CVIP HW3

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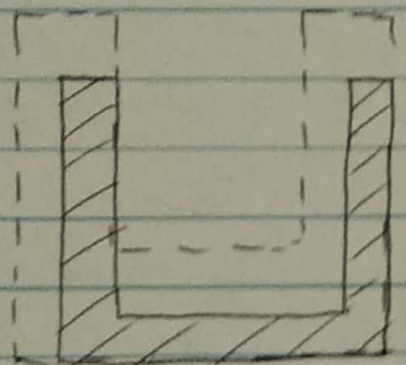
CSE 573 Homework 3

Q.1

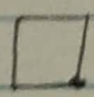
We are given one image as the reference Image on which the operations are to carried out. Now we will see the morphological operations :

(a)



(a)

Structuring Element Used : Square Structuring element



B

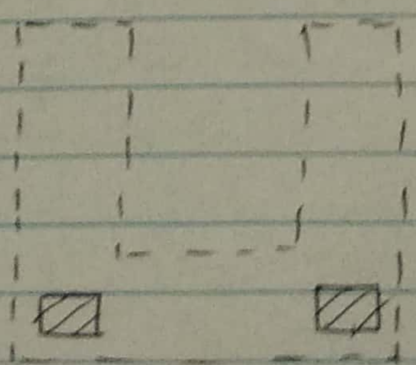
Original Img

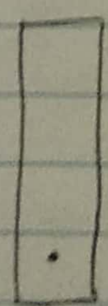
A

We will use a square structuring image with origin or anchor element at bottom right. We will erode the original image with this structuring element to produce the result.

$(a) = A \ominus B$

(b)





B

Structuring Element Used :

Tall structuring element Rectangular Shaped.

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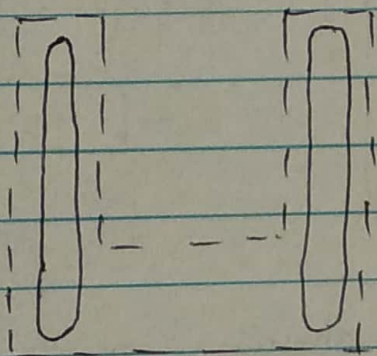
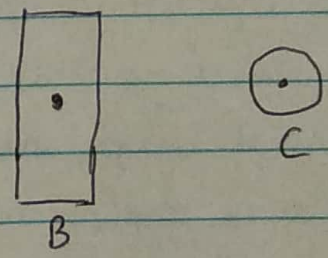
We will use a rectangular tall structuring element which has anchor element at the bottom of the structuring element. We will erode the original image with the structuring element.

(b) ~~A~~ $A \ominus B$

(c)



Structuring Element Used: Rectangular Structuring Element and Circular Element



We want to create image (c) so we will create a rectangular structuring element with the anchor ~~pt~~ element in the middle of the structuring element. We erode image with B

Intermediate Result = $A \ominus B$

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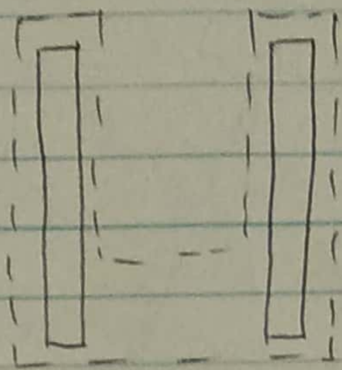
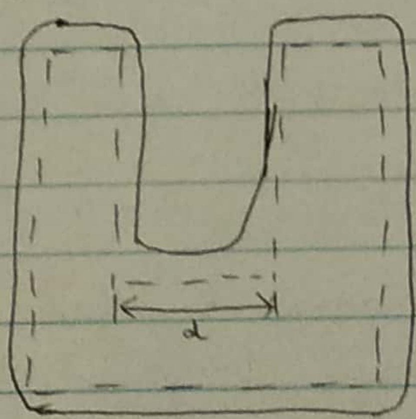
Now result is



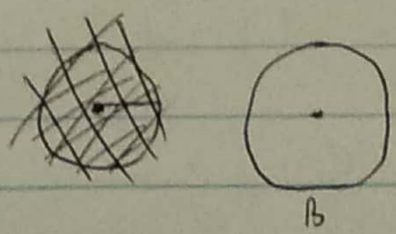
Now we use a smaller circular structuring element which has it anchor element in the centre of the circle We use this so as to give the resultant image a rounded boundary. We dilate intermediate with C.

(c) = Intermediate Result ⊕ C

(d)



Structuring Element Used :



To obtain such a image, we will use a circular structuring element with its anchor element at the centre. The diameter of structuring element (B) should not be more than distance d. We dilate original image with B.

(d) = A ⊕ B

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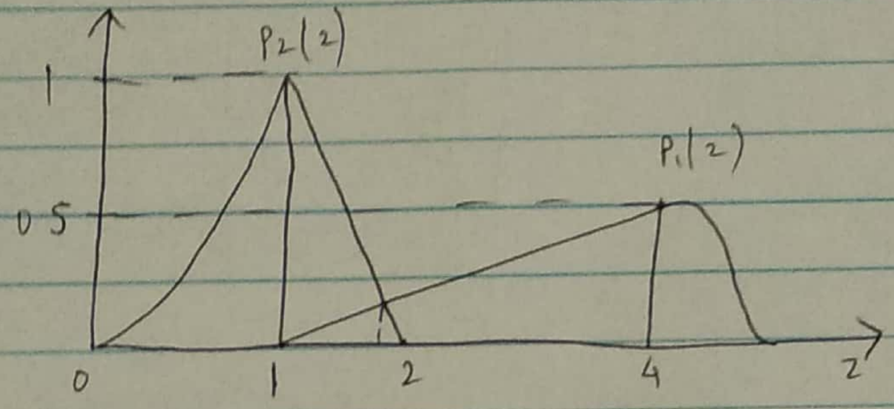
Q.2



In the above diagram, we ~~are~~ are given the gray level probability density function wherein $p_1(z)$ corresponds to objects $p_2(z)$ to the background.
Since we are unable to get the equation of the curve in $p_1(z)$ & $p_2(z)$, to solve this sum we will assume $P_1 = P_2$.
Then the optimum threshold is where the curve $p_1(z)$ and $p_2(z)$ intersect.

So, we estimate the optimal threshold between the object and the background pixels to be $t = 1.8$

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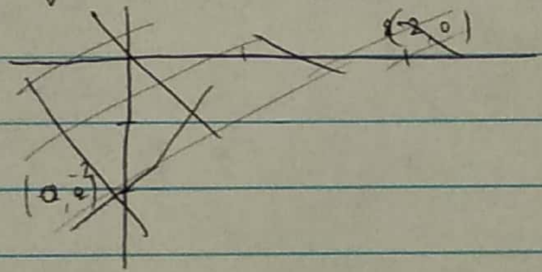
Q.3

(a)

(1) $y = x - 2$

We consider $x = 0$ & get $(0, -2)$ & $y = 0$ and get $(2, 0)$

We get line as follows :



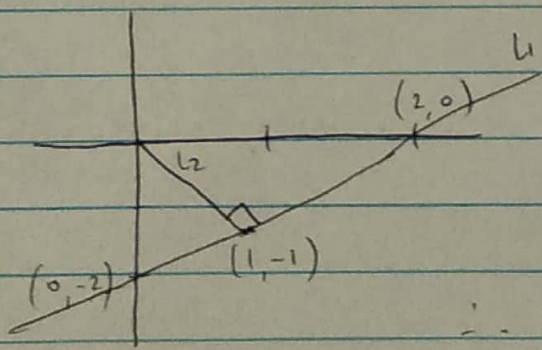
We find slope of line l_1

$$\text{slope} = \frac{2}{2} = 1$$

Since l_1 ~~is~~ perpendicular to l_2 we have

$$l_1 \times l_2 = -1$$

$$\therefore \text{slope } l_2 = -1$$



$\therefore$ We get equation $y = -x$

From this we get point $(1, -1)$.

$$\text{Now} = r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{1^2 + (-1)^2}$$

$$r = \sqrt{1 + 1}$$

$$r = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-1}{1}\right) = \tan^{-1}(-1)$$

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(a) $\theta = \dfrac{-\pi}{4}$

Now substituting $x = r\cos\theta$ and $y = r\sin\theta$ in main equation.

$$y = x - 2$$

$$r\overset{\sin}{\cancel{\cos}}\theta = r\overset{\cos}{\cancel{\sin}}\theta - 2$$

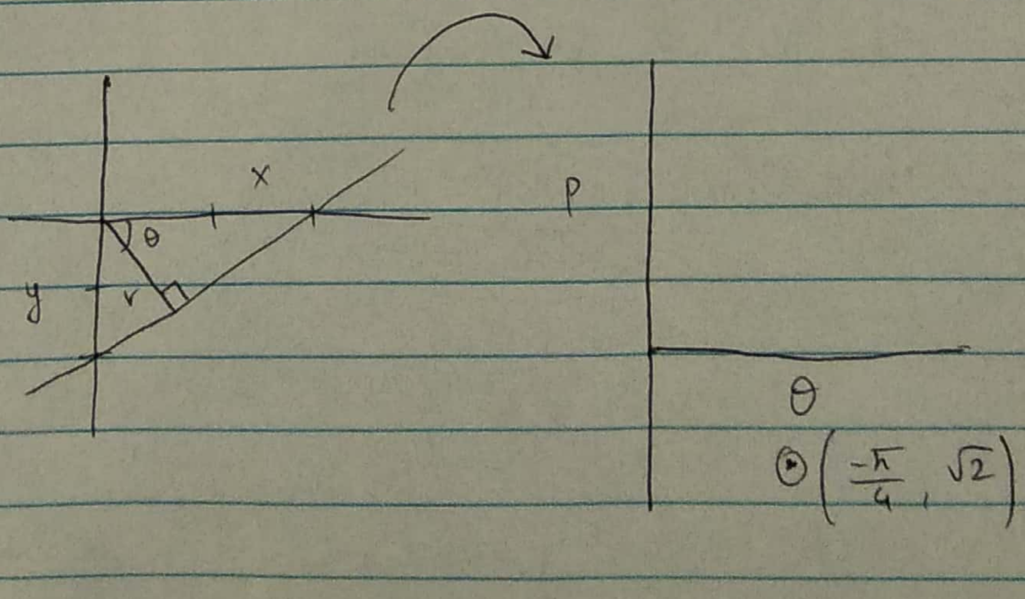
$$r\overset{\sin}{\cancel{\cos}}\theta - r\overset{\cos}{\cancel{\sin}}\theta = -2$$

$$r(\cos\theta - \sin\theta) = 2$$

$$\boxed{r = \left(\frac{2}{\cos\theta - \sin\theta}\right)}$$

$r = \sqrt{2}$ and $\theta = \dfrac{-\pi}{4}$ satisfy above equation. So

$$r = \left(\frac{2}{\cos\theta - \sin\theta}\right)$$



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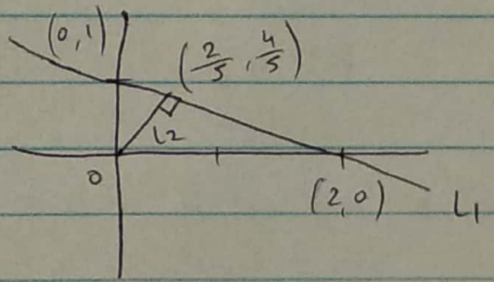
Q.3 (a)

(2) $y = 1 - \frac{x}{2}$

$2y = 2 - x$

Putting $x = 0$ we get $(0, 1)$ & putting $y = 0$ we get $(2, 0)$



Now slope of L_1 is

Slope $= \frac{-1}{2} = -\frac{1}{2}$

Now since L_1 is perpendicular to L_2 we have $L_1 \times L_2 = -1$

$\therefore$ Slope of $L_2 = 2$

Now equation for L_2 is $y = mx + c$ which is

$y = 2x. \longrightarrow (1)$

Now substitute (1) in main equation

$y = 1 - \frac{x}{2}$

$2x + \frac{x}{2} = 1$

$\frac{5x}{2} = 1$

$x = \frac{2}{5}$ Similarly $y = \frac{4}{5}$

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Now $r = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$

$= \sqrt{\frac{4}{25} + \frac{16}{25}} = \sqrt{\frac{20}{25}}$

$r = \frac{2\sqrt{5}}{5}$

Now $\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4/5}{2/5}\right)$

$\theta = \tan^{-1}(2) = 63.43°$.

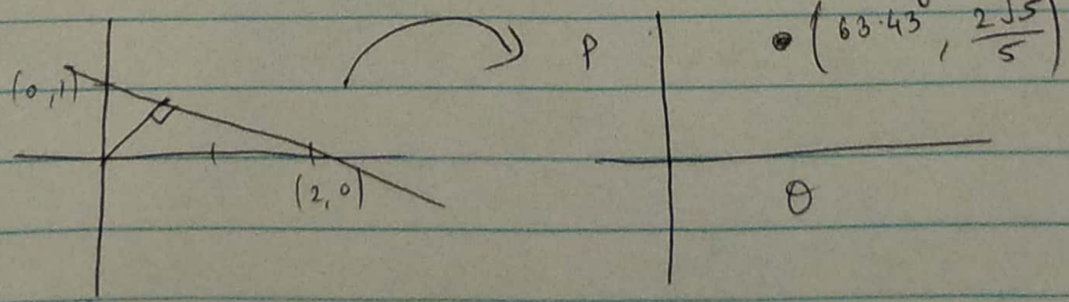
Substituting $x = r\cos\theta$ & $y = r\sin\theta$ in main equation we have.

$r\sin\theta = 1 - \frac{r\cos\theta}{2}$

$2r\sin\theta = 2 - r\cos\theta$

$2r\sin\theta + r\cos\theta = 2$

$$\boxed{r = \left(\frac{2}{2\sin\theta + \cos\theta}\right)}$$



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Q.3

(b) Show that if you use a line equation $x \cos\theta + y \sin\theta = p$ each image (x, y) results in a sinusoid in (p, θ) hough space. Relate the amplitude and phase of sinusoid to (x, y). Does period of sinusoid vary with image point (x, y).

We are given a line equation $x \cos\theta + y \sin\theta = p$ where (x, y) represents image points.

Now

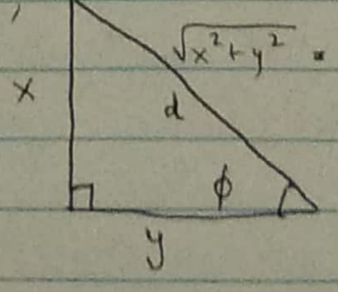
$$p = x \cos\theta + y \sin\theta \rightarrow (1)$$

We want to prove that line equation (1) varies sinusoidally.

We divide whole of equation (1) by $\sqrt{x^2+y^2}$

$$\frac{p}{\sqrt{x^2+y^2}} = \frac{x \cos\theta}{\sqrt{x^2+y^2}} + \frac{y \sin\theta}{\sqrt{x^2+y^2}} \rightarrow (4)$$

Now assuming,



By hypotenuse we can say that $d = \sqrt{x^2+y^2}$

Considering angle ϕ, $\sin\phi = \frac{\text{opposite side}}{\text{hypotenuse}}$

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$$\sin\phi = \frac{x}{\sqrt{x^2+y^2}} \longrightarrow (2)$$

Similarly $\cos\phi = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

$$\cos\phi = \frac{y}{\sqrt{x^2+y^2}} \longrightarrow (3)$$

Substituting (2) & (3) in (4)

$$\frac{p}{\sqrt{x^2+y^2}} = \sin\phi\cos\theta + \cos\phi\sin\theta$$

$\cancel{\frac{p}{\sqrt{x^2+y^2}}}$ = It is of the form $\sin A\cos B + \cos A\sin B$

$$= \sin(A+B)$$

$$\frac{p}{\sqrt{x^2+y^2}} = \sin(\phi+\theta)$$

$$\boxed{p = \sqrt{x^2+y^2}\,\sin(\theta+\phi)} \longrightarrow (5)$$

We have proved that line equation $x\cos\theta + y\sin\theta = p$ varies sinusoidally.

Now comparing with equation of sinusoidal wave

$$y(t) = A\sin(\omega t + \phi) \longrightarrow (6)$$

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We have by comparing 5 & 6,

$$A = \sqrt{x^2 + y^2} \quad \text{Amplitude}$$

~~$\phi = \sin\theta =$~~

$$\sin\phi = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\phi = \sin^{-1}\left(\frac{x}{\sqrt{x^2 + y^2}}\right) \quad \text{Phase}$$

$$w = 2\pi f \quad \text{Frequency}$$

$$\theta = wt$$

We can say that frequency does not vary with input (x, y). Thus we have related the amplitude and phase of the sinusoid to point (x, y).

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