

Let $X^{(1)}$ and $X^{(2)}$ have nonsingular form $\begin{matrix} C & X^{(1)} \\ (p \times p) & (p \times 1) \end{matrix}$ and $\begin{matrix} D & X^{(2)} \\ (q \times q) & (q \times 1) \end{matrix}$.

Then the covariance matrix is given below,

$$\text{Cov} \left(\begin{bmatrix} CX^{(1)} \\ DX^{(2)} \end{bmatrix} \right) = \begin{bmatrix} C\Sigma_{11}C' & C\Sigma_{11}D' \\ D\Sigma_{11}C' & D\Sigma_{11}D' \end{bmatrix}$$

Consider the linear transformation given below,

$$a_1' (CX^{(1)}) = a_1' X^{(1)} \text{ with } a_1' = a_1' C$$

$$b_1' (DX^{(2)}) = b_1' X^{(2)} \text{ with } b_1' = b_1' D$$

Then using result 10.1,

The correlation will be as given below,

$$\max_{a,b} \text{Corr}(a_1' X^{(1)}, b_1' X^{(2)}) = \rho_1^*$$

It will attain maximum by the linear combinations (first canonical variate pair) as given below,

$$a_1' = e' \Sigma_{11}^{-1/2} C^{-1}$$

$$b_1' = f' \Sigma_{22}^{-1/2} D^{-1}$$

Hence the canonical correlations are invariant under nonsingular transformations.