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4. ~~Let $U = X_1(X_3+X_4)+X_2$ is a s~~

Q1 Take statistic $T = \sum_{i=1}^{4} X_i$

$$h(x, y, \theta) = \frac{\prod_{i=1}^{4} f(x_i, \theta)}{\prod_{i=1}^{4} f(y, \theta)} = \left(\frac{p}{1-p}\right)^{\sum_{i=1}^{4} x_i - \sum_{i=1}^{4} y_i}$$

$h(x, y, \theta)$ will become free from $p \iff$

$\sum_{i=1}^{4} x_i = \sum_{i=1}^{4} y_i \iff T(x) = T(y)$

$\therefore$ T is a minimal sufficient stat.

Now let $U = X_1(X_3+X_4)+X_2$ is a sufficient stat. We know That $T = \sum_{i=1}^{4} X_i$ is a minimal sufficient statistics.

Then T must be a f^n of U. This means given any value of U, T must be obtained uniquely consider $\{U=0\}$

$\{U=0\}$ is a union of following 3 events

$S_1 = \{X_1 = 0, X_3+X_4 = 0, X_2 = 0\}$

$S_2 = \{X_1 = 0, X_3+X_4 = 1, X_2 = 0\}$

$S_3 = \{X_i = 1, X_3+X_4 = 0, X_2 = 0)$

T for s_1 set $= 0$

T " s_2 " $= 1$

T " s_3 " $= 1$

For $U = 0$ there are 2 observed values of T. $\therefore$ ~~to~~ T is not a f^n of U.

$\therefore$ U is not a suff. stats.

(2)

ABHISHEK AGRAHARI

$X_1, \ldots X_5 \overset{iid}{\sim} N(20, 25\theta^2), \theta > 0$

PDF of $X_i = \frac{1}{\sqrt{2\pi 25\theta^2}} e^{-\frac{(x-2\theta)^2}{2 \times 25\theta^2}}$

$= \frac{1}{5\theta\sqrt{2\pi}} e^{-\frac{(x-2\theta)^2}{50\theta^2}}$

$$h(x, y, \theta) = \frac{\prod_{i=1}^{5} \frac{1}{5\theta\sqrt{2\pi}} e^{-\frac{[x_i^2 - 4\theta x_i + 4\theta^2]}{50\theta^2}}}{\prod_{i=1}^{5} \frac{1}{5\theta\sqrt{2\pi}} e^{-\frac{(y_i^2 - 4\theta y_i + 4\theta^2)}{50\theta^2}}}$$

$$= e^{-\left[\frac{\left(\sum_{i=1}^{5} x_i^2 - \sum_{i=1}^{5} y_i^2\right) - 4\theta\left(\sum_{i=1}^{5} x_i - \sum_{i=1}^{5} Y_i\right)}{50\theta^2}\right]}$$

Consider a statistic $T(x) = \left(\sum_{i=1}^{5} x_i, \sum_{i=1}^{5} x_i^2\right)$

If $T(x) = T(y) \Rightarrow \sum_{i=1}^{5} x_i^2 = \sum_{i=1}^{5} y_i^2$ and $\sum_{i=1}^{5} x_i = \sum_{i=1}^{5} y_i$

$\therefore h(x, y, \theta) = 1$ in this case and it does not involve θ.

Now let $h(x, y, \theta)$ does not involve θ and $T(x) \neq T(y)$

$\because h(x, y, \theta)$ does not involve θ. coefficient of $\frac{\cancel{4\theta}4\theta}{5\theta^2} = \frac{4}{5\theta}$ in the numerator have to be 0.

$\therefore \sum_{i=1}^{5} x_i = \sum_{i=1}^{5} y_i$ — (1)

Similarly coefficient of $\frac{1}{50\theta^2}$ have to be 0.

$\therefore \sum_{i=1}^{5} x_i^2 = \sum_{i=1}^{5} y_i^2$ — (2)

By (1) and (2) $T(x) = T(y)$ which is a contradiction

$\therefore h(x, y, \theta)$ does not involve $\iff T(x) = T(y)$

$\therefore T$ is a minimal sufficient.

② It can be ~~easily~~ proved that there ~~st~~ is a one to one correspondance between $\left(\sum_{i=1}^{5} x_i, \sum_{i=1}^{5} x_i^2\right)$ and $(\bar{x}, s^2)$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{5} (x_i - \bar{x}^2)$$

$$E[\bar{x}^2] = \left(\sum_{i=1}^{5} E[x_i^2] + \sum_{i=1}^{5} \sum_{\substack{j=1 \\ j \neq i}}^{5} E[x_i x_j]\right) / 25$$

$$= \left(29\theta^2 \times 5 + (\cancel{n^2}\ 20) 4\theta^2\right) / 25$$

$$= \frac{145\theta^2 + 80\theta^2}{25} = (225\theta^2)/25 = 9\theta^2$$

$$E(s^2) = \frac{1}{n-1}\left(\sum_{i=1}^{5} E(x_i^2] - 5E[\bar{x}^2]\right)$$

$$\frac{1}{(n-1)}\left(5 \times 29\theta^2 - \cancel{5 \times 2}\ 9\theta^2 \times 5\right]$$

$$= \frac{1}{4}(145 - 45)\theta^2$$

$$= 25\theta^2$$

ABHISHEK AGRAHARI

$E\left[\frac{\bar{X}^2}{9}\right] = E\left[\frac{S^2}{25}\right]$

$\therefore E\left[\frac{\bar{X}^2}{9} - \frac{S^2}{25}\right] = 0$

now take $h(t) = \frac{\bar{X}^2}{9} - \frac{S^2}{25}$ $\qquad t = (\bar{X}, S^2)$

then $E[h(T)] = 0$ for all $\theta > 0$

However $P_\theta(h(T) = 0) = 0$ because h is a cont. f^n

$\therefore$ It is not a complete stat.
Because $\left(\sum_{i=1}^{5} X_i, \sum_{i=1}^{5} X_i^2\right)$ is a one to one f^n of T that is also not a complete stat.