

Indian Institute of echnology Guwahati
Statistical Inference and Multivariate Analysis (MA 324)
Problem Set 04

1. Let X_1, X_2, \dots, X_n be a random sample form $U(\theta_1, \theta_2)$, $-\infty < \theta_1 < \theta_2 < \infty$. Find moment estimator of (θ_1, θ_2) .
2. A sample (X_1, \dots, X_{10}) is drawn from a distribution with a probability density function

$$\frac{1}{2} \left(\frac{1}{\theta} e^{-x/\theta} + \frac{1}{10} e^{-x/10} \right), \quad 0 < x < \infty.$$

The sum of all 10 observations equals 150. Estimate θ by the method of moments.

3. Let X_1, \dots, X_n be a random sample form the probability density function

$$f(x; \theta) = \frac{1}{2} e^{-|x-\theta|} \quad \text{for } x \in \mathbb{R},$$

where $\theta \in \mathbb{R}$. Find the maximum likelihood estimator of θ .

4. Estimate the unknown parameter θ from a sample $\{3, 3, 3, 3, 3, 7, 7, 7\}$ drawn from a discrete distribution with the probability mass function

$$P(3) = \theta, \quad P(7) = 1 - \theta.$$

Obtain MME and MLE of θ .

5. Let X_1, \dots, X_n be a random sample form the probability mass function

$$f(x; \theta) = \begin{cases} \frac{1-\theta}{2} & \text{if } x = 1 \\ \frac{1}{2} & \text{if } x = 2 \\ \frac{\theta}{2} & \text{if } x = 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find the maximum likelihood estimator of $\theta \in (0, 1)$.

6. Let X_1, X_2, \dots, X_n be a sample from probability mass function

$$P(X = k) = \begin{cases} \frac{1}{N} & \text{if } k = 1, 2, \dots, N \\ 0 & \text{otherwise,} \end{cases}$$

where N is a positive integer. Find the maximum likelihood estimator of N .

7. Suppose we want to estimate the number of fishes in a pond. The following procedure is followed to perform the estimation. First M fishes are caught from the pond, tagged and returned to the

pond. Next n fishes are caught at random out of which x fishes are tagged. Based on this data, find the maximum likelihood estimator of the total number of fishes present in the pond. Hint: You should not differentiate the likelihood function with respect to N , as N is integer valued and $N \geq \max\{M, n\}$.

8. Let X_1, X_2, \dots, X_n be a random sample on the lifetime of an integrated circuit. Let the lifetime of the integrated circuit has the probability density function

$$f(x, \theta) = \begin{cases} 2\lambda x e^{-\lambda x^2} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Let $\tau > 0$ be a known time, and X be the number of integrated circuits that fail before τ . Find the maximum likelihood estimator of the variance of X .