

Indian Institute of Technology Guwahati
Statistical Inference and Multivariate Analysis (MA324)
Problem Set 09

1. Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$, with usual assumptions on ε . Show that

$$\text{Cov}(\hat{\beta}_0, \hat{\beta}_1) = -\frac{\bar{x}\sigma^2}{S_{xx}} \quad \text{and} \quad \text{Cov}(\bar{y}, \hat{\beta}_0) = 0.$$

2. Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$, with usual assumptions on ε . Show that

$$E(MS_R) = \sigma^2 + \beta_1^2 S_{xx} \quad \text{and} \quad E(MS_{Res}) = \sigma^2.$$

3. Suppose that we have fit a simple linear regression model $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1$, but the response is affected by a second variable x_2 such that the true regression is $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$. Is the least square estimator $\hat{\beta}_1$ in the simple linear regression model unbiased?
4. Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$, where ε 's are independent and identically $N(0, \sigma^2)$ random variables. Find the MLEs of β_0 , β_1 , and σ^2 . Is the MLE of σ^2 UE?
5. Suppose that we are fitting a straight line and wish to make standard error of the slope as small as possible. Suppose that the region of interest for x is $-1 \leq x \leq 1$. Where should the observations x_1, x_2, \dots, x_n be taken?
6. Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$, with usual assumptions on ε . Also assume that β_0 is known.
- (a) Find the LSE of β_1 .
 - (b) Find $\text{Var}(\hat{\beta}_1)$.
 - (c) Find a $100(1 - \alpha)\%$ confidence interval for β_1 . Is the interval narrower than the estimator for the case where both slope and intercept are unknown?