

Sorting

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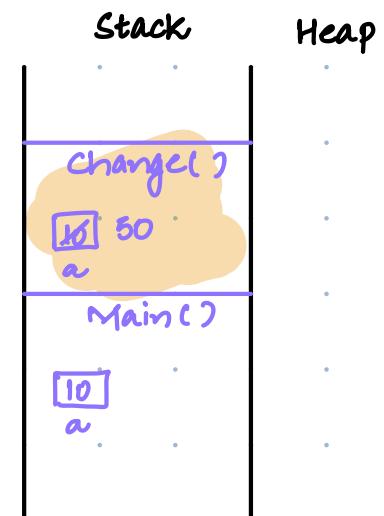
QUIZ

- What are the two types of memory with respect to program execution.

Stack
Heap

QUIZ

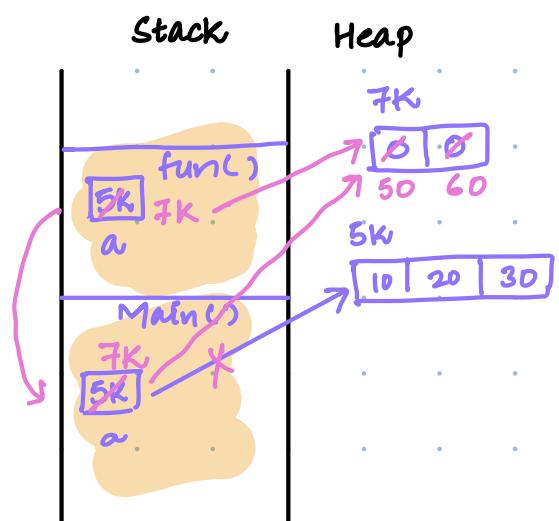
| | |
|---|---|
| Java <pre>static void change(int a) { a = 50; } public static void main(String args[]) { int a = 10; change(a); System.out.println(a); }</pre> | Python <pre>def change(a): a = 50 def main(): a = 10 change(a) print(a) if __name__ == "__main__": main()</pre> |
|---|---|



O/p : 10

QUIZ

| | |
|---|---|
| Java <pre>static int[] fun(int[]a) { a = new int[2]; a[0] = 50; a[1] = 60; return a; } public static void main(String args[]) { int[]a = {10,20,30}; a = fun(a); System.out.println(a[0]); }</pre> | Python <pre>def fun(a): a = [0, 0] a[0] = 50 a[1] = 60 return a def main(): a = [10, 20, 30] a = fun(a) print(a[0]) if __name__ == "__main__": main()</pre> |
|---|---|



O/p : 50

QUIZ

- What happens to the call stack when a called function returns?

Sorting

- Sorting is arrangement of data in particular order on the basis of some parameter.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 5 | 7 |
|---|---|---|---|---|

Parameter: Magnitude

Sorted in Inc order

| | | | | |
|---|---|---|---|---|
| 9 | 6 | 3 | 2 | 1 |
|---|---|---|---|---|

Parameter: Magnitude

Sorted in Dec order.

QUIZ

| | | | | |
|---|----|---|---|----|
| 1 | 13 | 9 | 6 | 12 |
|---|----|---|---|----|

No OF Factors:

1 2 3 4 6

13 → 1, 13

9 → 1, 3, 9

6 → 1, 2, 3, 6

Parameter: No of factors

Sorted in Inc order.

12 → 1, 2, 3, 4, 6, 12

Why sorting?

- Sorting is essential for organizing, analyzing, searching, and presenting data efficiently and effectively in various applications and contexts.



Question (Elements Removal)

Given N elements, at every step remove an array element.

Cost to remove an element = Sum of array of elements present in an array before removal.

Find minimum cost to remove all elements.

NOTE : First add the cost of removal and then remove it.

| 0 | 1 | 2 |
|---|---|---|
| 2 | 1 | 4 |

ans : 11

| 0 | 1 | 2 |
|---|---|---|
| 2 | 1 | 4 |

$$\begin{array}{r} 2+1+4=7 \\ 1+4=5 \\ \quad \quad \quad 4 \\ \hline \quad \quad \quad 16 \end{array}$$

| 0 | 1 | 2 |
|---|---|---|
| 2 | 1 | 4 |

$$\begin{array}{r} 2+1+4=7 \\ 1+4=5 \\ \hline \quad \quad \quad 1 \\ \hline \quad \quad \quad 13 \end{array}$$

| 0 | 1 | 2 |
|---|---|---|
| 2 | 1 | 4 |

$$\begin{array}{r} 4+1+2=7 \\ 1+2=3 \\ \quad \quad \quad 2 \\ \hline \quad \quad \quad 12 \end{array}$$

| 0 | 1 | 2 |
|---|---|---|
| 2 | 1 | 4 |

$$\begin{array}{r} 4+1+2=7 \\ 2+1=3 \\ \hline \quad \quad \quad 1 \\ \hline \quad \quad \quad 11 \end{array}$$

| 0 | 1 | 2 |
|---|---|---|
| 2 | 1 | 4 |

$$\begin{array}{r} 2+1+4=7 \\ 2+4=6 \\ \quad \quad \quad 4 \\ \hline \quad \quad \quad 17 \end{array}$$

| 0 | 1 | 2 |
|---|---|---|
| 2 | 1 | 4 |

$$\begin{array}{r} 2+1+4=7 \\ 2+4=6 \\ \hline \quad \quad \quad 2 \\ \hline \quad \quad \quad 15 \end{array}$$



• QUIZ •

| | | |
|---|---|---|
| 0 | 1 | 2 |
|  |  |  |

$$\begin{array}{r} 6 + 4 + 1 = 11 \\ 4 + 1 = 5 \\ \hline 11 \end{array}$$

QUIZ

| | | | |
|----|----|----|---|
| 0 | 1 | 2 | 3 |
| -3 | -2 | -1 | 0 |

$$5+3+1+(-3)=6$$

$$3 + 1 + (-3) = 1$$

$$\begin{array}{r} 1 - 3 = -2 \\ \cdot \quad \cdot \quad \cdot \\ \quad \quad \quad -3 \end{array}$$

2

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| a | b | c | d |

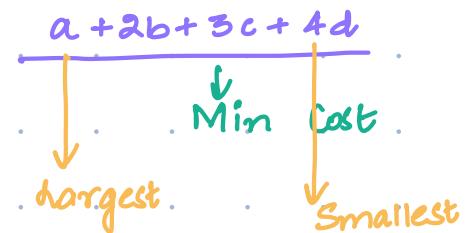
Remove a \rightarrow a + b + c + d

$$\therefore b \rightarrow b + c + d$$

$$f \circ c \rightarrow c + d$$

$d \rightarrow$ _____ d

Observations:



Order of elements: a > b > c > d

i^{th} index element is occurring $(i+1)$ times in sum.

Contribution of i th element : $(i+1)^* a[i]$

Total sum = Σ contribution of all elements.

$$= \sum_{i=0}^{N-1} (i+1) * a[i]$$

Approach: Sort the data in DEC. order.
Iterate on array & add contribution of each term.

```
int MinCost( int arr[], int n ) {  
    ans = 0 ;  
    // Sort in dec. order.  
    reverse_sort( arr ); → N log N  
  
    // Add contribution  
    for( i=0 ; i<n ; i++ ) {  
        ans += (i+1) * arr[i];  
    }  
    return ans ;  
}
```

T.C : $O(N \log N + N)$
 $= O(N \log N)$
S.C : $O(1)$

**Question (Noble Integers) { Distinct data }**

Given N array elements, calculate number of noble integers.

An element ele in arr [] is said to be noble if { count of smaller elements = ele itself }

| 0 | 1 | 2 | 3 | 4 | 5 |
|-------------------------|----|---|---|-----|---|
| 1 | -5 | 3 | 5 | -10 | 4 |
| No of Smaller elements: | 2 | 1 | 3 | 5 | 0 |

ans : 3

QUIZ

| 0 | 1 | 2 | 3 |
|-------------------------|---|---|---|
| -3 | 0 | 2 | 5 |
| No of Smaller elements: | 0 | 1 | 2 |

ans : 1

Brute Force Solution: For each element, find count. of smaller elements



```
int NobleCount( int arr[], int n) {
```

```
    ans = 0;
```

```
    for (i = 0; i < n; i++) {
```

```
        count = 0;
```

```
        for (j = 0; j < n; j++) {
```

```
            if (arr[j] < arr[i]) {
```

```
                count++;
```

```
y
```

```
y
```

```
            if (count == arr[i]) {
```

```
                ans++;
```

```
y
```

```
    return ans;
```

```
y
```

T.C : $O(N^2)$

S.C : $O(1)$

Observations:

| | | | | | |
|---|----|---|---|-----|---|
| 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | -5 | 3 | 5 | -10 | 4 |

↓ Sort

| | | | | | |
|-----|----|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 |
| -10 | -5 | 1 | 3 | 4 | 5 |

Total No of smaller elements = index

Optimised Approach: Sort the array in INC order.
IF $\text{arr}[i] == i$ \rightarrow Noble element

```
int NobleCount( int arr[], int n ) {
```

```
    // Sort
```

```
    arr. sort();
```

```
    ans = 0;
```

```
    for ( i = 0; i < n; i++ ) {
```

```
        if ( arr[i] == i ) {
```

```
            ans++;
```

```
        }
```

```
    }
```

T.C : $O(N \log N)$

S.C : $O(1)$



Question (Noble Integers) : { Data can repeat }

| 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 0 | 2 | 2 | 3 | 3 | 6 |

No of smaller elements:

| | | | | | |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 3 | 5 |
|---|---|---|---|---|---|

ans : 3

QUIZ

| 0 | 1 | 2 | 3 | 4 |
|-----|---|---|---|-----|
| -10 | 1 | 1 | 3 | 100 |

No of smaller elements:

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 4 |
|---|---|---|---|---|

ans : 3

QUIZ

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----|---|---|---|---|---|---|---|----|
| -10 | 1 | 1 | 2 | 4 | 4 | 4 | 8 | 10 |

No of smaller elements:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 4 | 4 | 4 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|

ans : 5

Observations:

Case I: IF curr element \neq prev element
No. of smaller elements = index

Case II: IF curr element $=$ prev element

No. of smaller elements = No of smaller elements
for curr element for prev element.

```
int NobleCount ( int arr[], int n ) {
```

```
    ans = 0;
```

```
    // Sort the array
```

```
    sort (arr);
```

```
    if (arr[0] == 0) ans++;
```

```
    int count = 0;
```

T.C: $O(N \log N)$

S.C: $O(1)$

```
    // Check for smaller elements
```

```
    for (i = 1; i < n; i++) {
```

```
        if (arr[i] != arr[i - 1]) {
```

```
            count = i;
```

```
y
```

```
        if (count == arr[i])
```

```
            ans++;
```

```
y
```

```
    return ans;
```

3



Selection Sort

Idea: Select the minimum element and send that elements to correct position by swapping.

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 5 | 6 | 4 | X |
| 2 | | 5 | |

i
0

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 2 | 6 | 4 | 5 |
| 4 | 6 | | |

1

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 2 | 4 | 6 | 5 |
| 5 | 6 | | |

2

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 2 | 4 | 5 | 6 |

3

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 2 | 4 | 5 | 6 |

| Iterate | Minval | Min-idx | Swap |
|---------|--------|---------|-----------------------|
| [0 3] | 2 | 3 | arr[Min-idx] & arr[0] |
| [1 3] | 4 | 2 | arr[Min-idx] & arr[1] |
| [2 3] | 5 | 3 | arr[Min-idx] & arr[2] |
| [3 3] | 6 | 3 | arr[Min-idx] & arr[3] |



</> Code

```
void SelectionSort( int arr[] ) {  
    n = arr.size();  
  
    for( i=0 ; i<n ; i++ ) {  
        int min_val = INT_MAX;  
        int min_idx = -1;  
  
        // Find Min value.  
        for( j=i ; j<n ; j++ ) {  
            if( arr[j] < min_val ) {  
                min_val = arr[j];  
                min_idx = j;  
            }  
        }  
  
        // Bring Min val to correct position by swapping  
        swap( arr[min_idx] , arr[i] );  
    }  
}
```

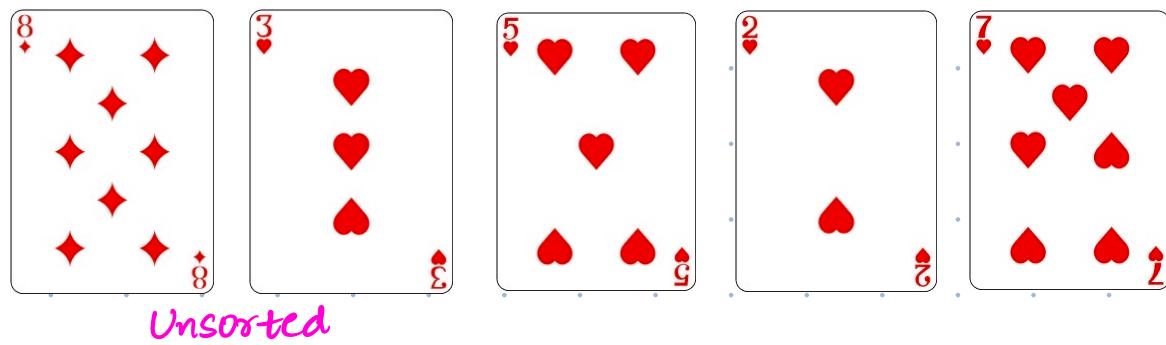
T.C : $O(N^2)$

S.C : $O(1)$

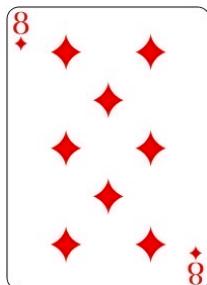
Insertion Sort

 (Arrangement of playing cards)

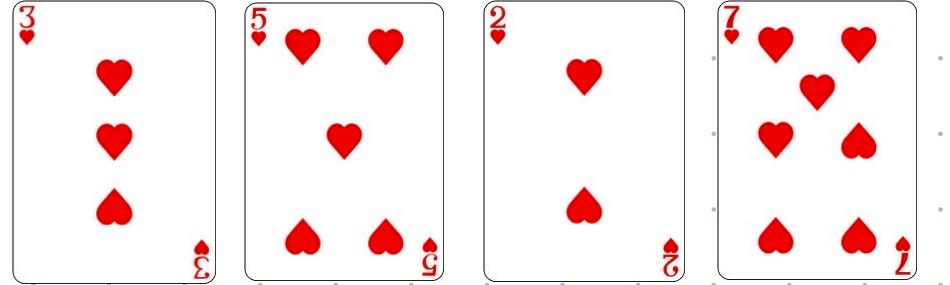
Pick a card from UNSORTED GROUP & put it in right place in SORTED GROUP.



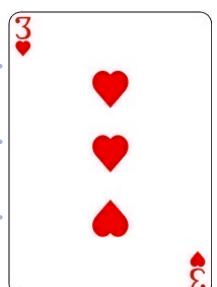
Unsorted



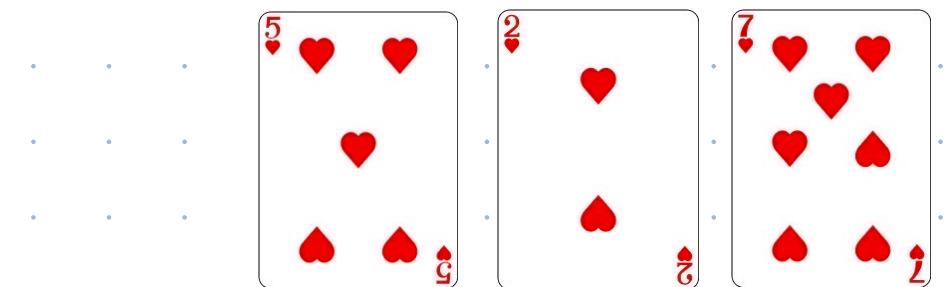
Sorted



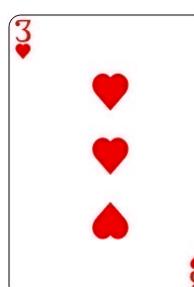
Unsorted



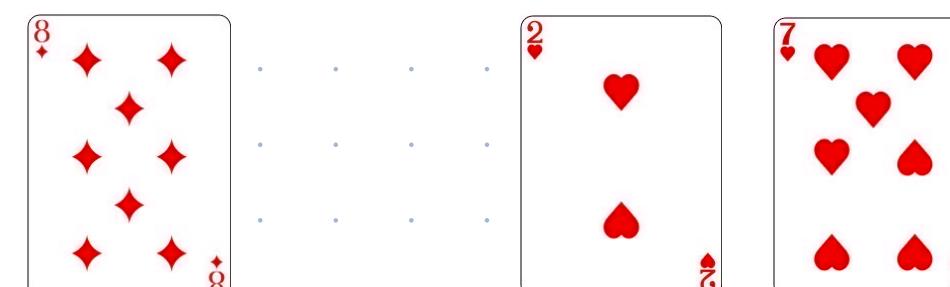
Sorted



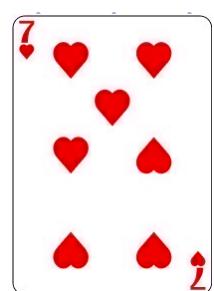
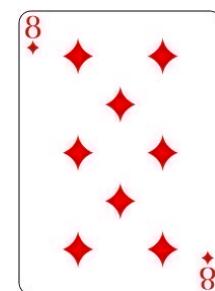
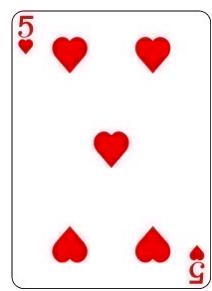
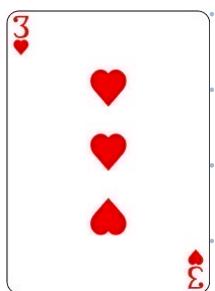
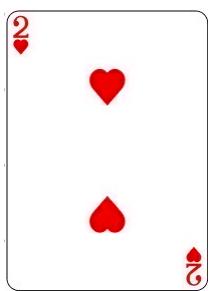
Unsorted



Sorted

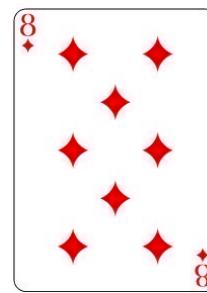
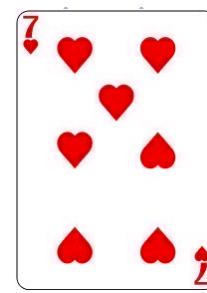
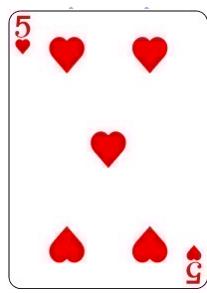
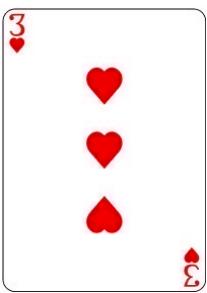
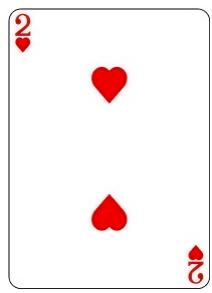


Unsorted



Sorted

Unsorted



Sorted



i



</> Code

```
void Insertionsort (int [] arr, int n) {
```

```
    for (i=1 ; i<n ; i++) {
```

```
        int key = arr[i];
```

```
        int j = i-1;
```

```
        while (j >= 0 && arr[j] > key) {
```

```
            arr[j+1] = arr[j];
```

```
            j--;
```

// Place key at its correct position.

```
        arr[j+1] = key;
```

y

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|

| | | | | |
|----|----|---|---|---|
| 23 | 10 | 5 | 2 | 1 |
|----|----|---|---|---|

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|

| | | | | |
|---|---|---|----|----|
| 1 | 2 | 5 | 10 | 23 |
|---|---|---|----|----|

T.C. : $O(N^2)$ in
Reverse Case.

T.C. : $O(N)$ in Best
Case

(when data is
already SORTED)

