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CS 111 Homework 3 - Vibrations Report Abhishek Ambastha

In mechanical systems involving masses and springs, vibrations often occur due to displacement from equilibrium positions. Each mass in such a system will move in response to the forces applied by the connected springs. So, this report outlines the equations, eigenvalue relation, motion and plot description.

$m_i \ddot{x}_i = \sum F_i$ and $F = -kx$ RIGHT is positive motion

$a = \frac{d^2x}{dt^2} \rightarrow a_i = \frac{d^2x_i}{dt_i^2}$

leftword force

diff. between two displacements

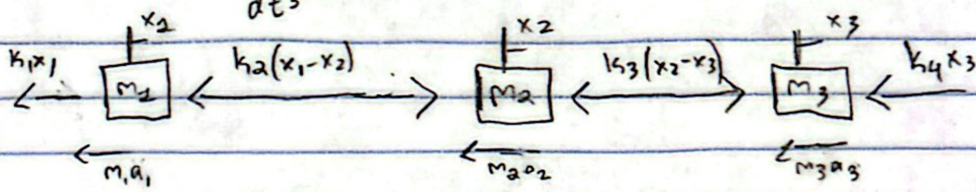
$$m_1 a_1 = m_1 \frac{d^2x_1}{dt^2} = -k_1x_1 - \overbrace{k_2(x_1 - x_2)}$$

rightword

$$m_2 a_2 = \frac{m_2 d^2x_2}{dt^2} = k_2(x_1 - x_2) - k_3(x_2 - x_3)$$

leftward

$$\frac{m_2 d^2x_3}{dt^3} = k_3(x_2 - x_3) - k_4x_3$$



Natural Frequencies, Amplitudes → Eigenvalue, Eigenvector

$x_i(t) = A_i \sin(wt)$

$x_i'(t) = A_i w \cos(wt)$

$x_i''(t) = -A_i w^2 \sin(wt) = -w^2 x_i(t)$

diagonals

$-w^2MA + kA = 0$

$\underbrace{(k - w^2M) = 0}$

$Ma + kx = 0$

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

$$k = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3+k_4 \end{bmatrix}$$

In this format of the equation, it becomes a generalized eigenvalue problem where $\lambda = w^2$ represents the squared natural frequencies of the system, and eigenvectors A give the relative amplitudes of each mass in the mode. This then becomes the mode shapes.

As $\lambda = w^2$ then $w = \sqrt{\lambda}$ so this relates natural frequency to the eigenvalue. Additionally, the eigenvectors associated with each eigenvalue define the mode shapes which are relative amplitudes of each mass in that mode of vibration. So when the system vibrates in a specific mode, each mass oscillates at the same frequency but with amplitudes defined by eigenvector entries.

Plot Visuals

Essentially the plots show $x_i(t) = A_i \sin(wt)$ for a time interval showing how each mass oscillates in that mode. In real-world though, a system's initial displacement may not correspond to a single mode shape but rather a combination of many. Total displacement is sum of displacements due to each mode

$$x(t) = \sum_j A_j \sin(w_j t)$$, A_j is eigenvector and w_j is natural frequency for mode j.

This is why the superposition creates a more complex pattern as the mass oscillates due to combined effects of all nodes.

For each mode j, plot $x_i(t) = A_{ij} \sin(w_j t)$ where A_{ij} is the amplitude of mass i in mode j.

In this case, each natural frequency represents a unique way in which the masses oscillate together. At lower frequencies, the system tends to move in unison with large displacements, whereas at higher frequencies, more complex oscillations emerge. The mode shapes (which are the eigenvectors) are like the distribution of energy across masses at each frequency.

This analysis reveals that the natural frequencies and corresponding mode shapes govern vibration behavior with each mode having a characteristic frequency and amplitude pattern.