-3}$ for all $x \in [x_0, x_2]$. Let $p_2(x)$ be the quadratic polynomial interpolating $f(x)$ at the evenly spaced points x_0, $x_1 = x_0 + h$, $x_2 = x_0 + 2h$. Determine computable bounds for these errors

$$|f'(x_i) - p_2'(x_i)|, \quad i = 0, 1, 2.$$

[3 marks]

5. Use inverse interpolation to find an approximate root of $x^3 - 6x - 11 = 0$ in the interval $[3, 4]$. Consider only four sets of data. **[3 marks]**

6. Consider a differentiation rule of the form

$$f'(x_0) = c_1 f(x_0) + c_2 f(x_1) + c_3 f(x_2),$$

where $x_1 = x_0 + h$ and $x_2 = x_0 + 2h$. Find the values of c_1, c_2 and c_3 so that the rule is exact for all polynomials of degree ≤ 2. **[3 marks]**

7. Let $x_0 < x_1 < \cdots < x_N$ be a partition of $[x_0, x_N]$ with $x_N = x_0 + Nh$, h being the step size. Determine N so that the composite trapezoidal rule

$$C_T = \frac{h}{2}\left[f(x_0) + f(x_N) + 2\sum_{i=1}^{N-1} f(x_i)\right]$$

gives the value of $\displaystyle\int_0^1 e^{-x^2}\, dx$ correct to six digits after the decimal point. Assume that e^{-x^2} can be calculated accurately. **[3 marks]**

******* Paper Ends *******