## ADA(B) Homework Assignment 4

Deadline: April 25 (Sunday) 11.59 pm.

The theory assignment has to be done in a team of at most two members, as already selected by you. The solutions are to be typed either as a word document or latex-ed and uploaded as pdf on GC. We shall strictly not accept solutions written in any other form. Remember that both team members need to upload the HW solution on GC.

Collaboration across teams or seeking help from any sources other than the lectures, notes and texts mentioned on the homepage will be considered an act of plagiarism.

Some questions below are from the text by Jeff Erickson. You can find an online copy here.

For all the questions (unless mentioned otherwise), you need to show the following.

**Algorithm.** This can be in the form of pseudocode or clear and brief precise description in English or a particular syntax.

**Runtime.** Formally argue the best possible asymptotic runtime.

**Correctness.** You should give precise arguments for why your algorithm works. Ideally you should use proof techniques like induction, contradiction etc.

You may use any algorithm or claim from class lectures as a black box.

**Problem 1** You are a paleontologist and you have collected a lot of fossil records. You have identified a list of animal species  $s_1, s_2, \ldots, s_n$ . Your goal is to identify possible origin and extinction times for these species with the help of your fossil records.

The fossil records give you pieces of information of the following kind - For some pairs  $(s_i, s_j)$ ,  $i \neq j$  of species, the fossil records tell you that:

- 1. Species  $s_i$  became extinct before species  $s_i$  originated
- 2. Or, species  $s_i$  and  $s_j$  overlapped i.e. there was some time t when species  $s_i$  and species  $s_j$  both roamed the earth

You want to determine an ordering of origin and extinction times of the species which satisfy all the pieces of information from the fossil records; or determine that no such ordering exists.

For example, suppose you have three species  $s_1$ ,  $s_2$ ,  $s_3$ . You are given that

1.  $s_1$  became extinct before  $s_3$  originated,

- 2.  $s_2$  and  $s_3$  overlapped
- 3.  $s_1$  and  $s_2$  overlapped

Denoting the origin and extinction times of species i by  $o_i$  and  $e_i$  respectively, the following (not unique) ordering satisfies all the given information

Give an efficient algorithm for this problem. Justify its correctness and analyze its time complexity.

**Problem 2** Solve problem 12 from the above text *with possibly negative edge weights*. i.e. in the original problem, all edges have positive weights, but in your problem, edges could possibly have negative weights.