16-720 Computer Vision: Homework 3
Lucas-Kanade Tracking and Background Subtraction

Abhishek Bhatia (abhatia1@andrew.cmu.edu)

3/2/2016

1) Lucas-Kanade Tracking

1.1)

$$\min_{u,v} J(u,v) = \sum_{x,y \in R_t} \left(I_{t+1}(x+u, y+v) - I_t(x,y) \right)^2 \quad \text{— (1)}$$

Since our scenario consists of two-dimensional tracking with pure translation, we define the warp $W(x;p)$ as

$$W(x;p) = \begin{bmatrix} 1 & 0 & u \\ 0 & 1 & v \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x+u \\ y+v \end{bmatrix}$$

With this, (1) can be written as

$$\sum_{x,y \in R_t} \left(I_{t+1}(W(x;p+\Delta p)) - I_t(x,y) \right)^2$$

$$\Rightarrow \sum_{x} \left[I_{t+1}(W(x;p+\Delta p)) - I_t(x) \right]^2 \quad \text{— (2)}$$

This non-linear expression in (2) is linearized by performing a first order Taylor expansion of $I_{t+1}(W(x;p+\Delta p))$ to give:

$$\sum_{x} \left[I_{t+1}(W(x;p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - I_t(x) \right]^2 \quad \text{— (3)}$$

where. $\nabla I = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$ { gradient of I evaluated at $W(x;p)$ }

$$\frac{\partial W}{\partial p} = \begin{bmatrix} \frac{\partial W_x}{\partial u} & \frac{\partial W_x}{\partial v} \\ \frac{\partial W_y}{\partial u} & \frac{\partial W_x}{\partial v} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, ③ can also be represented as

$$\min_{\Delta p} (A \Delta p - b)^2$$

where, $A = \nabla I \frac{\partial W}{\partial p}$ | $b = - I_{t+1}(W(x:p)) + I_t(x)$

$$A^T A = \left(\nabla I \frac{\partial W}{\partial p}\right)^T \left(\nabla I \frac{\partial W}{\partial p}\right) \quad \{\text{ Hessian matrix }\}$$

$\nabla I \frac{\partial W}{\partial p}$: steepest descent

Equation ③ is a least squares problem and has a ~~a~~ closed form solution which can be derived by taking the partial derivative of ③ wrt to Δp and setting it equal to zero.

$$\Rightarrow \boxed{\Delta p = H^{-1} \sum_x \left[\nabla I \frac{\partial W}{\partial p}\right]^T \left[I_t(x) - I_{t+1}(W(x:p))\right]}$$

Hence, the Hessian $A^T A$ must be <u>Invertible</u> so that the template offset can be calculated reliably.

1.3) Lucas-Kanade tracking

Figure 1: Tracking performance (image + bounding rectangle) for Lucas-Kanade at frames 1, 100, 200, 300 and 400

1.4) Lucas-Kanade tracking with template correction

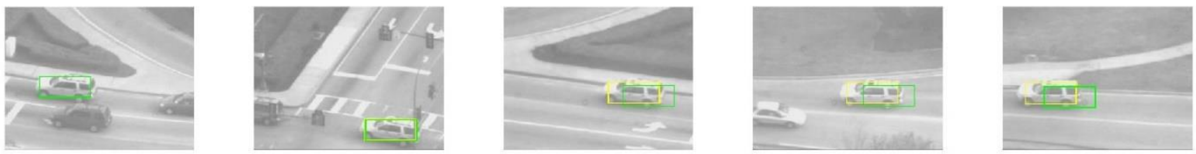

Figure 2: Tracking performance (image + bounding rectangle) for Lucas-Kanade with template correction at frames 1, 100, 200, 300 and 400 (bounding box in yellow defining the template correction)

2) Lucas-Kanade Tracking with Appearance Basis

2.1) Appearance Basis

2.1) Appearance Basis :

$$I_{t+1} = I_t + \sum_{c=1}^{K} w_c B_c \quad -(1)$$

where, $\{B_c\}_{c=1}^{k}$ are the given set of K orthogonal image bases of same size

I_{t+1} is the new frame and I_t is the previous frame

$w = [w_1, \dots w_K]^T$ are weights with which each bases is weighted

From (1)

$$I_{t+1} - I_t = [w_1, w_2 \dots w_K] \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_K \end{bmatrix}$$

$\Rightarrow$ for each c, we have

$$w_c = (I_{t+1} - I_t)(B_c)^{-1}$$

Given that the image bases $\{B_c\}_{c=1}^{k}$ are orthogonal

$$\Rightarrow (B_c)^{-1} = B_c^T$$

Hence, $\boxed{w_c = (I_{t+1} - I_t) B_c^T}$

2.3)

Figure 3: Tracking performance (image + bounding rectangle) for Lucas-Kanade with Appearance Basis at frames 1, 200, 300, 350 and 400 (yellow and green bounding boxes coincided because both algorithms gave the same result)

Figure 4: Tracking performance (image + bounding rectangle) for Lucas-Kanade with Appearance Basis at frames 1, 50, 100, 150 and 200 for second video dataset provided on piazza (bounding box with green defining the results with Lucas-Kanade Appearance Basis)

3) Affine Motion Subtraction

3.3)

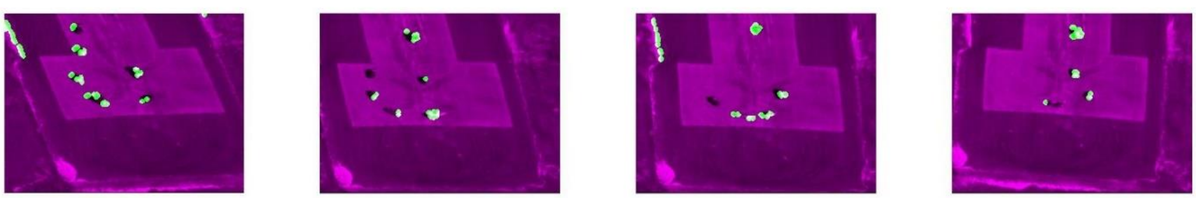

Figure 5: Tracking performance (image + bounding rectangle) for Lucas-Kanade, Moving object detection at frames 30, 60, 90, and 120