MA 351 : Theory of Computation (Midsem)

Indian Institute of Technology Guwahati

Date: 28/02/2022, Time: 2:00pm - 4:00pm

Write Name and Roll No. at the beginning of answer-script

1. Draw the **minimal DFA**s for the following lanuages over alphabet $\{0, 1\}$.
   No marks unless the DFAs are **minimal**

   (a) Set of all strings that start and end with same symbol. 3

   (b) $(\mathbf{01} + \mathbf{10})^*$. 3

   (c) Set of all strings (interpreted as binary representations of integers) which are divisible by 4. ϵ should be considered as the integer 0. 4

2. For any language L over alphabet Σ, we define $\frac{1}{2}L = \{w \mid \exists x \in \Sigma^* \text{ s.t. } wx \in L \text{ and } |w| = |x|\}$.
   Also, recall that for any language L we defined the binary relation R_L over Σ^* as follows:
   xR_Ly if for all $z \in \Sigma^*$, $xz \in L \Leftrightarrow yz \in L$.
   Now, prove or disprove that, *If R_L has infinite index then $R_{\frac{1}{2}L}$ also has infinite index.* 10
   Write "**Proving**" or "**Disproving**" at the beginning of your answer. Otherwise you get zero.

3. Let L be the set of all strings over $\{a, b\}$ which contain more a than b.
   Prove or disprove that, *L is regular* 10
   Write "**Proving**" or "**Disproving**" at the beginning of your answer. Otherwise you get zero.

4. A, B and X are formal languages (not necessarily regular) s.t. $X = AX \cup B$.
   Also, $A, B \neq \phi$ and $A, B \neq \{\epsilon\}$.
   Prove or disprove that, $X = A^*B$. 10
   Write "**Proving**" or "**Disproving**" at the beginning of your answer. Otherwise you get zero.