ECE 513: Computer Assignment 4

Image Restoration using 2-D Wiener Filtering

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1 Introduction

Image restoration is the removal or reduction of degradation's which are included during the acquisition of images e.g.; Noise, pixel value errors, out of focus blurring or camera motion blurring using prior knowledge of the degradation phenomenon. This means it deals with the modelling of the degradation and applying the process (inverse) to reconstruct the image[1]. Similar to the enhancement technique, the function of the restoration technique is to improve the image. In this assignment, the 'Lena' image is used for learning the properties of 2-D Wiener Filter for motion blur removal, Noise removal and Image Restoration.The Image restoration is given by:

$$g(x, y) = H[f(x, y)] + \eta(x, y) \tag{1}$$

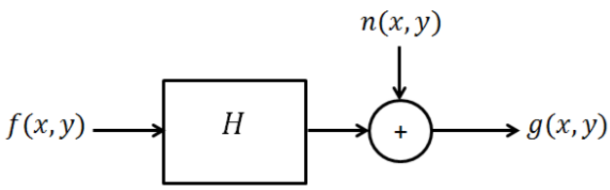

Figure 1: General Image Restoration system

In 1 $f(x, y)$ is the original Image, $\eta(x, y)$ is the restored image, $H[.]$ is the degradation function which is imposed onto the original image.

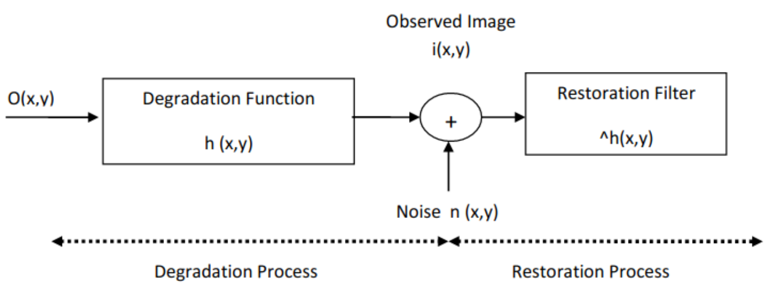

Figure 2: Image Restoration process.

2 Theory

2.1 Image Restoration

There are two types of image restoration filters

- Deterministic-Based Filter.
- Stochastic-Based Filter.

2.1.1 Deterministic-Based Filter

In this type of restoration the image values can be restored either using space or frequency domain. This method can be used if previous knowledge concerning degradation is known[2]. These methods ignore effects of noise and statistics of the image,e.g., inverse filter and Least Squares (LS) filter. The output of the model is fully determined based on the initial Conditions.

2.1.2 Stochastic-Based Filter

In this type of restoration the image values can be restored either using statistical information (auto-correlation function, covariance function,variance, mean etc.) of the noise and image[2]. The same set of parameter values and initial conditions will lead to an ensemble of different outputs.e.g., 2-D Wiener filter and 2-D Kalman filter.

In this assignment we are using 2-d Wiener filter which is a Stochastic-Based Filter.

2.1.3 2-d Wiener filter

The Wiener filter is a linear spatially invariant filter in which the point spread function $h(n_1, n_2)$ is chosen such that it minimizes the mean-squared error (MSE) between the ideal and the restored image. This criterion attempts to make the restoration error between the ideal image and the restored image as small as possible on the average:

$$MSE = E[(f(n_1, n_2) - \hat{f}(n_1, n_2))^2] \tag{2}$$

In 2 where $\hat{f}(n_1, n_2)$ is the restored image and $f(n_1, n_2)$ is the actual image.

The Wiener filter incorporates both the degradation function and statistical characteristics of noise into the restoration process. Wiener filter uses the 1^{st} and 2^{nd}. To get the idea of the 2-D wiener filter we start with a 1-D wiener filter[2].

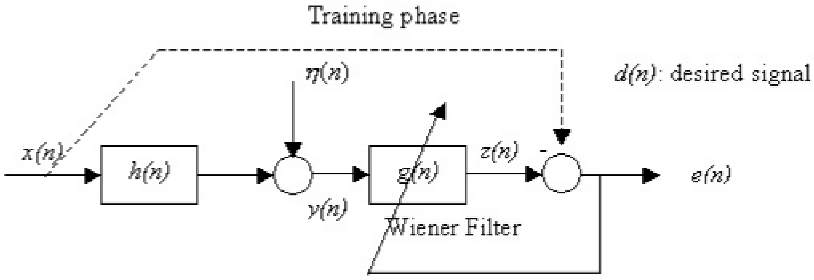

Figure 3: Wiener Filter

The general function for the wiener filter transfer function is:

$$G(e^{j\Omega}) = \frac{S_{dy}(e^{j\Omega})}{S_{yy}(e^{j\Omega})} \tag{3}$$

In 3 $S_{dy}(e^{j\Omega}) = DTFT r_{dy}(K)$ is the Cross-power spectrum,
$S_{dy}(e^{j\Omega}) = DTFT r_{dy}(K)$ is the power spectrum
The following is the Transfer function of a 1-D transfer function.

$$G(e^{j\Omega}) = \frac{H_*(e^{j\Omega})S_{xx}(e^{j\Omega})}{|H(e^{j\Omega})|^2 S_{xx}(e^{j\Omega}) + S_{\eta\eta}(e^{j\Omega})} \tag{4}$$

The following is the Transfer function of a 1-D transfer function.

$$G(k,l) = \frac{H_*(k,l)S_{xx}(k,l)}{|H(k,l)|^2 S_{xx}(k,l) + S_{\eta\eta}(k,l)} \tag{5}$$

where H(k, l) = 2-D DFT $h(m,n)$, and $S_{xx}(k,l)$ and $S_{\eta\eta}(k,l)$ are power spectra of the original image x(m, n) and additive noise, $\eta(m,n)$, respectively.

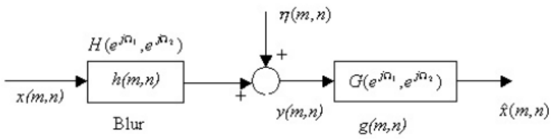

Figure 4: 2-D Wiener Filtering

Using wiener filter is used in de-blurring, noise removal and reconstruction of the image in this assignment. The MATLAB function 'deconvwnr' is used for de-blurring and removing noise from the 'Lena' image. For this assignment we simulate a horizontal motion blur to the 'Lena' image by creating a filtering function. We add this motion blur using 'fspecial' function[3].

2.2 Noise Removal

Noise removal is done using the 2-d wiener filer with the MATLAB function 'wiener2' which is also used to estimate the variance of the output image. The function take two inputs: The noisy Image and a Kernel block size of mxn. We then formulate an algorithm that using the power spectrum of the original image and the noisy image for finding the variance of the noisy image. The variance of the image is found by calculating the estimate of the local mean and variance around each pixel. This process is done before the filtering is done[4].

$$\mu = \frac{1}{NM} \sum_{n_1, n_2 \epsilon \eta} a(n_1, n_2) \tag{6}$$

and

$$\sigma^2 = \frac{1}{NM} \sum_{n_1, n_2 \epsilon \eta} a(n_1, n_2) - \mu^2 \tag{7}$$

where η is the N-by-M local neighbourhood of each pixel in the image A. Wiener2 then creates a pixelwise Wiener filer using these estimates.

$$b(n_1, n_2) = \nu + \frac{\sigma^2 - v^2}{\sigma^2}(a(n_1, n_2) - \mu) \tag{8}$$

where v^2 is the noise variance estimated by the filter if known.

2.3 Signal to Noise Ratio:

Since the reconstructed image has some Noise in it the SNR for the original image and Reconstructed image is calculated using the following formula.[5]

$$SNR = 10log_{10}\frac{\sigma_o^2}{\sigma_e^2} \tag{9}$$

In 9 the σ_o^2 is the variance of the original image and σ_e^2 is the variance of the error image. Using this SNR value we can calculate the loss in the compression of the image as well as the quality of the image.

3 Results

In this Assignment we use the 'Lena' image. Using the MATLAB function 'fspecial' we create a correlation kernel, which is the appropriate form to use with 'imfilter' using which we create a motion blur on the image.

The following is the horizontal motion blur applied image.

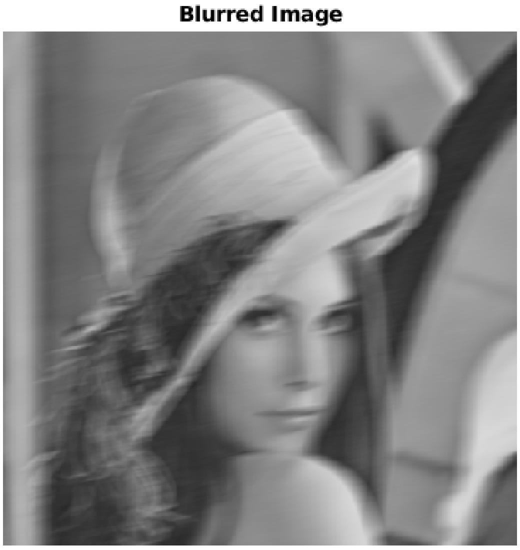

Figure 5: Blurred Image

The 'fspecial' function uses motion blur with linear motion 21 and angular motion 11. The imfilter uses a convolution and circular Padding which implicitly assume that the input array is periodic.

3.1 Image De-blurring

The MATLAB function 'deconvwnr' is used for de-blurring the blurred image using the appropriate PSF used for blurring.

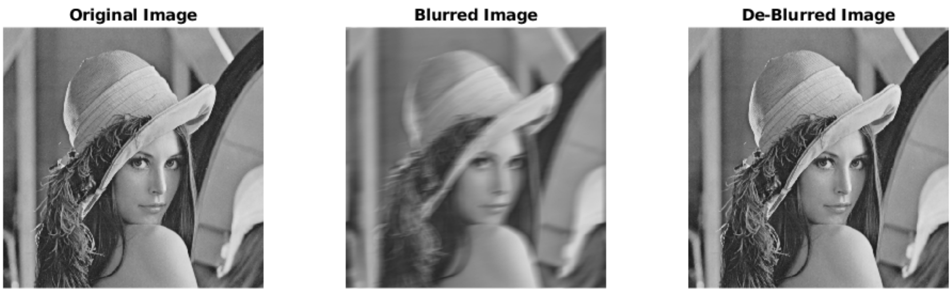

Figure 6: De-Blurred Image

It can be seen that the Wiener filter gives good reconstructed image for a motion blurred image. The filter size used is small, with linear camera motion of 21 and angular motion of 11. As we increase the filter size, it is observed that the the reconstructed image still has some blurring. This is due to the assumption of Wide-sense stationary i.e. it has constant (and finite) mean and variance, and a correlation function which is a function of time shift only.

3.2 Noise Removal

For this part of the Assignment we add a 5dB white Gaussian noise to the 'Lena' image. This image is then used with the 2D wiener filter. The following is the image with 5db noise

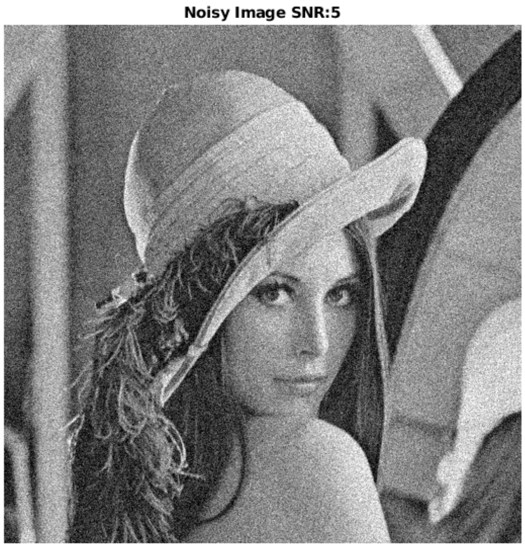

Figure 7: Lena Image with 5dB noise

The noisy image is the given as an input to the wiener 2 filter with a noise-to-signal ratio as zero. The following are the results of the Wiener filter.

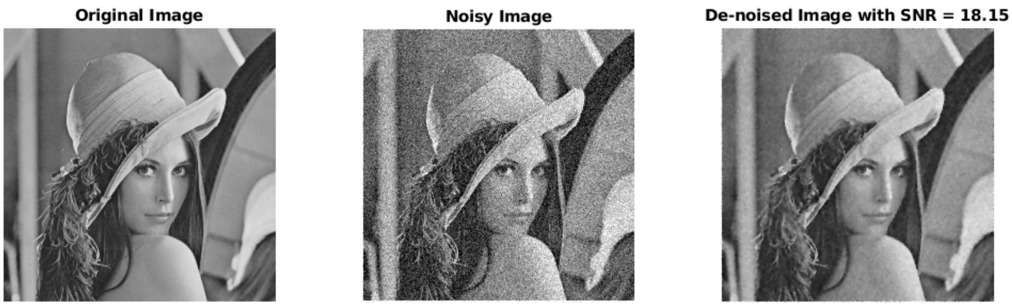

Figure 8: Noise removal using Wiener filter.

To compare the Noise removal efficiency of the wiener2 filter we also use a spatial filter and the results are as follows:

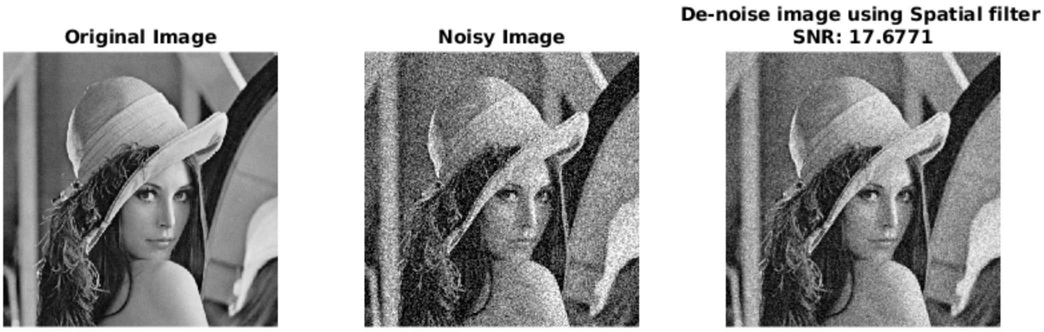

Figure 9: Noise removal using spatial filter.

We can see that the wiener filter does not show any visible noise elements due to the edge smearing, patchiness. It can also be seen that the snr of wiener filter 18.15dB is higher than the snr of the spatial filter 17.67dB so the wiener filter performs better than the spatial filter. We also use a algorithm that find the variance of the noisy image which can be used in the reconstruction of the image.

3.3 Image Recontruction

For this part we use the horizontal motion blurred image and add a 5dB noise to it. The noise-to-signal ratio is calculated using the algorithm. The following is the output of the Reconstruction.

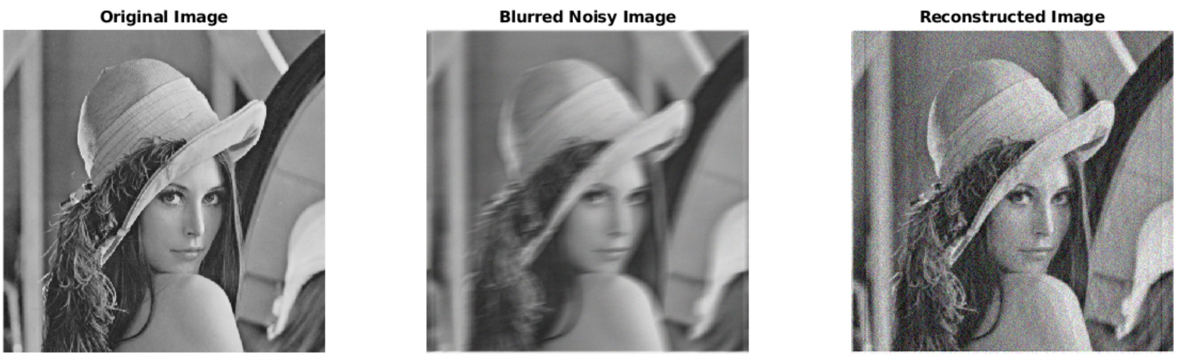

Figure 10: Reconstruction using Wiener filter

We can see that the reconstructed image have some noise in it along with the edge smearing artifact and blur. This is due to the factor that Wiener filter assume a wide-spread stationarity of the image field. The quality of the reconstructed image degrades as the motion blur increases.

4 Discussion

The reconstruction of the image using the Wiener filter gives better solution than the other enhancement techniques. The following table show the Snr value of the reconstructed image for different linear (LIN) and angular (THETA) motion added to the image keeping the White Gaussian noise constant

LIN	THETA	SNR
21	11	14.5547
31	21	13.9856
41	31	12.9789

Table 1: SNR Comparison of reconstructed image.

We can see that the snr decreases as the Linear and Angular motion increases. from this we can see that the Wiener filter better but the overall the smearing artifacts persists which is due to the wide-sense stationarity. The results of noise removal of Wiener filter was also better than the spatial filter.

5 Conclusion

In this assignment we studied the performance of Wiener filter for de-blurring, noise removal and total reconstruction. The Wiener filter was also compared with spatial filter from which we infer that Wiener filter is better than the spatial filter. Even though it is not the best performing filter it is better qualities than most of the filters.

6 References

[1] Rani, Shilpa Jindal, Sonika Kaur, Bhavneet. (2016). A Brief Review on Image Restoration Techniques. International Journal of Computer Applications. 150. 30-33. 10.5120/ijca2016911623.

[2] Prof. M.R.Azimi, Lecture Notes.

[3] MATLAB Documentation/deconvwnr

[4] MATLAB Documentation/wiener2.

[5] https://en.wikipedia.org/wiki/Signal-to-noiseratio