

2D Matrices

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Which algorithm would you choose as most efficient to find the maximum subarray sum efficiently?

- Brute Force algorithm
- Kadane's Algorithm
- Dijkstra's Algorithm
- Moore's Voting algorithm

In the context of finding the maximum subarray sum, what does the 'carry forward' technique imply?

- Using a stack to store elements
- Using recursion for sum calculation
- Continuously adding elements while maintaining the maximum sum
- Using two pointers for sum calculation

How can the time complexity be optimized for performing multiple increment operation queries from index i to last index in an array?

- by using stack
- by using prefix sums
- by using recursion
- by using a queue

What is the optimized time complexity for handling multiple range queries using the prefix sum technique?

- O(Q + N)
- O(Q * N)
- O(Q^2^)
- O(N^2^)

In the problem of rainwater trapping, what does the water stored over a building depend on?

- the width of the building
- the height of the building
- the minimum of the maximum heights on either side of the building
- the total number of buildings

$$\text{water} = \min(l_{\max}, r_{\max}) - h[i]$$



Search element “k” in row-wise & column-wise sorted 2D array

- 5	- 2	1	13	Search (6) → True
- 4	0	3	14	Search (15) → False
- 3	2	5	18	Search (0) → True
2	6	10	20	

N * M

Quiz : 1



BF Idea

- Iterate on all elements & check if current element = k.

T.C : O(N * M)

S.C : O(1).



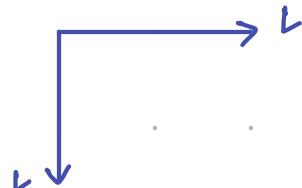
Optimisation Approach

Observation Matrix is Row-wise & Col-wise sorted.

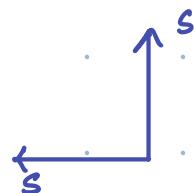
	0	1	2	3
0	5	-2	1	13
1	-4	0	3	14
2	-3	2	5	18
3	2	6	10	20

Search (6)

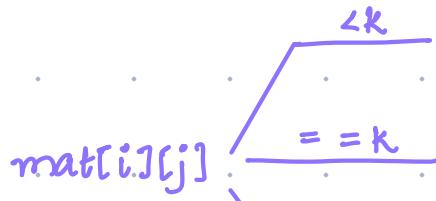
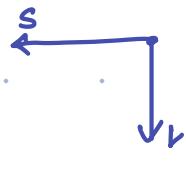
Case I : Start from TOP LEFT X



Case II : Start from BOTTOM RIGHT X



Case III : Start from TOP RIGHT

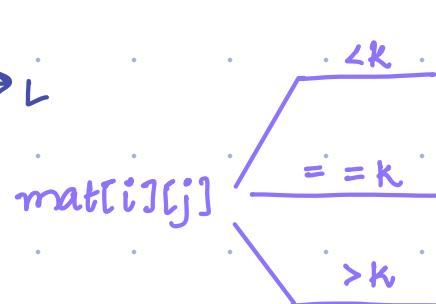


larger elements are in column.
Discard Row & Search in column.

Return TRUE

Smaller elements are in Row.
Discard col & Search in Row.

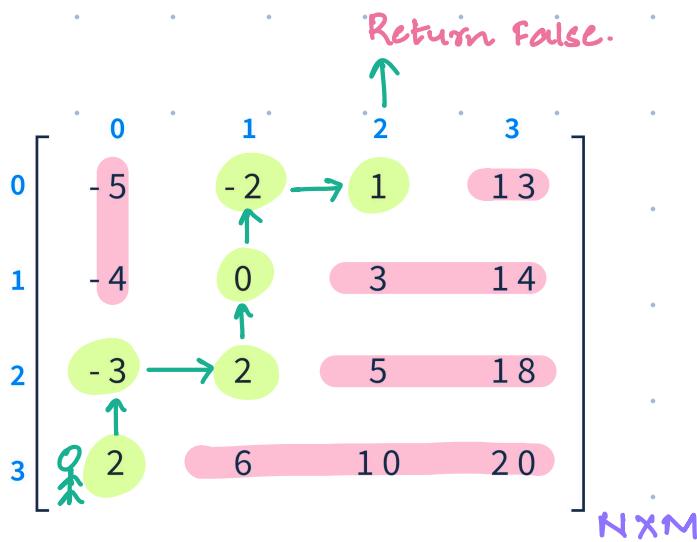
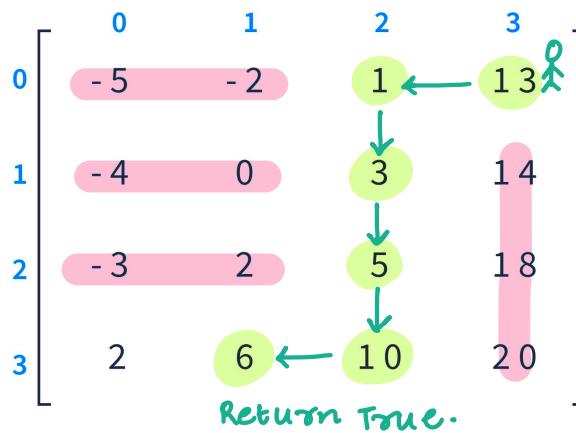
Case IV : Start from BOTTOM LEFT



Larger elements are in Row.
Discard col & Search in Row .

Return TRUE

Smaller elements are in Column.
Discard Row & Search in Column.



</> Code

// Starting from TOP RIGHT

```
i=0, j=M-1;  
while(i < n && j > 0){  
    if(mat[i][j] == k) return True;  
    else if(mat[i][j] < k) // Move in downward direction  
        i++;  
    else  
        j--; // Move in leftward direction  
}  
return False;
```

T.C: O(N+M)
S.C: O(1)



Print Boundary Elements

Given a matrix of $N * N$ i.e. $\text{Mat}[N][N]$, print boundary elements in clockwise direction.

$\text{mat}[N][N]$

	0	1	2	3	4
0	1	2	3	4	5
1	6	7	8	9	10
2	11	12	13	14	15
3	16	17	18	19	20
4	21	22	23	24	25

$\text{mat}[5][5]$

$N = 5$

4 ele \rightarrow
4 ele \downarrow
4 ele \leftarrow
4 ele \uparrow

o/p $\rightarrow [\underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \underline{10}, \underline{15}, \underline{20}, \underline{25}, \underline{24}, \underline{23}, \underline{22}, \underline{21}, \underline{16}, \underline{11}, \underline{6}]$

Quiz : 3

1	2	3
4	5	6
7	8	9

$N=3$

O/p : 1, 2, 3, 6, 9, 8, 7, 9, 4

2 ele \rightarrow
2 ele \downarrow
2 ele \leftarrow
2 ele \uparrow

Approach

$N * N$
 $(N-1)$ ele \rightarrow
 $(N-1)$ ele \downarrow
 $(N-1)$ ele \leftarrow
 $(N-1)$ ele \uparrow

$N * M$

$(M-1)$ ele \rightarrow
 $(N-1)$ ele \downarrow
 $(M-1)$ ele \leftarrow
 $(N-1)$ ele \uparrow

T.C : O(N+M)
S.C : O(1)



< / > Code

Void print Boundary(arr[N][N]) {

i = 0, j = 0;

// (N-1) ele →

for (K = 1; K < N; K++) {

print(arr[i][j]);

j++;

// i = 0, j = (N-1) print (N-1) ele ↓

for (K = 1; K < N; K++) {

print(arr[i][j]);

i++;

// i = (N-1), j = (N-1) print (N-1) ele ←

for (K = 1; K < N; K++) {

print(arr[i][j]);

j--;

// i = (N-1), j = 0 print (N-1) ele ↑

for (K = 1; K < N; K++) {

print(arr[i][j]);

i--;

y

$$T.C \approx O(4 * (N-1))$$

$$= O(N)$$

$$S.C \approx O(1)$$

Quiz : 3

1	2	3
4	5	6
7	8	9

N=3

i = 0	j = 0	K = 1
X	X	X
X	X	X
X	X	X
0	0	0
X	X	X
X	X	X
X	X	X
3	3	3



Spiral Matrix (Clockwise)

mat[N][N]

0	1	2	3	4	5
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

i	j	N	(N-1)
0	0	6	5
1	1	4	3
2	2	2	1
3	3	0	STOP

o/p → [1 , 2 , 3 , 4 , 5 , 6 , 12 , 18 , 24 , 30 , 36 , 35 , 34 , 33 , 32 , 31 , 25 , 19 ,
 13 , 7 , 8 , 9 , 10 , 11 , 17 , 23 , 29 , 28 , 27 , 26 , 20 , 14 , 15 , 16 , 22 , 21]

0	1	2	3	4
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

i	j	N	(N-1)
0	0	5	4
1	1	3	2
2	2	1	0

O/P: 1 2 3 4 5 10 15 20 25 24 23 22 21 16 11 6 7 8 9
 14 19 18 17 12 13.



Quiz : 4

0	1	2	3
0	13	14	12
1	9	1	2
2	0	4	3
3	10	5	6

O/P :

13 14 12 8 7 0 11 6 5 10 0 9
1 2 3 4



Idea

Print boundary elements for matrices of decreasing size. (size dec by N-2)

</> Code

$i = 0, j = 0 ;$

while($N > 1$) {

 || $(N-1)$ ele —→

```
    for(k=1 ; k<N ; k++) {  
        | print(arr[i][j]);  
        |     j++;  
    }
```

 || $i=0, j=(N-1)$ print $(N-1)$ ele

```
    for(k=1 ; k<N ; k++) {  
        | print(arr[i][j]);  
        |     i++;  
    }
```

 || $i=(N-1), j=(N-1)$ print $(N-1)$ ele ←

```
    for(k=1 ; k<N ; k++) {  
        | print(arr[i][j]);  
        |     j--;  
    }
```

 || $i=(N-1), j=0$ print $(N-1)$ ele ↑

```
    for(k=1 ; k<N ; k++) {  
        | print(arr[i][j]);  
        |     i--;  
    }
```

 || $i=0, j=0$

 i++, j++, N=N-2;

 y
 if(N == 1)
 print(arr[i][j]);

T.C : $O(N^2)$

S.C : $O(1)$

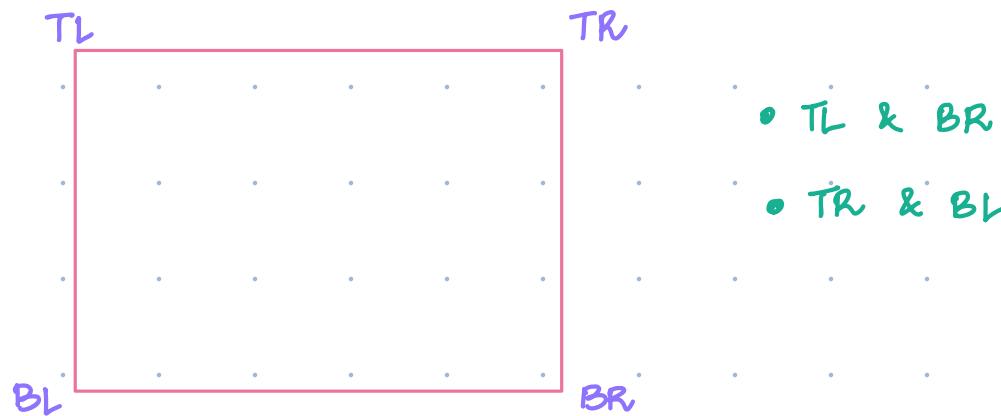
Sub - Matrix

Contiguous part of a matrix.

- Single cell is a sub matrix.
- Entire matrix is also a sub matrix.

	0	1	2	3
0	1	2	3	4
1	5	6	7	8
2	9	10	11	12
3	13	14	15	16

How can we uniquely identify a rectangle?





Sum of all Sub-matrices Sum

Given mat[N] [M]. Find sum of all sub-matrix sums.

	0	1	2
0	4	9	6
1	5	-1	2

$$\begin{array}{cccccc} 4 & & 13 & 19 & 9 & 17 & 25 \\ [4] & [4 \ 9] & [4 \ 9 \ 6] & [4 \ 5] & [4 \ 9 \ -1] & [4 \ 9 \ 6] \\ & & & & & & \\ & & & & & & \end{array}$$

$$\begin{array}{cccccc} 9 & & 15 & 8 & 16 & 6 & 8 \\ [9] & [9 \ 6] & [9] & [9 \ 6 \ 2] & [6] & [6] \\ & & & & & & \\ & & & & & & \end{array}$$

$$\begin{array}{cccccc} 5 & & 4 & 6 & -1 & 1 & 2 \\ [5] & [5 \ -1] & [5 \ -1 \ 2] & [-1] & [-1 \ 2] & [2] \\ & & & & & & \\ & & & & & & \end{array}$$

$$\text{Sum} = 166$$



BF Idea

Generate all submatrices, find their sum & calculate total sum.

T.C.: $O(N^3M^3)$ S.C.: $O(1)$

HW

Total NO OF Submatrices for matrix of size $N \times M$ = $\frac{NM(N+1)(M+1)}{4}$



Idea

Contribution Technique

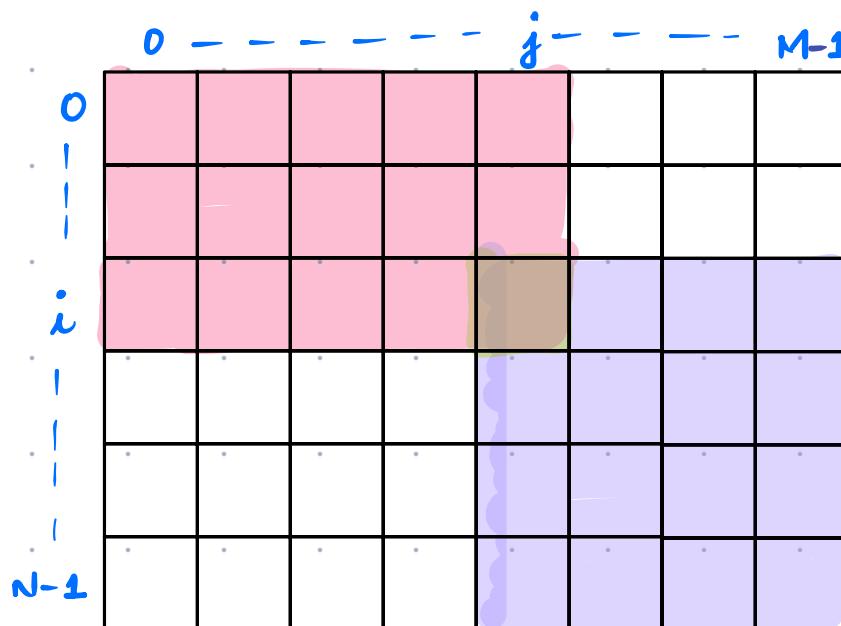
Quiz : 5

	0	1	2	3
0	TL	TL	TV	
1	TL	TL	TL	
2	TL	TL	TV BR	BR
3			BR	BR
4			BR	BR

- In how many sub - matrices $(2, 2)$ will be present?

$$\text{Options for } TL * \text{ Options for } BR = 9 * 6 = 54$$

- In how many sub - matrices (i, j) will be $= (i+1) * (j+1) * (N-i) * (M-j)$



Options for TL :
 $[0 \dots i] [0 \dots j]$
 $(i+1) (j+1)$

Options for BR :
 $[i \dots N-1] [j \dots M-1]$
 $(N-i) (M-j)$
 $= (N-i) * (M-j)$



</> Code

```
ans = 0 ;  
for (i=0 ; i<N ; i++) {  
    for (j=0 ; j<M ; j++) {  
        top_left = (i+1) * (j+1) ;  
        bottom_right = (N-i) * (M-j) ;  
        contribution = arr[i][j] * top_left * bottom_right ;  
        ans += contribution ;  
    }  
}  
return ans ;
```

T.C : O(N*M)
S.C : O(1)