Splines

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February 21, 2019

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Problem Statement

Finding best fit for a data

Figure 1

Suppose we are given few points

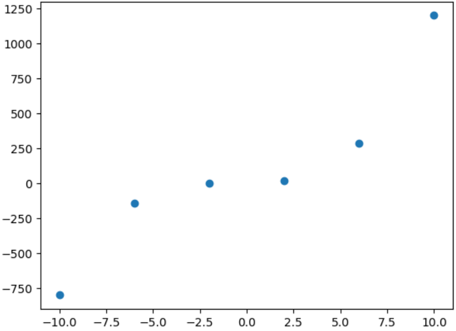


Figure 2

They can be joined linearly

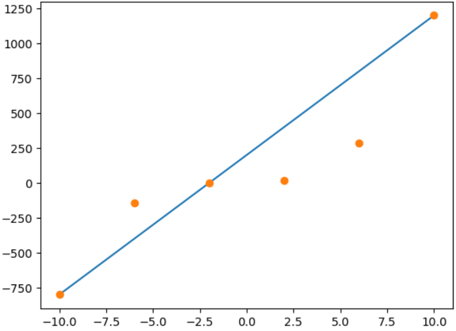


Figure 3

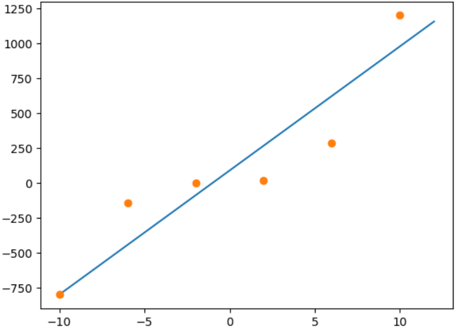


Figure 4

In a better way

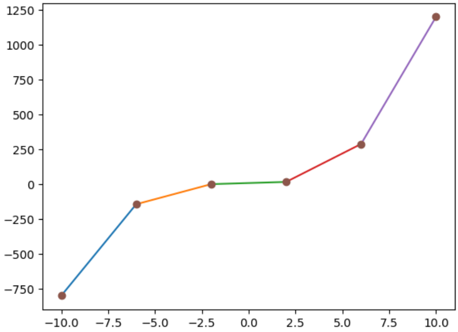
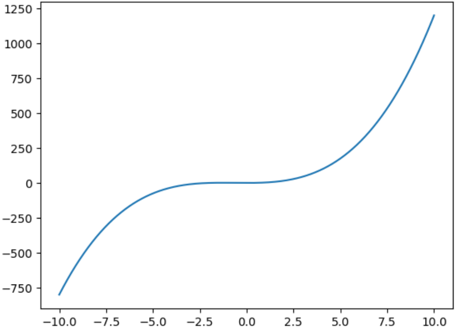


Figure 5



Options that we have

1. nth order polynomial

2. Lagrange's coefficient polynomial

$$\sum_{k=0}^{N}[Y_k L_{n,k}(x)] \tag{1}$$

$$L_{n,k}(x) = \frac{(x - x_0)(x - x_1)...(x - x_{k-1})(x - x_k)...(x - x_n)}{(x_k - x_0)(x_k - x_1)....(x_k - x_{k-1})(x_k - x_{k+1})....(x_k - x_n)} \tag{2}$$

3. Splines

Types of splines

Piecewise cubic spines

Smoothing splines

Cubic splines

Each pair of adjacent known points are assumed to be lying on a cubic curve.

At each known point, the two cubic parts from both sides are chosen such that the overall function is twice differentiabe at that point.

At the first and last given point, it is assumed that the doube derivative is zero.

Between any two given points, the cubic function is expressed as

$S_k(x) = S_{k,0} + S_{k,1}(x - x_k) + S_{k,2}(x - x_k)^2 + S_{k,3}(x - x_k)^3$

How to find the coefficients

On comparing we get

$$s_k(x_k) = y_k \tag{3}$$

$$s_k(x_{k+1}) = s_{k+1}(x_{k+1}) \tag{4}$$

$$s_k^{'}(x_{k+1}) = s_{k+1}^{'}(x_{k+1}) \tag{5}$$

$$s_k^{''}(x_{k+1}) = s_{k+1}^{''}(x_{k+1}) \tag{6}$$

$$s_k^{''}(x_0) = 0 \tag{7}$$

$$s_n^{''}(x_n) = 0 \tag{8}$$