CS685: Data Mining
Data Discretization

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- Reduces number of possible values of an attribute
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- Discretization often leads to conceptualization where each distinct group represents a particular concept

Concept Hierarchy

- Discrete intervals or concepts can be generated at multiple levels
- Concept hierarchy captures more basic concepts towards the top and finer details towards the leaves
- Example
  - Higher level: dark, medium, light
  - Middle level: Gray value > 160, $80 - 160$, < 80
  - Lower level: actual gray value
- Can be used to represent knowledge hierarchies as well
- Ontology
- Example
  - WordNet: ontology for English words
  - Gene Ontology: three separate ontologies that capture different attributes of a gene

Methods of Discretization

- For numeric data
  - Binning and histogram analysis
  - Entropy-based discretization
  - Chi-square merging
  - Clustering
  - Intuitive partitioning

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  - Entropy $= 0$
- When is entropy maximum?
  - When every p_i is $1/n$
  - Most random
  - Entropy $= \log_2 n$
  - For $n = 2$, it is 1

Entropy-based Discretization

- Supervised
- Top-down

Entropy-based Discretization

- Supervised
- Top-down
- For attribute A, choose n partitions $D_1, D_2, \ldots, D_n$ for the dataset D
- If D_i has instances from m classes $C_1, C_2, \ldots, C_m$, then entropy of partition D_i is

$$entropy(D_i) = -\sum_{j=1}^{m} (p_{j_i} \log_2 p_{j_i})$$

where p_{j_i} is *probability* of class C_j in partition D_i

$$p_{j_i} = \frac{|C_j \in D_i|}{|D_i|}$$

Choosing Partitions

- Choose $n-1$ partition values $s_1, s_2, \ldots, s_{n-1}$ such that $\forall v \in D_i,\ s_{i-1} < v \leq s_i$
  - Implicitly, s_0 is minimum and s_n is maximum
- How to choose these partition values?

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  - Implicitly, s_0 is minimum and s_n is maximum
- How to choose these partition values?
- Information gain and expected information requirement

Information Gain

- When D is split into n partitions, the expected information requirement is defined as

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- Keep on partitioning into two parts
- Stopping criterion

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- Keep on partitioning into two parts
- Stopping criterion
  - Number of categories greater than a threshold
  - Expected information gain is below a threshold

Example

- Dataset D consists of two classes 1 and 2

Class 1	10, 14, 22, 28
Class 2	26, 28, 34, 36, 38

- Probabilities of two classes are $p_1 = 4/9$ and $p_2 = 5/9$

$$entropy(D) = -p_1 \log_2 p_1 - p_2 \log_2 p_2 = 0.99$$

- How to choose a splitting point?

Splitting Point 1

- Suppose splitting point $s = 24$

Class 1	10, 14, 22
Class 2	

Class 1	28
Class 2	26, 28, 34, 36, 38

- Then, probabilities of classes per partition are

$$p_{1_1} = 3/3 \qquad p_{2_1} = 0/3 \qquad\qquad p_{1_2} = 1/6 \qquad p_{2_2} = 5/6$$

- Entropies are

$$entropy(D_1) = -(3/3)\log_2(3/3) - (0/3)\log_2(0/3) = 0.00$$
$$entropy(D_2) = -(1/6)\log_2(1/6) - (5/6)\log_2(5/6) = 0.65$$

- Expected information requirement and entropy gain are

$$\begin{aligned} info(D) &= (|D_1|/|D|)entropy(D_1) + (|D_2|/|D|)entropy(D_2) \\ &= (3/9) \times 0.00 + (6/9) \times 0.65 = 0.43 \\ gain(D) &= entropy(D) - info(D) \\ &= 0.99 - 0.43 = 0.56 \end{aligned}$$

Splitting Point 2

- Suppose splitting point $s = 31$

Class 1	10, 14, 22, 28
Class 2	26, 28

Class 1	
Class 2	34, 36, 38

Splitting Point 2

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Class 1	10, 14, 22, 28
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Class 1	
Class 2	34, 36, 38

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$$p_{1_1} = 4/6 \qquad p_{2_1} = 2/6 \qquad\qquad p_{1_2} = 0/3 \qquad p_{2_2} = 3/3$$

- Entropies are

$$entropy(D_1) = -(4/6)\log_2(4/6) - (2/6)\log_2(2/6) = 0.92$$
$$entropy(D_2) = -(0/3)\log_2(0/3) - (3/3)\log_2(3/3) = 0.00$$

- Expected information requirement and entropy gain are

$$\begin{aligned} info(D) &= (|D_1|/|D|)entropy(D_1) + (|D_2|/|D|)entropy(D_2) \\ &= (6/9) \times 0.92 + (3/9) \times 0.00 = 0.61 \\ gain(D) &= entropy(D) - info(D) \\ &= 0.99 - 0.61 = 0.38 \end{aligned}$$

Example (contd.)

- So, 24 is a better splitting point than 31
- What is the optimal splitting point?

Example (contd.)

- So, 24 is a better splitting point than 31
- What is the optimal splitting point?
  - Exhaustive algorithm
  - Tests all possible $n - 1$ partitions

Chi-Square Test

- Statistical test
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- Statistical test
- Pearson's chi-square test
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- Chi-square statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

  where O_i is the observed frequency, E_i is the expected frequency and k is the number of possible outcomes
- Chi-square statistic *asymptotically* approaches the chi-square distribution
- Chi-square distribution is characterized by a single parameter: degrees of freedom
- Here, degrees of freedom is $k - 1$

Chi-square Test for Independence

- Two distributions with k frequencies each
- Chi-square statistic

$$\chi^2 = \sum_{i=1}^{2} \sum_{j=1}^{k} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

- Degrees of freedom is $(2-1) \times (k-1)$

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- Degrees of freedom is $(2 - 1) \times (k - 1)$
- The value of the statistic is compared against the chi-square distribution with $df = k - 1$
- Choose a significance level, say, 0.05
- If the statistic obtained is *less* than the theoretical level, then conclude that the two distributions are *independent* at the chosen level of significance

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- Choose a significance level, say, 0.05
- If the statistic obtained is *less* than the theoretical level, then conclude that the two distributions are *independent* at the chosen level of significance
- Lower chi-square corresponds to higher p-value
- Null hypothesis is that the two distributions are independent

Test for Independence: Example

- Null hypothesis H_0 : Variables x, y are *independent*
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- Does riding cycles depend on age?
- Contingency table

	18-24	25-34	35-49	50-64	Total
Yes	60	54	46	41	201
No	40	44	53	57	194
Total	100	98	99	98	395

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  - $x_{1.} = 201/395 = 0.50$, $y_{.1} = 100/395 = 0.25$
  - Expected $c_{1,1} = 0.50 * 0.25 * 395 = 50.89$; Observed is 60
- Chi-square with degrees of freedom $(4-1)*(2-1) = 3$

$$\chi^2_{(3)} = (50.89 - 60)^2/50.89 + \cdots = 8.006$$

- *P-value* (from chi-square distribution table) is 0.046
  - At 5% level of significance (but not 1%), cycling *does* depend on age

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- Otherwise, the difference in frequencies is statistically significant, and they should not be merged
- Keep on merging intervals with *lowest* chi-square value(s)
- Stopping criterion
  - When lowest chi-square value is greater than threshold chosen at a particular level of significance
  - Number of categories greater than a threshold

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- Dataset D consists of two classes 1 and 2

Class 1	1, 7, 8, 9, 37, 45, 46, 59
Class 2	3, 11, 23, 39

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- Test the first merging, i.e., the first two values: $(1, C_1)$ and $(3, C_2)$
- *Contingency table* is

	C_1	C_2	
I_1	1	0	1
I_2	0	1	1
	1	1	2

$$\chi^2 = \frac{(1-1/2)^2}{1/2} + \frac{(0-1/2)^2}{1/2} + \frac{(1-1/2)^2}{1/2} + \frac{(0-1/2)^2}{1/2} = 2$$

Example (contd.)

- Test the second merging, i.e., the second and third values: $(3, C_2)$ and $(7, C_1)$

Example (contd.)

- Test the second merging, i.e., the second and third values: $(3, C_2)$ and $(7, C_1)$
- χ^2 is again 2
- Test the third merging, i.e., the third and fourth values: $(7, C_1)$ and $(8, C_1)$
- Contingency table is

	C_1	C_2	
I_1	1	0	1
I_2	1	0	1
	2	0	2

$$\chi^2 = \frac{(1-1)^2}{1} + \frac{(0-0)^2}{0} + \frac{(1-1)^2}{1} + \frac{(0-0)^2}{0} = 0$$

Merging

Data	1	3	7	8	9	11	23	37	39	45	46	59

Merging

Data	1		3		7		8		9		11		23		37		39		45		46		59
χ^2		2		2		0		0		2		0		2		2		2		0		0	

Merging

Data	1		3		7		8		9		11		23		37		39		45		46		59
χ^2		2		2		0		0		2		0		2		2		2		0		0	

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Data	1		3		7, 8, 9		11, 23		37		39		45, 46, 59
χ^2		2		4		5		3		2		4	

Merging

Data	1		3		7		8		9		11		23		37		39		45		46		59
χ^2		2		2		0		0		2		0		2		2		2		0		0	

Data	1		3		7, 8, 9		11, 23		37		39		45, 46, 59
χ^2		2		4		5		3		2		4	

- Continue till a threshold
- From chi-square distribution table, chi-square value at significance level 0.1 with degrees of freedom 1 is 2.70 (for significance level 0.05, chi-square value is 3.84)
- So, when none of the chi-square values is less than 2.70, stop

Merging

Data	1		3		7		8		9		11		23		37		39		45		46		59
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Data	1, 3		7, 8, 9		11, 23		37, 39		45, 46, 59
χ^2		1.875		5		1.33		1.875	

Merging

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χ^2		2		2		0		0		2		0		2		2		2		0		0	

Data	1		3		7, 8, 9		11, 23		37		39		45, 46, 59
χ^2		2		4		5		3		2		4	

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Data	1, 3		7, 8, 9		11, 23, 37, 39		45, 46, 59
χ^2		1.875		3.93		3.93	

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χ^2		2		2		0		0		2		0		2		2		2		0		0	

Data	1		3		7, 8, 9		11, 23		37		39		45, 46, 59
χ^2		2		4		5		3		2		4	

- Continue till a threshold
- From chi-square distribution table, chi-square value at significance level 0.1 with degrees of freedom 1 is 2.70 (for significance level 0.05, chi-square value is 3.84)
- So, when none of the chi-square values is less than 2.70, stop

Data	1, 3		7, 8, 9		11, 23		37, 39		45, 46, 59
χ^2		1.875		5		1.33		1.875	

Data	1, 3		7, 8, 9		11, 23, 37, 39		45, 46, 59
χ^2		1.875		3.93		3.93	

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Merging

Data	1		3		7		8		9		11		23		37		39		45		46		59
χ^2		2		2		0		0		2		0		2		2		2		0		0	

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χ^2		2.72		3.93	

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- To avoid outliers, it is applied for data that represents the *majority*, i.e., 5th percentile to 95th percentile

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  - (2000, 5000) into 3 partitions: (2000, 3000), (3000, 4000), (4000, 5000)