Indian Institute of echnology Guwahati
Statistical Inference and Multivariate Analysis (MA 324)
Problem Set 06

1. Let $\phi(\cdot)$ be a most powerful level α test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. Then show that $\beta(\theta_0) \leq \beta(\theta_1)$, where $\beta(\cdot)$ is the power function of the most powerful test.

2. Let $X_1,\, X_2,\, \ldots,\, X_n$ be a random sample form a $N(\mu,\, \sigma^2)$ distribution, where σ is known.
   (a) Find MP level α test for $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$, where $\mu_1 < \mu_0$.
   (b) Find UMP level α test for $H_0 : \mu = \mu_0$ against $H_1 : \mu < \mu_0$.

3. Let $X_1,\, X_2,\, \ldots,\, X_n$ be a random sample from the PDF
$$f(x,\, \delta,\, b) = \frac{1}{b\Gamma(\delta)} x^{\delta-1} e^{-\frac{x}{b}} \quad \text{if } x > 0,$$
where both $b > 0$ and $\delta > 0$ are unknown. Derive MP level α test for $H_0 : b = b_0,\, \delta = \delta^*$ against $H_1 : b = b_1,\, \delta = \delta^*$, where $b_1 > b_0$.

4. Let $X_1,\, X_2,\, \ldots,\, X_n$ be a random sample from a $\mathrm{P}(\lambda)$, where $\lambda > 0$. Find the most powerful level α test for $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$.

5. Let X_1 and X_2 be a random sample of size two from a probability density function $f(x)$, $x \in \mathbb{R}$. Consider the following two functions
$$f_0(x) = \frac{3}{64} x^2 I_{(0,\,4)}(x) \qquad \text{and} \qquad f_1(x) = \frac{3}{16}\sqrt{x} I_{(0,\,4)}(x).$$
Determine the most powerful level α test for testing $H_0 : f(x) = f_0(x)$ against $H_1 : f(x) = f_1(x)$.

6. Let $X_1,\, X_2$ be independent random variables distributed as $N(\mu,\, \sigma^2)$ and $N(\mu,\, 4\sigma^2)$, respectively. Suppose that $\mu \in \mathbb{R}$ is unknown, but $\sigma > 0$ is known. Derive MP level α test for $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1\, (> \mu_0)$. Note that the random variables are independent but not identically distributed.

7. Let $X_1,\, X_2,\, \ldots,\, X_n$ be a random sample form a one-parameter exponential family as given in Definition 2.16. Suppose that $b_1(\theta)$ is nondecreasing function of θ. Then show that $\{f(x, \theta)\}$ has MLR property in $R(\boldsymbol{X})$.

8. Let $X_1,\, X_2,\, \ldots,\, X_n$ be a random sample from a $\mathrm{P}(\lambda)$, where $\lambda > 0$. Find the UMP level α test for $H_0 : \lambda \leq \lambda_0$ against $H_1 : \lambda > \lambda_0$.