

# One Dimensional Array

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## Maximum Subarray Sum

Given an integer array, find the maximum subarray sum out of all the subarrays.

$$N(N+1)/2$$

0	1	2	3	4	5	6
-2	3	4	-1	5	-10	7

$$\text{Sum} = 11$$

0	1	2	3	4	5	6
-3	4	6	8	-10	2	7

$$\text{Sum} = 18$$

## Quiz : 1

For the given array A, what is the maximum subarray sum ?

$$A[] = \{ \color{green}{4, 5, 2, 1, 6} \}$$

$$\text{Sum} = 18$$

## Quiz : 2

For the given array A, what is the maximum subarray sum ?

$$A[] = \{ -4, -3, -6, -9, \color{yellow}{-2} \}$$

$$-2 > -9$$

$$\text{ans} : -2$$



## BF Idea

Consider all the subarrays. Find their sum & max.

</> Code

```
ans = INT_MIN;  
  
for(s=0 ; s<n ; s++) {  
  
    for(e=s ; e<n ; e++) {  
  
        // Calculate sum  
  
        int sum=0;  
  
        for(i=s ; i<=e ; i++) {  
  
            sum += arr[i];  
        }  
  
        ans = Math.Max(ans,sum);  
    }  
  
    return ans;  
}
```

T.C:  $O(N^3)$   
S.C:  $O(1)$



### Idea-1 Using psum[]

// Calculate psum[]

ans = INT-MIN;

for (s=0; s < n; s++) {

    for (e=s; e < n; e++) {

        // Calculate sum

        int sum=0;

        if (s == 0) sum = psum[e];

        else sum = psum[e] - psum[s-1];

        ans = Math.Max(ans, sum);

}

return ans;

T.C: O(N<sup>2</sup>)

S.C: O(N)

### Idea-2 Using Carry Forward

int ans = INT-MIN;

for (s=0; s < n; s++) {

    int sum=0;

    for (e=s; e < n; e++) {

        sum += arr[e];

        ans = Math.max(ans, sum);

}

return ans;

T.C: O(N<sup>2</sup>)

S.C: O(1)



## Optimised Approach

### KADANE'S algorithm

Case I: All elements are POSITIVE

0	1	2	3	4
4	1	2	6	3

ans : Sum of all the elements.

Case II: All elements are NEGATIVE

0	1	2	3	4
-4	-2	-1	-9	-7

ans : Max element of array

Case III:

-ve	+ve	-ve
-20	5	-6
-2	5	5

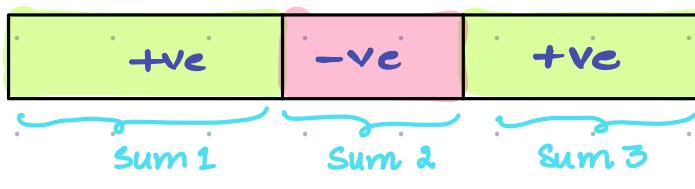
ans : Sum of all Positive elements.

Case IV:





Case I :



10      -3      5

10      -12      5

Sum 1 + Sum 2

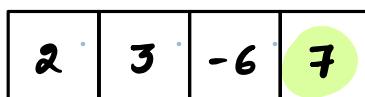
70      Consider these (-ve) elements in sum b'coz in future if some (+ve) elements come, they will increase the sum.

<0      Ignore (-ve) elements.



Sum 0    2    5    1    7

Ans    2    5    5    7  
INT-MIN



Sum 0    2    5    -1    7

Ans    2    5    5    7  
INT-MIN



-2	-3	-1	-4
----	----	----	----

Sum       $\begin{matrix} 0 \\ -2 \\ -3 \\ -1 \\ -4 \end{matrix}$

Ans      -2    -2    -1    -1

INT-MIN

## Quiz : 3

Tell the output of the below example after running the Kadane's Algorithm on that example

$A[] = \{-2, 3, 4, -1, 5, -10, 7\}$

-2	3	4	-1	5	-10	7
----	---	---	----	---	-----	---

Sum       $\begin{matrix} 0 \\ -2 \\ 3 \\ 7 \\ 6 \\ 11 \\ 1 \\ 8 \end{matrix}$

Ans      -2    3    7    7    11    11    11

INT-MIN

</> Code

```
int maxSubarraySum (int arr[], int n) {  
    sum = 0, ans = INT_MIN;  
    for (i=0; i<n; i++) {  
        sum += arr[i];  
        if (sum > ans)  
            ans = sum; }  
        ans = Math.Max(ans, sum);  
        if (sum < 0)  
            sum = 0; }  
    return ans; }
```

T.C : O(N)

S.C : O(1)

## Zero Queries - 1

Given an array where every element is 0. Find the final array after performing multiple queries.

Query (i, x): Add x to all numbers from index i to N-1.

Eg:  $N = 7, Q = 3$

Queries :

i	x
1	3
4	2
3	1

0	1	2	3	4	5	6
0	0	0	0	0	0	0
	+3	+3	+3	+3	+3	+3
				+2	+2	+2
				+1	+1	+1
0	3	3	4	6	6	6

Quiz : 4

**Return the final array after performing the queries Note:**

- **Query (i, x): Add x to all the numbers from index i to N-1**
- **0-based Indexing**

```
A = [0, 0, 0, 0, 0]
Query(1, 3)
Query(0, 2)
Query(4, 1)
```

0	1	2	3	4
0	0	0	0	0
	+3	+3	+3	+3
+2	+2	+2	+2	+2
				+1
2	5	5	5	6



## BF Idea

For every query, iterate from index  $i$  to  $(N-1)$  & add value " $x$ " to all the elements.

$$T.C : O(Q * N)$$

$$S.C : O(1)$$

## Observations :

arr[ ]:	0	1	2	3	4	5
	0	0	2	0	0	0

psum[ ]:	0	0	2	2	2	2
	0	0	2	2	2	2

arr[ ]:	0	1	2	3	4	5	6	7	8
	0	0	2	0	0	0	3	0	0

psum[ ]:	0	0	2	2	2	2	5	5	5
	0	0	2	2	2	2	5	5	5

$$2+3 \quad 2+3 \quad 2+3$$

arr[ ]:	0	1	2	3	4	5	6	7	8	9	10	11
	0	0	1	0	2	0	0	6	0	3	0	0

psum[ ]:	0	0	1	1	3	3	3	9	9	12	12	12
	0	0	1	1	3	3	3	9	9	12	12	12

$$1+2 \quad 1+2 \quad 1+2 \quad 1+2 \quad 1+2 \quad 1+2 \quad 1+2 \\ +6 \quad +6 \quad +6 \quad +6 \quad +6 \quad +6 \quad +6$$



## Optimised Approach:

Add  $x$  in  $\text{arr}[i]$  & then calculate prefix sum.

$$N = 7, Q = 3$$

Queries :

i	x
1	3
4	2
3	1

0	1	2	3	4	5	6
0	0	0	0	0	0	0
	3		1		2	

psum[]:

0	3	3	4	6	6	6
---	---	---	---	---	---	---

</> Code

```
for(i=0 ; i< queries.length ; i++) {
```

```
    int index = queries[i][0];
```

T.C : O(Q+N)

```
    int x = queries[i][1];
```

S.C : O(1)

```
    arr[index] += x;
```

y

// Calculate psum[].

```
psum[0] = arr[0];
```

```
for(i=1 ; i< n ; i++) {
```

```
    psum[i] = psum[i-1] + arr[i]; // arr[i] = arr[i-1] + arr[i]
```

y

```
return psum[i];
```

return arr[];



## Zero Queries -2

Given an array where every element is 0. Find the final array after performing multiple queries.

Query (i, j, x): Add x to all numbers from index i to j. ( $i \leq j$ )

Eg:  $N = 7, Q = 3$

Queries:

$i$	$j$	$x$
1	3	2
2	5	3
5	6	-1

0	1	2	3	4	5	6
0	0	0	0	0	0	0
	+2	+2	+2			
		+3	+3	+3	+3	
						-1
0	2	5	5	3	2	-1



## Quiz : 5

Find the final array after performing the given queries on array of size 8.

i j x

1 4 3

0 5 -1

2 2 4

4 6 3

	0	1	2	3	4	5	6	7
i	0	0	0	0	0	0	0	0
j								
1	+3	+3	+3	+3	+3			
4	-1	-1	-1	-1	-1	-1		
		+4			+3	+3	+3	
6								
3								

-1	2	6	2	5	2	3	0
----	---	---	---	---	---	---	---



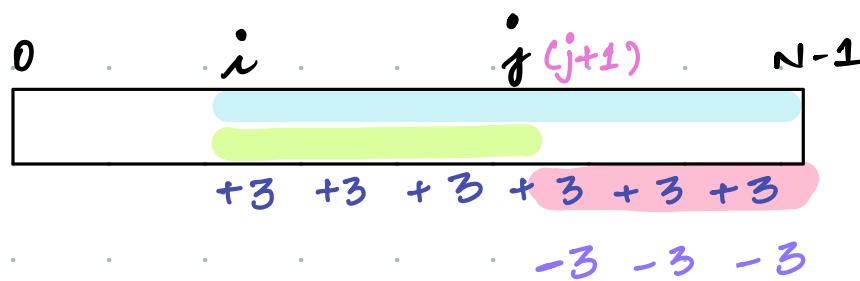
## BF Idea

For every query, iterate from index  $i$  to  $j$  & add value " $x$ " to all the elements.

T.C :  $O(Q * N)$ S.C :  $O(1)$



## Observations :



## Optimised Approach:

Add value  $x$  in  $\text{arr}[i]$  & add  $-x$  in  $\text{arr}[j+1]$ .  
Calculate  $\text{psum}[]$ .

$$N = 7, Q = 3$$

### Queries:

$i$	$j$	$x$
1	3	2
2	5	3
5	6	-1

0	1	2	3	4	5	6
0	0	0	0	0	0	0
	+2				-2	
		+3				-3
						-1

$e == N-1$   
Nothing to add  
at  $(j+1)$

arr[]:	0	2	3	0	-2	-1	-3
--------	---	---	---	---	----	----	----

psum[]:	0	2	5	5	3	2	-1
---------	---	---	---	---	---	---	----



&lt;/&gt; Code

```
int RangeSum(int arr[], int n, int queries[ ][ ]){
```

```
    for (q=0; q < queries.length; q++) {
```

```
        int i = queries[q][0];
```

```
        int j = queries[q][1];
```

```
        int x = queries[q][2];
```

T.C: O(Q+N)

S.C: O(1)

```
        arr[i] += x;
```

```
        if (j != N-1)
```

```
            arr[j+1] -= x;
```

y

// calculate psum

```
for (i=1; i < n; i++) {
```

```
    arr[i] = arr[i-1] + arr[i];
```

y  
return arr;

y



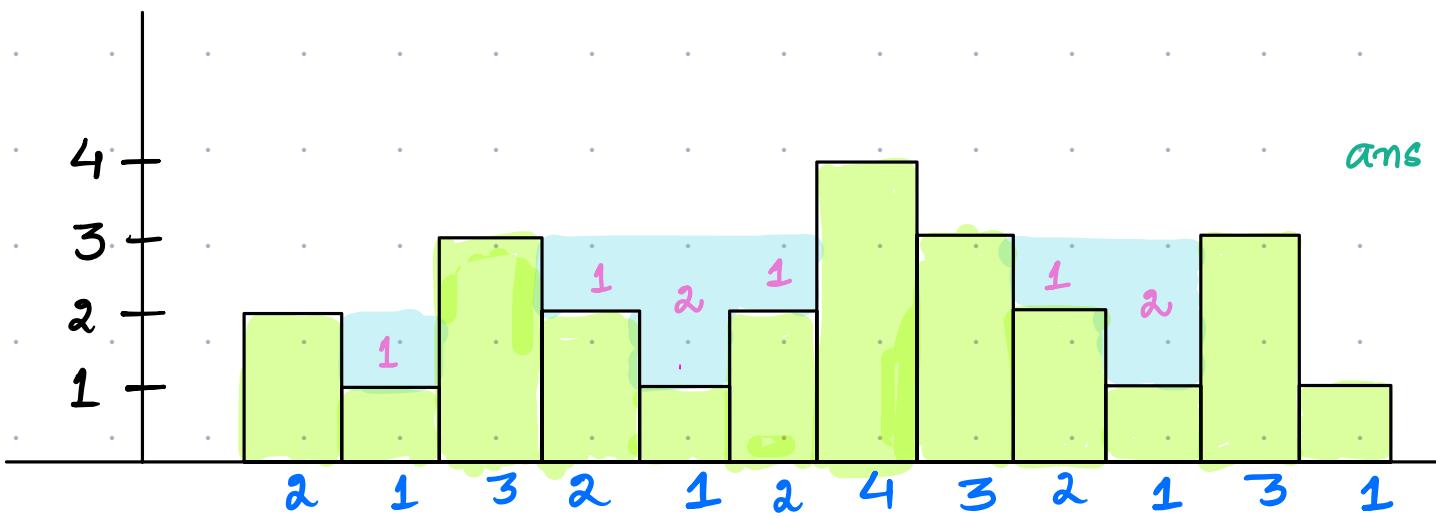
## Problem: Trapping Rain Water

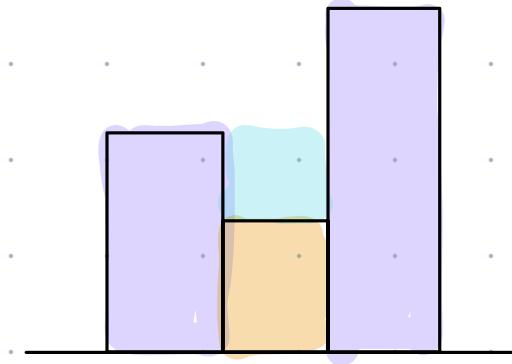
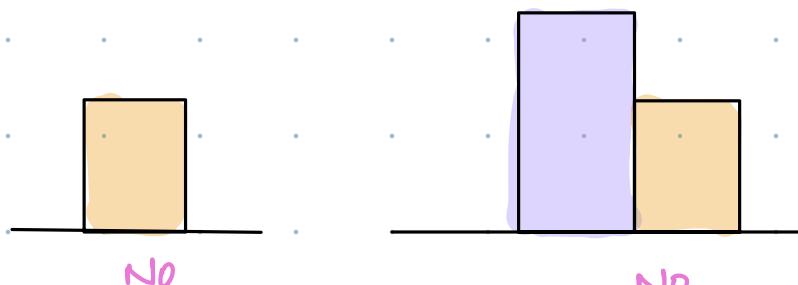
Given N buildings with height of each building. Find the rain water trapped between the buildings.

Eg:

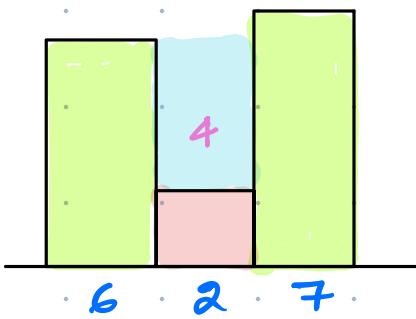
0	1	2	3	4	5	6	7	8	9	10	11
2	1	3	2	1	2	4	3	2	1	3	1

$N=12$





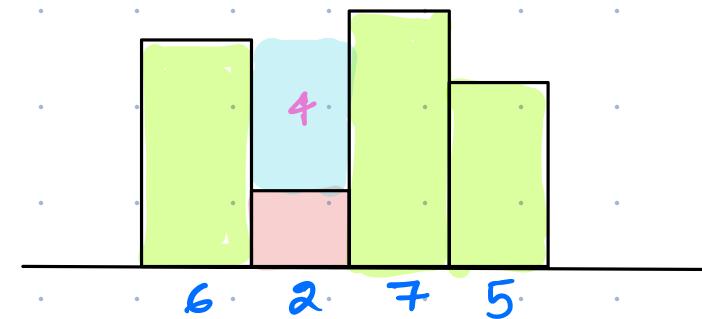
Water can be trapped  
if we have taller  
buildings on both sides :



$$l_{\max} : 6$$

$$r_{\max} : 7$$

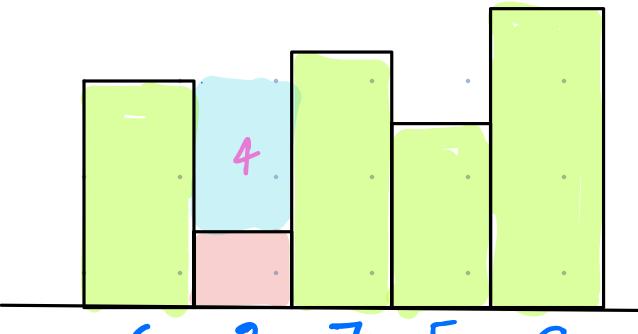
$$6 - 2 = 4$$



$$l_{\max} : 6$$

$$r_{\max} : 7$$

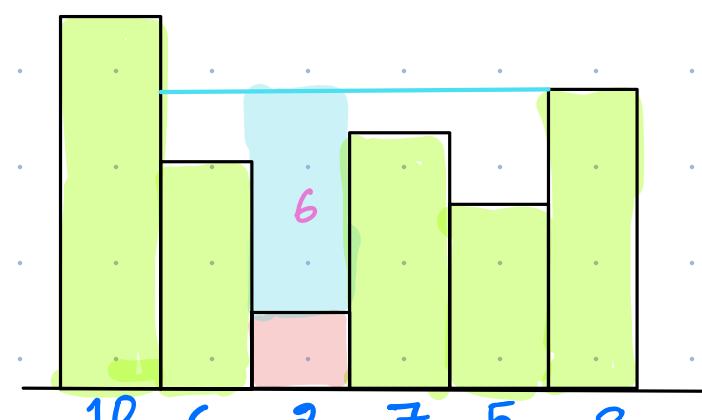
$$6 - 2 = 4$$



$$l_{\max} : 6$$

$$r_{\max} : 8$$

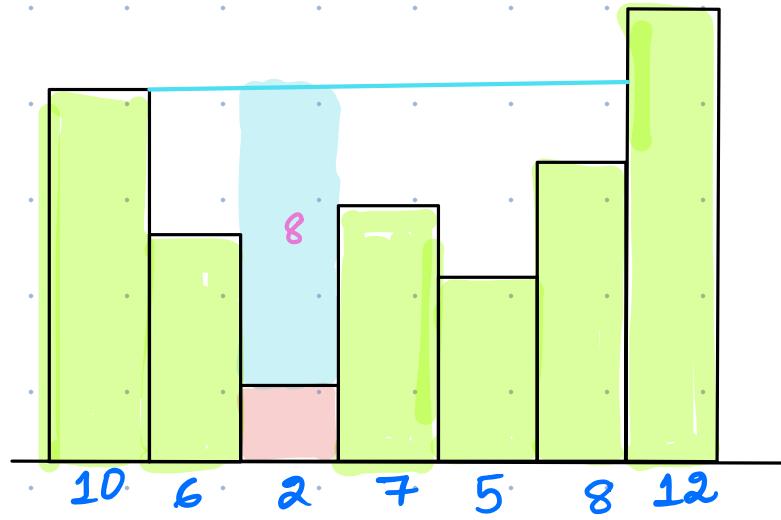
$$6 - 2 = 4$$



$$l_{\max} : 10$$

$$r_{\max} : 8$$

$$8 - 2 = 6$$



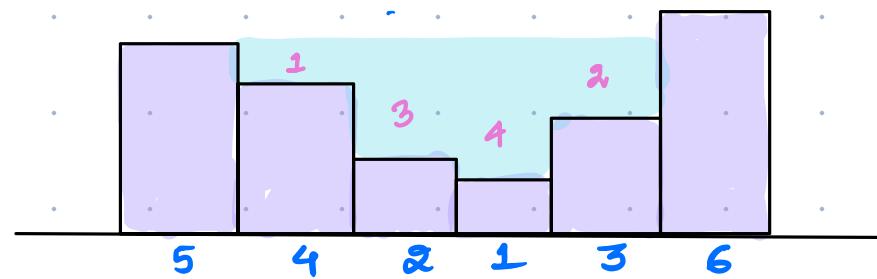
$$l_{\max} : 10$$

$$r_{\max} : 12$$

$$10 - 2 = 8$$

## Approach:

$$\text{Water Accumulated} = \min(l_{\max}, r_{\max}) - \text{ht of building}$$

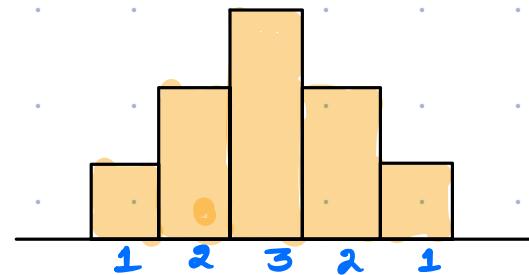


$l_{max}$	5	5	5	5	5	6
$r_{max}$	6	6	6	6	6	6
$\min(l_{max}, r_{max})$	5	5	5	5	5	6
Water on Building	0	1	3	4	2	0

ans :  $1 + 3 + 4 + 2 = 10$

## Quiz : 6

Given N buildings with height of each building, find the rain water trapped between the buildings.  $A = [1, 2, 3, 2, 1]$



$l_{max}$	1	2	3	3	3
$r_{max}$	3	3	3	2	1
$\min(l_{max}, r_{max})$	1	2	3	2	1
Water on Building	0	0	0	0	0

ans : 0



&lt;/&gt; Code

```
ans=0;  
for (i=0; i<n; i++) {  
    //Find lmax : Max from index [0...i]  
    //Find rmax : Max from index [i...N-1]  
    int water = Math.min(lmax, rmax) - h[i];  
    ans+=water;  
}  
return ans;
```

T.C : O(N<sup>2</sup>)  
S.C : O(1)



**Optimised Approach:** Precompute  $l_{max}$  &  $r_{max}$

$l_{max}[n]$      $r_{max}[n]$

// Compute  $l_{max}$

$l_{max}[0] = h[0];$

for ( $i=1$ ;  $i < n$ ;  $i++$ ) {

$l_{max}[i] = \text{Math.Max}(l_{max}[i-1], arr[i]);$

}

T.C : O(N)

S.C : O(N)

// Compute  $r_{max}$

$r_{max}[N-1] = h[N-1];$

for ( $i=N-2$ ;  $i > 0$ ;  $i--$ ) {

$r_{max}[i] = \text{Math.Max}(r_{max}[i+1], arr[i]);$

}

$ans = 0;$

for ( $i=0$ ;  $i < n$ ;  $i++$ ) {

$\text{water} = \text{Math.Min}(l_{max}[i], r_{max}[i]) - h[i];$

$ans += \text{water};$

}

return  $ans;$