JEE matrix problem through Python

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Problem Statement

Find the equation of the tangent to the circle, at the point

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

whose centre is the point of intersection of the straight lines
$(2\ 1)\mathbf{x} = 3$
$(1\ -1)\mathbf{x} = 1$

Figure

The figure for the above problem from plotting is as follows.

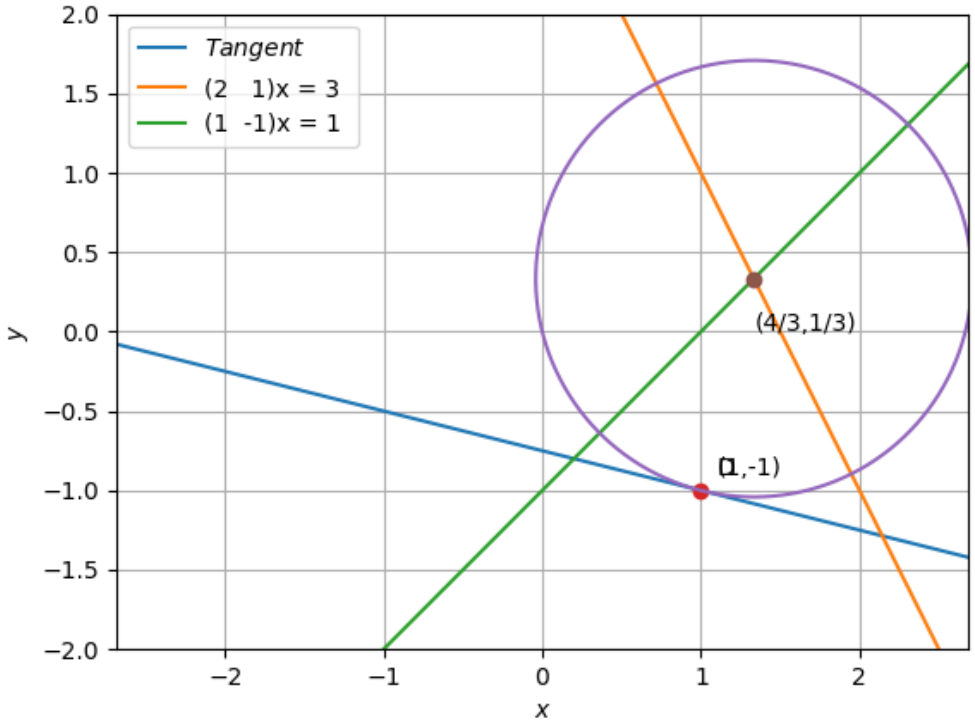

Figure: Tangent on Circle

Steps to solve

Find point of intersection of the two given lines

Find normal vector of the required tangent

Write in matrix format

Solution

Finding point of intersection of the two given lines The given lines are

$$(2\ 1)\mathbf{x} = 3$$

$$(1\ -1)\mathbf{x} = 1 \tag{1}$$

This can be written as the matrix equation

$$\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

The point of intersection can be found by multiplying both sides of the equation by the inverse matrix.

$$\mathbf{x} = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & -2/3 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

So the required intersection point is

$$\mathbf{x} = \begin{pmatrix} 4/3 \\ 1/3 \end{pmatrix}$$

The normal vector of the tangent will be the direction vector of the line joining the centre of circle C(4/3 , 1/3) and point of contact P(1,-1). So, the direction vector will be given by : C-P

$$\begin{pmatrix} 4/3 \\ 1/3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 4/3 \end{pmatrix}$$

The equation of the line is of the form:

$$\begin{pmatrix} 1/3 \\ 4/3 \end{pmatrix} \begin{pmatrix} \mathbf{X\text{-}P} \end{pmatrix} = \begin{pmatrix} 0 \end{pmatrix}$$

Where P is

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Code

The link of the code can be found here - Matrix code