Agenda

1. Log Basics + Iteration Problems
2. Comparing Iterations using Graph
3. Time Complexity - Definition and Notations (Asymptotic Analysis - Big O)
4. Why do we get TLE ?

Class starts at 7:05 IST

CREATING A ROUTINE

Routines can improve overall health, well-being, and productivity.

There are many starting points to get into a routine, but the number one rule is to make it work for you. A routine is unique to everyone, just because a routine works for your friend doesn't mean it's the best for you to take.

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Log Basics

Q. What is the meaning of LOG ?

Inverse of exponential function.

Q. How to read the statement "$\log_b(a)$"?

Base

To what value should we raise "b" such that we get "a".

If $\log_b a = c$ then $b^c = a$

1. $\log_2 64 =$

$2^x = 64$

$2^x = 2^6 \Rightarrow x = 6$

2. $\log_3 27 =$

$3^x = 27$

$3^x = 3^3 \Rightarrow x = 3$

3. $\log_2 32 =$

$2^x = 32$

$2^x = 2^5 \Rightarrow x = 5$

4. $\log_2 10 =$

$2^x = 10 \Rightarrow x = 3$

$2^3 < 10 < 2^4$

8 16

5. $\log_2 40 =$

$2^x = 40 \Rightarrow x = 5, ---$

$2^5 < 40 < 2^6$

32 64 $\Rightarrow x = 5$

Generalising :

$\log_2 N = k$

$N = 2^k$

6. $\log_2 2^6 =$ $2^x = 2^6$

$x = 6$

7. $\log_3 3^5 =$ $3^x = 3^5$

$x = 5$

Generalising :

$$\log_a a^N = N$$

< Question > : Given a positive integer N. How many times do we need to divide it by 2 until it reaches 1?

N = 100

$100 \longrightarrow 50 \longrightarrow 25 \longrightarrow 12 \longrightarrow 6 \longrightarrow 3 \longrightarrow 1$

ans : 6

N = 324

$324 \longrightarrow 162 \longrightarrow 81 \longrightarrow 40 \longrightarrow 20 \longrightarrow 10 \longrightarrow 5 \longrightarrow 2 \longrightarrow 1$

ans : 8

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Generalising :

$$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \; - - - - \; - - - \; 1$$

$$\frac{N}{2^0} \rightarrow \frac{N}{2^1} \rightarrow \frac{N}{2^2} \rightarrow \frac{N}{2^3} \; - - - - - - \; \frac{N}{2^k}$$

k steps to reach 1 from N

$$N/2^k = 1 \Rightarrow N = 2^k \Rightarrow \log_2 N = k$$

QUIZ

N = 9

$$9 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

$$\log_2 9 \Rightarrow 2^3 < 9 < 2^4$$
$$\Rightarrow 8 < 9 < 16$$
$$\Rightarrow x = 3.\ldots$$
$$\Rightarrow x = 3$$

ans : 3

QUIZ

N = 27

$$27 \rightarrow 13 \rightarrow 6 \rightarrow 3 \rightarrow 1$$

$$\log_2 27$$
$$2^4 < 27 < 2^5$$
$$2^4 < 2^x < 2^5$$

$$x = 4$$

ans: 4

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Quiz- 3

How many iterations will be there in this loop ?

```
N > 0
i = N;
while(i > 1)
{
    i=i/2;
}
```

$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \; - - - - - \; 1$

ans : $\log_2 N$

$O(\log_2 N)$

Quiz- 4

How many iterations will be there in this loop

```
for(i=1; i<N; i=i*2)
{
    ...
}
```

$N = 32$

$32 \underset{\times 2}{\overset{\div 2}{\rightleftarrows}} 16 \underset{\times 2}{\overset{\div 2}{\rightleftarrows}} 8 \underset{\times 2}{\overset{\div 2}{\rightleftarrows}} 4 \underset{\times 2}{\overset{\div 2}{\rightleftarrows}} 2 \underset{\times 2}{\overset{\div 2}{\rightleftarrows}} 1$

ans : $\log_2 N$

$O(\log_2 N)$

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Amazon

Quiz- 5

How many iterations will be there in this loop ?

```
N>=0
for(i=0; i<=N; i = i*2)
{
    ...
}
```

i_{before}	iteration	i_{after}
0	1	$0*2=0$
0	2	$0*2=0$
0	3	0
0	4	0
	⋮	
0	∞	0

ans : ∞

Quiz- 6

How many iterations will be there in this loop

```
for(i=1; i<=10; i++){
    for(j=1; j<=N; j++){
    /      ......../
    }
}
```

for(i = 1 → 10)
for (j = 1 ; j ≤ N ; j++) {
| =
3 =

i	j	No of iterations
1	[1, N]	N
2	[1, N]	N
3	[1, N]	N
⋮	⋮	⋮
10	[1, N]	N

10 times

$\overbrace{N+N+N \; - - \; N}$

ans : $10*N$

$O(N)$

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Quiz- 7

How many iterations will be there in this loop

```
for(i=1; i<=N; i++){
    for(j=1; j<=N; j++){
        ...
    }
}
```

i	j	No of iterations
1	[1, N]	N
2	[1, N]	N
3	[1, N]	N
⋮	⋮	⋮
N	[1, N]	N

ans: $\underbrace{N + N + N - - - - N}_{N \text{ times}}$

$= N * N = N^2$

$O(N^2)$

Quiz- 8

How many iterations will be there in this loop

```
for(i=1; i <= N; i++){
    for(j=1; j <= N; j = j*2){
        ...
    }
}
```

i	j	No of iterations
1	[1, N] * 2	$\log_2 N$
2	[1, N] * 2	$\log_2 N$
3	⋮	⋮
N	[1, N] * 2	$\log_2 N$

ans: $\underbrace{\log_2 N + \log_2 N - - - \log_2 N}_{N \text{ times}}$

$= N * \log_2 N$

$O(N \log_2 N)$

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Quiz- 9

How many iterations will be there in this loop ?

```
for(i = 1; i <= 4; i++) {
    for(j = 1; j <= i ; j++) {
        //print(i+j)
    }
}
```

i	j [1, i]	No of iterations
1	[1, 1]	1
2	[1, 2]	2
3	[1, 3]	3
4	[1, 4]	4

ans : $1 + 2 + 3 + 4 = 10$

$O(1)$

Quiz- 10

How many Iterations will be there in this loop ?

```
for(i = 1; i <= N; i++) {
    for(j = 1; j <= i ; j++) {
        //print(i+j)
    }
}
```

i	j [1, i]	No of iterations
1	[1, 1]	1
2	[1, 2]	2
3	[1, 3]	3
⋮	⋮	⋮
N	[1, N]	N

ans : $1+2+3 - - - - N$

$= \frac{N(N+1)}{2} = \frac{N^2+N}{2}$

$= \frac{N^2}{2} + N/2$

$O(N^2)$

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Quiz- 11

How many iterations will be there in this loop

```
for(i=1; i<=N; i++){
    for(j=1; j<=(2^i); j++)
    {
        ...
    }
}
```

i	j $[1, 2^i]$	No of iterations
1	$[1, 2^1] = [1, 2]$	2
2	$[1, 2^2] = [1, 4]$	4
3	$[1, 2^3] = [1, 8]$	8
4	$[1, 2^4] = [1, 16]$	16
⋮	⋮	⋮
N	$[1, 2^N]$	2^N

ans : $\overset{a}{2} + \overset{ar}{4} + \overset{ar^2}{8} + \overset{ar^3}{16} - - - - - \overset{ar^N}{2^N}$

$a = 2$

$r = 4/2 = 2$

Sum of N terms of GP $= \dfrac{a(r^N - 1)}{r - 1}$

$= \dfrac{2(2^N - 1)}{2 - 1}$

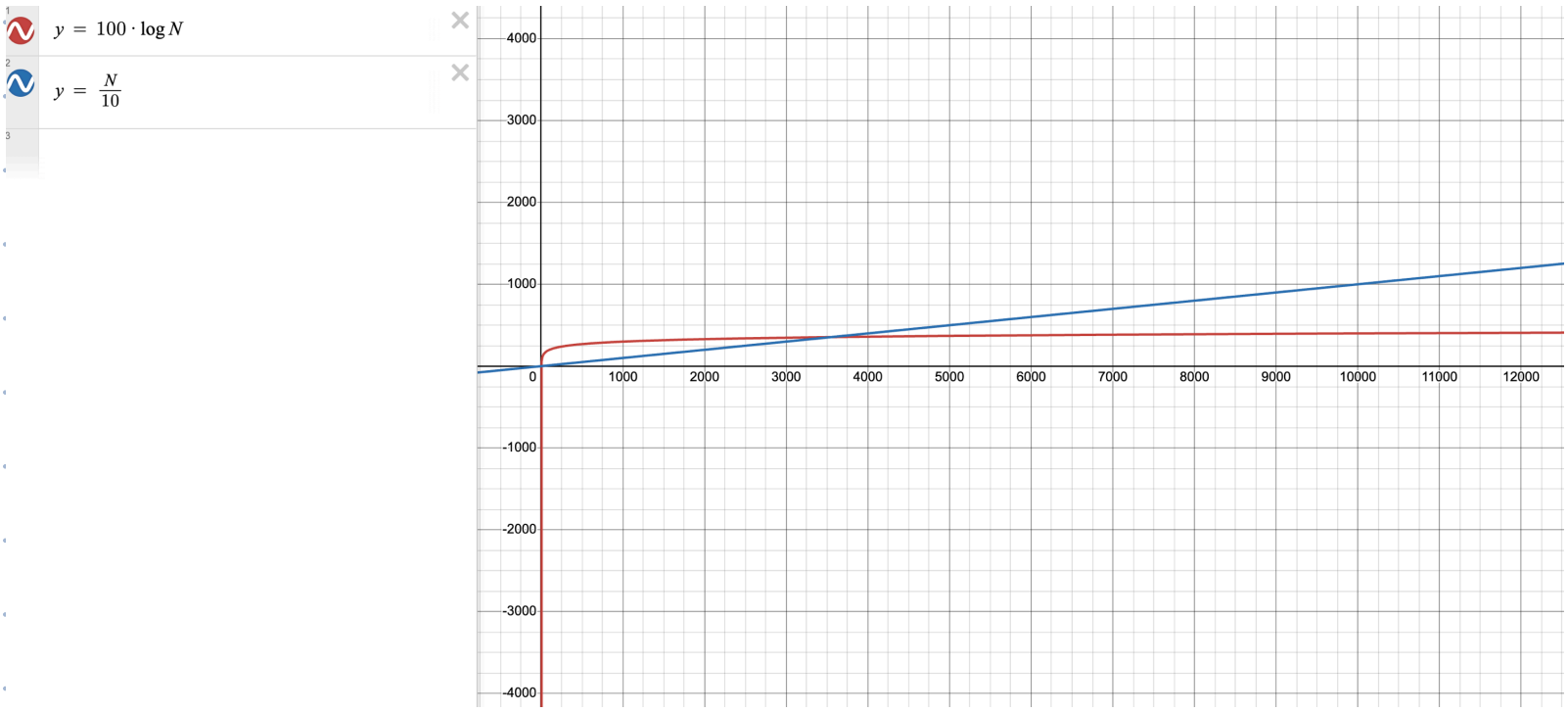
$= 2(2^N - 1)$

$2 \cdot 2^N - 2$

$O(2^N)$

Comparing Iterations of two algorithms using Graphs

	Bharadwaj's Algo (Algo1)	Daksh's Algo (Algo2)
No of iterations :	100*logN	N/10



Till 3550, algo 2 is better (BLUE one)

After 3550, algo 1 (RED) is better.

- India vs Australia (world cup final) → 6 crore
- Baby Shark YT video → 15B

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Asymptotic analysis of Algorithms

Asymptotic Analysis or Big(O) simply means analysing performance of algorithms for larger inputs.

Steps for Big O calculation are as follows:

- Calculate Iterations based on Input Size
- Ignore Lower Order Terms
- Ignore Constant Coefficients

Comparsion Order: (Valid for large value of N)

log(N) < sqrt(N) < N < N log(N) < N sqrt(N) < N^2 < N^3 < 2^(N) < N! < N^N

Using an example

N = 36

$5 < 6 < 36 < 36*5 < 36*6 < 36^2 < 36^3 < 2^{36} < 36! < 36^{36}$

What is the BigO notation time complexity of following expressions?

1) $4N^2 + 3N + 1 \rightarrow O(N^2)$

2) $4N^2 + 3N + 6 \text{ sqrt}(N) + 9 \log(N) + 10 \rightarrow O(N^2)$

3) $100 \log N \rightarrow O(\log N)$

4) $N/10 \rightarrow O(N)$

Quiz 12: $4N + 3 N \log N + 1 \rightarrow O(N \log N)$

Quiz 13: $4N \log N + 3N \text{ sqrt}(N) + 10^6 \rightarrow O(N \text{ sqrt } N)$

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Why do we ignore lower order terms?

No of Iterations → $N^2 + 10.N$

N	Total Iterations = $N^2 + 10N$	Lower Order Term $10N$	Contribution of Lower Order Term in Total Iterations = $\frac{10N}{N^2+10N} * 100$
10	$10^2 + 10 * 10 = 200$	$10 * 10 = 100$	50 %
100	$(10^2)^2 + 10.10^2 = 10^4 + 10^3$	$10.10^2 = 10^3$	~ 9%
1000	$(10^3)^2 + 10.10^3 = 10^6 + 10^4$	$10.10^3 = 10^4$	~ 1%

Conclusion: As Input size increases, contribution of lower order term decreases.

Why to neglect co-efficient / constants?

Algo 1	Algo 2	Winner
$10 \log N$	N	$10 \log N$
$100 \log N$	N	$100 \log N$
$9N$	N^2	$9N$
$10N$	$N^2/10$	$10N$

Conclusion: Growth rate of an algo remains same irrespective of the value of const.

Issues with Big(O)

Issue 1

We cannot always say that one algorithm will always be better than the other algorithm.

Algo 1 : $10^3 N \rightarrow O(N)$

Algo 2 : $N^2 \rightarrow O(N^2)$

Input Size (N)	Algo 1 ($10^3 N$)	Algo 2 (N^2)	Optimised
10	10^4	10^2	Algo 2
100	10^5	10^4	Algo 2
10^3	10^6	10^6	Both same
10^3+1	$10^3(10^3+1)$ $= 10^6 + 10^3$	$(10^3+1)^2$ $= 10^6+10^3+10^3+1$	Algo 1
10^4	10^7	10^8	Algo 1

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Issue 2

If 2 algorithms have same higher order terms, then Big O is not capable to identify algorithm with higher iterations.

```
for(int i=1; i<=N; i++) {
    if(i%2 != 0) {
     c = c+1;
    }
}
```

N iterations

T.C : O(N)

```
for(int i=1; i<=N; i=i+2) {
     c = c+1;
}
```

$\frac{N}{2}$ iterations

T.C : O(N)

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Time Limit Exceeded (T.L.E)

Nishant → Amazon
(Online Assessment)

Q1 → Approach → Implement → Submit → TLE

Is it necessary to write the entire code and then test it to determine its correctness?

Can we assess the logic's viability before writing any code?

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Online Editors and Why T.L.E occurs?

- Codes are executed on online servers of various platforms such as Codechef, Codeforces, etc.

- The processing speed of their server machines is 1 GHz which means they can perform 10^9 instructions per second.

  + − * / initialisation, assignment
  < == |

- Generally, codes should be executed in 1 second.

```
int CountFactors (int N) {
    int count = 0;

    for (i = 1 ; i ≤ N ; i++) {
        if (N % i == 0) {
            count++;
        }
    }

    return count;
}
```

1 iteration → 5 instructions
N iterations → 5N "

Total instructions: 5N + 3.

Conclusion: Calculating exact no. of instructions is tedious, so we make approximation in terms of no. of iterations.

Approximation 1 : 1 iteration = 10 instructions

10^9 instruction $\longrightarrow$ 1 sec

$10^8 \times 10$ " $\longrightarrow$ 1 sec

10^8 iterations $\longrightarrow$ 1 sec

Approximation 2 : 1 iteration = 100 instructions

10^9 instructions $\longrightarrow$ 1 sec

$10^7 \times 100$ " $\longrightarrow$ 1 sec

10^7 iterations $\longrightarrow$ 1 sec

Conclusion : Our code can have $10^7 - 10^8$ iterations to be able to run in 1 sec