Indian Institute of Technology Guwahati
Statistical Inference and Multivariate Analysis (MA324)
Problem Set 08

1. A sample of size 25 from a normal population with variance 81 produced a sample mean 81.2. Find a 95% confidence interval for the mean. Ans: [77.67, 84.73].

2. A sample of size 25 from a normal population produced a sample mean 81.2 and sample variance 81. Find a 95% confidence interval for the mean. Ans: [77.48, 84.96].

3. Let $\overline{X}$ be the mean of a random sample of size n from N(μ, 16). Find the smallest sample size n such that $(\overline{X} - 1, \overline{X} + 1)$ is a 90% confidence interval for μ. Ans: 44.

4. Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be independent random samples from N(μ_1, σ^2) and N(μ_2, σ^2), respectively. Find a $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$ when (a) σ is known, and (b) σ is unknown.

5. Two independent samples, each of size 7, from normal populations with common unknown variance σ^2 produced sample means 4.8 and 5.4 and sample variances 8.38 and 7.62, respectively. Find a 95% confidence interval for the difference between the means of samples 1 and 2. Ans: [-3.89, 2.69].

6. Let $X_1, X_2, \ldots, X_n$ be identically and independently distributed random variables with common probability density function

$$f(x;\theta) = e^{-(x-\theta)} I_{(\theta,\infty)}(x).$$

   Find a pivot and construct $100(1-\alpha)\%$ lower, upper, and symmetric confidence interval. Also find the smallest length $100(1-\alpha)\%$ confidence interval for θ based on the pivot.

7. For a sample of size one from the population

$$f(x;\theta) = \frac{2}{\theta^2}(\theta - x) I_{(0,\theta)}(x).$$

   Find the $100(1-\alpha)\%$ symmetric confidence interval for θ using pivotal technique.