Indian Institute of Technology Guwahati
Statistical Inference and Multivariate Analysis (MA324)
Problem Set 09

1. Consider the multiple linear regression model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon.$$

Using the procedure for testing a general linear hypothesis, show how to test

(a) $H_0 : \beta_1 = \beta_2,\ \beta_3 = \beta_4$.

(b) $H_0 : \beta_1 - 2\beta_2 = 4\beta_3,\ \beta_1 + 2\beta_2 = 0$.

2. Prove that the matrices H and $I - H$ are idempotent.

3. Show that $Var(\widehat{\boldsymbol{y}}) = \sigma^2 H$.

4. Consider the multiple linear regression model $\boldsymbol{y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}$. Show that LSE of $\boldsymbol{\beta}$ can be written as $\widehat{\boldsymbol{\beta}} = \boldsymbol{\beta} + R\boldsymbol{\varepsilon}$, where $R = (X'X)^{-1} X'$.

5. Prove that R^2 is the square of the correlation between $\boldsymbol{y}$ and $\widehat{\boldsymbol{y}}$.