# JEE matrix problem through Python

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# Problem Statement

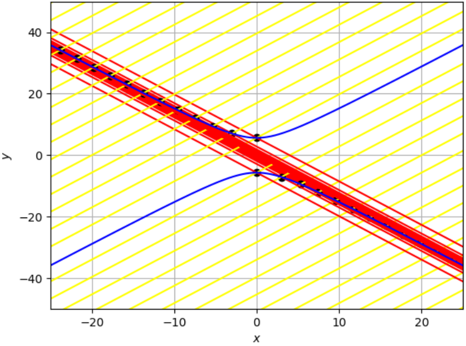
Find the locus of the point of intersection of the lines:

$$\begin{pmatrix} \sqrt{2} & -1 \end{pmatrix} \mathbf{x} + 4\sqrt{2}k = 0$$

$$\begin{pmatrix} \sqrt{2}k & k \end{pmatrix} \mathbf{x} - 4\sqrt{2} = 0$$

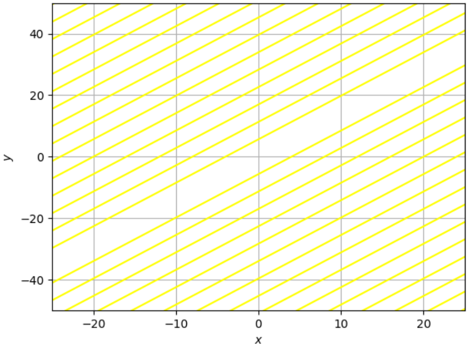
# Figure 1

The figure for the above problem from plotting is as follows.



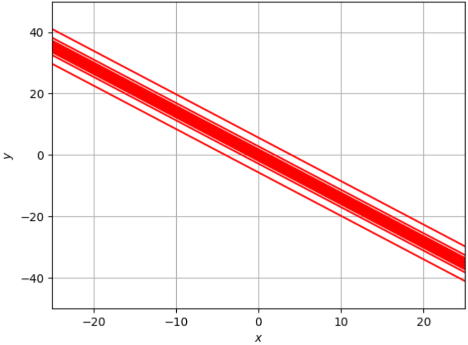
# Figure 2

The family of first equation of lines.



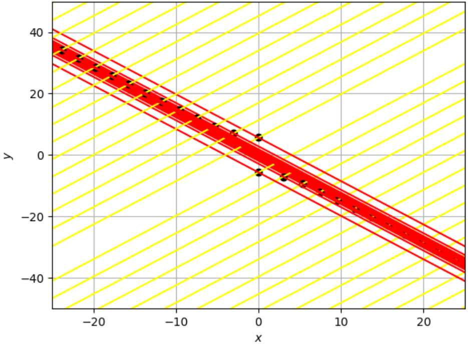
# Figure 3

The family of second equation of lines.



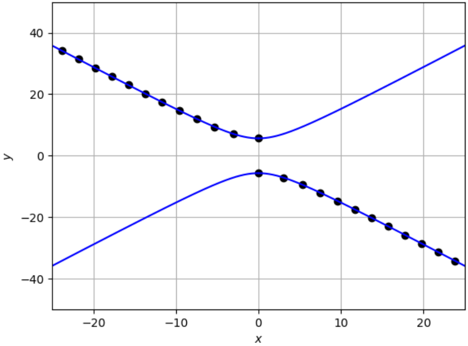
# Figure 4

Their intersection points.



# Figure 5

Final locus.



# Steps to solve

**Find value of k from first equation**

**Find transpose of k and use it in second equation.**

**Simplify the eqaution**

**Check the condition for conic.**

# Solution

**Find value of k from first equation**

The given lines are:

$$\begin{pmatrix} \sqrt{2} & -1 \end{pmatrix} \mathbf{x} + 4\sqrt{2}k = 0$$

$$\begin{pmatrix} \sqrt{2}k & k \end{pmatrix} \mathbf{x} - 4\sqrt{2} = 0$$

So from first line:

$$k = -\begin{pmatrix} \sqrt{2} & -1 \end{pmatrix} x \,/\, 4\sqrt{2}$$

So,

$$k^T = -\, x^T \begin{pmatrix} \sqrt{2} \\ -1 \end{pmatrix} / \, 4\sqrt{2}$$

And Second equation can be written as

$$k \begin{pmatrix} \sqrt{2} & 1 \end{pmatrix} \mathbf{x} = 4\sqrt{2}$$

Since k is a scalar, so

$$k^T = k$$

So putting value of k in second equation we get:

$$x^T \begin{pmatrix} \sqrt{2} \\ -1 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 1 \end{pmatrix} x = -\, 32$$

$$x^T \begin{pmatrix} 2 & \sqrt{2} \\ -\sqrt{2} & -1 \end{pmatrix} x + 32 = 0$$

This can be written as:

$$x^T \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} x + x^T \begin{pmatrix} 0 & \sqrt{2} \\ -\sqrt{2} & 0 \end{pmatrix} x + 32 = 0$$

And

$$x^T \begin{pmatrix} 0 & \sqrt{2} \\ -\sqrt{2} & 0 \end{pmatrix} x \ = 0$$

So

$$x^T \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} x \ + \ 32 \ = 0$$

General equation of conic section:

$$x^T \ \ V\, x \ + \ 2 \ \ u^T\, x + \ F \ = 0$$

# Final Conclusion

On comparing our equation with general equation

We get det(V) as -2 which is less than 0.

So the conic is a hyperbola

So the locus of the intersection point is a hyperbola.

# Code

The link of the code can be found here - Matrix code