Assignment - Parameter Estimation

1) R S size $= n$

given population = Normal

mean $= \theta_1$, variance $= \theta_2$

def^n

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

for θ_1 => Likelihood function

$$L(\mu) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

taking log on both sides

$$\log(L(\mu)) = -n\log(\sigma\sqrt{2\pi}) - \sum_{i=1}^{n}\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$$

diff w.r.t θ_1 and put $= 0$

$$\Rightarrow \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) \Rightarrow \boxed{\frac{n\bar{x}-n\theta_1}{\sigma^2} = 0} \text{ — } 1$$

Simplifying 1 we get $\boxed{\theta = \bar{x}}$ → Answer

for θ_2

$$\boxed{var(\hat{\theta}) = \frac{1}{E\left(\left[\frac{-\partial^2 \ln L(\theta)}{\partial \theta^2}\right]\right)}} \text{ — } 2$$

using 2a, we know that

$$\frac{\partial \log(L(\theta_1))}{\partial(\theta_1)} = \frac{n}{\sigma^2}(\bar{x}-\mu)$$

$$\frac{d^2 \log(L(\theta_1))}{d\,\theta_1^2} = \frac{-n}{\sigma^2} \quad \text{--- (3)}$$

Substituting (3) in (2) we get

$$\boxed{\text{variance} = \theta_2 = \frac{\sigma^2}{n}}$$

(2). $B(m, \theta)$ $\qquad \theta \in [0, 1]$

$$pmf = P(X = x) = {}^{m}C_{x}\, \theta^{x} (1-\theta)^{m-x}$$

x = no. of success in m trials
θ = prob. of success
$1-\theta$ = pro. of failure

likelihood $f^n \Rightarrow L(\theta) = \prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$

$$\ln(L(\theta)) = \sum_{i=1}^{n} \ln({}^{m}C_{x_i}) + \sum_{i=1}^{n} x_i \ln(\theta) + \sum_{i=1}^{n} (m-x_i) \ln(1-\theta)$$

$$\frac{d}{d\theta}(\ln(L(\theta))) = \sum_{i=1}^{n} \frac{x_i}{\theta} - \frac{(n-x_i)}{1-\theta} = 0$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} = \frac{\sum_{i=1}^{n} (m-x_i)}{1-\theta}$$

$$\frac{\sum_{i=1}^{m} x_i}{\theta} = \frac{mn - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\theta \cdot (nm - \Sigma x_i) = (1-\theta) \, \Sigma x_i$$

$$\theta \cdot nm = \Sigma x_i$$

$$\boxed{\theta = \frac{\Sigma x_i}{nm} = \frac{\bar{x}}{m}}$$

Abhishek Beniwal
C01
102103159